Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/59-3.2.1-f+g-x-^m-A+B-log-e-a+b-x-over-c+d-x-^n-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [314]. This is test number [59].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	95.86 (301)	4.14 (13)
Maxima	75.80 (238)	24.20 (76)
Maple	71.34 (224)	28.66 (90)
Fricas	66.88 (210)	33.12 (104)
Mupad	63.69 (200)	36.31 (114)
Giac	60.83 (191)	39.17 (123)
Sympy	36.31 (114)	63.69 (200)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

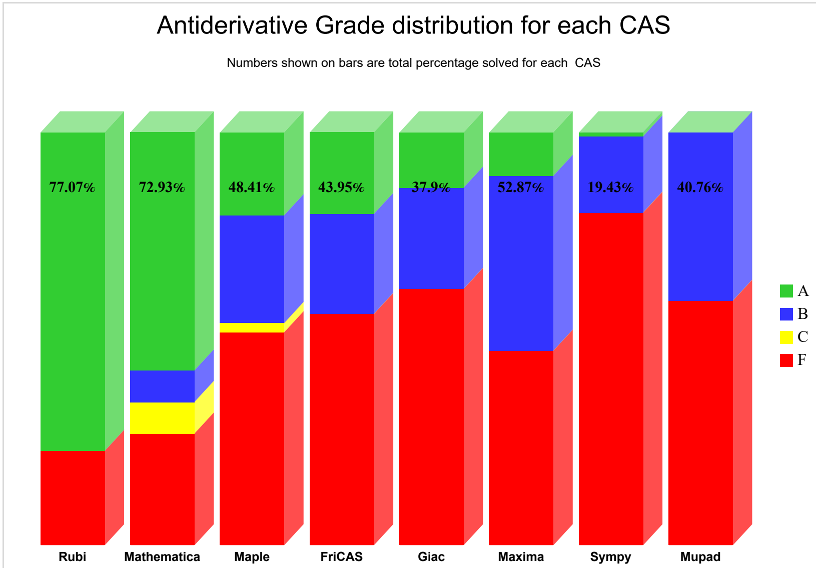
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

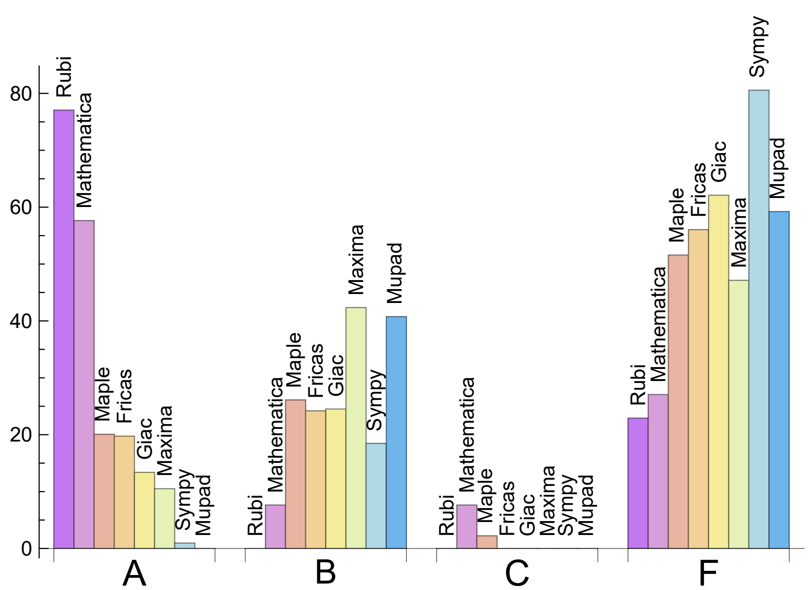
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.070	0.000	0.000	22.930
Mathematica	57.643	7.643	7.643	27.070
Maple	20.064	26.115	2.229	51.592
Fricas	19.745	24.204	0.000	56.051
Giac	13.376	24.522	0.000	62.102
Maxima	10.510	42.357	0.000	47.134
Sympy	0.955	18.471	0.000	80.573
Mupad	0.000	40.764	0.000	59.236

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	13	100.00	0.00	0.00
Maxima	76	100.00	0.00	0.00
Maple	90	98.89	1.11	0.00
Fricas	104	92.31	7.69	0.00
Mupad	114	0.00	100.00	0.00
Giac	123	95.12	4.88	0.00
Sympy	200	19.00	69.00	12.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.17
Maxima	0.29
Mathematica	0.67
Fricas	1.25
Mupad	3.64
Giac	5.05
Maple	12.86
Sympy	17.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	206.85	1.00	149.00	1.00
Fricas	368.95	2.32	163.50	2.05
Mathematica	398.74	1.56	144.00	1.06
Mupad	429.54	2.05	160.00	1.54
Sympy	451.38	4.54	274.50	3.66
Maple	636.55	2.78	218.00	1.64
Maxima	692.84	4.16	428.00	3.08
Giac	980.03	5.14	224.00	1.65

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

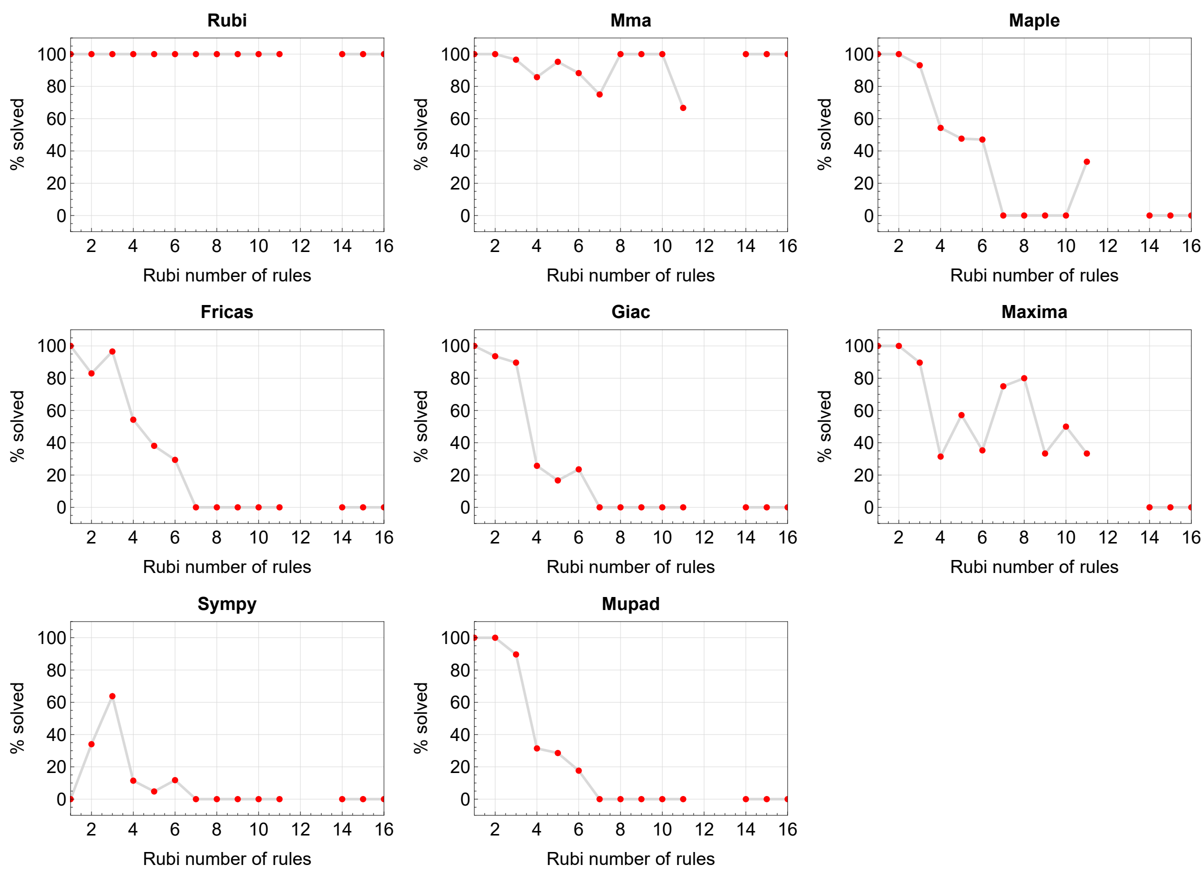


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

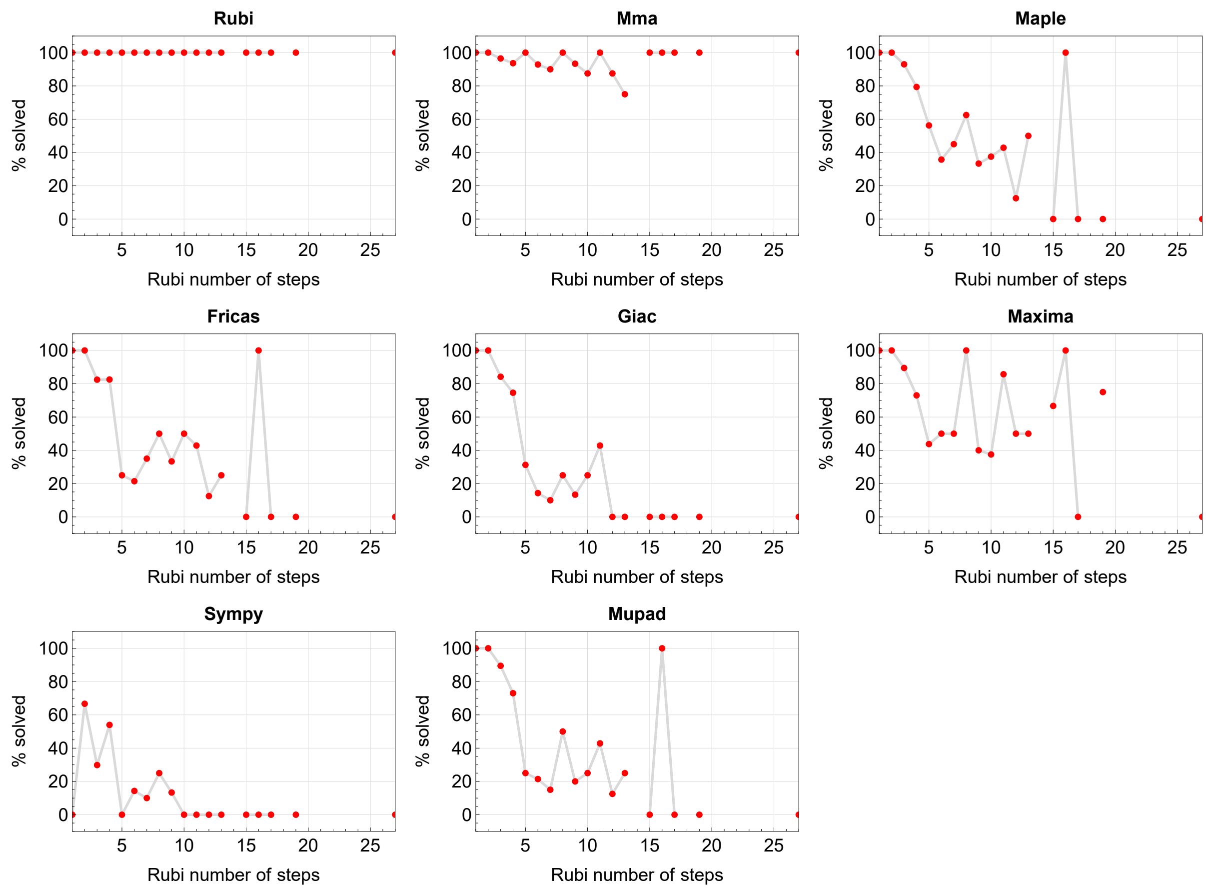


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

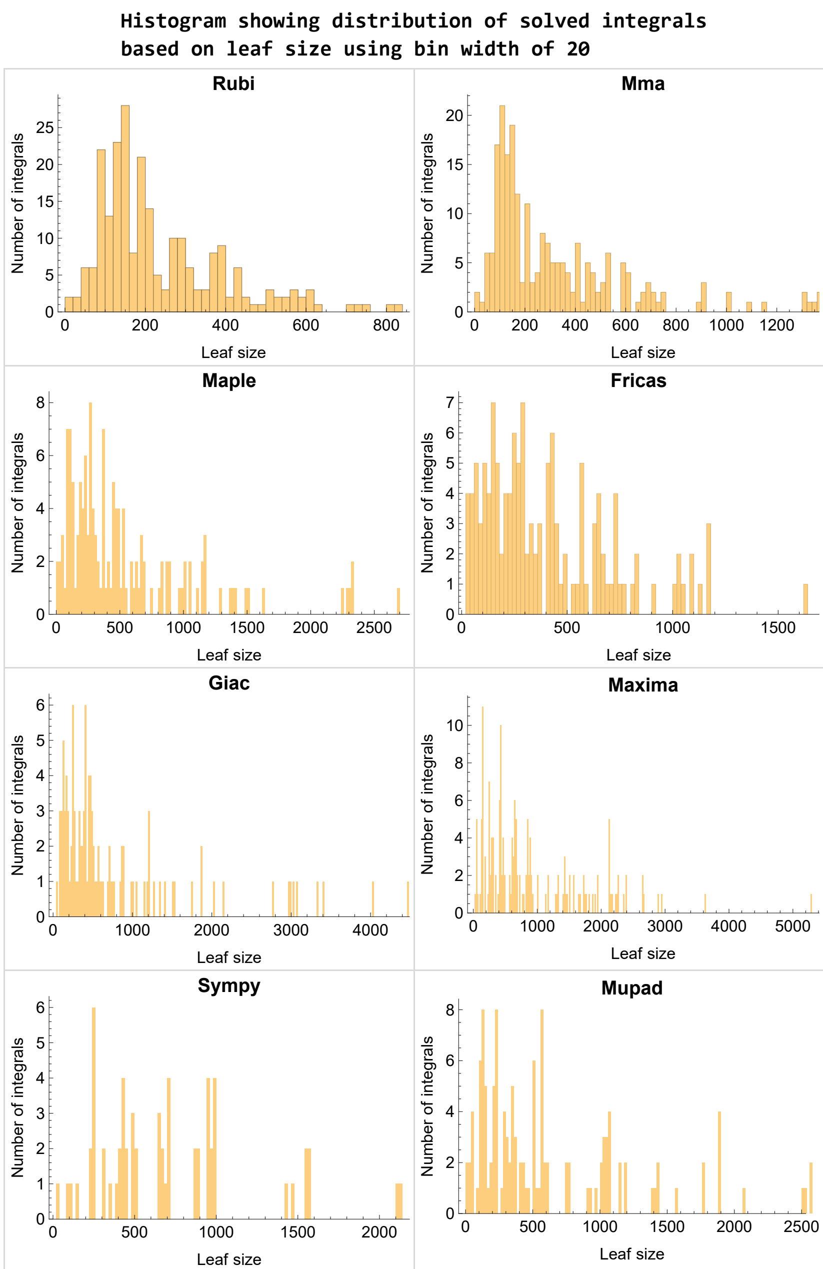


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

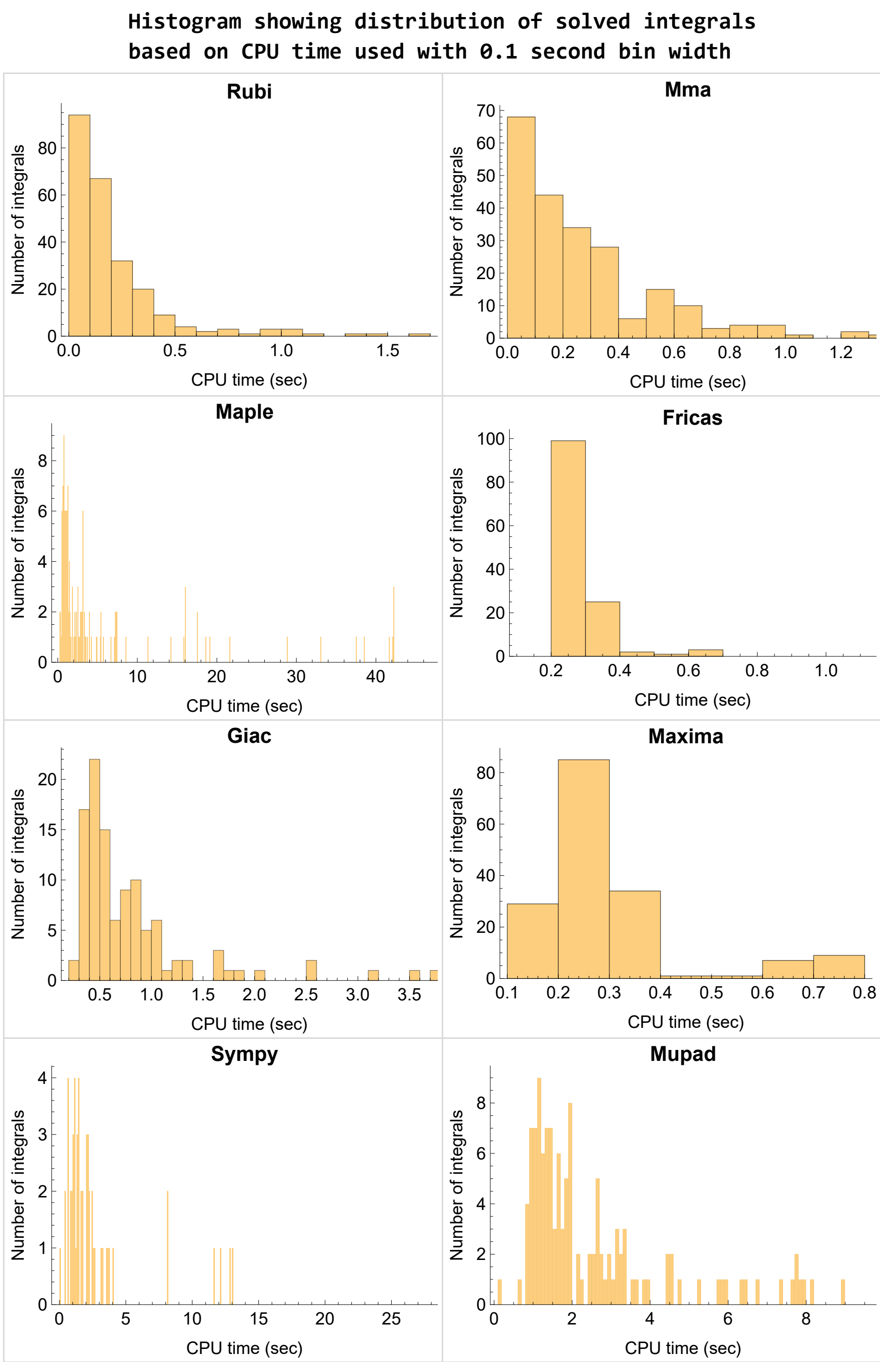


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

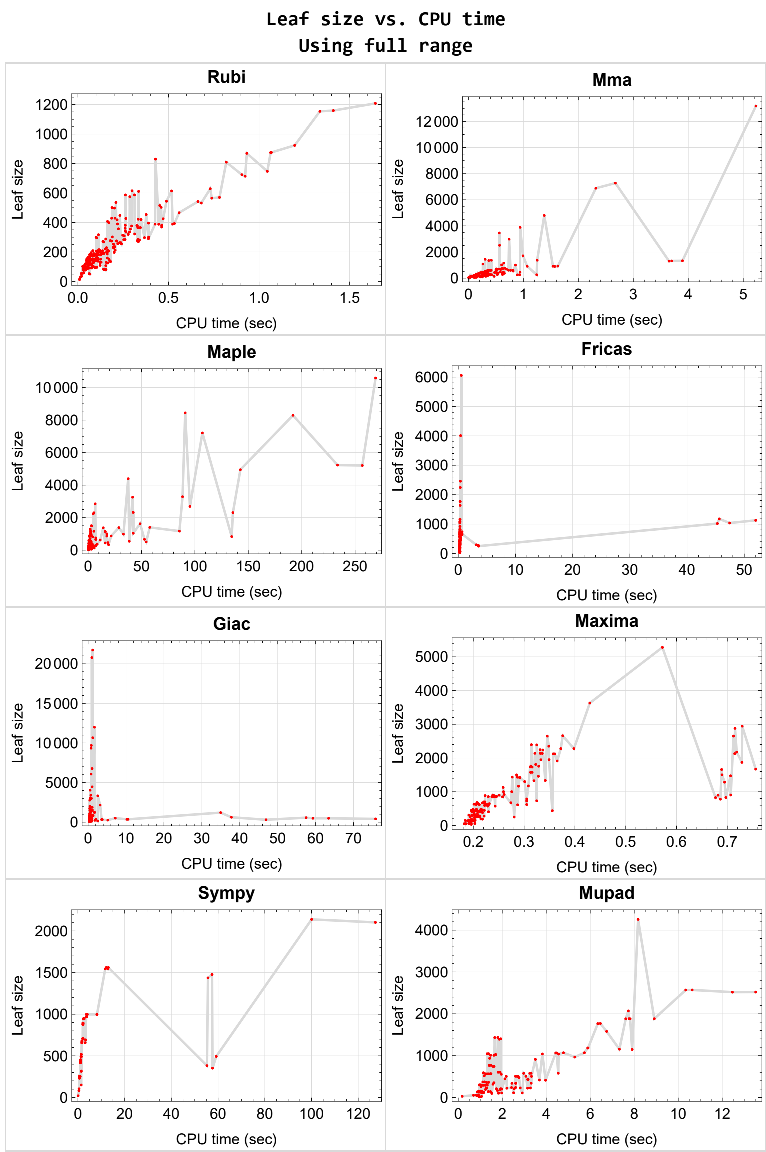


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {151, 156, 157, 158, 298, 303, 305}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

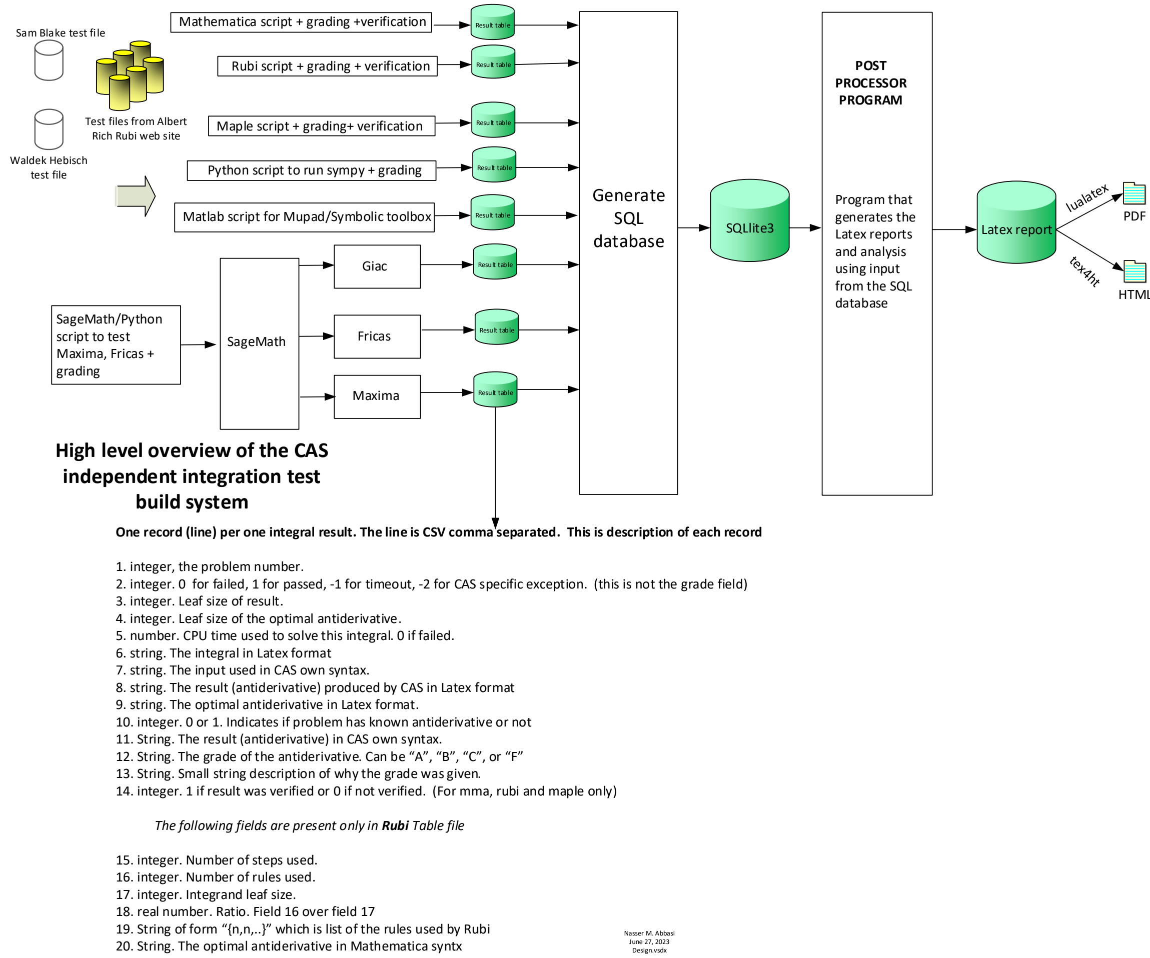
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 222, 223, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311 }

B grade { 14, 71, 72, 101, 106, 147, 156, 157, 158, 159, 164, 165, 166, 167, 168, 244, 245, 276, 277, 306, 307, 308, 309, 310 }

C grade { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F normal fail { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 6, 7, 34, 35, 43, 61, 90, 91, 93, 94, 102, 103, 106, 107, 108, 112, 113, 117, 118, 121, 122, 124, 125, 133, 134, 152, 175, 176, 177, 178, 179, 180, 187, 188, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 249, 262, 263, 264, 265, 266, 297 }

B grade { 1, 2, 3, 4, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 44, 45, 46, 57, 58, 59, 60, 63, 64, 65, 66, 88, 89, 92, 95, 96, 101, 104, 105, 119, 120, 123, 126, 127, 135, 136, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 181, 186, 189, 190, 235, 236, 237, 238, 239, 244, 267, 268, 269, 270, 271, 293, 294, 295, 296, 299, 300, 301, 302 }

C grade { 151, 156, 157, 158, 298, 303, 305 }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 304 }

F(-2) exception fail { }

Fricas

A grade { 4, 6, 7, 15, 22, 23, 27, 34, 35, 43, 50, 51, 55, 60, 61, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 122, 124, 125, 133, 134, 135, 136, 152, 172, 176, 178, 179, 187, 188, 189, 194, 195, 204, 206, 207, 215, 216, 217, 229, 230, 232, 233, 234, 249, 262, 265, 266, 296, 297 }

B grade { 1, 2, 3, 8, 9, 16, 17, 18, 28, 29, 30, 31, 32, 36, 37, 44, 45, 46, 56, 57, 58, 59, 63, 64, 88, 89, 90, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 180, 181, 190, 199, 200, 201, 202, 203, 208, 209, 218, 231, 236, 237, 263, 264, 268, 269, 293, 294, 295, 299, 300 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 65, 66, 238, 239, 270, 271, 301, 302 }

F(-2) exception fail { }

Maxima

A grade { 4, 7, 32, 34, 35, 57, 58, 59, 60, 61, 63, 64, 91, 94, 150, 152, 153, 176, 179, 206, 230, 231, 232, 233, 234, 236, 249, 266, 293, 295, 296, 297, 299 }

B grade { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 300, 301, 302, 303, 304 }

C grade { }

F normal fail { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 6, 7, 15, 16, 17, 34, 35, 43, 44, 45, 55, 56, 93, 94, 102, 103, 104, 105, 122, 125, 136, 150, 152, 153, 178, 179, 187, 188, 189, 199, 200, 204, 206, 207, 218, 263, 264, 265, 266, 296, 297, 299 }

B grade { 1, 2, 3, 4, 8, 9, 18, 29, 30, 31, 32, 33, 36, 37, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 95, 96, 106, 107, 108, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 176, 177, 180, 181, 190, 201, 202, 203, 208, 209, 215, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 268, 269, 270, 271, 295, 300, 301, 302 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 38, 39, 40, 41, 42, 50, 51, 62, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 186, 194, 195, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 67, 262, 293, 294, 303, 309 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-2) exception fail { }

Sympy

A grade { 61, 234, 249 }

B grade { 4, 7, 32, 35, 60, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade { }

F normal fail { 5, 14, 15, 16, 17, 22, 33, 42, 43, 44, 45, 50, 70, 92, 101, 106, 107, 108, 112, 113, 117, 123, 132, 140, 141, 145, 151, 159, 177, 186, 194, 195, 199, 205, 214, 222, 223, 227 }

F(-1) timedout fail { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 18, 23, 27, 28, 29, 30, 31, 34, 36, 37, 38, 39, 40, 41, 46, 51, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 79, 80, 81, 86, 87, 97, 98, 99, 100, 105, 114, 118, 128, 129, 130, 131, 136, 142, 146, 152, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 190, 200, 210, 211, 212, 213, 218, 228, 229, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 290, 291, 292, 299, 300, 301, 302, 307, 308, 313, 314 }

F(-2) exception fail $\{ 147, 148, 149, 150, 156, 157, 158, 164, 165, 166, 293, 294, 295, 296, 297, 298, 303, 304, 305, 306, 309, 310, 311, 312 \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	146	1004	676	569	0	4462	1046
N.S.	1	1.00	0.78	5.34	3.60	3.03	0.00	23.73	5.56
time (sec)	N/A	0.089	0.079	17.529	0.207	0.337	0.000	1.060	1.378

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	755	479	426	0	3034	588
N.S.	1	1.00	0.79	4.84	3.07	2.73	0.00	19.45	3.77
time (sec)	N/A	0.067	0.072	7.215	0.215	0.317	0.000	0.813	1.142

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	103	528	309	296	0	1866	303
N.S.	1	1.00	0.83	4.26	2.49	2.39	0.00	15.05	2.44
time (sec)	N/A	0.052	0.041	2.894	0.197	0.281	0.000	0.810	0.962

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	73	276	156	160	352	880	134
N.S.	1	1.00	0.85	3.21	1.81	1.86	4.09	10.23	1.56
time (sec)	N/A	0.035	0.029	1.066	0.191	0.274	57.622	0.485	0.897

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	101	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	79	115	131	137	103	0	88	112
N.S.	1	1.18	1.72	1.96	2.04	1.54	0.00	1.31	1.67
time (sec)	N/A	0.034	0.043	3.125	0.189	0.280	0.000	0.758	2.516

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	114	271	259	265	2139	224	222
N.S.	1	1.00	0.75	1.79	1.72	1.75	14.17	1.48	1.47
time (sec)	N/A	0.082	0.101	7.393	0.205	0.268	100.059	0.782	1.292

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	145	440	432	482	0	381	349
N.S.	1	1.00	0.79	2.40	2.36	2.63	0.00	2.08	1.91
time (sec)	N/A	0.097	0.112	15.869	0.200	0.276	0.000	0.663	1.622

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	162	1043	651	733	0	541	603
N.S.	1	1.00	0.75	4.85	3.03	3.41	0.00	2.52	2.80
time (sec)	N/A	0.118	0.136	42.201	0.208	0.308	0.000	0.882	1.812

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	535	0	2945	0	0	0	0
N.S.	1	1.00	1.35	0.00	7.44	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.326	0.000	0.729	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	335	411	0	2175	0	0	0	0
N.S.	1	1.00	1.23	0.00	6.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.226	0.000	0.718	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	303	0	1501	0	0	0	0
N.S.	1	1.00	1.11	0.00	5.48	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.164	0.000	0.690	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	215	0	828	0	0	0	0
N.S.	1	1.00	1.10	0.00	4.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.136	0.000	0.697	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	537	0	0	0	0	0	0
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.814	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	330	296	430	258	0	174	238
N.S.	1	1.00	2.43	2.18	3.16	1.90	0.00	1.28	1.75
time (sec)	N/A	0.063	0.331	3.136	0.216	0.272	0.000	0.989	2.608

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	463	672	861	651	0	478	506
N.S.	1	1.00	1.61	2.33	2.99	2.26	0.00	1.66	1.76
time (sec)	N/A	0.154	0.288	7.388	0.246	0.287	0.000	1.039	3.078

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	612	1146	1432	1164	0	840	1038
N.S.	1	1.00	1.37	2.56	3.20	2.60	0.00	1.88	2.32
time (sec)	N/A	0.232	0.415	16.000	0.287	0.317	0.000	1.321	4.562

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	615	700	2326	2136	1762	0	1206	1769
N.S.	1	1.00	1.14	3.78	3.47	2.87	0.00	1.96	2.88
time (sec)	N/A	0.299	0.556	42.228	0.336	0.332	0.000	1.694	6.460

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	1.06
time (sec)	N/A	0.025	0.408	0.053	0.431	0.253	34.232	27.573	0.758

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	1.06
time (sec)	N/A	0.013	0.128	0.099	0.441	0.265	16.309	16.773	0.672

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	1.06
time (sec)	N/A	0.030	0.151	0.210	0.342	0.249	16.193	9.085	0.617

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	94	0	0	62	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.093	0.144	0.000	0.000	0.262	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	172	0	0	149	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.200	0.282	0.000	0.000	0.280	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	1.06
time (sec)	N/A	0.023	0.429	0.053	0.478	0.273	44.805	1.456	1.310

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	1.06
time (sec)	N/A	0.015	0.463	0.018	0.488	0.264	58.784	0.789	0.835

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	1.06
time (sec)	N/A	0.029	0.278	0.076	0.357	0.282	130.893	0.681	0.701

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	146	0	0	274	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.101	0.161	0.000	0.000	0.273	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	314	314	254	0	0	755	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.247	0.400	0.000	0.000	0.291	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	146	864	676	572	0	1876	1045
N.S.	1	1.00	0.78	4.60	3.60	3.04	0.00	9.98	5.56
time (sec)	N/A	0.087	0.072	17.540	0.207	0.327	0.000	1.090	1.314

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	652	479	429	0	1402	588
N.S.	1	1.00	0.79	4.18	3.07	2.75	0.00	8.99	3.77
time (sec)	N/A	0.064	0.060	7.138	0.199	0.297	0.000	0.836	1.141

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	101	463	309	297	0	990	303
N.S.	1	1.00	0.81	3.73	2.49	2.40	0.00	7.98	2.44
time (sec)	N/A	0.049	0.040	2.921	0.191	0.287	0.000	0.629	1.023

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	250	156	162	382	580	134
N.S.	1	1.00	0.86	2.91	1.81	1.88	4.44	6.74	1.56
time (sec)	N/A	0.036	0.026	1.069	0.195	0.276	55.240	0.441	0.871

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	101	0	0	0	0	566	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	7.08	0.00
time (sec)	N/A	0.151	0.031	0.000	0.000	0.000	0.000	57.494	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	114	81	136	105	0	91	113
N.S.	1	1.00	1.12	0.79	1.33	1.03	0.00	0.89	1.11
time (sec)	N/A	0.030	0.040	3.108	0.194	0.271	0.000	0.598	1.064

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	115	272	259	266	2103	207	221
N.S.	1	1.00	0.76	1.80	1.72	1.76	13.93	1.37	1.46
time (sec)	N/A	0.078	0.089	7.432	0.194	0.276	127.349	0.927	1.265

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	146	440	433	483	0	405	349
N.S.	1	1.00	0.80	2.40	2.37	2.64	0.00	2.21	1.91
time (sec)	N/A	0.092	0.100	16.009	0.205	0.280	0.000	0.676	1.623

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	162	1043	652	735	0	684	603
N.S.	1	1.00	0.75	4.85	3.03	3.42	0.00	3.18	2.80
time (sec)	N/A	0.115	0.129	42.246	0.209	0.306	0.000	0.848	1.902

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	544	544	533	0	2880	0	0	0	0
N.S.	1	1.00	0.98	0.00	5.29	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.311	0.000	0.715	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	454	454	409	0	2129	0	0	0	0
N.S.	1	1.00	0.90	0.00	4.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.226	0.000	0.715	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	303	0	1473	0	0	0	0
N.S.	1	1.00	0.84	0.00	4.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.154	0.000	0.706	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	216	0	825	0	0	0	0
N.S.	1	1.00	0.98	0.00	3.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	0.124	0.000	0.677	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	268	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.527	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	331	294	428	263	0	175	237
N.S.	1	1.00	2.03	1.80	2.63	1.61	0.00	1.07	1.45
time (sec)	N/A	0.055	0.257	3.121	0.200	0.277	0.000	0.987	2.422

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	464	672	861	654	0	407	505
N.S.	1	1.00	1.46	2.12	2.72	2.06	0.00	1.28	1.59
time (sec)	N/A	0.113	0.258	7.409	0.240	0.285	0.000	0.967	2.183

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	612	1146	1435	1167	0	776	1040
N.S.	1	1.00	1.43	2.67	3.34	2.72	0.00	1.81	2.42
time (sec)	N/A	0.186	0.382	16.052	0.277	0.294	0.000	1.308	3.823

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	700	2326	2138	1768	0	1265	1765
N.S.	1	1.00	1.31	4.34	3.99	3.30	0.00	2.36	3.29
time (sec)	N/A	0.210	0.510	42.183	0.332	0.317	0.000	1.685	6.344

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	1.06
time (sec)	N/A	0.027	0.165	0.053	0.437	0.257	24.070	27.328	0.703

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	1.06
time (sec)	N/A	0.015	0.117	0.097	0.428	0.265	14.064	15.913	0.670

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	1.06
time (sec)	N/A	0.029	0.217	0.205	0.329	0.270	18.380	9.260	0.648

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	62	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.079	0.125	0.000	0.000	0.269	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	174	0	0	147	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.158	0.260	0.000	0.000	0.261	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	1.06
time (sec)	N/A	0.030	0.429	0.054	0.474	0.263	45.489	0.997	1.316

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	1.06
time (sec)	N/A	0.014	0.470	0.019	0.484	0.261	58.587	0.691	0.894

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	1.06
time (sec)	N/A	0.030	0.256	0.074	0.366	0.260	139.335	0.628	0.757

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	180	0	0	291	0	146	0
N.S.	1	1.00	1.17	0.00	0.00	1.89	0.00	0.95	0.00
time (sec)	N/A	0.082	0.145	0.000	0.000	0.278	0.000	0.362	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	288	0	0	770	0	325	0
N.S.	1	1.00	1.12	0.00	0.00	3.01	0.00	1.27	0.00
time (sec)	N/A	0.220	0.347	0.000	0.000	0.281	0.000	0.420	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	285	1160	631	736	0	11996	1433
N.S.	1	1.00	0.78	3.19	1.73	2.02	0.00	32.96	3.94
time (sec)	N/A	0.342	0.390	5.459	0.201	0.663	0.000	1.674	1.659

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	219	976	443	521	0	6772	766
N.S.	1	1.00	0.93	4.15	1.89	2.22	0.00	28.82	3.26
time (sec)	N/A	0.197	0.184	2.960	0.196	0.414	0.000	1.050	1.405

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	146	617	282	334	0	3408	371
N.S.	1	1.00	0.93	3.93	1.80	2.13	0.00	21.71	2.36
time (sec)	N/A	0.101	0.094	1.631	0.198	0.329	0.000	0.721	1.117

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	120	366	150	179	493	1215	153
N.S.	1	1.00	1.04	3.18	1.30	1.56	4.29	10.57	1.33
time (sec)	N/A	0.062	0.081	0.814	0.184	0.272	59.226	0.566	0.933

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	82	52	63	150	243	52
N.S.	1	1.00	1.00	1.46	0.93	1.12	2.68	4.34	0.93
time (sec)	N/A	0.021	0.008	0.218	0.182	0.292	1.388	0.332	0.694

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	122	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	109	364	142	294	0	461	140
N.S.	1	1.00	1.20	4.00	1.56	3.23	0.00	5.07	1.54
time (sec)	N/A	0.059	0.089	3.514	0.191	3.144	0.000	0.546	1.365

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	1376	355	1175	0	2994	430
N.S.	1	1.00	0.91	7.24	1.87	6.18	0.00	15.76	2.26
time (sec)	N/A	0.144	0.296	14.207	0.202	45.636	0.000	0.865	3.156

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	264	3257	852	0	0	9692	1182
N.S.	1	1.00	0.93	11.51	3.01	0.00	0.00	34.25	4.18
time (sec)	N/A	0.253	0.424	41.680	0.237	0.000	0.000	0.852	5.886

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	359	5206	1761	0	0	21743	2569
N.S.	1	1.00	0.93	13.42	4.54	0.00	0.00	56.04	6.62
time (sec)	N/A	0.446	0.613	256.759	0.313	0.000	0.000	1.213	10.342

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	923	923	757	0	2651	0	0	0	0
N.S.	1	1.00	0.82	0.00	2.87	0.00	0.00	0.00	0.00
time (sec)	N/A	1.197	0.662	0.000	0.712	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	565	565	506	0	1659	0	0	0	0
N.S.	1	1.00	0.90	0.00	2.94	0.00	0.00	0.00	0.00
time (sec)	N/A	0.739	0.361	0.000	0.689	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	290	362	0	899	0	0	0	0
N.S.	1	1.00	1.25	0.00	3.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.203	0.000	0.682	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	226	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	1441	0	0	0	0	0	0
N.S.	1	1.00	4.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	418	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	389	615	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	0.809	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	747	747	918	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.046	1.627	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1208	1208	1329	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.642	3.893	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	43	31	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.34	0.97	1.06	1.06
time (sec)	N/A	0.022	0.170	0.055	0.434	0.279	28.897	26.492	0.723

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	32	29	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.97	1.07	1.07
time (sec)	N/A	0.013	0.126	0.095	0.430	0.254	16.379	16.024	0.667

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.005	0.014	0.065	0.328	0.260	4.036	11.363	0.589

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	39	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.22	0.00	1.06	1.06
time (sec)	N/A	0.029	0.557	0.213	0.336	0.262	0.000	17.042	0.656

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	63	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.00	1.06	1.06
time (sec)	N/A	0.028	0.529	0.073	0.336	0.251	0.000	26.964	0.690

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	87	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	2.72	0.00	1.06	1.06
time (sec)	N/A	0.022	5.458	0.075	0.356	0.263	0.000	35.221	0.687

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	342	71	32	34	34
N.S.	1	1.00	1.06	1.00	10.69	2.22	1.00	1.06	1.06
time (sec)	N/A	0.028	0.437	0.056	0.496	0.264	51.515	1.117	1.346

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	249	60	31	32	32
N.S.	1	1.00	1.07	1.00	8.30	2.00	1.03	1.07	1.07
time (sec)	N/A	0.014	0.325	0.017	0.514	0.263	60.304	0.777	0.823

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	195	54	22	26	26
N.S.	1	1.00	1.08	1.00	8.12	2.25	0.92	1.08	1.08
time (sec)	N/A	0.004	0.276	0.089	0.362	0.264	22.704	1.022	0.618

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	500	79	32	34	34
N.S.	1	1.00	1.06	1.00	15.62	2.47	1.00	1.06	1.06
time (sec)	N/A	0.029	0.587	0.074	0.378	0.258	146.187	1.176	0.783

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	752	120	0	34	34
N.S.	1	1.00	1.06	1.00	23.50	3.75	0.00	1.06	1.06
time (sec)	N/A	0.026	1.114	0.077	0.385	0.264	0.000	1.344	1.018

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	1001	161	0	34	34
N.S.	1	1.00	1.06	1.00	31.28	5.03	0.00	1.06	1.06
time (sec)	N/A	0.020	18.026	0.078	0.393	0.267	0.000	2.181	2.173

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	142	444	623	431	969	4036	1009
N.S.	1	1.00	0.79	2.47	3.46	2.39	5.38	22.42	5.61
time (sec)	N/A	0.070	0.072	1.115	0.206	0.305	3.516	0.531	1.470

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	315	439	318	706	2776	566
N.S.	1	1.00	0.81	2.11	2.95	2.13	4.74	18.63	3.80
time (sec)	N/A	0.064	0.064	0.883	0.204	0.283	2.043	0.447	1.254

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	207	280	222	491	1742	290
N.S.	1	1.00	0.84	1.75	2.37	1.88	4.16	14.76	2.46
time (sec)	N/A	0.049	0.040	0.700	0.213	0.263	1.314	0.422	1.127

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	69	109	144	125	253	869	126
N.S.	1	1.00	0.85	1.35	1.78	1.54	3.12	10.73	1.56
time (sec)	N/A	0.033	0.026	0.553	0.195	0.277	0.869	0.375	0.991

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	95	229	0	0	0	0	0
N.S.	1	1.00	1.19	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	0.035	1.149	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	75	105	87	132	83	233	125	104
N.S.	1	1.19	1.67	1.38	2.10	1.32	3.70	1.98	1.65
time (sec)	N/A	0.032	0.038	0.696	0.199	0.263	0.626	0.360	1.833

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	110	229	255	217	422	262	209
N.S.	1	1.00	0.76	1.59	1.77	1.51	2.93	1.82	1.45
time (sec)	N/A	0.072	0.079	0.986	0.209	0.255	1.057	0.404	1.734

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	361	428	406	656	416	339
N.S.	1	1.00	0.81	2.06	2.45	2.32	3.75	2.38	1.94
time (sec)	N/A	0.092	0.101	1.203	0.212	0.282	1.625	0.461	2.440

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	158	474	647	629	944	573	577
N.S.	1	1.00	0.77	2.30	3.14	3.05	4.58	2.78	2.80
time (sec)	N/A	0.113	0.121	2.155	0.223	0.264	2.408	0.494	2.974

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	365	511	0	2389	0	0	0	0
N.S.	1	1.00	1.40	0.00	6.55	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	0.306	0.000	0.325	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	391	0	1732	0	0	0	0
N.S.	1	1.00	1.27	0.00	5.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.214	0.000	0.316	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	287	0	1165	0	0	0	0
N.S.	1	1.00	1.13	0.00	4.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.142	0.000	0.289	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	203	0	611	0	0	0	0
N.S.	1	1.00	1.13	0.00	3.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.112	0.000	0.287	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	458	464	0	0	0	0	0
N.S.	1	1.00	3.58	3.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.939	1.184	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	314	176	416	150	434	192	222
N.S.	1	1.00	2.49	1.40	3.30	1.19	3.44	1.52	1.76
time (sec)	N/A	0.058	0.287	0.684	0.214	0.262	1.092	0.399	2.210

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	443	485	848	367	894	440	507
N.S.	1	1.00	1.65	1.81	3.16	1.37	3.34	1.64	1.89
time (sec)	N/A	0.143	0.285	1.053	0.243	0.263	2.110	0.426	2.603

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	582	892	1419	672	1544	725	1064
N.S.	1	1.00	1.39	2.13	3.39	1.61	3.69	1.73	2.55
time (sec)	N/A	0.216	0.405	1.339	0.291	0.274	11.684	0.514	4.432

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	575	666	1179	2123	1035	0	1014	1881
N.S.	1	1.00	1.16	2.05	3.69	1.80	0.00	1.76	3.27
time (sec)	N/A	0.286	0.589	2.322	0.357	0.283	0.000	0.561	7.614

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	114	30	158	30	0	1203	25
N.S.	1	1.04	4.07	1.07	5.64	1.07	0.00	42.96	0.89
time (sec)	N/A	0.014	0.044	0.844	0.204	0.251	0.000	34.962	1.058

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	61	22	0	320	15
N.S.	1	1.00	1.00	1.00	4.07	1.47	0.00	21.33	1.00
time (sec)	N/A	0.009	0.021	0.530	0.192	0.264	0.000	3.748	0.945

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	59	22	0	322	13
N.S.	1	1.00	1.00	1.31	4.54	1.69	0.00	24.77	1.00
time (sec)	N/A	0.009	0.018	0.532	0.203	0.266	0.000	3.572	0.979

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	1.06
time (sec)	N/A	0.021	0.562	0.866	0.221	0.257	8.352	13.898	1.855

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	1.07
time (sec)	N/A	0.014	0.119	0.766	0.213	0.250	4.428	11.092	1.951

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	1.06
time (sec)	N/A	0.027	0.217	0.671	0.227	0.242	2.656	9.026	2.229

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	52	61	0	47	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.067	0.137	1.850	0.000	0.249	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	89	117	0	130	0	0	0
N.S.	1	1.00	0.83	1.09	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.154	0.254	2.807	0.000	0.257	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	305	79	0	34	34
N.S.	1	1.00	1.06	1.00	9.53	2.47	0.00	1.06	1.06
time (sec)	N/A	0.021	0.930	1.062	0.225	0.254	0.000	0.888	4.301

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	227	61	274	32	32
N.S.	1	1.00	1.07	1.00	7.57	2.03	9.13	1.07	1.07
time (sec)	N/A	0.012	0.480	1.592	0.222	0.246	15.893	1.184	4.586

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	162	83	90	34	34
N.S.	1	1.00	1.06	1.00	5.06	2.59	2.81	1.06	1.06
time (sec)	N/A	0.024	0.401	0.691	0.230	0.278	1.582	0.662	5.580

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	113	0	199	0	0	0
N.S.	1	1.00	0.84	1.10	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	0.085	0.158	2.639	0.000	0.252	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	136	182	0	570	0	0	0
N.S.	1	1.00	0.64	0.86	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	0.206	0.468	3.918	0.000	0.268	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	144	446	885	454	998	490	1025
N.S.	1	1.00	0.79	2.45	4.86	2.49	5.48	2.69	5.63
time (sec)	N/A	0.065	0.065	1.201	0.223	0.330	3.610	59.292	1.707

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	122	318	647	341	707	355	567
N.S.	1	1.00	0.81	2.11	4.28	2.26	4.68	2.35	3.75
time (sec)	N/A	0.056	0.066	0.883	0.222	0.285	2.039	10.233	1.456

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	98	210	437	243	517	246	296
N.S.	1	1.00	0.82	1.75	3.64	2.02	4.31	2.05	2.47
time (sec)	N/A	0.042	0.038	0.704	0.356	0.276	1.390	1.813	1.303

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	108	250	148	250	129	120
N.S.	1	1.00	0.92	1.38	3.21	1.90	3.21	1.65	1.54
time (sec)	N/A	0.031	0.027	0.530	0.280	0.261	0.840	0.538	1.114

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	264	0	0	0	0	0
N.S.	1	1.00	1.06	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.031	0.704	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	77	111	93	187	110	255	187	108
N.S.	1	1.18	1.71	1.43	2.88	1.69	3.92	2.88	1.66
time (sec)	N/A	0.042	0.035	0.855	0.196	0.273	0.650	0.367	1.975

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	109	239	307	238	418	268	206
N.S.	1	1.00	0.79	1.73	2.22	1.72	3.03	1.94	1.49
time (sec)	N/A	0.065	0.080	1.214	0.199	0.270	1.028	0.330	1.945

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	140	379	480	430	677	477	341
N.S.	1	1.00	0.79	2.14	2.71	2.43	3.82	2.69	1.93
time (sec)	N/A	0.076	0.079	1.569	0.206	0.272	1.664	0.337	2.618

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	162	527	699	654	947	419	579
N.S.	1	1.00	0.78	2.53	3.36	3.14	4.55	2.01	2.78
time (sec)	N/A	0.083	0.118	2.708	0.221	0.287	2.416	0.663	3.313

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	377	523	0	2650	0	0	0	0
N.S.	1	1.00	1.39	0.00	7.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.318	0.000	0.346	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	402	0	1948	0	0	0	0
N.S.	1	1.00	1.26	0.00	6.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	0.225	0.000	0.333	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	298	0	1326	0	0	0	0
N.S.	1	1.00	1.17	0.00	5.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.154	0.000	0.319	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	207	0	727	0	0	0	0
N.S.	1	1.00	1.10	0.00	3.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.116	0.000	0.305	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	259	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.931	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	321	188	574	200	454	380	228
N.S.	1	1.00	2.47	1.45	4.42	1.54	3.49	2.92	1.75
time (sec)	N/A	0.065	0.272	0.940	0.228	0.272	1.177	0.723	2.817

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	451	490	1001	410	879	0	503
N.S.	1	1.00	1.66	1.80	3.68	1.51	3.23	0.00	1.85
time (sec)	N/A	0.138	0.290	1.514	0.259	0.273	2.076	0.000	2.680

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	595	1019	1575	719	1561	0	1069
N.S.	1	1.00	1.39	2.38	3.67	1.68	3.64	0.00	2.49
time (sec)	N/A	0.205	0.399	2.112	0.313	0.261	12.152	0.000	4.783

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	680	1486	2279	1084	0	874	1883
N.S.	1	1.00	1.16	2.53	3.88	1.85	0.00	1.49	3.21
time (sec)	N/A	0.262	0.581	3.497	0.372	0.295	0.000	1.067	7.763

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	1.06
time (sec)	N/A	0.023	0.103	0.718	0.217	0.246	14.055	0.526	2.267

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	1.06
time (sec)	N/A	0.014	0.081	0.628	0.220	0.290	4.835	0.577	2.450

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	1.06
time (sec)	N/A	0.027	0.104	0.679	0.220	0.268	4.859	0.423	2.643

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	91	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	149	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	308	125	0	36	36
N.S.	1	1.00	1.06	1.00	9.06	3.68	0.00	1.06	1.06
time (sec)	N/A	0.022	0.321	0.663	0.227	0.269	0.000	0.643	6.234

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	230	107	558	34	34
N.S.	1	1.00	1.06	1.00	7.19	3.34	17.44	1.06	1.06
time (sec)	N/A	0.016	0.239	0.625	0.231	0.269	22.193	0.529	6.396

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	166	129	156	36	36
N.S.	1	1.00	1.06	1.00	4.88	3.79	4.59	1.06	1.06
time (sec)	N/A	0.024	0.151	0.615	0.223	0.246	2.862	0.473	7.564

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	150	147	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	266	263	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	364	832	671	563	0	507	936
N.S.	1	1.00	2.13	4.87	3.92	3.29	0.00	2.96	5.47
time (sec)	N/A	0.065	0.540	134.474	0.220	0.263	0.000	7.178	1.428

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	273	664	467	417	0	363	520
N.S.	1	1.00	1.92	4.68	3.29	2.94	0.00	2.56	3.66
time (sec)	N/A	0.046	0.323	52.803	0.215	0.282	0.000	2.068	1.184

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	194	462	294	282	0	241	262
N.S.	1	1.00	1.72	4.09	2.60	2.50	0.00	2.13	2.32
time (sec)	N/A	0.038	0.193	18.612	0.203	0.275	0.000	0.830	1.033

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	126	265	154	163	0	131	127
N.S.	1	1.00	1.50	3.15	1.83	1.94	0.00	1.56	1.51
time (sec)	N/A	0.028	0.101	5.399	0.192	0.263	0.000	0.410	0.911

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	129	523	0	0	0	0	0
N.S.	1	1.00	1.63	6.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.071	2.591	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	89	130	116	107	0	111	97
N.S.	1	1.00	0.92	1.34	1.20	1.10	0.00	1.14	1.00
time (sec)	N/A	0.043	0.063	5.723	0.194	0.273	0.000	0.299	1.525

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	334	230	296	0	243	192
N.S.	1	1.00	0.88	2.44	1.68	2.16	0.00	1.77	1.40
time (sec)	N/A	0.057	0.203	19.185	0.202	0.273	0.000	0.325	1.239

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	143	504	400	540	0	454	317
N.S.	1	1.00	0.86	3.04	2.41	3.25	0.00	2.73	1.91
time (sec)	N/A	0.065	0.235	54.431	0.198	0.285	0.000	0.289	1.579

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	165	2309	618	820	0	718	555
N.S.	1	1.00	0.85	11.84	3.17	4.21	0.00	3.68	2.85
time (sec)	N/A	0.073	0.219	135.602	0.212	0.291	0.000	0.314	1.956

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	322	1709	10586	1871	0	0	0	0
N.S.	1	1.00	5.31	32.88	5.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.993	269.189	0.729	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	263	1149	7208	1284	0	0	0	0
N.S.	1	1.00	4.37	27.41	4.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.642	107.027	0.695	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	195	195	656	4394	779	0	0	0	0
N.S.	1	1.00	3.36	22.53	3.99	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	0.521	37.570	0.686	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	269	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	236	299	449	339	0	0	200
N.S.	1	1.00	1.83	2.32	3.48	2.63	0.00	0.00	1.55
time (sec)	N/A	0.087	0.268	7.258	0.222	0.281	0.000	0.000	1.984

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	332	871	899	919	0	0	444
N.S.	1	1.00	1.21	3.18	3.28	3.35	0.00	0.00	1.62
time (sec)	N/A	0.186	0.353	21.602	0.238	0.303	0.000	0.000	2.122

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	432	1400	1500	1635	0	0	911
N.S.	1	1.00	1.01	3.28	3.51	3.83	0.00	0.00	2.13
time (sec)	N/A	0.265	0.475	57.746	0.285	0.329	0.000	0.000	3.506

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	1011	4946	2238	2458	0	0	1579
N.S.	1	1.00	1.72	8.43	3.81	4.19	0.00	0.00	2.69
time (sec)	N/A	0.313	0.604	142.586	0.337	0.371	0.000	0.000	6.745

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	809	809	6885	0	0	0	0	0	0
N.S.	1	1.00	8.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	2.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	614	614	4802	0	0	0	0	0	0
N.S.	1	1.00	7.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	1.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	376	2984	0	0	0	0	0	0
N.S.	1	1.00	7.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.740	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	2513	0	0	0	0	0	0
N.S.	1	1.00	13.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	524	543	1129	825	0	0	474
N.S.	1	1.00	2.85	2.95	6.14	4.48	0.00	0.00	2.58
time (sec)	N/A	0.105	0.505	38.576	0.259	0.287	0.000	0.000	2.761

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	693	1622	2246	2244	0	0	966
N.S.	1	1.00	1.78	4.16	5.76	5.75	0.00	0.00	2.48
time (sec)	N/A	0.218	0.692	48.694	0.331	0.356	0.000	0.000	5.298

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	1003	2687	3630	4008	0	0	2069
N.S.	1	1.00	1.64	4.40	5.94	6.56	0.00	0.00	3.39
time (sec)	N/A	0.335	0.856	95.365	0.429	0.410	0.000	0.000	7.731

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	830	830	1370	8292	5280	6057	0	0	4257
N.S.	1	1.00	1.65	9.99	6.36	7.30	0.00	0.00	5.13
time (sec)	N/A	0.428	1.251	191.847	0.572	0.528	0.000	0.000	8.175

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	62	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.100	0.000	0.000	0.000	0.285	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	142	442	619	433	969	2030	1008
N.S.	1	1.00	0.79	2.46	3.44	2.41	5.38	11.28	5.60
time (sec)	N/A	0.077	0.069	1.155	0.227	0.348	3.778	0.475	1.621

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	314	436	320	706	1506	566
N.S.	1	1.00	0.81	2.11	2.93	2.15	4.74	10.11	3.80
time (sec)	N/A	0.063	0.057	0.928	0.203	0.294	2.219	0.448	1.355

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	206	278	223	491	1056	290
N.S.	1	1.00	0.84	1.75	2.36	1.89	4.16	8.95	2.46
time (sec)	N/A	0.053	0.036	0.786	0.221	0.276	1.478	0.400	1.227

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	69	111	143	127	253	627	126
N.S.	1	1.00	0.85	1.37	1.77	1.57	3.12	7.74	1.56
time (sec)	N/A	0.038	0.027	0.645	0.198	0.278	0.935	0.456	1.077

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	95	149	0	0	0	617	0
N.S.	1	1.00	1.17	1.84	0.00	0.00	0.00	7.62	0.00
time (sec)	N/A	0.155	0.034	1.358	0.000	0.000	0.000	37.793	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	99	86	81	134	87	231	116	106
N.S.	1	1.55	1.34	1.27	2.09	1.36	3.61	1.81	1.66
time (sec)	N/A	0.031	0.035	0.733	0.198	0.270	0.652	0.583	1.849

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	128	225	255	221	422	234	208
N.S.	1	1.00	0.89	1.56	1.77	1.53	2.93	1.62	1.44
time (sec)	N/A	0.082	0.063	1.043	0.203	0.269	1.173	0.449	1.932

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	330	428	412	656	447	339
N.S.	1	1.00	0.81	1.89	2.45	2.35	3.75	2.55	1.94
time (sec)	N/A	0.085	0.096	1.269	0.208	0.276	1.770	0.437	2.610

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	166	437	647	637	944	751	578
N.S.	1	1.00	0.81	2.12	3.14	3.09	4.58	3.65	2.81
time (sec)	N/A	0.103	0.127	2.204	0.224	0.269	2.667	0.831	3.251

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	503	503	512	0	2395	0	0	0	0
N.S.	1	1.00	1.02	0.00	4.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.459	0.322	0.000	0.314	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	420	392	0	1735	0	0	0	0
N.S.	1	1.00	0.93	0.00	4.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.218	0.000	0.312	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	335	290	0	1172	0	0	0	0
N.S.	1	1.00	0.87	0.00	3.50	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.152	0.000	0.308	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	203	0	619	0	0	0	0
N.S.	1	1.00	1.00	0.00	3.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.114	0.000	0.305	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	252	346	0	0	0	0	0
N.S.	1	1.00	1.97	2.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.890	1.417	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	314	180	416	154	430	174	223
N.S.	1	1.00	2.05	1.18	2.72	1.01	2.81	1.14	1.46
time (sec)	N/A	0.049	0.270	0.730	0.217	0.269	1.208	0.395	3.152

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	444	485	847	373	892	369	507
N.S.	1	1.00	1.50	1.64	2.86	1.26	3.01	1.25	1.71
time (sec)	N/A	0.104	0.256	1.094	0.242	0.288	2.271	0.444	2.720

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	582	836	1420	680	1544	714	1064
N.S.	1	1.00	1.46	2.10	3.56	1.70	3.87	1.79	2.67
time (sec)	N/A	0.170	0.345	1.407	0.292	0.271	12.818	0.708	4.475

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	666	1112	2122	1045	0	1194	1880
N.S.	1	1.00	1.34	2.23	4.26	2.10	0.00	2.40	3.78
time (sec)	N/A	0.202	0.502	2.376	0.360	0.291	0.000	0.562	7.813

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	1.06
time (sec)	N/A	0.022	0.539	1.092	0.222	0.261	2.564	14.765	2.020

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	1.07
time (sec)	N/A	0.013	0.115	0.909	0.224	0.252	2.042	11.642	2.312

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	1.06
time (sec)	N/A	0.026	0.324	0.780	0.223	0.255	2.920	9.754	3.064

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	55	0	50	0	0	0
N.S.	1	1.00	0.94	1.04	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.062	0.105	3.164	0.000	0.250	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	89	126	0	129	0	0	0
N.S.	1	1.00	0.82	1.16	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.122	0.216	3.355	0.000	0.246	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	309	79	400	34	34
N.S.	1	1.00	1.06	1.00	9.66	2.47	12.50	1.06	1.06
time (sec)	N/A	0.022	0.933	1.293	0.227	0.251	11.034	0.696	5.577

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	231	61	275	32	32
N.S.	1	1.00	1.07	1.00	7.70	2.03	9.17	1.07	1.07
time (sec)	N/A	0.012	0.458	1.158	0.227	0.252	6.975	1.196	6.859

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	166	83	92	34	34
N.S.	1	1.00	1.06	1.00	5.19	2.59	2.88	1.06	1.06
time (sec)	N/A	0.024	0.270	1.046	0.265	0.259	1.711	0.709	8.663

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	107	0	208	0	142	0
N.S.	1	1.00	0.85	1.03	0.00	2.00	0.00	1.37	0.00
time (sec)	N/A	0.078	0.141	3.787	0.000	0.283	0.000	0.586	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	135	268	0	584	0	291	0
N.S.	1	1.00	0.85	1.69	0.00	3.67	0.00	1.83	0.00
time (sec)	N/A	0.162	0.317	4.999	0.000	0.253	0.000	0.537	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	144	296	882	457	998	487	1024
N.S.	1	1.00	0.79	1.63	4.85	2.51	5.48	2.68	5.63
time (sec)	N/A	0.067	0.064	1.289	0.241	0.297	4.024	63.426	1.686

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	122	233	645	343	707	358	567
N.S.	1	1.00	0.81	1.54	4.27	2.27	4.68	2.37	3.75
time (sec)	N/A	0.062	0.051	0.990	0.231	0.287	2.190	10.537	1.488

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	98	185	436	245	517	243	296
N.S.	1	1.00	0.82	1.54	3.63	2.04	4.31	2.02	2.47
time (sec)	N/A	0.046	0.034	0.776	0.214	0.290	1.475	1.797	1.320

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	113	250	149	250	132	120
N.S.	1	1.00	0.92	1.45	3.21	1.91	3.21	1.69	1.54
time (sec)	N/A	0.032	0.027	0.650	0.213	0.256	0.916	0.702	1.164

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	162	0	0	0	0	0
N.S.	1	1.00	1.05	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.028	0.743	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	89	93	187	110	253	187	108
N.S.	1	1.00	0.87	0.91	1.83	1.08	2.48	1.83	1.06
time (sec)	N/A	0.026	0.033	0.836	0.218	0.265	0.696	0.417	2.908

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	128	162	306	240	418	268	206
N.S.	1	1.00	0.92	1.17	2.20	1.73	3.01	1.93	1.48
time (sec)	N/A	0.064	0.062	1.173	0.209	0.273	1.137	0.392	2.541

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	140	211	480	432	677	477	341
N.S.	1	1.00	0.79	1.19	2.71	2.44	3.82	2.69	1.93
time (sec)	N/A	0.074	0.073	1.410	0.215	0.283	1.783	0.380	3.315

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	162	232	699	658	947	424	579
N.S.	1	1.00	0.78	1.12	3.36	3.16	4.55	2.04	2.78
time (sec)	N/A	0.082	0.102	2.570	0.223	0.277	2.589	0.409	4.546

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	515	515	524	0	2660	0	0	0	0
N.S.	1	1.00	1.02	0.00	5.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.316	0.000	0.376	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	422	422	402	0	1950	0	0	0	0
N.S.	1	1.00	0.95	0.00	4.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.210	0.000	0.349	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	343	343	298	0	1333	0	0	0	0
N.S.	1	1.00	0.87	0.00	3.89	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.147	0.000	0.342	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	195	0	730	0	0	0	0
N.S.	1	1.00	0.92	0.00	3.46	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.111	0.000	0.325	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	259	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	1.242	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	322	188	573	200	450	388	227
N.S.	1	1.00	2.05	1.20	3.65	1.27	2.87	2.47	1.45
time (sec)	N/A	0.055	0.270	0.962	0.243	0.273	1.198	0.653	3.279

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	452	490	1001	413	877	0	504
N.S.	1	1.00	1.51	1.64	3.35	1.38	2.93	0.00	1.69
time (sec)	N/A	0.100	0.277	1.418	0.276	0.280	2.186	0.000	3.328

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	595	694	1576	721	1561	0	1069
N.S.	1	1.00	1.46	1.71	3.87	1.77	3.84	0.00	2.63
time (sec)	N/A	0.164	0.357	1.847	0.316	0.277	13.024	0.000	5.733

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	680	886	2278	1088	0	883	1882
N.S.	1	1.00	1.36	1.77	4.55	2.17	0.00	1.76	3.76
time (sec)	N/A	0.188	0.515	3.063	0.398	0.291	0.000	1.095	8.911

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	1.06
time (sec)	N/A	0.023	0.102	0.881	0.230	0.266	3.564	0.503	3.022

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	1.06
time (sec)	N/A	0.014	0.079	0.734	0.226	0.253	2.774	0.515	3.536

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	1.06
time (sec)	N/A	0.028	0.103	0.849	0.236	0.256	3.758	0.453	4.969

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	312	125	792	36	36
N.S.	1	1.00	1.06	1.00	9.18	3.68	23.29	1.06	1.06
time (sec)	N/A	0.022	0.309	0.783	0.246	0.294	16.082	0.644	9.728

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	234	107	559	34	34
N.S.	1	1.00	1.06	1.00	7.31	3.34	17.47	1.06	1.06
time (sec)	N/A	0.012	0.236	0.737	0.233	0.263	10.733	0.562	11.962

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	170	129	158	36	36
N.S.	1	1.00	1.06	1.00	5.00	3.79	4.65	1.06	1.06
time (sec)	N/A	0.028	0.155	0.726	0.272	0.274	2.389	0.555	13.930

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.169	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	62	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.105	0.000	0.000	0.000	0.283	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	279	594	593	636	1436	10664	1392
N.S.	1	1.00	0.79	1.67	1.67	1.79	4.05	30.04	3.92
time (sec)	N/A	0.290	0.383	1.207	0.222	0.637	55.682	1.233	1.865

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	215	412	415	445	998	6073	741
N.S.	1	1.00	0.95	1.81	1.83	1.96	4.40	26.75	3.26
time (sec)	N/A	0.197	0.172	1.030	0.211	0.384	8.132	0.793	1.587

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	265	262	280	658	3076	356
N.S.	1	1.00	0.95	1.77	1.75	1.87	4.39	20.51	2.37
time (sec)	N/A	0.091	0.088	0.931	0.205	0.308	3.120	0.607	1.289

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	114	122	140	150	318	1145	144
N.S.	1	1.00	1.05	1.12	1.28	1.38	2.92	10.50	1.32
time (sec)	N/A	0.060	0.070	0.768	0.201	0.263	1.421	0.495	1.081

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	51	54	56	83	406	47
N.S.	1	1.00	1.00	0.98	1.04	1.08	1.60	7.81	0.90
time (sec)	N/A	0.020	0.008	0.631	0.185	0.276	0.434	0.443	0.852

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	372	0	0	0	0	0
N.S.	1	1.00	0.82	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.042	3.980	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	105	242	138	255	0	511	166
N.S.	1	1.00	1.21	2.78	1.59	2.93	0.00	5.87	1.91
time (sec)	N/A	0.053	0.076	1.188	0.196	3.546	0.000	0.498	1.781

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	169	694	351	1017	0	2969	417
N.S.	1	1.00	0.92	3.79	1.92	5.56	0.00	16.22	2.28
time (sec)	N/A	0.114	0.291	1.894	0.208	45.301	0.000	0.523	3.689

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	260	1504	848	0	0	9339	1154
N.S.	1	1.00	0.95	5.47	3.08	0.00	0.00	33.96	4.20
time (sec)	N/A	0.214	0.393	3.046	0.258	0.000	0.000	0.784	7.324

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	355	2850	1757	0	0	20791	2518
N.S.	1	1.00	0.94	7.52	4.64	0.00	0.00	54.86	6.64
time (sec)	N/A	0.326	0.529	6.654	0.329	0.000	0.000	0.991	12.462

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	874	874	733	0	2140	0	0	0	0
N.S.	1	1.00	0.84	0.00	2.45	0.00	0.00	0.00	0.00
time (sec)	N/A	1.064	0.628	0.000	0.321	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	532	532	486	0	1300	0	0	0	0
N.S.	1	1.00	0.91	0.00	2.44	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	0.334	0.000	0.301	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	346	0	673	0	0	0	0
N.S.	1	1.00	1.28	0.00	2.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.194	0.000	0.274	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	214	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	1348	818	0	0	0	0	0
N.S.	1	1.00	4.87	2.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.370	4.263	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	402	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.290	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	369	595	0	0	0	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.767	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	714	714	894	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.923	1.571	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1159	1159	1301	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.410	3.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	32	29	20	103	28
N.S.	1	1.00	0.86	0.83	0.91	0.83	0.57	2.94	0.80
time (sec)	N/A	0.020	0.006	0.279	0.196	0.285	0.073	0.347	0.178

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	42	31	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.45	1.07	1.07	1.07
time (sec)	N/A	0.021	0.170	1.440	0.226	0.255	8.776	13.778	1.754

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	31	29	29	29
N.S.	1	1.00	1.07	1.00	1.07	1.15	1.07	1.07	1.07
time (sec)	N/A	0.012	0.116	2.559	0.228	0.256	4.695	10.803	1.884

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	25	19	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.19	0.90	1.10	1.10
time (sec)	N/A	0.004	0.013	1.006	0.211	0.280	0.848	11.526	0.986

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	38	31	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.31	1.07	1.07	1.07
time (sec)	N/A	0.023	0.644	1.079	0.237	0.260	2.742	17.436	1.889

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	62	32	31	31
N.S.	1	1.00	1.07	1.00	1.07	2.14	1.10	1.07	1.07
time (sec)	N/A	0.022	1.377	0.934	0.233	0.274	166.168	27.733	5.187

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	86	0	31	31
N.S.	1	1.00	1.07	1.00	1.07	2.97	0.00	1.07	1.07
time (sec)	N/A	0.020	21.204	1.453	0.226	0.270	0.000	38.331	7.908

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	318	69	0	31	31
N.S.	1	1.00	1.07	1.00	10.97	2.38	0.00	1.07	1.07
time (sec)	N/A	0.020	0.648	1.088	0.234	0.268	0.000	0.748	4.066

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	225	58	337	29	29
N.S.	1	1.00	1.07	1.00	8.33	2.15	12.48	1.07	1.07
time (sec)	N/A	0.012	0.410	0.799	0.246	0.260	22.021	1.202	4.513

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	171	52	158	23	23
N.S.	1	1.00	1.10	1.00	8.14	2.48	7.52	1.10	1.10
time (sec)	N/A	0.004	0.223	0.766	0.220	0.260	8.146	0.959	1.963

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	452	77	0	31	31
N.S.	1	1.00	1.07	1.00	15.59	2.66	0.00	1.07	1.07
time (sec)	N/A	0.020	1.435	0.609	0.247	0.266	0.000	0.936	5.479

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	688	118	0	31	31
N.S.	1	1.00	1.07	1.00	23.72	4.07	0.00	1.07	1.07
time (sec)	N/A	0.020	1.550	0.780	0.238	0.264	0.000	1.068	22.057

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	921	159	0	31	31
N.S.	1	1.00	1.07	1.00	31.76	5.48	0.00	1.07	1.07
time (sec)	N/A	0.019	48.734	2.677	0.256	0.284	0.000	1.733	31.878

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	282	599	855	660	1477	0	1403
N.S.	1	1.00	0.79	1.68	2.39	1.85	4.14	0.00	3.93
time (sec)	N/A	0.224	0.371	0.663	0.230	0.674	57.415	0.000	1.932

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	217	416	623	468	998	410	743
N.S.	1	1.00	0.95	1.82	2.72	2.04	4.36	1.79	3.24
time (sec)	N/A	0.158	0.166	0.573	0.231	0.388	8.165	75.813	1.628

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	142	270	419	301	692	257	362
N.S.	1	1.00	0.93	1.78	2.76	1.98	4.55	1.69	2.38
time (sec)	N/A	0.071	0.087	0.517	0.209	0.334	3.270	5.199	1.378

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	118	123	246	174	314	139	133
N.S.	1	1.00	1.13	1.18	2.37	1.67	3.02	1.34	1.28
time (sec)	N/A	0.053	0.070	0.391	0.204	0.285	1.408	0.702	1.110

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	54	57	80	104	82	50
N.S.	1	1.00	1.00	1.00	1.06	1.48	1.93	1.52	0.93
time (sec)	N/A	0.020	0.018	0.303	0.191	0.266	0.479	0.320	0.931

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	119	396	0	0	0	0	0
N.S.	1	1.00	0.83	2.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.041	3.368	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	108	245	192	279	0	454	191
N.S.	1	1.00	1.20	2.72	2.13	3.10	0.00	5.04	2.12
time (sec)	N/A	0.052	0.075	0.670	0.215	3.499	0.000	0.521	1.904

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	172	629	405	1036	0	486	412
N.S.	1	1.00	0.98	3.59	2.31	5.92	0.00	2.78	2.35
time (sec)	N/A	0.104	0.279	1.221	0.230	47.453	0.000	0.502	3.969

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	263	1293	900	0	0	1359	1147
N.S.	1	1.00	0.95	4.67	3.25	0.00	0.00	4.91	4.14
time (sec)	N/A	0.179	0.381	2.043	0.251	0.000	0.000	0.844	7.905

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	358	2299	1809	0	0	2159	2520
N.S.	1	1.00	0.94	6.03	4.75	0.00	0.00	5.67	6.61
time (sec)	N/A	0.262	0.514	5.498	0.323	0.000	0.000	3.176	13.522

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	869	869	746	0	2351	0	0	0	0
N.S.	1	1.00	0.86	0.00	2.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.933	0.615	0.000	0.349	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	542	542	497	0	1458	0	0	0	0
N.S.	1	1.00	0.92	0.00	2.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	0.335	0.000	0.328	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	351	0	786	0	0	0	0
N.S.	1	1.00	1.25	0.00	2.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.181	0.000	0.305	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	220	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	1370	0	0	0	0	0	0
N.S.	1	1.00	4.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.417	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	409	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	381	603	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.729	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	724	724	909	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.905	1.538	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1154	1154	1317	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.336	3.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	65	85	33	33
N.S.	1	1.00	1.06	1.00	1.06	2.10	2.74	1.06	1.06
time (sec)	N/A	0.023	0.111	0.747	0.237	0.333	21.419	0.565	2.262

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	54	83	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.86	2.86	1.07	1.07
time (sec)	N/A	0.013	0.080	0.635	0.223	0.328	6.770	0.561	2.350

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	48	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	2.09	0.96	1.09	1.09
time (sec)	N/A	0.004	0.023	0.533	0.220	0.323	1.636	0.476	1.109

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	61	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.00	1.06	1.06
time (sec)	N/A	0.022	0.062	0.611	0.233	0.314	0.000	0.513	2.406

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	85	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	2.74	0.00	1.06	1.06
time (sec)	N/A	0.022	0.062	0.650	0.237	0.311	0.000	1.197	7.127

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	109	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	3.52	0.00	1.06	1.06
time (sec)	N/A	0.022	0.067	0.702	0.249	0.283	0.000	0.511	12.412

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	321	115	0	33	33
N.S.	1	1.00	1.06	1.00	10.35	3.71	0.00	1.06	1.06
time (sec)	N/A	0.019	0.466	0.683	0.226	0.326	0.000	0.585	5.851

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	228	104	729	31	31
N.S.	1	1.00	1.07	1.00	7.86	3.59	25.14	1.07	1.07
time (sec)	N/A	0.011	0.245	0.579	0.227	0.329	26.300	0.578	6.344

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	174	98	333	25	25
N.S.	1	1.00	1.09	1.00	7.57	4.26	14.48	1.09	1.09
time (sec)	N/A	0.004	0.143	0.529	0.218	0.292	8.939	0.477	2.493

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	455	123	0	33	33
N.S.	1	1.00	1.06	1.00	14.68	3.97	0.00	1.06	1.06
time (sec)	N/A	0.023	0.283	0.540	0.235	0.313	0.000	0.761	8.003

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	690	164	0	33	33
N.S.	1	1.00	1.06	1.00	22.26	5.29	0.00	1.06	1.06
time (sec)	N/A	0.020	0.348	0.573	0.253	0.348	0.000	1.455	32.232

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	924	205	0	33	33
N.S.	1	1.00	1.06	1.00	29.81	6.61	0.00	1.06	1.06
time (sec)	N/A	0.019	0.356	0.757	0.262	0.333	0.000	1.091	48.710

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	463	1170	671	805	0	0	1434
N.S.	1	1.00	1.27	3.21	1.84	2.21	0.00	0.00	3.93
time (sec)	N/A	0.331	0.637	85.504	0.218	0.339	0.000	0.000	1.800

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	314	984	467	571	0	0	767
N.S.	1	1.00	1.33	4.17	1.98	2.42	0.00	0.00	3.25
time (sec)	N/A	0.204	0.386	33.088	0.217	0.318	0.000	0.000	1.491

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	204	623	294	365	0	304	372
N.S.	1	1.00	1.29	3.94	1.86	2.31	0.00	1.92	2.35
time (sec)	N/A	0.110	0.239	11.353	0.204	0.324	0.000	46.925	1.192

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	124	371	154	192	0	153	154
N.S.	1	1.00	1.07	3.20	1.33	1.66	0.00	1.32	1.33
time (sec)	N/A	0.071	0.108	3.158	0.198	0.323	0.000	2.533	1.031

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	83	59	59	0	58	53
N.S.	1	1.00	1.00	1.46	1.04	1.04	0.00	1.02	0.93
time (sec)	N/A	0.031	0.009	0.476	0.196	0.322	0.000	0.387	0.823

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	150	521	0	0	0	0	0
N.S.	1	1.00	1.01	3.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.068	2.521	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	117	368	151	250	0	169	141
N.S.	1	1.00	0.98	3.07	1.26	2.08	0.00	1.41	1.18
time (sec)	N/A	0.077	0.114	8.523	0.203	3.578	0.000	0.368	1.462

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	178	1385	382	1127	0	531	431
N.S.	1	1.00	0.93	7.25	2.00	5.90	0.00	2.78	2.26
time (sec)	N/A	0.137	0.348	28.812	0.218	52.004	0.000	0.520	3.177

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	273	3286	920	0	0	1530	1183
N.S.	1	1.00	0.96	11.57	3.24	0.00	0.00	5.39	4.17
time (sec)	N/A	0.260	0.620	88.437	0.261	0.000	0.000	1.138	5.901

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	366	5231	1912	0	0	3325	2570
N.S.	1	1.00	0.94	13.45	4.92	0.00	0.00	8.55	6.61
time (sec)	N/A	0.426	0.654	233.550	0.365	0.000	0.000	2.553	10.632

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	570	570	906	8443	1671	0	0	0	0
N.S.	1	1.00	1.59	14.81	2.93	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	1.070	90.914	0.756	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	472	0	903	0	0	0	0
N.S.	1	1.00	1.61	0.00	3.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.580	180.000	0.707	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	137	217	2240	0	0	0	0	0
N.S.	1	1.00	1.58	16.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	0.098	4.852	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	1082	0	0	0	0	0	0
N.S.	1	1.00	3.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	3460	0	0	0	0	0	0
N.S.	1	1.00	16.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	393	393	13182	0	0	0	0	0	0
N.S.	1	1.00	33.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	5.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	875	875	7279	0	0	0	0	0	0
N.S.	1	1.00	8.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.068	2.674	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	466	466	3890	0	0	0	0	0	0
N.S.	1	1.00	8.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.559	0.940	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	378	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	425	425	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	629	629	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.731	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [309] had the largest ratio of [.4848000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	33	0.091
2	A	4	3	1.00	33	0.091
3	A	4	3	1.00	33	0.091
4	A	4	3	1.00	31	0.097
5	A	5	5	1.00	33	0.152
6	A	2	2	1.18	33	0.061
7	A	4	3	1.00	33	0.091
8	A	4	3	1.00	33	0.091
9	A	4	3	1.00	33	0.091
10	A	8	5	1.00	35	0.143
11	A	7	5	1.00	35	0.143
12	A	6	5	1.00	35	0.143
13	A	5	5	1.00	33	0.152
14	A	4	4	1.00	35	0.114
15	A	3	3	1.00	35	0.086
16	A	7	4	1.00	35	0.114
17	A	9	4	1.00	35	0.114
18	A	11	4	1.00	35	0.114
19	N/A	0	0	1.00	35	0.000
20	N/A	0	0	1.00	33	0.000
21	N/A	0	0	1.00	35	0.000
22	A	3	3	1.00	35	0.086
23	A	7	4	1.00	35	0.114

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	N/A	0	0	1.00	35	0.000
25	N/A	0	0	1.00	33	0.000
26	N/A	0	0	1.00	35	0.000
27	A	4	4	1.00	35	0.114
28	A	9	5	1.00	35	0.143
29	A	4	3	1.00	33	0.091
30	A	4	3	1.00	33	0.091
31	A	4	3	1.00	33	0.091
32	A	4	3	1.00	31	0.097
33	A	5	5	1.00	33	0.152
34	A	3	2	1.00	33	0.061
35	A	4	3	1.00	33	0.091
36	A	4	3	1.00	33	0.091
37	A	4	3	1.00	33	0.091
38	A	19	8	1.00	35	0.229
39	A	15	8	1.00	35	0.229
40	A	11	8	1.00	35	0.229
41	A	7	7	1.00	33	0.212
42	A	4	4	1.00	35	0.114
43	A	4	3	1.00	35	0.086
44	A	8	6	1.00	35	0.171
45	A	6	5	1.00	35	0.143
46	A	5	5	1.00	35	0.143
47	N/A	0	0	1.00	35	0.000
48	N/A	0	0	1.00	33	0.000
49	N/A	0	0	1.00	35	0.000
50	A	3	3	1.00	35	0.086
51	A	7	5	1.00	35	0.143
52	N/A	0	0	1.00	35	0.000
53	N/A	0	0	1.00	33	0.000
54	N/A	0	0	1.00	35	0.000
55	A	4	4	1.00	35	0.114
56	A	10	6	1.00	35	0.171
57	A	3	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	3	2	1.00	30	0.067
59	A	3	2	1.00	30	0.067
60	A	3	2	1.00	28	0.071
61	A	3	2	1.00	22	0.091
62	A	7	4	1.00	30	0.133
63	A	3	3	1.00	30	0.100
64	A	3	2	1.00	30	0.067
65	A	3	2	1.00	30	0.067
66	A	3	2	1.00	30	0.067
67	A	15	10	1.00	32	0.312
68	A	12	10	1.00	32	0.312
69	A	9	8	1.00	30	0.267
70	A	6	6	1.00	24	0.250
71	A	9	5	1.00	32	0.156
72	A	4	4	1.00	32	0.125
73	A	9	8	1.00	32	0.250
74	A	12	10	1.00	32	0.312
75	A	15	10	1.00	32	0.312
76	N/A	0	0	1.00	32	0.000
77	N/A	0	0	1.00	30	0.000
78	N/A	0	0	1.00	24	0.000
79	N/A	0	0	1.00	32	0.000
80	N/A	0	0	1.00	32	0.000
81	N/A	0	0	1.00	32	0.000
82	N/A	0	0	1.00	32	0.000
83	N/A	0	0	1.00	30	0.000
84	N/A	0	0	1.00	24	0.000
85	N/A	0	0	1.00	32	0.000
86	N/A	0	0	1.00	32	0.000
87	N/A	0	0	1.00	32	0.000
88	A	4	3	1.00	30	0.100
89	A	4	3	1.00	30	0.100
90	A	4	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	3	1.00	28	0.107
92	A	5	5	1.00	30	0.167
93	A	2	2	1.19	30	0.067
94	A	4	3	1.00	30	0.100
95	A	4	3	1.00	30	0.100
96	A	4	3	1.00	30	0.100
97	A	8	5	1.00	32	0.156
98	A	7	5	1.00	32	0.156
99	A	6	5	1.00	32	0.156
100	A	5	5	1.00	30	0.167
101	A	4	4	1.00	32	0.125
102	A	3	3	1.00	32	0.094
103	A	7	4	1.00	32	0.125
104	A	9	4	1.00	32	0.125
105	A	11	4	1.00	32	0.125
106	A	1	1	1.04	29	0.034
107	A	1	1	1.00	18	0.056
108	A	1	1	1.00	20	0.050
109	N/A	0	0	1.00	32	0.000
110	N/A	0	0	1.00	30	0.000
111	N/A	0	0	1.00	32	0.000
112	A	3	3	1.00	32	0.094
113	A	7	4	1.00	32	0.125
114	N/A	0	0	1.00	32	0.000
115	N/A	0	0	1.00	30	0.000
116	N/A	0	0	1.00	32	0.000
117	A	4	4	1.00	32	0.125
118	A	9	5	1.00	32	0.156
119	A	4	3	1.00	32	0.094
120	A	4	3	1.00	32	0.094
121	A	4	3	1.00	32	0.094
122	A	4	3	1.00	30	0.100
123	A	5	5	1.00	32	0.156
124	A	2	2	1.18	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	3	1.00	32	0.094
126	A	4	3	1.00	32	0.094
127	A	4	3	1.00	32	0.094
128	A	8	5	1.00	34	0.147
129	A	7	5	1.00	34	0.147
130	A	6	5	1.00	34	0.147
131	A	5	5	1.00	32	0.156
132	A	4	4	1.00	34	0.118
133	A	3	3	1.00	34	0.088
134	A	7	4	1.00	34	0.118
135	A	9	4	1.00	34	0.118
136	A	11	4	1.00	34	0.118
137	N/A	0	0	1.00	34	0.000
138	N/A	0	0	1.00	32	0.000
139	N/A	0	0	1.00	34	0.000
140	A	3	3	0.97	34	0.088
141	A	7	4	0.98	34	0.118
142	N/A	0	0	1.00	34	0.000
143	N/A	0	0	1.00	32	0.000
144	N/A	0	0	1.00	34	0.000
145	A	4	4	0.98	34	0.118
146	A	9	5	0.99	34	0.147
147	A	3	2	1.00	31	0.065
148	A	3	2	1.00	31	0.065
149	A	3	2	1.00	31	0.065
150	A	3	2	1.00	29	0.069
151	A	5	5	1.00	31	0.161
152	A	3	2	1.00	31	0.065
153	A	3	2	1.00	31	0.065
154	A	3	2	1.00	31	0.065
155	A	3	2	1.00	31	0.065
156	A	8	6	1.00	33	0.182
157	A	7	6	1.00	33	0.182
158	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	5	5	1.00	33	0.152
160	A	4	4	1.00	33	0.121
161	A	8	5	1.00	33	0.152
162	A	10	5	1.00	33	0.152
163	A	12	5	1.00	33	0.152
164	A	27	15	1.00	33	0.454
165	A	17	14	1.00	33	0.424
166	A	11	9	1.00	31	0.290
167	A	6	6	1.00	33	0.182
168	A	5	4	1.00	33	0.121
169	A	10	5	1.00	33	0.152
170	A	13	5	1.00	33	0.152
171	A	16	5	1.00	33	0.152
172	A	4	4	1.00	36	0.111
173	A	4	3	1.00	30	0.100
174	A	4	3	1.00	30	0.100
175	A	4	3	1.00	30	0.100
176	A	4	3	1.00	28	0.107
177	A	5	5	1.00	30	0.167
178	A	3	2	1.55	30	0.067
179	A	4	3	1.00	30	0.100
180	A	4	3	1.00	30	0.100
181	A	4	3	1.00	30	0.100
182	A	19	8	1.00	32	0.250
183	A	15	8	1.00	32	0.250
184	A	11	8	1.00	32	0.250
185	A	7	7	1.00	30	0.233
186	A	4	4	1.00	32	0.125
187	A	4	3	1.00	32	0.094
188	A	8	6	1.00	32	0.188
189	A	6	5	1.00	32	0.156
190	A	5	5	1.00	32	0.156
191	N/A	0	0	1.00	32	0.000
192	N/A	0	0	1.00	30	0.000
193	N/A	0	0	1.00	32	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	3	3	1.00	32	0.094
195	A	7	5	1.00	32	0.156
196	N/A	0	0	1.00	32	0.000
197	N/A	0	0	1.00	30	0.000
198	N/A	0	0	1.00	32	0.000
199	A	4	4	1.00	32	0.125
200	A	10	6	1.00	32	0.188
201	A	4	3	1.00	32	0.094
202	A	4	3	1.00	32	0.094
203	A	4	3	1.00	32	0.094
204	A	4	3	1.00	30	0.100
205	A	5	5	1.00	32	0.156
206	A	3	2	1.00	32	0.062
207	A	4	3	1.00	32	0.094
208	A	4	3	1.00	32	0.094
209	A	4	3	1.00	32	0.094
210	A	19	8	1.00	34	0.235
211	A	15	8	1.00	34	0.235
212	A	11	8	1.00	34	0.235
213	A	7	7	1.00	32	0.219
214	A	4	4	1.00	34	0.118
215	A	4	3	1.00	34	0.088
216	A	8	6	1.00	34	0.176
217	A	6	5	1.00	34	0.147
218	A	5	5	1.00	34	0.147
219	N/A	0	0	1.00	34	0.000
220	N/A	0	0	1.00	32	0.000
221	N/A	0	0	1.00	34	0.000
222	A	3	3	1.00	34	0.088
223	A	7	5	1.00	34	0.147
224	N/A	0	0	1.00	34	0.000
225	N/A	0	0	1.00	32	0.000
226	N/A	0	0	1.00	34	0.000
227	A	4	4	1.00	34	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	10	6	1.00	34	0.176
229	A	4	4	1.00	36	0.111
230	A	3	2	1.00	27	0.074
231	A	3	2	1.00	27	0.074
232	A	3	2	1.00	27	0.074
233	A	3	2	1.00	25	0.080
234	A	3	2	1.00	19	0.105
235	A	7	4	1.00	27	0.148
236	A	3	3	1.00	27	0.111
237	A	3	2	1.00	27	0.074
238	A	3	2	1.00	27	0.074
239	A	3	2	1.00	27	0.074
240	A	15	10	1.00	29	0.345
241	A	12	10	1.00	29	0.345
242	A	9	8	1.00	27	0.296
243	A	6	6	1.00	21	0.286
244	A	9	5	1.00	29	0.172
245	A	4	4	1.00	29	0.138
246	A	9	8	1.00	29	0.276
247	A	12	10	1.00	29	0.345
248	A	15	10	1.00	29	0.345
249	A	3	3	1.00	14	0.214
250	N/A	0	0	1.00	29	0.000
251	N/A	0	0	1.00	27	0.000
252	N/A	0	0	1.00	21	0.000
253	N/A	0	0	1.00	29	0.000
254	N/A	0	0	1.00	29	0.000
255	N/A	0	0	1.00	29	0.000
256	N/A	0	0	1.00	29	0.000
257	N/A	0	0	1.00	27	0.000
258	N/A	0	0	1.00	21	0.000
259	N/A	0	0	1.00	29	0.000
260	N/A	0	0	1.00	29	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
261	N/A	0	0	1.00	29	0.000
262	A	3	2	1.00	29	0.069
263	A	3	2	1.00	29	0.069
264	A	3	2	1.00	29	0.069
265	A	3	2	1.00	27	0.074
266	A	3	2	1.00	21	0.095
267	A	7	4	1.00	29	0.138
268	A	3	3	1.00	29	0.103
269	A	3	2	1.00	29	0.069
270	A	3	2	1.00	29	0.069
271	A	3	2	1.00	29	0.069
272	A	15	10	1.00	31	0.323
273	A	12	10	1.00	31	0.323
274	A	9	8	1.00	29	0.276
275	A	6	6	1.00	23	0.261
276	A	9	5	1.00	31	0.161
277	A	4	4	1.00	31	0.129
278	A	9	8	1.00	31	0.258
279	A	12	10	1.00	31	0.323
280	A	15	10	1.00	31	0.323
281	N/A	0	0	1.00	31	0.000
282	N/A	0	0	1.00	29	0.000
283	N/A	0	0	1.00	23	0.000
284	N/A	0	0	1.00	31	0.000
285	N/A	0	0	1.00	31	0.000
286	N/A	0	0	1.00	31	0.000
287	N/A	0	0	1.00	31	0.000
288	N/A	0	0	1.00	29	0.000
289	N/A	0	0	1.00	23	0.000
290	N/A	0	0	1.00	31	0.000
291	N/A	0	0	1.00	31	0.000
292	N/A	0	0	1.00	31	0.000
293	A	3	2	1.00	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	3	2	1.00	31	0.065
295	A	3	2	1.00	31	0.065
296	A	3	2	1.00	29	0.069
297	A	3	2	1.00	23	0.087
298	A	7	4	1.00	31	0.129
299	A	3	2	1.00	31	0.065
300	A	3	2	1.00	31	0.065
301	A	3	2	1.00	31	0.065
302	A	3	2	1.00	31	0.065
303	A	13	11	1.00	33	0.333
304	A	10	9	1.00	31	0.290
305	A	6	6	1.00	25	0.240
306	A	10	6	1.00	33	0.182
307	A	5	5	1.00	33	0.152
308	A	10	9	1.00	33	0.273
309	A	19	16	1.00	33	0.485
310	A	13	11	1.00	31	0.355
311	A	6	6	1.00	25	0.240
312	A	12	7	1.00	33	0.212
313	A	6	6	1.00	33	0.182
314	A	13	11	1.00	33	0.333

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \frac{B(bc-ad)^4g^4nx}{5d^4}-\frac{B(bc-ad)^3g^4n(a+bx)^2}{10bd^3} \\ &+\frac{B(bc-ad)^2g^4n(a+bx)^3}{15bd^2}-\frac{B(bc-ad)g^4n(a+bx)^4}{20bd} \\ &+\frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}-\frac{B(bc-ad)^5g^4n\log(c+dx)}{5bd^5}\end{aligned}$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*n*(b*x+a)^2/b/d^3+1/
15*B*(-a*d+b*c)^2*g^4*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*n*(b*x+a)^4/b
/d+1/5*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/5*B*(-a*d+b*c)^5*g
^4*n*ln(d*x+c)/b/d^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bg^4nx(bc-ad)^4}{5d^4} \\ &-\frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4n(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4n(a+bx)^4(bc-ad)}{20bd}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2)/(10
*b*d^3) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g
^4*n*(a + b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(5*b) - (B*(b*c - a*d)^5*g^4*n*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}-\frac{(B(bc-ad)n)\int\frac{(ag+bgx)^5}{(a+bx)(c+dx)}\,dx}{5bg}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}-\frac{(B(bc-ad)g^4n)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}\\
&\quad -\frac{(B(bc-ad)g^4n)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{B(bc-ad)^4 g^4 n x}{5d^4} - \frac{B(bc-ad)^3 g^4 n (a+bx)^2}{10bd^3} \\
&\quad + \frac{B(bc-ad)^2 g^4 n (a+bx)^3}{15bd^2} - \frac{B(bc-ad) g^4 n (a+bx)^4}{20bd} \\
&\quad + \frac{g^4 (a+bx)^5 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{B(bc-ad)^5 g^4 n \log(c+dx)}{5bd^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$
\int (ag+bgx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx
$$

$$
= \frac{g^4 \left((a+bx)^5 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{B(bc-ad)n(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^{}}{12d^5}\right.}{5b}
$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c)
 + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(
12*d^5)))/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(176) = 352.

Time = 17.53 (sec) , antiderivative size = 1004, normalized size of antiderivative = 5.34

method	result	size
parallelrisch	Expression too large to display	1004

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*d^5*g^4*n+36*B*a^4*b*c*d^4*g^4
*n^2+60*B*a^3*b^2*c^2*d^3*g^4*n^2-90*B*a^2*b^3*c^3*d^2*g^4*n^2+54*B*a*b^4*c
^4*d*g^4*n^2-180*A*a^4*b*c*d^4*g^4*n+12*B*x*b^5*c^4*d*g^4*n^2+60*A*x*a^4*b*
d^5*g^4*n+12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^4*n+3*B*x^4*a*b^4*d^
5*g^4*n^2-3*B*x^4*b^5*c*d^4*g^4*n^2+60*A*x^4*a*b^4*d^5*g^4*n+16*B*x^3*a^2*b
^3*d^5*g^4*n^2+4*B*x^3*b^5*c^2*d^3*g^4*n^2+120*A*x^3*a^2*b^3*d^5*g^4*n+36*B
*x^2*a^3*b^2*d^5*g^4*n^2-6*B*x^2*b^5*c^3*d^2*g^4*n^2+120*A*x^2*a^3*b^2*d^5*
g^4*n+48*B*x*a^4*b*d^5*g^4*n^2-120*B*x*a^3*b^2*c*d^4*g^4*n^2+120*B*x*a^2*b^
3*c^2*d^3*g^4*n^2-60*B*x*a*b^4*c^3*d^2*g^4*n^2+60*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^4*b*c*d^4*g^4*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^2*d^3*g^4*n+
120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^4*n-60*B*ln(e*((b*x+a)/(d
```

```
*x+c))^n)*a*b^4*c^4*d*g^4*n-60*B*ln(b*x+a)*a^4*b*c*d^4*g^4*n^2+120*B*ln(b*x
+a)*a^3*b^2*c^2*d^3*g^4*n^2-120*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^4*n^2+60*B*ln
(b*x+a)*a*b^4*c^4*d*g^4*n^2+60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^
4*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^4*n-20*B*x^3*a*b^4*c*
d^4*g^4*n^2+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*d^5*g^4*n-60*B*x^2*
a^2*b^3*c*d^4*g^4*n^2+30*B*x^2*a*b^4*c^2*d^3*g^4*n^2-48*B*a^5*d^5*g^4*n^2-1
2*B*b^5*c^5*g^4*n^2-60*A*a^5*d^5*g^4*n+12*A*x^5*b^5*d^5*g^4*n+12*B*ln(e*((b
*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+12*B*ln(b*x+a)*a^5*d^5*g^4*n^2-12*B*ln(b*x+
a)*b^5*c^5*g^4*n^2)/d^5/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(176) = 352$.

Time = 0.34 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.03

$$\int (ag+bgx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4n\log\left(bx+a\right) - 12\left(Bb^5c^5 - 5\,Bab^4c^4d + 10\,Ba^2b^3c^3d^2 - 10\,Ba^3b^2c^2d^3 + 5\,Ba^4\right.}{}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*n*log(b*x + a) - 12*(B*b^5*c^
5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4
*b*c*d^4)*g^4*n*log(d*x + c) + 3*(20*A*a*b^4*d^5*g^4 - (B*b^5*c*d^4 - B*a*b
^4*d^5)*g^4*n)*x^4 + 4*(30*A*a^2*b^3*d^5*g^4 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c
*d^4 + 4*B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*a^3*b^2*d^5*g^4 - (B*b^5*c^3*d
^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*g^4*n)*x^2 +
 12*(5*A*a^4*b*d^5*g^4 + (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^
2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^5*g^4*x^
5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5*g^4
*x^2 + 5*B*a^4*b*d^5*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*a*b^4*d^
5*g^4*n*x^4 + 10*B*a^2*b^3*d^5*g^4*n*x^3 + 10*B*a^3*b^2*d^5*g^4*n*x^2 + 5*B
*a^4*b*d^5*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(176) = 352$.

Time = 0.21 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= \frac{1}{5}\,Bb^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&+\frac{1}{5}\,Ab^4g^4x^5 + Bab^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&+Aab^3g^4x^4 + 2\,Ba^2b^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&+2\,Aa^2b^2g^4x^3 + 2\,Ba^3bg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + 2\,Aa^3bg^4x^2 \\
&+\frac{1}{60}\,Bb^4g^4n\left(\frac{12\,a^5\log\left(bx+a\right)}{b^5}-\frac{12\,c^5\log\left(dx+c\right)}{d^5}-\frac{3\left(b^4cd^3-ab^3d^4\right)x^4-4\left(b^4c^2d^2-a^2b^2d^4\right)x^3+6\,(b}{b^4d^4}\right. \\
&-\frac{1}{6}\,Bab^3g^4n\left(\frac{6\,a^4\log\left(bx+a\right)}{b^4}-\frac{6\,c^4\log\left(dx+c\right)}{d^4}+\frac{2\left(b^3cd^2-ab^2d^3\right)x^3-3\left(b^3c^2d-a^2bd^3\right)x^2+6\,(b^3c^3}{b^3d^3}\right. \\
&+Ba^2b^2g^4n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{\left(b^2cd-abd^2\right)x^2-2\left(b^2c^2-a^2d^2\right)x}{b^2d^2}\right) \\
&-2\,Ba^3bg^4n\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&+Ba^4g^4n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right) \\
&+Ba^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + Aa^4g^4x\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
[Out] 1/5*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^4*g^4*x^
5 + B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^3*g^4*x^
4 + 2*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*a^2*b^
```

```
2*g^4*x^3 + 2*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*
a^3*b*g^4*x^2 + 1/60*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*a^2*b^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*a^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^4*g
^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4462 vs. $2(176) = 352$.

Time = 1.06 (sec) , antiderivative size = 4462, normalized size of antiderivative = 23.73

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^10*c^6*g^4*n - 6*B*a*b^9*c^5*d*g^4*n - 5*(b*x + a)*B*b^9*c^6*
d*g^4*n/(d*x + c) + 15*B*a^2*b^8*c^4*d^2*g^4*n + 30*(b*x + a)*B*a*b^8*c^5*d
^2*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 20*B*
a^3*b^7*c^3*d^3*g^4*n - 75*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) - 60
*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^7*c^6*d
^3*g^4*n/(d*x + c)^3 + 15*B*a^4*b^6*c^2*d^4*g^4*n + 100*(b*x + a)*B*a^3*b^6
*c^3*d^4*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*b^
6*c^6*d^4*g^4*n/(d*x + c)^4 - 6*B*a^5*b^5*c*d^5*g^4*n - 75*(b*x + a)*B*a^4*
b^5*c^2*d^5*g^4*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x
+ c)^2 - 150*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 30*(b*x + a)
^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*b^4*d^6*g^4*n + 30*(b*x + a)*B
*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d
*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 75*(b*x +
 a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 5*(b*x + a)*B*a^6*b^3*d^7*g^4*n
/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 - 150*(b*x +
a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 100*(b*x + a)^4*B*a^3*b^3*c^3*d^
7*g^4*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x + c)^2 + 60*(
b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 75*(b*x + a)^4*B*a^4*b^2*c^2
*d^8*g^4*n/(d*x + c)^4 - 10*(b*x + a)^3*B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 30*
(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)^4 + 5*(b*x + a)^4*B*a^6*d^10*g^4*
```

n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*x + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x + c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) + (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 113*(b*x + a)*B*b^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 678*(b*x + a)*B*a*b^8*c^5*d^2*g^4*n/(d*x + c) + 196*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1695*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) - 1176*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 156*(b*x + a)^3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 2260*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c)^2 + 936*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 48*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n - 1695*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 3920*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 288*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*d^6*g^4*n + 678*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 3120*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 720*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 113*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 960*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 196*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x + c)^2 + 936*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 720*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 156*(b*x + a)^3*B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 288*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)^4 + 48*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 + 12*B*b^10*c^6*g^4*log(e) - 72*B*a*b^9*c^5*d*g^4*log(e) - 60*(b*x + a)*B*b^9*c^6*d*g^4*log(e)/(d*x + c) + 180*B*a^2*b^8*c^4*d^2*g^4*log(e) + 360*(b*x + a)*B*a*b^8*c^5*d^2*g^4*log(e)/(d*x + c) + 120*(b*x + a)^2*B*b^8*c^6*d^2*g^4*log(e)/(d*x + c)^2 - 240*B*a^3*b^7*c^3*d^3*g^4*log(e) - 900*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*log(e)/(d*x + c) - 720*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*log(e)/(d*x + c)^2 - 120*(b*x + a)^3*B*b^7*c^6*d^3*g^4*log(e)/(d*x + c)^3 + 180*B*a^4*b^6*c^2*d^4*g^4*log(e) + 1200*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4*log(e)/(d*x + c) + 1800*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*log(e)/(d*x + c)^2 + 720*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*log(e)/(d*x + c)^3 + 60*(b*x + a)^4*B*b^6*c^6*d^4*g^4*log(e)/(d*x + c)^4 - 72*B*a^5*b^5*c*d^5*g^4*log(e) - 900*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*log(e)/(d*x + c) - 2400*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*log(e)/(d*x + c)^2 - 1800*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*log(e)/(d*x + c)^3 - 360*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*log(e)/(d*x + c)^4 + 12*B*a^6*b^4*d^6*g^4*log(e) + 360*(b*x + a)*B*a^5*b^4*c*d^6*g^4*log(e)/(d*x + c) + 1800*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*log(e)/(d*x + c)^2 + 2400*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*log(e)/(d*x + c)^3 + 900*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*log(e)/(d*x + c)^4 - 60*(b*x + a)*B*a^6*b^3*d^7*g^4*log(e)/(d*x + c) - 720*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*log(e)/(d*x + c)^2 - 1800*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*log(e)/(d*x + c)^3 - 1200*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*log(e)/(d*x + c)^4 + 120*(b*x + a)^2*B*a^6*b^2*d^8*g^4*log(e)/(d*x + c)^2 + 720*

```
(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*log(e)/(d*x + c)^3 + 900*(b*x + a)^4*B*a^4*
b^2*c^2*d^8*g^4*log(e)/(d*x + c)^4 - 120*(b*x + a)^3*B*a^6*b*d^9*g^4*log(e)
/(d*x + c)^3 - 360*(b*x + a)^4*B*a^5*b*c*d^9*g^4*log(e)/(d*x + c)^4 + 60*(b
*x + a)^4*B*a^6*d^10*g^4*log(e)/(d*x + c)^4 + 12*A*b^10*c^6*g^4 - 72*A*a*b^
9*c^5*d*g^4 - 60*(b*x + a)*A*b^9*c^6*d*g^4/(d*x + c) + 180*A*a^2*b^8*c^4*d^
2*g^4 + 360*(b*x + a)*A*a*b^8*c^5*d^2*g^4/(d*x + c) + 120*(b*x + a)^2*A*b^8
*c^6*d^2*g^4/(d*x + c)^2 - 240*A*a^3*b^7*c^3*d^3*g^4 - 900*(b*x + a)*A*a^2*
b^7*c^4*d^3*g^4/(d*x + c) - 720*(b*x + a)^2*A*a*b^7*c^5*d^3*g^4/(d*x + c)^2
 - 120*(b*x + a)^3*A*b^7*c^6*d^3*g^4/(d*x + c)^3 + 180*A*a^4*b^6*c^2*d^4*g^
4 + 1200*(b*x + a)*A*a^3*b^6*c^3*d^4*g^4/(d*x + c) + 1800*(b*x + a)^2*A*a^2
*b^6*c^4*d^4*g^4/(d*x + c)^2 + 720*(b*x + a)^3*A*a*b^6*c^5*d^4*g^4/(d*x + c
)^3 + 60*(b*x + a)^4*A*b^6*c^6*d^4*g^4/(d*x + c)^4 - 72*A*a^5*b^5*c*d^5*g^4
 - 900*(b*x + a)*A*a^4*b^5*c^2*d^5*g^4/(d*x + c) - 2400*(b*x + a)^2*A*a^3*b
^5*c^3*d^5*g^4/(d*x + c)^2 - 1800*(b*x + a)^3*A*a^2*b^5*c^4*d^5*g^4/(d*x +
c)^3 - 360*(b*x + a)^4*A*a*b^5*c^5*d^5*g^4/(d*x + c)^4 + 12*A*a^6*b^4*d^6*g
^4 + 360*(b*x + a)*A*a^5*b^4*c*d^6*g^4/(d*x + c) + 1800*(b*x + a)^2*A*a^4*b
^4*c^2*d^6*g^4/(d*x + c)^2 + 2400*(b*x + a)^3*A*a^3*b^4*c^3*d^6*g^4/(d*x +
c)^3 + 900*(b*x + a)^4*A*a^2*b^4*c^4*d^6*g^4/(d*x + c)^4 - 60*(b*x + a)*A*a
^6*b^3*d^7*g^4/(d*x + c) - 720*(b*x + a)^2*A*a^5*b^3*c*d^7*g^4/(d*x + c)^2
- 1800*(b*x + a)^3*A*a^4*b^3*c^2*d^7*g^4/(d*x + c)^3 - 1200*(b*x + a)^4*A*a
^3*b^3*c^3*d^7*g^4/(d*x + c)^4 + 120*(b*x + a)^2*A*a^6*b^2*d^8*g^4/(d*x + c
)^2 + 720*(b*x + a)^3*A*a^5*b^2*c*d^8*g^4/(d*x + c)^3 + 900*(b*x + a)^4*A*a
^4*b^2*c^2*d^8*g^4/(d*x + c)^4 - 120*(b*x + a)^3*A*a^6*b*d^9*g^4/(d*x + c)^
3 - 360*(b*x + a)^4*A*a^5*b*c*d^9*g^4/(d*x + c)^4 + 60*(b*x + a)^4*A*a^6*d^
10*g^4/(d*x + c)^4)/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*x + c) + 10*(b*x + a)
^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x + c)^3 + 5*(b*x + a)^4
*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) + 12*(B*b^6*c^6*g^4*n -
6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g
^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n
)*log(b - (b*x + a)*d/(d*x + c))/(b*d^5) - 12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*
c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*
B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x
 + a)/(d*x + c))/(b*d^5))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.56

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^2 \left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c}{d}\right.}{10\,b\,d} - \frac{a\,c\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d} + \frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d} \right)$$

$$- x^3 \left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d} - \frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d} + \frac{A\,a\,b^3\,c\,g^4}{3\,d} \right)$$

$$+ \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4 + \frac{B\,b^4\,g^4\,x^5}{5}\right) + x \left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{d} \right.$$

$$- \frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$+ \frac{a\,c\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (
A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/d) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n -
B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^4*g^4*x^5)/5 + B*a^4*
g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x*((a^
3*g^4*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b*c*n))/d - ((5*a*d + 5*b*c)*((
(5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4
*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(5*b*d)
- (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*g
^4*(5*a*d + 5*b*c))/(5*d)))/(b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/d))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n
 - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5
*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c
*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/
(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(B*b^4*c^5*g^4
*n + 5*B*a^4*c*d^4*g^4*n - 5*B*a*b^3*c^4*d*g^4*n - 10*B*a^3*b*c^2*d^3*g^4*n
 + 10*B*a^2*b^2*c^3*d^2*g^4*n))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5*g^4*n*
log(a + b*x))/(5*b)
```

3.2 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+bx)^3}{12bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}+\frac{B(bc-ad)^4g^3n\log(c+dx)}{4bd^4}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*n*(b*x+a)^2/b/d^2-1/
12*B*(-a*d+b*c)*g^3*n*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/b+1/4*B*(-a*d+b*c)^4*g^3*n*ln(d*x+c)/b/d^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}+\frac{Bg^3n(bc-ad)^4\log(c+dx)}{4bd^4} - \frac{Bg^3nx(bc-ad)^3}{4d^3}+\frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3n(a+bx)^3(bc-ad)}{12bd}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*n*x)/d^3 + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)/(8
*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A
```

```
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) + (B*(b*c - a*d)^4*g^3*n*Log[c +
 d*x])/(4*b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} - \frac{(B(bc-ad)n)\int\frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{4bg}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} - \frac{(B(bc-ad)g^3n)\int\frac{(a+bx)^3}{c+dx}\,dx}{4b}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}\\
&\quad - \frac{(B(bc-ad)g^3n)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{4b}\\
&= -\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+bx)^3}{12bd}\\
&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}+\frac{B(bc-ad)^4g^3n\log(c+dx)}{4bd^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx))}{6d^4}\right)}{4b}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(146) = 292.

Time = 7.22 (sec) , antiderivative size = 755, normalized size of antiderivative = 4.84

method	result
parallelrisch	$\frac{-36B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^2c^2d^2g^3n-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^3n+6B\ln(bx+a)a^4d^4g^3n^2+6B\ln(bx+a)b^4c^4g^3n^2+9B\,a^3bc\,d^3g}{}$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-36*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^3*n-6*B*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*c^4*g^3*n+6*B*ln(b*x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+a)*b^
4*c^4*g^3*n^2+9*B*a^3*b*c*d^3*g^3*n^2+24*B*a^2*b^2*c^2*d^2*g^3*n^2-21*B*a*b
^3*c^3*d*g^3*n^2-60*A*a^3*b*c*d^3*g^3*n+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b
^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2+24*A*x^3*a
*b^3*d^4*g^3*n+9*B*x^2*a^2*b^2*d^4*g^3*n^2+3*B*x^2*b^4*c^2*d^2*g^3*n^2+36*A
*x^2*a^2*b^2*d^4*g^3*n+18*B*x*a^3*b*d^4*g^3*n^2-6*B*x*b^4*c^3*d*g^3*n^2+24*
A*x*a^3*b*d^4*g^3*n+24*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*g^3*n-24*B*l
n(b*x+a)*a^3*b*c*d^3*g^3*n^2+36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^3*n^2-24*B*ln
(b*x+a)*a*b^3*c^3*d*g^3*n^2+24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*g^
3*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4*g^3*n-12*B*x^2*a*b^3*c*d
^3*g^3*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d^4*g^3*n-36*B*x*a^2*b^2*
c*d^3*g^3*n^2+24*B*x*a*b^3*c^2*d^2*g^3*n^2+24*B*ln(e*((b*x+a)/(d*x+c))^n)*a
^3*b*c*d^3*g^3*n-18*B*a^4*d^4*g^3*n^2+6*B*b^4*c^4*g^3*n^2-24*A*a^4*d^4*g^3*
n+6*A*x^4*b^4*d^4*g^3*n)/d^4/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(146) = 292$.

Time = 0.32 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.73

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3n\log(bx+a)+6\,(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)g^3n\log(dx+c)}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4*c^4 -
 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d*x + c
) + 2*(12*A*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*
A*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*d^4)*g^3
*n)*x^2 + 6*(4*A*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2
*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b^3*d^4
*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log(e) + 6*(B*b^4
*d^4*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^2 + 4*B*
a^3*b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.21 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{1}{4}\,Bb^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}\,Ab^3g^3x^4+Bab^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Aab^2g^3x^3+\frac{3}{2}\,Ba^2bg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}\,Aa^2bg^3x^2$$
$$-\frac{1}{24}\,Bb^3g^3n\left(\frac{6\,a^4\log\left(bx+a\right)}{b^4}-\frac{6\,c^4\log\left(dx+c\right)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-\cdots}{b^3d^3}\right.$$
$$+\frac{1}{2}\,Bab^2g^3n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}\,Ba^2bg^3n\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Ba^3g^3n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)$$
$$+Ba^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^3g^3x$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*g^3*x^
4 + B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*g^3*x^
3 + 3/2*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*
b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3034 vs. 2(146) = 292.

Time = 0.81 (sec) , antiderivative size = 3034, normalized size of antiderivative = 19.45

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] -1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*d^
2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B*a^
3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 30*(
b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5*d^3*
g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*c^2*d^
4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 20
*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*n - 20*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 4
*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b^2*c*d^6*g
^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 6*(
b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^4*b*c*d^7*g^3
*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*log((b*x + a)/(
d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/
(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4
) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 110*
B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
 225*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c
^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 380*(b*x + a)*B*a^3*b
^4*c^2*d^4*g^3*n/(d*x + c) + 450*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x +
 c)^2 + 90*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5
*g^3*n - 190*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 450*(b*x + a)^2*B*
a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n
/(d*x + c)^3 + 38*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 225*(b*x + a)^2
*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 180*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*
n/(d*x + c)^3 - 45*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 90*(b*x + a)
^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 18*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x +
c)^3 + 6*B*b^8*c^5*g^3*log(e) - 30*B*a*b^7*c^4*d*g^3*log(e) - 24*(b*x + a)*
B*b^7*c^5*d*g^3*log(e)/(d*x + c) + 60*B*a^2*b^6*c^3*d^2*g^3*log(e) + 120*(b
*x + a)*B*a*b^6*c^4*d^2*g^3*log(e)/(d*x + c) + 36*(b*x + a)^2*B*b^6*c^5*d^2
*g^3*log(e)/(d*x + c)^2 - 60*B*a^3*b^5*c^2*d^3*g^3*log(e) - 240*(b*x + a)*B
*a^2*b^5*c^3*d^3*g^3*log(e)/(d*x + c) - 180*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3
*log(e)/(d*x + c)^2 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^3*log(e)/(d*x + c)^3 +
 30*B*a^4*b^4*c*d^4*g^3*log(e) + 240*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3*log(e)
/(d*x + c) + 360*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*log(e)/(d*x + c)^2 + 120
*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*log(e)/(d*x + c)^3 - 6*B*a^5*b^3*d^5*g^3*l
og(e) - 120*(b*x + a)*B*a^4*b^3*c*d^5*g^3*log(e)/(d*x + c) - 360*(b*x + a)^
2*B*a^3*b^3*c^2*d^5*g^3*log(e)/(d*x + c)^2 - 240*(b*x + a)^3*B*a^2*b^3*c^3*
d^5*g^3*log(e)/(d*x + c)^3 + 24*(b*x + a)*B*a^5*b^2*d^6*g^3*log(e)/(d*x + c
) + 180*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*log(e)/(d*x + c)^2 + 240*(b*x + a)^
3*B*a^3*b^2*c^2*d^6*g^3*log(e)/(d*x + c)^3 - 36*(b*x + a)^2*B*a^5*b*d^7*g^3
*log(e)/(d*x + c)^2 - 120*(b*x + a)^3*B*a^4*b*c*d^7*g^3*log(e)/(d*x + c)^3
```

```
+ 24*(b*x + a)^3*B*a^5*d^8*g^3*log(e)/(d*x + c)^3 + 6*A*b^8*c^5*g^3 - 30*A*
a*b^7*c^4*d*g^3 - 24*(b*x + a)*A*b^7*c^5*d*g^3/(d*x + c) + 60*A*a^2*b^6*c^3
*d^2*g^3 + 120*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x + c) + 36*(b*x + a)^2*A*b
^6*c^5*d^2*g^3/(d*x + c)^2 - 60*A*a^3*b^5*c^2*d^3*g^3 - 240*(b*x + a)*A*a^2
*b^5*c^3*d^3*g^3/(d*x + c) - 180*(b*x + a)^2*A*a*b^5*c^4*d^3*g^3/(d*x + c)^
2 - 24*(b*x + a)^3*A*b^5*c^5*d^3*g^3/(d*x + c)^3 + 30*A*a^4*b^4*c*d^4*g^3 +
 240*(b*x + a)*A*a^3*b^4*c^2*d^4*g^3/(d*x + c) + 360*(b*x + a)^2*A*a^2*b^4*
c^3*d^4*g^3/(d*x + c)^2 + 120*(b*x + a)^3*A*a*b^4*c^4*d^4*g^3/(d*x + c)^3 -
 6*A*a^5*b^3*d^5*g^3 - 120*(b*x + a)*A*a^4*b^3*c*d^5*g^3/(d*x + c) - 360*(b
*x + a)^2*A*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 240*(b*x + a)^3*A*a^2*b^3*c^3
*d^5*g^3/(d*x + c)^3 + 24*(b*x + a)*A*a^5*b^2*d^6*g^3/(d*x + c) + 180*(b*x
+ a)^2*A*a^4*b^2*c*d^6*g^3/(d*x + c)^2 + 240*(b*x + a)^3*A*a^3*b^2*c^2*d^6*
g^3/(d*x + c)^3 - 36*(b*x + a)^2*A*a^5*b*d^7*g^3/(d*x + c)^2 - 120*(b*x + a
)^3*A*a^4*b*c*d^7*g^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^5*d^8*g^3/(d*x + c)^
3)/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/(d*x +
c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4) + 6*(
B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B
*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b +
(b*x + a)*d/(d*x + c))/(b*d^4) - 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n
 + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^
4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a)/(d*x + c))/(b*d^4))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right) - x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)\,(4\,a\,d+4\,b\,c)}{8\,b\,d} - \frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right) + \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right) + x\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)\,(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}\right.}{4\,b\,d} + \frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,d} - \frac{a\,c\,\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right) + \frac{\ln(c+d\,x)\,\left(-4\,B\,n\,a^3\,c\,d^3\,g^3+6\,B\,n\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,n\,a\,b^2\,c^3\,d\,g^3+B\,n\,b^3\,c^4\,g^3\right)}{4\,d^4} + \frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,n\,\ln(a+b\,x)}{4\,b}$$

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*g^3
*(4*a*d + 4*b*c))/(12*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n -
 B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b
*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c*
g^3)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3
*x + (3*B*a^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((b^2
*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d +
4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a
```

```
*d*n - B*b*c*n))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*g^3*(8*A*a*d + 12*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*d) - (a*c*((b^2*g^3*(16*A*a*d + 4*A*b*c +
 B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))/(b*d)) + (
log(c + d*x)*(B*b^3*c^4*g^3*n - 4*B*a^3*c*d^3*g^3*n - 4*B*a*b^2*c^3*d*g^3*n
 + 6*B*a^2*b*c^2*d^2*g^3*n))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*n*log
(a + b*x))/(4*b)
```

3.3 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx &= \frac{B(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2}{6bd} \\ &+ \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{B(bc-ad)^3g^2n\log(c+dx)}{3bd^3}\end{aligned}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*n*x/d^2-1/6*B*(-a*d+b*c)*g^2*n*(b*x+a)^2/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/3*B*(-a*d+b*c)^3*g^2*n*ln(d*
x+c)/b/d^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx &= \frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b} - \frac{Bg^2n(bc-ad)^3\log(c+dx)}{3bd^3} \\ &+ \frac{Bg^2nx(bc-ad)^2}{3d^2} - \frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2)/(6*b*
d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b
*c - a*d)^3*g^2*n*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{(B(bc-ad)n)\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{3bg}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{\left(B(bc-ad)g^2n\right)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{\left(B(bc-ad)g^2n\right)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= \frac{B(bc-ad)^2g^2nx}{3d^2}-\frac{B(bc-ad)g^2n(a+bx)^2}{6bd}\\
&\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{B(bc-ad)^3g^2n\log(c+dx)}{3bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d
)*n*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/(2*d^3)))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(116) = 232.

Time = 2.89 (sec) , antiderivative size = 528, normalized size of antiderivative = 4.26

method	result
parallelrisch	$\frac{-6B\ln(bx+a)a^2bc\,d^2g^2n^2+6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^2d^3g^2n+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b\,d^3g^2n-6Bxa\,b^2c\,d^2g^2n^2+6B\ln\left(e\left(\frac{bx+a}{dx+c}\right.\right.}{}$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-6*B*ln(b*x+a)*a^2*b*c*d^2*g^2*n^2+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^2*d^3*g^2*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^3*g^2*n-6*B*x*a*b^2*
c*d^2*g^2*n^2+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*c*d^2*g^2*n-6*B*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^2*c^2*d*g^2*n+6*B*ln(b*x+a)*a*b^2*c^2*d*g^2*n^2-4*B*a^
3*d^3*g^2*n^2-2*B*b^3*c^3*g^2*n^2-6*A*a^3*d^3*g^2*n+2*B*x^3*ln(e*((b*x+a)/(
d*x+c))^n)*b^3*d^3*g^2*n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2+6*
A*x^2*a*b^2*d^3*g^2*n+4*B*x*a^2*b*d^3*g^2*n^2+2*B*x*b^3*c^2*d*g^2*n^2+6*A*x
*a^2*b*d^3*g^2*n+B*a^2*b*c*d^2*g^2*n^2+5*B*a*b^2*c^2*d*g^2*n^2-12*A*a^2*b*c
*d^2*g^2*n+2*A*x^3*b^3*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*
n+2*B*ln(b*x+a)*a^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{2\,Ab^3d^3g^2x^3 + 2\,Ba^3d^3g^2n\log(bx+a) - 2\,(Bb^3c^3 - 3\,Bab^2c^2d + 3\,Ba^2bcd^2)g^2n\log(dx+c) + (6\,Aab^2d^3g}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*n*log(b*x + a) - 2*(B*b^3*c^3 -
3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*n*log(d*x + c) + (6*A*a*b^2*d^3*g^2
- (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*a^2*b*d^3*g^2 + (B*b^3*c^
2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*
B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3
+ 3*B*a*b^2*d^3*g^2*n*x^2 + 3*B*a^2*b*d^3*g^2*n*x)*log((b*x + a)/(d*x + c))
)/(b*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{3}\,Bb^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ab^2g^2x^3$$
$$+ Babg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aabg^2x^2$$
$$+\frac{1}{6}\,Bb^2g^2n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$- Babg^2n\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+ Ba^2g^2n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)$$
$$+ Ba^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^2g^2x$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*g^2*x^
3 + B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*g^2*x^2 +
1/6*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*g^2*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1866 vs. 2(116) = 232.

Time = 0.81 (sec) , antiderivative size = 1866, normalized size of antiderivative = 15.05

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*g
^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2*g
^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3*b^
3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*x +
```

```
a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x + a)
*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(
d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^2*B*a^3
*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2)*log
((b*x + a)/(d*x + c))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a
)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + (3*B*b^6*c^4*g^2*n -
 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^
2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 4*(b*x
 + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 42*(b*
x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a*b^3*c^3*d^3*g
^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a^3*b^2*c*d^4*g^2
*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 7*(b*x
+ a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x
+ c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*B*b^6*c^4*g^2*log(e)
 - 8*B*a*b^5*c^3*d*g^2*log(e) - 6*(b*x + a)*B*b^5*c^4*d*g^2*log(e)/(d*x + c
) + 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 24*(b*x + a)*B*a*b^4*c^3*d^2*g^2*log(
e)/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^4*d^2*g^2*log(e)/(d*x + c)^2 - 8*B*a^3
*b^3*c*d^3*g^2*log(e) - 36*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*log(e)/(d*x + c)
 - 24*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*log(e)/(d*x + c)^2 + 2*B*a^4*b^2*d^4*
g^2*log(e) + 24*(b*x + a)*B*a^3*b^2*c*d^4*g^2*log(e)/(d*x + c) + 36*(b*x +
a)^2*B*a^2*b^2*c^2*d^4*g^2*log(e)/(d*x + c)^2 - 6*(b*x + a)*B*a^4*b*d^5*g^2
*log(e)/(d*x + c) - 24*(b*x + a)^2*B*a^3*b*c*d^5*g^2*log(e)/(d*x + c)^2 + 6
*(b*x + a)^2*B*a^4*d^6*g^2*log(e)/(d*x + c)^2 + 2*A*b^6*c^4*g^2 - 8*A*a*b^5
*c^3*d*g^2 - 6*(b*x + a)*A*b^5*c^4*d*g^2/(d*x + c) + 12*A*a^2*b^4*c^2*d^2*g
^2 + 24*(b*x + a)*A*a*b^4*c^3*d^2*g^2/(d*x + c) + 6*(b*x + a)^2*A*b^4*c^4*d
^2*g^2/(d*x + c)^2 - 8*A*a^3*b^3*c*d^3*g^2 - 36*(b*x + a)*A*a^2*b^3*c^2*d^3
*g^2/(d*x + c) - 24*(b*x + a)^2*A*a*b^3*c^3*d^3*g^2/(d*x + c)^2 + 2*A*a^4*b
^2*d^4*g^2 + 24*(b*x + a)*A*a^3*b^2*c*d^4*g^2/(d*x + c) + 36*(b*x + a)^2*A*
a^2*b^2*c^2*d^4*g^2/(d*x + c)^2 - 6*(b*x + a)*A*a^4*b*d^5*g^2/(d*x + c) - 2
4*(b*x + a)^2*A*a^3*b*c*d^5*g^2/(d*x + c)^2 + 6*(b*x + a)^2*A*a^4*d^6*g^2/(
d*x + c)^2)/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/
(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3
*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^
4*g^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b*d^3) - 2*(B*b^4*c^4*g^2*n - 4*B*
a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a
^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b
*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-\frac{\ln(c+d\,x)\,\left(3\,B\,n\,a^2\,c\,d^2\,g^2-3\,B\,n\,a\,b\,c^2\,d\,g^2+B\,n\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,n\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n)
)/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b*c*g^2)/d) + x^2*((b*g^2*(9*A*a*d + 3*
A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*g^2*(3*a*d + 3*b*c))/(6*d)) - (log
(c + d*x)*(B*b^2*c^3*g^2*n + 3*B*a^2*c*d^2*g^2*n - 3*B*a*b*c^2*d*g^2*n))/(3
*d^3) + (A*b^2*g^2*x^3)/3 + (B*a^3*g^2*n*log(a + b*x))/(3*b)
```

3.4 $\int (ag+bgx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ag+bgx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-ad)^2 gn \log(c+dx)}{2bd^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
+1/2*B*(-a*d+b*c)^2*g*n*ln(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2547, 21, 45}

$$\int (ag+bgx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{g(a+bx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}+\frac{Bgn(bc-ad)^2 \log(c+dx)}{2bd^2}-\frac{Bgnx(bc-ad)}{2d}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/d + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(2*b) + (B*(b*c - a*d)^2*g*n*Log[c + d*x])/(2*b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(B(bc-ad)n)\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{2bg}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(B(bc-ad)gn)\int\frac{a+bx}{c+dx}\,dx}{2b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(B(bc-ad)gn)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx}{2b}\\
&= -\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-ad)^2gn\log(c+dx)}{2bd^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d)*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(80) = 160.

Time = 1.07 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.21

method	result
parallelrisch	$\frac{B\,x^{2} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) b^{2} d^{2} g n+A\,x^{2} b^{2} d^{2} g n+B\ln\left(bx+a\right) a^{2} d^{2} g\,n^{2}-2 B\ln\left(bx+a\right) a b c d g\,n^{2}+B\ln\left(bx+a\right) b^{2} c^{2} g\,n^{2}+2 B x\ln\left(e\left(\frac{bx+}{dx+}\right.\right.}{}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n+B*ln(b*x
+a)*a^2*d^2*g*n^2-2*B*ln(b*x+a)*a*b*c*d*g*n^2+B*ln(b*x+a)*b^2*c^2*g*n^2+2*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g*n^
2+2*A*x*a*b*d^2*g*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*g*n-B*ln(e*((b*x+
a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g*n^2+B*b^2*c^2*g*n^2-2*A*a^2*d^2*g*n-
3*A*a*b*c*d*g*n)/b/d^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)\,dx = \frac{Ab^{2}d^{2}gx^{2}+Ba^{2}d^{2}gn\log\left(bx+a\right)+\left(Bb^{2}c^{2}-2\,Babcd\right)gn\log\left(dx+c\right)+\left(2\,Aabd^{2}g-\left(Bb^{2}cd-Babd^{2}\right)gn\right.}{2\,bd^{2}}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*n*log(b*x + a) + (B*b^2*c^2 - 2*B*a*b*c*
d)*g*n*log(d*x + c) + (2*A*a*b*d^2*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (B*
b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*a*b*d^2*
g*n*x)*log((b*x + a)/(d*x + c)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(73) = 146$.

Time = 57.62 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.09

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} agx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Ax+\frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+Bnx+Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)\right) \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2} \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b}+\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b}-\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{d}+\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c+dx}+\frac{b}{c+}\right.\right.\end{cases}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Piecewise((a*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(A*x +
 B*c*log(e*(a/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n)), Eq(b
, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b) - B
*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x**2*
log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*n*
log(c/d + x)/(2*b) + B*a**2*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b)
 - B*a*c*g*n*log(c/d + x)/d + B*a*g*n*x/2 + B*a*g*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n) + B*b*c**2*g*n*log(c/d + x)/(2*d**2) - B*b*c*g*n*x/(2*d) +
 B*b*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}Bbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Abgx^2 -\frac{1}{2}Bbgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) +Bagn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bagx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aagx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*g*x^2 - 1/2*
B*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*g*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(80) = 160$.

Time = 0.48 (sec) , antiderivative size = 880, normalized size of antiderivative = 10.23

$$\int (ag + bgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx =$$

$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{2\,(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6\,(bx+}{}\right.}{b^2d^2 - \frac{2\,(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}}\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(d
*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c)
 - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x + a)*
B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*
d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^4*c^3*g*n - 3*B*a*b^3*c
^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*
(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a
^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) + B*b^4*c^3*g*
log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(e)/(d*x +
 c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*log(e)/(d*
x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*c*d^3*g*log(e)/(d*x + c
) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^4*c^3*g - 3*A*a*b^3*c^2*
d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*g + 6*(b*x +
a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - 6*(b*x + a)*A*a^2*b*c*d^3*
g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d
^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^3*c^3*g*n - 3*B*a*b^2*c^
2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x +
c))/(b*d^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n -
B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2))*(b*c/(b*c - a*d)^2 - a*d/(
b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)$$
$$+\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g\,n-2\,B\,a\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,n\,\ln(a+b\,x)}{2\,b}$$

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d*n - B*b*c*n))/(2*d) - (A*g*(2*a*d + 2*b*c)
)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b*g*x^2)/2 + B*a*g*x) + (log(
c + d*x)*(B*b*c^2*g*n - 2*B*a*c*d*g*n))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*
n*log(a + b*x))/(2*b)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{Bn\operatorname{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g+B*n*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2541, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \frac{Bn\operatorname{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(b*g)) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2541

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d
)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
a*g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{(B(bc-ad)n)\int\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx, x, a+bx\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} - \frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} - \frac{(B(bc-ad)n)\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{Bn\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$$

$$= \frac{\log(g(a+bx))\left(-Bn\log(g(a+bx))+2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2Bn\,\text{PolyLog}\left(2\right.}{2bg}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
[Out] (Log[g*(a + b*x)]*(-(B*n*Log[g*(a + b*x)]) + 2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*n*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(2*b*g)
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{bgx+ag}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="fricas
")
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="maxima
")
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/(b*g) +
integrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d
*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x), x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx=-\frac{Bn}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -B*n/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^2/(
b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2549, 2341}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx=-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{Bn(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*n*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) - ((c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\ &= -\frac{Bn(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg(ag+bgx)} + \frac{B(bc-ad)n\left(-\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g*(a*g + b*g*x))) + (B*(b*c - a
*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

method	result	size
parallelrisch	$-\frac{Bab^2d^2n^2-Bb^3cdn^2+Aab^2d^2n-Ab^3cdn-Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^2n-B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3cdn}{g^2(bx+a)b^3dn(ad-cb)}$	131

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n-B*x*ln(e*((b*x+a)
/(d*x+c))^n)*b^3*d^2*n-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n)/g^2/(b*x+a)/b
^3/d/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbc-Bad)\log(e)+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) + (B*b*d*n*x +
 B*b*c*n)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(67) = 134$.

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.04

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -Bn\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2} - \frac{A}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(Bn+B\log(e)+A)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*n*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (B*n + B*log(e)
+ A)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\,n}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B*n)/(b^2*g^2*x + a*b*g^2) - (B*log(e*((a + b*x)/(c + d*x))^n))/(b*(
a*g^2 + b*g^2*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(
b*g^2*(a*d - b*c))
```

3.7 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2g^3}$$

```
[Out] -1/4*B*n/b/g^3/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*n*ln(
b*x+a)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a
)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bdn}{2bg^3(a+bx)(bc-ad)}-\frac{Bn}{4bg^3(a+bx)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(B*n)/(b*g^3*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*g^3*(a + b*x)) +
(B*d^2*n*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c
 + d*x))^n])/(2*b*g^3*(a + b*x)^2) - (B*d^2*n*Log[c + d*x])/(2*b*(b*c - a*d
)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^2}\,dx}{2bg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2} + \frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{2bg^3}$$

$$= -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2g^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx))}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(-3*a*d
+ b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
 + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 7.39 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{3B\,a^2b^3d^3n^2+B\,b^5c^2d\,n^2+2A\,a^2b^3d^3n+2A\,b^5c^2dn-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^4d^3n-4B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^4c\,d^2n-4Ba\,b^4c\,d^2}{4g^3(bx+a)^2(a^2d^2-2abcd+b}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*B*a^2*b^3*d^3*n^2+B*b^5*c^2*d*n^2+2*A*a^2*b^3*d^3*n+2*A*b^5*c^2*d*n
-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-4*B*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^4*c*d^2*n-4*B*a*b^4*c*d^2*n^2-4*A*a*b^4*c*d^2*n-2*B*x^2*ln(e*((b*x+a)/(
d*x+c))^n)*b^5*d^3*n+2*B*x*a*b^4*d^3*n^2-2*B*x*b^5*c*d^2*n^2+2*B*ln(e*((b*x
+a)/(d*x+c))^n)*b^5*c^2*d*n)/g^3/(b*x+a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/
d/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx+(Bb^2c^2-4\,Babcd+3\,Ba^2d^2)n+2\,(Bb^2c^2-2}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d
 + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 -
2*B*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3
*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2139 vs. $2(133) = 266$.

Time = 100.06 (sec) , antiderivative size = 2139, normalized size of antiderivative = 14.17

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

```
[Out] Piecewise((zoo*(A + B*log(0**n*e))/(g**3*x**2), Eq(a, 0) & Eq(b, 0)), (-A*d
**2/(2*b**3*c**2*g**3 + 4*b**3*c*d*g**3*x + 2*b**3*d**2*g**3*x**2) - B*d**2
*log(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*b**3*c**2*g**3 + 4*b**3*
c*d*g**3*x + 2*b**3*d**2*g**3*x**2), Eq(a, b*c/d)), ((A*x + B*c*log(e*(a/(c
 + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n))/(a**3*g**3), Eq(b, 0)
), (-2*A*a**2*d**2/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2
*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**2*b**
3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c
**2*g**3*x**2) + 4*A*a*b*c*d/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8
*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x +
4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2
+ 4*b**5*c**2*g**3*x**2) - 2*A*b**2*c**2/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*
c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c
*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*
d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) - 3*B*a**2*d**2*n/(4*a**4*b*d**2*g**3
- 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 -
16*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x
 - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) + 4*B*a*b*c*d*n/(4*a**4*
b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*
c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4
*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) + 4*B*a*b*c*
d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(4*a**4*b*d**2*g**3 - 8*a**3*b**2
*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*
c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c
*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) - 2*B*a*b*d**2*n*x/(4*a**4*b*d**2*g**
```

```
3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3
- 16*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3
*x - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) + 4*B*a*b*d**2*x*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g*
*3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**
3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3
*x**2 + 4*b**5*c**2*g**3*x**2) - B*b**2*c**2*n/(4*a**4*b*d**2*g**3 - 8*a**3
*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*
b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b
**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) - 2*B*b**2*c**2*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3
*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**
2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 + 4*b
**5*c**2*g**3*x**2) + 2*B*b**2*c*d*n*x/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*
d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d
*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*
g**3*x**2 + 4*b**5*c**2*g**3*x**2) + 2*B*b**2*d**2*x**2*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b*
*2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**2*b
**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 + 4*b**5
*c**2*g**3*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{1}{4}\,Bn\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2}{(b^3c^2-}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x +
 a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{Bbn-\frac{4\,(bx+a)Bdn}{dx+c}+2\,Bb\log\left(e\right)-\frac{4\,(bx+a)Bd\log(e)}{dx+c}+2\,Ab-\frac{4\,(bx+c}{dx+}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*
x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) + (B*b*n -
4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log(e)/(d*x +
c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 -
(b*x + a)^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log(e*((a +
 b*x)/(c + d*x))^n))/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x)) - (B*d^2*n
*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x
)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2)
```

3.8 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2} - \frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3g^4}$$

```
[Out] -1/9*B*n/b/g^4/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2*n/b
/(-a*d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-
B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3+1/3*B*d^3*n*ln(d*x+c)/b/(-a*d+
b*c)^3/g^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3n\log(a+bx)}{3bg^4(bc-ad)^3} + \frac{Bd^3n\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2} + \frac{Bdn}{6bg^4(a+bx)^2(bc-ad)}-\frac{Bn}{9bg^4(a+bx)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*(B*n)/(b*g^4*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2)
- (B*d^2*n)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*n*Log[a + b*x])/(3*b
*(b*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b*g^4*(a +
b*x)^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^3}\,dx}{3bg}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)}{3bg^4}\\
&= -\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}\\
&\quad-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3g^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]

[Out] -1/18*(6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(11*a^2
*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(
a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(
b*g^4*(a + b*x)^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(174) = 348$.

Time = 15.87 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{11B\,a^3b^4d^4n^2-2B\,b^7c^3d\,n^2+6A\,a^3b^4d^4n-6A\,b^7c^3dn-18B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^6d^4n-18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^5d^4n-18B\cdots}{\cdots}$

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)

[Out] -1/18*(11*B*a^3*b^4*d^4*n^2-2*B*b^7*c^3*d*n^2+6*A*a^3*b^4*d^4*n-6*A*b^7*c^3
*d*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n-18*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b^5*d^4*n-18*B*x*a*b^6*c*d^3*n^2-18*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^5*c*d^3*n+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n+6*B*x^2*a
*b^6*d^4*n^2-6*B*x^2*b^7*c*d^3*n^2+15*B*x*a^2*b^5*d^4*n^2+3*B*x*b^7*c^2*d^2
*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d*n-18*B*a^2*b^5*c*d^3*n^2+9*B*a
*b^6*c^2*d^2*n^2-18*A*a^2*b^5*c*d^3*n+18*A*a*b^6*c^2*d^2*n-6*B*x^3*ln(e*((b
*x+a)/(d*x+c))^n)*b^7*d^4*n)/g^4/(b*x+a)^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/n/b^5/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(171) = 342.

Time = 0.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.63

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(Bb^3c^2d - 6\,Bab^2cd^2 + 5\,B}{18\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fric
as")
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*
a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a
^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*l
og(e) + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b
^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c)))/((
b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6
*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^
5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c
^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(171) = 342.

Time = 0.20 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2}\right. - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)} - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*
d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g
^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(171) = 342$.

Time = 0.66 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.08

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\left(\frac{6\left(Bb^2n-\frac{3(bx+a)Bbdn}{dx+c}+\frac{3(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{2Bb^2n-\frac{9(bx+a)Bbdn}{dx+c}+\frac{18(bx+a)^2Bd^2n}{(dx+c)^2}+6Bb^2\cdots}{\cdots}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/(
d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 -
 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^
3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*B*d^2*n/(d
*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*x + c) + 18*(b*x
+ a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*A*b*d/(d*x + c) +
18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*
(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))
*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*c
)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c
^2*n)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i +
b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(3*b*g^4*(a + b*x)^3) - (B*b*d^2*n*x^2)/(3*g^4*(a*d - b*c)^2*(
a + b*x)^3) + (7*B*a*c*d*n)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*
d^2*n)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*n*x)/(6*g^4*(a*d -
 b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.9 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3n}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4g^5}$$

```
[Out] -1/16*B*n/b/g^5/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2*n
/b/(-a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*
d^4*n*ln(b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
^5/(b*x+a)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{2547, 21, 46\}$

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4n\log(a+bx)}{4bg^5(bc-ad)^4} - \frac{Bd^4n\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2} + \frac{Bdn}{12bg^5(a+bx)^3(bc-ad)}-\frac{Bn}{16bg^5(a+bx)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/16*(B*n)/(b*g^5*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3
) - (B*d^2*n)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3*n)/(4*b*(b*c - a
*d)^3*g^5*(a + b*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A
+ B*Log[e*((a + b*x)/(c + d*x))^n])/(4*b*g^5*(a + b*x)^4) - (B*d^4*n*Log[c
+ d*x])/(4*b*(b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^4}\,dx}{4bg}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^4}{(bc-ad)^5(a+bx)}\right.}{4bg^5}\\
&= -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3}\\
&\quad-\frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3n}{4b(bc-ad)^3g^5(a+bx)}\\
&\quad+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4g^5}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4g^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$$
$$=\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}+\frac{Bn\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4) + (B*n*((-3*(b*c - a
*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2
)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d
^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(204) = 408$.

Time = 42.20 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(25*B*x^4*a^6*b^3*c*d^4*n^2-48*B*x^4*a^5*b^4*c^2*d^3*n^2+36*B*x^4*a^4*
b^5*c^3*d^2*n^2-16*B*x^4*a^3*b^6*c^4*d*n^2+12*A*x^4*a^6*b^3*c*d^4*n-48*A*x^
4*a^5*b^4*c^2*d^3*n+72*A*x^4*a^4*b^5*c^3*d^2*n-48*A*x^4*a^3*b^6*c^4*d*n+88*
B*x^3*a^7*b^2*c*d^4*n^2-180*B*x^3*a^6*b^3*c^2*d^3*n^2+144*B*x^3*a^5*b^4*c^3
*d^2*n^2-64*B*x^3*a^4*b^5*c^4*d*n^2+48*A*x^3*a^7*b^2*c*d^4*n-192*A*x^3*a^6*
b^3*c^2*d^3*n+288*A*x^3*a^5*b^4*c^3*d^2*n-192*A*x^3*a^4*b^5*c^4*d*n+108*B*x
^2*a^8*b*c*d^4*n^2-240*B*x^2*a^7*b^2*c^2*d^3*n^2+210*B*x^2*a^6*b^3*c^3*d^2*
n^2-96*B*x^2*a^5*b^4*c^4*d*n^2-192*A*x*a^8*b*c^2*d^3*n+48*B*x^3*ln(e*((b*x+
a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c*d
^4*n+3*B*x^4*a^2*b^7*c^5*n^2+12*A*x^4*a^2*b^7*c^5*n+12*B*x^3*a^3*b^6*c^5*n^
2+48*A*x^3*a^3*b^6*c^5*n+18*B*x^2*a^4*b^5*c^5*n^2+72*A*x^2*a^4*b^5*c^5*n+48
*B*x*a^9*c*d^4*n^2+12*B*x*a^5*b^4*c^5*n^2+48*A*x*a^9*c*d^4*n+48*A*x*a^5*b^4
*c^5*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2*d^3*n-12*B*ln(e*((b*x+a)/(d*x
+c))^n)*a^6*b^3*c^5*n+72*A*x^2*a^8*b*c*d^4*n-288*A*x^2*a^7*b^2*c^2*d^3*n+43
2*A*x^2*a^6*b^3*c^3*d^2*n-288*A*x^2*a^5*b^4*c^4*d*n+48*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^9*c*d^4*n-120*B*x*a^8*b*c^2*d^3*n^2+120*B*x*a^7*b^2*c^3*d^2*n^2
-60*B*x*a^6*b^3*c^4*d*n^2+288*A*x*a^7*b^2*c^3*d^2*n-192*A*x*a^6*b^3*c^4*d*n
-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^2*n+48*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^7*b^2*c^4*d*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n)/g^5
/(b*x+a)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/
n/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(201) = 402$.

Time = 0.31 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.41

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 + 6\,(Bb^4c^2 \cdots}{48\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2 \cdots}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*
d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3
*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a
*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^
4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^
2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c
^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^
8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4
)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^
4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(201) = 402.

Time = 0.21 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.03

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{48}Bn\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 2}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b
^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^
5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(201) = 402$.

Time = 0.88 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.52

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}\left(\frac{12\left(Bb^3n-\frac{4(bx+a)Bb^2dn}{dx+c}+\frac{6(bx+a)^2Bbd^2n}{(dx+c)^2}-\frac{4(bx+a)^3Bd^3n}{(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4}-\frac{3(bx+a)^4ab^2c^2dg^5}{(dx+c)^4}+\frac{3(bx+a)^4a^2bcd^2g^5}{(dx+c)^4}-\frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}}+\frac{3Bb^3n-\frac{16(bx+a)Bb^2dn}{dx+c}+\frac{36(b}{}}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/48*(12*(B*b^3*n - 4*(b*x + a)*B*b^2*d*n/(d*x + c) + 6*(b*x + a)^2*B*b*d^
2*n/(d*x + c)^2 - 4*(b*x + a)^3*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c
))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*
x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*
g^5/(d*x + c)^4) + (3*B*b^3*n - 16*(b*x + a)*B*b^2*d*n/(d*x + c) + 36*(b*x
+ a)^2*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*n/(d*x + c)^3 + 12*B*b^
3*log(e) - 48*(b*x + a)*B*b^2*d*log(e)/(d*x + c) + 72*(b*x + a)^2*B*b*d^2*l
og(e)/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*log(e)/(d*x + c)^3 + 12*A*b^3 - 48
*(b*x + a)*A*b^2*d/(d*x + c) + 72*(b*x + a)^2*A*b*d^2/(d*x + c)^2 - 48*(b*x
 + a)^3*A*d^3/(d*x + c)^3)/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x +
a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^
4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a
*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,c\ldots\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\ldots\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\ldots}$$

$$-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{4\,b\,\left(a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4\right)}$$

$$-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,\left(a\,d-b\,c\right)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{\left(a\,d-b\,c\right)^4}\right)}{2\,b\,g^5\,\left(a\,d-b\,c\right)^4}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*
x^2 + 4*a^3*b*g^5*x)) - (B*d^4*n*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8
*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g
^5*(a*d - b*c)^4)
```

3.10 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 396

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&\quad+\frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad+\frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad-\frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} \\
&\quad+\frac{B(bc-ad)^4g^4n(a+bx)\left(12A+13Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} \\
&\quad+\frac{B(bc-ad)^5g^4n\left(12A+25Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30bd^5} \\
&\quad+\frac{2B^2(bc-ad)^5g^4n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}
$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*
g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/30*B*(-a*d+b*c)^2*g^4*n
*(b*x+a)^3*(4*A+B*n+4*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/60*B*(-a*d+b*c)^
3*g^4*n*(b*x+a)^2*(12*A+7*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+1/30*B*
(-a*d+b*c)^4*g^4*n*(b*x+a)*(12*A+13*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d
^4+1/30*B*(-a*d+b*c)^5*g^4*n*(12*A+25*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))*l
n((-a*d+b*c)/b/(d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*n^2*polylog(2,d*(b*x
+a)/b/(d*x+c))/b/d^5
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{Bg^4n(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+25Bn\right)}{30bd^5}$$
$$+\frac{Bg^4n(a+bx)(bc-ad)^4\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+13Bn\right)}{30bd^4}$$
$$-\frac{Bg^4n(a+bx)^2(bc-ad)^3\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+7Bn\right)}{60bd^3}$$
$$+\frac{Bg^4n(a+bx)^3(bc-ad)^2\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{30bd^2}$$
$$-\frac{Bg^4n(a+bx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10bd}$$
$$+\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b}+\frac{2B^2g^4n^2(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/10*(B*(b*c - a*d)*g^4*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b
) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3*(4*A + B*n + 4*B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2*(12*A + 7*B*n
 + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*
n*(a + b*x)*(12*A + 13*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(30*b*d^
4) + (B*(b*c - a*d)^5*g^4*n*(12*A + 25*B*n + 12*B*Log[e*((a + b*x)/(c + d*x
))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^
4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
```

```
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{x^4(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)^5 g^4 n\right) \text{Subst}\left(\int \frac{x^4(A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)g^4 n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{\left(B(bc-ad)^5 g^4 n\right) \text{Subst}\left(\int \frac{x^3(4A+Bn+4B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10bd}$$

$$= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} - \frac{(B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{x^2(4Bn+3(4A+Bn)+12B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30bd^2}$$

$$= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} + \frac{(B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{x(12Bn+2(4Bn+3(4A+Bn))+24B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{60bd^3}$$

$$= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4n(a+bx)\left(12A+13Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} - \frac{(B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{36Bn+2(4Bn+3(4A+Bn))+24B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{60bd^4}$$

$$= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4n(a+bx)\left(12A+13Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} + \frac{B(bc-ad)^5g^4n\left(12A+25Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30bd^5} - \frac{(2B^2(bc-ad)^5g^4n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{5bd^5}$$

$$= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4n(a+bx)\left(12A+13Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} + \frac{B(bc-ad)^5g^4n\left(12A+25Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30bd^5} + \frac{2B^2(bc-ad)^5g^4n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.35

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \frac{g^4\left((a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{B(bc-ad)n\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-12d^2(bc-ad)^2(a+\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

[Out] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2945 vs. $2(381) = 762$.

Time = 0.73 (sec) , antiderivative size = 2945, normalized size of antiderivative = 7.44

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b^4*g^
4*x^5 + 2*A*B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b^3*g^4*x^4 + 4*A*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A^2*a^2*b^2*g^4*x^3 + 4*A*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 2*A^2*a^3*b*g^4*x^2 + 1/30*A*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*a^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + A^2*a^4*g^4*x - 1/30*((25*g^4*n^2 + 12*g^4*n*log(e))*b^4*c^5 - (113*
g^4*n^2 + 60*g^4*n*log(e))*a*b^3*c^4*d + 4*(49*g^4*n^2 + 30*g^4*n*log(e))*a
^2*b^2*c^3*d^2 - 12*(13*g^4*n^2 + 10*g^4*n*log(e))*a^3*b*c^2*d^3 + 12*(4*g^
4*n^2 + 5*g^4*n*log(e))*a^4*c*d^4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4*
n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d
^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*
d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5
) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 12*B^2*a^5*d^5*g^4*n^2*log(b*x
+ a)^2 - 6*(b^5*c*d^4*g^4*n*log(e) - (g^4*n*log(e) + 10*g^4*log(e)^2)*a*b^4
```

*d^5)*B^2*x^4 + 2*((g^4*n^2 + 4*g^4*n*log(e))*b^5*c^2*d^3 - 2*(g^4*n^2 + 10
*g^4*n*log(e))*a*b^4*c*d^4 + (g^4*n^2 + 16*g^4*n*log(e) + 60*g^4*log(e)^2)*
a^2*b^3*d^5)*B^2*x^3 - ((7*g^4*n^2 + 12*g^4*n*log(e))*b^5*c^3*d^2 - 3*(9*g^
4*n^2 + 20*g^4*n*log(e))*a*b^4*c^2*d^3 + 3*(11*g^4*n^2 + 40*g^4*n*log(e))*a
^2*b^3*c*d^4 - (13*g^4*n^2 + 72*g^4*n*log(e) + 120*g^4*log(e)^2)*a^3*b^2*d^
5)*B^2*x^2 + 24*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d
^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B^2*log(b*
x + a)*log(d*x + c) - 12*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*
b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B
^2*log(d*x + c)^2 + 2*((13*g^4*n^2 + 12*g^4*n*log(e))*b^5*c^4*d - (59*g^4*n
^2 + 60*g^4*n*log(e))*a*b^4*c^3*d^2 + 6*(17*g^4*n^2 + 20*g^4*n*log(e))*a^2*
b^3*c^2*d^3 - (79*g^4*n^2 + 120*g^4*n*log(e))*a^3*b^2*c*d^4 + (23*g^4*n^2 +
48*g^4*n*log(e) + 30*g^4*log(e)^2)*a^4*b*d^5)*B^2*x + 2*(12*a*b^4*c^4*d*g^
4*n^2 - 54*a^2*b^3*c^3*d^2*g^4*n^2 + 94*a^3*b^2*c^2*d^3*g^4*n^2 - 77*a^4*b*
c*d^4*g^4*n^2 + (25*g^4*n^2 + 12*g^4*n*log(e))*a^5*d^5)*B^2*log(b*x + a) +
12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*
x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*log((b*x + a)^n)^
2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*
g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*log((d*x + c)
^n)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5*log(e) + 12*B^2*a^5*d^5*g^4*n*log(b*x + a
) - 3*(b^5*c*d^4*g^4*n - (g^4*n + 20*g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^
5*c^2*d^3*g^4*n - 5*a*b^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*log(e))*a^2*b^3
*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c
*d^4*g^4*n - 2*(3*g^4*n + 10*g^4*log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4
*d*g^4*n - 5*a*b^4*c^3*d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c*
d^4*g^4*n + (4*g^4*n + 5*g^4*log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n -
5*a*b^4*c^4*d*g^4*n + 10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n
+ 5*a^4*b*c*d^4*g^4*n)*B^2*log(d*x + c))*log((b*x + a)^n) - 2*(12*B^2*b^5*d
^5*g^4*x^5*log(e) + 12*B^2*a^5*d^5*g^4*n*log(b*x + a) - 3*(b^5*c*d^4*g^4*n
- (g^4*n + 20*g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4*n - 5*a*b
^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*log(e))*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*
c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c*d^4*g^4*n - 2*(3*g^4*n
+ 10*g^4*log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4*n - 5*a*b^4*c^3*
d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c*d^4*g^4*n + (4*g^4*n +
5*g^4*log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n - 5*a*b^4*c^4*d*g^4*n +
10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n + 5*a^4*b*c*d^4*g^4*n)*
B^2*log(d*x + c) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B
^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x
)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^5)

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^4\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.11 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 335

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} \\
&\quad-\frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} \\
&\quad-\frac{B(bc-ad)^4g^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12bd^4} \\
&\quad-\frac{B^2(bc-ad)^4g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g
^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(-a*d+b*c)^2*g^3*n*
(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/12*B*(-a*d+b*c)^3
*g^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/12*B*(-a*d
+b*c)^4*g^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d
*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/
d^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2549, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=-\frac{Bg^3n(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{12bd^4}$$
$$-\frac{Bg^3n(a+bx)(bc-ad)^3\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{12bd^3}$$
$$+\frac{Bg^3n(a+bx)^2(bc-ad)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{12bd^2}$$
$$-\frac{Bg^3n(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6bd}$$
$$+\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b}-\frac{B^2g^3n^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*g^3*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*d) + (g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b)
 + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(6*A + 5*B*n + 6
*B*Log[e*((a + b*x)/(c + d*x))^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*n*(6*
A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*
x))])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(
c + d*x))])/(2*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^4 g^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} \\
&= -\frac{B(bc-ad)g^3 n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad + \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd} \\
&= -\frac{B(bc-ad)g^3 n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^2 g^3 n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} \\
&\quad - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{12bd^2}
\end{aligned}$$

$$= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} - \frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} + \frac{(B(bc-ad)^4g^3n)\operatorname{Subst}\left(\int \frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{12bd^3}$$

$$= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} - \frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} - \frac{B(bc-ad)^4g^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12bd^4} + \frac{(B^2(bc-ad)^4g^3n^2)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd^4}$$

$$= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} - \frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} - \frac{B(bc-ad)^4g^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12bd^4} - \frac{B^2(bc-ad)^4g^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+\cdots\right.}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(
a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])))/(3*d^4)))/(4*b)
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
```

```
3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*
g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(322) = 644$.

Time = 0.72 (sec) , antiderivative size = 2175, normalized size of antiderivative = 6.49

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*g^
3*x^4 + 2*A*B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b^2*g^3*x^3 + 3*A*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/2*A^2*a^2*b*g^3*x^2 - 1/12*A*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A^2*a^3*g^3*x + 1/12*((11*g^3*n^2 + 6*g^3*n*log(e))*b^3*c^4 - 2*(19*g^
3*n^2 + 12*g^3*n*log(e))*a*b^2*c^3*d + 9*(5*g^3*n^2 + 4*g^3*n*log(e))*a^2*b
*c^2*d^2 - 6*(3*g^3*n^2 + 4*g^3*n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 +
 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 -
 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(
b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*
B^2*b^4*d^4*g^3*x^4*log(e)^2 - 3*B^2*a^4*d^4*g^3*n^2*log(b*x + a)^2 - 2*(b^
4*c*d^3*g^3*n*log(e) - (g^3*n*log(e) + 6*g^3*log(e)^2)*a*b^3*d^4)*B^2*x^3 +
```

```
((g^3*n^2 + 3*g^3*n*log(e))*b^4*c^2*d^2 - 2*(g^3*n^2 + 6*g^3*n*log(e))*a*b
^3*c*d^3 + (g^3*n^2 + 9*g^3*n*log(e) + 18*g^3*log(e)^2)*a^2*b^2*d^4)*B^2*x^
2 - 6*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2
- 4*a^3*b*c*d^3*g^3*n^2)*B^2*log(b*x + a)*log(d*x + c) + 3*(b^4*c^4*g^3*n^2
 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^
2)*B^2*log(d*x + c)^2 - ((5*g^3*n^2 + 6*g^3*n*log(e))*b^4*c^3*d - (17*g^3*n
^2 + 24*g^3*n*log(e))*a*b^3*c^2*d^2 + (19*g^3*n^2 + 36*g^3*n*log(e))*a^2*b^
2*c*d^3 - (7*g^3*n^2 + 18*g^3*n*log(e) + 12*g^3*log(e)^2)*a^3*b*d^4)*B^2*x
- (6*a*b^3*c^3*d*g^3*n^2 - 21*a^2*b^2*c^2*d^2*g^3*n^2 + 26*a^3*b*c*d^3*g^3*
n^2 - (11*g^3*n^2 + 6*g^3*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*
d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a
^3*b*d^4*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d
^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*log((d*x +
c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) + 6*B^2*a^4*d^4*g^3*n*log(b*x + a)
- 2*(b^4*c*d^3*g^3*n - (g^3*n + 12*g^3*log(e))*a*b^3*d^4)*B^2*x^3 + 3*(b^4*
c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*n + 3*(g^3*n + 4*g^3*log(e))*a^2*b^2*d^4)
*B^2*x^2 - 6*(b^4*c^3*d*g^3*n - 4*a*b^3*c^2*d^2*g^3*n + 6*a^2*b^2*c*d^3*g^3
*n - (3*g^3*n + 4*g^3*log(e))*a^3*b*d^4)*B^2*x + 6*(b^4*c^4*g^3*n - 4*a*b^3
*c^3*d*g^3*n + 6*a^2*b^2*c^2*d^2*g^3*n - 4*a^3*b*c*d^3*g^3*n)*B^2*log(d*x +
 c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) + 6*B^2*a^4*d^4*g^3*n
*log(b*x + a) - 2*(b^4*c*d^3*g^3*n - (g^3*n + 12*g^3*log(e))*a*b^3*d^4)*B^2
*x^3 + 3*(b^4*c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*n + 3*(g^3*n + 4*g^3*log(e)
)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3*n - 4*a*b^3*c^2*d^2*g^3*n + 6*a^2
*b^2*c*d^3*g^3*n - (3*g^3*n + 4*g^3*log(e))*a^3*b*d^4)*B^2*x + 6*(b^4*c^4*g
^3*n - 4*a*b^3*c^3*d*g^3*n + 6*a^2*b^2*c^2*d^2*g^3*n - 4*a^3*b*c*d^3*g^3*n)
*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^
2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*log((b*x + a)^n))*log((d*x +
 c)^n))/(b*d^4)
```

Giac [F]

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.12 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 274

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd}\\&+\frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b}\\&+\frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2}\\&+\frac{B(bc-ad)^3g^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3}\\&+\frac{2B^2(bc-ad)^3g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/3*g
^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/3*B*(-a*d+b*c)^2*g^2*n*(
b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2
*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3
+2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2549, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\frac{Bg^2n(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{3bd^3}+\frac{Bg^2n(a+bx)(bc-ad)^2\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{3bd^2}-\frac{Bg^2n(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b}+\frac{2B^2g^2n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^2*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b)
 + (B*(b*c - a*d)^2*g^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c +
d*x))^n]))/(3*b*d^2) + (B*(b*c - a*d)^3*g^2*n*(2*A + 3*B*n + 2*B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^3) + (2*B^2*(b
*c - a*d)^3*g^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)^3g^2n\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}\\
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} + \frac{\left(B(bc-ad)^3g^2n\right)\text{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}\\
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} + \frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2} - \frac{\left(B(bc-ad)^3g^2n\right)\text{Subst}\left(\int \frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&+ \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&+ \frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2} \\
&+ \frac{B(bc-ad)^3g^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\
&- \frac{(2B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3} \\
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&+ \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&+ \frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2} \\
&+ \frac{B(bc-ad)^3g^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\
&+ \frac{2B^2(bc-ad)^3g^2n^2\,\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.11

$$
\int(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-d^2(a+bx)^2\left(A+B\log\left(\right.\right.\right.}{}\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*
(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[c + d
*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]

$$\int (bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\int (bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(263) = 526$.

Time = 0.69 (sec) , antiderivative size = 1501, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*g^
2*x^3 + 2*A*B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*
g^2*x^2 + 1/3*A*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
a*b*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*a^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^2*g^2*
x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^2*g^2*x - 1/3*((3*g^2*n^2
+ 2*g^2*n*log(e))*b^2*c^3 - (7*g^2*n^2 + 6*g^2*n*log(e))*a*b*c^2*d + 2*(2*g
^2*n^2 + 3*g^2*n*log(e))*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*g^2
*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - B^2*a^3*d^3*g^2*n
^2*log(b*x + a)^2 - (b^3*c*d^2*g^2*n*log(e) - (g^2*n*log(e) + 3*g^2*log(e)^
2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*
b*c*d^2*g^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*g^2*n^2 - 3*a*b^2
*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(d*x + c)^2 + ((g^2*n^2 + 2*
g^2*n*log(e))*b^3*c^2*d - 2*(g^2*n^2 + 3*g^2*n*log(e))*a*b^2*c*d^2 + (g^2*n
^2 + 4*g^2*n*log(e) + 3*g^2*log(e)^2)*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2
*n^2 - 5*a^2*b*c*d^2*g^2*n^2 + (3*g^2*n^2 + 2*g^2*n*log(e))*a^3*d^3)*B^2*lo
g(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d
^3*g^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x
^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*g^2*x^3*log
(e) + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) - (b^3*c*d^2*g^2*n - (g^2*n + 6*g^2*
log(e))*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2*n - 3*a*b^2*c*d^2*g^2*n + (2*
g^2*n + 3*g^2*log(e))*a^2*b*d^3)*B^2*x - 2*(b^3*c^3*g^2*n - 3*a*b^2*c^2*d*g
^2*n + 3*a^2*b*c*d^2*g^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) - (2*B^2*b^3
*d^3*g^2*x^3*log(e) + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) - (b^3*c*d^2*g^2*n -
 (g^2*n + 6*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2*n - 3*a*b^2*c
*d^2*g^2*n + (2*g^2*n + 3*g^2*log(e))*a^2*b*d^3)*B^2*x - 2*(b^3*c^3*g^2*n -
 3*a*b^2*c^2*d*g^2*n + 3*a^2*b*c*d^2*g^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d
^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((b*x + a)
^n))*log((d*x + c)^n))/(b*d^3)
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
```

```
ac")
[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.13 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 196

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b}-\frac{B(bc-ad)^2gn\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2}-\frac{B^2(bc-ad)^2gn^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-B*(-a*d+b*c)^2*g*n*(A+B*n+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*n^2*po
lylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{2549, 2381, 2384, 2354, 2438\}$

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= -\frac{Bgn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{bd^2}$$
$$-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd}$$
$$+\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b}-\frac{B^2gn^2(bc-ad)^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*d
)) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b) - (B*(b
*c - a*d)^2*g*n*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d
)/(b*(c + d*x))])/(b*d^2) - (B^2*(b*c - a*d)^2*g*n^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^2 g\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{\left(B(bc-ad)^2 gn\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}$$

$$= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} + \frac{\left(B(bc-ad)^2 gn\right)\text{Subst}\left(\int \frac{A+Bn+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}$$

$$= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{B(bc-ad)^2 gn\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} + \frac{\left(B^2(bc-ad)^2 gn^2\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}$$

$$= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{B(bc-ad)^2 gn\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} - \frac{B^2(bc-ad)^2 gn^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A\right.\right.}{}\right.}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)*n
*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c - a
*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2
*b)
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(193) = 386$.

Time = 0.70 (sec) , antiderivative size = 828, normalized size of antiderivative = 4.22

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = ABbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2bgx^2 - ABbgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + 2ABagn\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + 2ABagx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + A^2agx + \frac{((gn^2+gn\log(e))bc^2 - (gn^2+2gn\log(e))acd)B^2\log(dx+c)}{d^2} + \frac{(b^2c^2gn^2 - 2abcdgn^2 + a^2d^2gn^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} - \frac{B^2a^2d^2gn^2\log(bx+a)^2 - B^2b^2d^2gx^2\log(e)^2 + 2(b^2c^2gn^2 - 2abcdgn^2)B^2\log(bx+a)\log(dx+c) - (b^2}{}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
[Out] A*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*g*x^2 - A*B*
b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*g*x*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + A^2*a*g*x + ((g*n^2 + g*n*log(e))*b*c^2 - (g
*n^2 + 2*g*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*a*b*c
*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*g*n^2*l
og(b*x + a)^2 - B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c^2*g*n^2 - 2*a*b*c*d*g
*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2
*log(d*x + c)^2 + 2*(b^2*c*d*g*n*log(e) - (g*n*log(e) + g*log(e)^2)*a*b*d^2
)*B^2*x + 2*(a*b*c*d*g*n^2 - (g*n^2 + g*n*log(e))*a^2*d^2)*B^2*log(b*x + a)
 - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n)^2 - (B^2*b^2*d^
```

```
2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((d*x + c)^n)^2 - 2*(B^2*b^2*d^2*g*x^2*log(
e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g*log(e))*a*b*d
^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c))*log((b*x + a)^n
) + 2*(B^2*b^2*d^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g
*n - (g*n + 2*g*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*
log(d*x + c) + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.14 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 138

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2549, 2379, 2421, 6724}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \frac{2Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bg} + \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/(b*g)) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/(b*g) + (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*
(a + b*x))])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}\\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg}\\
&\quad+\frac{(2Bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}
\end{aligned}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2n^2\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 537 vs. 2(138) = 276.

Time = 0.81 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.89

$$\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}dx$$

$$= \frac{3\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] (3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 + 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*n^2*(Log[a/b + x]^3
 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Lo
g[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*
(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(L
og[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d
)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - in
tegrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((
b*x + a)^n)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*lo
g(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*l
og(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^
2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x), x)
```

3.15 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 136

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=-\frac{2B^2n^2(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2549, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=-\frac{2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2n^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2*n^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B*n*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= -\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(2Bn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= -\frac{2B^2n^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)} \\
&\quad - \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n
*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*
x)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(136) = 272.

Time = 3.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.18

method	result
parallelrisch	$-\frac{-A^2b^3cdn+2B^2a\,b^2d^2n^3-2B^2b^3cd\,n^3+A^2a\,b^2d^2n+2ABa\,b^2d^2n^2-2AB\,b^3cd\,n^2-B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3d^2n-2B^2x\ln\left(e\left(\cdots\right.\right.}{g^2(bx-\cdots}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -(-A^2*b^3*c*d*n+2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n+2*A*
B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^2
*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^2*n^2-B^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^3*c*d*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n^2-2*A*B*x*ln(e*((
b*x+a)/(d*x+c))^n)*b^3*d^2*n-2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n)/g^2
/(b*x+a)/b^3/d/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{A^2bc-A^2ad+2\,(B^2bc-B^2ad)n^2+(B^2bc-B^2ad)\log\left(e\right)^2+(B^2bdn^2x+B^2bcn^2)\log\left(\frac{bx+a}{dx+c}\right)^2+2\,(AB$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(e
)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b*c
 - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n + (B^2*b*d*n*x
 + B^2*b*c*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b*c*n^2 + A*B*b*c*n
 + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d
)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\int \frac{A^2}{a^2+2abx+b^2x^2}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^2+2abx+b^2x^2}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^2+2abx+b^2x^2}\,dx}{g^2}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
```

```
[Out] (Integral(A**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2 + 2*a*b*x + b**2*x**2)
, x))/g**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(136) = 272$.

Time = 0.22 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.16

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -2\,ABn\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \left(2\,n\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) - \frac{((bdx+ad)\log}{} \right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{b^2g^2x+abg^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2} - \frac{A^2}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -2*A*B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - (2*n*(1/(b^2*g^2*x + a*b*g^2) + d*l
og(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2))*l
og(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + a*d)*log(b*x + a)^2 + (b*
d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) +
2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2*c*g^2
 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - B^2*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)^2/(b^2*g^2*x + a*b*g^2) - 2*A*B*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.99 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2} + \frac{2\left(B^2n^2+B^2n\log(e)+ABn\right)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2} + \frac{(2\,B^2n^2+2\,B^2n\log}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2) + 2*(B^2*n^2
 + B^2*n*log(e) + A*B*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*g^2)
 + (2*B^2*n^2 + 2*B^2*n*log(e) + B^2*log(e)^2 + 2*A*B*n + 2*A*B*log(e) + A^
2)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,b^2\,g^2+a\,b\,g^2}+\frac{2\,A\,B}{x\,b^2\,g^2+a\,b\,g^2}\right) - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A+B\,n\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(b^2*g^2*x + a*b*g^2) - log(e*((a + b*x)/(c +
 d*x))^n)^2*(B^2/(b*(a*g^2 + b*g^2*x)) - (B^2*d)/(b*g^2*(a*d - b*c))) - log
(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(b^2*g^2*x + a*b*g^2) + (2*A*B)/(b^2
*g^2*x + a*b*g^2)) - (B*d*n*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2
))*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*g^2*(a*d - b*c))
```

3.16 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2B^2dn^2(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bBn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+
b*c)^2/g^3/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+
b*c)^2/g^3/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2549, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2} + \frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{bB^2n^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2} + \frac{2B^2dn^2(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (2*B^2*d*n^2*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*n^2*(c + d*x
)^2)/(4*(b*c - a*d)^2*g^3*(a + b*x)^2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*n*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
+ (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g^3
*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(
b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
```

```
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{x^3} - \frac{d(A+B\log(ex^n))^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2} + \frac{(bBn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{(2Bdn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{2B^2dn^2(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bBn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-4d^2(a+bx)^2\log}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[
a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a
*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a
 + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(282) = 564.

Time = 7.39 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.33

method	result
parallelrisch	$-\frac{7B^2a^2b^3d^3n^3+B^2b^5c^2dn^3+2A^2a^2b^3d^3n+2A^2b^5c^2dn-8AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^4cd^2n-8ABab^4cd^2n^2+4ABxab^4d^3n^2-4A}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4*(7*B^2*a^2*b^3*d^3*n^3+B^2*b^5*c^2*d*n^3+2*A^2*a^2*b^3*d^3*n+2*A^2*b^5
*c^2*d*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n-8*A*B*a*b^4*c*d^2*n^
2+4*A*B*x*a*b^4*d^3*n^2-4*A*B*x*b^5*c*d^2*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^
n)^2*a*b^4*c*d^2*n-8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n^2+4*A*B*ln
(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5
*d^3*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*d^3*n-8*B^2*x*ln(e*((b*x+a
```

```
)/(d*x+c))^n)*a*b^4*d^3*n^2-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^2*n^2
-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)^2*b^5*d^3*n-6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^3*n^2+6*B^2*x
*a*b^4*d^3*n^3-6*B^2*x*b^5*c*d^2*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*
c^2*d*n+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n^2-8*B^2*a*b^4*c*d^2*n^3
+6*A*B*a^2*b^3*d^3*n^2+2*A*B*b^5*c^2*d*n^2-4*A^2*a*b^4*c*d^2*n)/g^3/(b*x+a)
^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(282) = 564$.

Time = 0.29 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.26

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\,A^2b^2c^2 - 4\,A^2abcd + 2\,A^2a^2d^2 + (B^2b^2c^2 - 8\,B^2abcd + 7\,B^2a^2d^2)n^2 + 2\,(B^2b^2c^2 - 2\,B^2abcd + B^2a^2d}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*
a*b*c*d + 7*B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2
)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 -
2*B^2*a*b*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^2*c^2 - 4*A*B*a*b
*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 + 2*(A*B*b^2
*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2
 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d + 3*B^2
*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B
^2*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*b^2*c^2 - 4*B^2*a
*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 -
2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*
x)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3
*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\frac{\int \frac{A^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx}{g^3}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

```
[Out] (Integral(A**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(282) = 564$.

Time $= 0.25$ (sec) , antiderivative size $= 861$, normalized size of antiderivative $= 2.99$

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\frac{1}{2}ABn\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2}{(b^3c^2-}\right.$$
$$+\frac{1}{4}\left(2\,n\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3c}\right.\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}$$
$$-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
- a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3)) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^
3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) +
 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x + c) + a
```

```
/(d*x + c))^n) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*
d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*l
og(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d
^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^
2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3
+ a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2
*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^3*g^3*x
^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^
2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 1.04 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(B^2bn^2-\frac{2\,(bx+a)B^2dn^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{2\left(B^2bn^2-\frac{4\,(bx+a)B^2dn^2}{dx+c}+2\,B^2bn\log\left(e\right)-\frac{4\,(bx+a)B^2dn\log(e}{dx+c}\right.}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg}{(dx+c)^2}}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] -1/4*(2*(B^2*b*n^2 - 2*(b*x + a)*B^2*d*n^2/(d*x + c))*log((b*x + a)/(d*x +
c))^2/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) +
 2*(B^2*b*n^2 - 4*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*B^2*b*n*log(e) - 4*(b*x
 + a)*B^2*d*n*log(e)/(d*x + c) + 2*A*B*b*n - 4*(b*x + a)*A*B*d*n/(d*x + c))
*log((b*x + a)/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*
d*g^3/(d*x + c)^2) + (B^2*b*n^2 - 8*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*B^2*b
*n*log(e) - 8*(b*x + a)*B^2*d*n*log(e)/(d*x + c) + 2*B^2*b*log(e)^2 - 4*(b*
x + a)*B^2*d*log(e)^2/(d*x + c) + 2*A*B*b*n - 8*(b*x + a)*A*B*d*n/(d*x + c)
 + 4*A*B*b*log(e) - 8*(b*x + a)*A*B*d*log(e)/(d*x + c) + 2*A^2*b - 4*(b*x +
 a)*A^2*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3
/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,\left(3\,b\,B^2\,n^2+2\,A\,b\,B\,n\right)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}+\frac{B^2\,d^2\left(\frac{b\,g^3\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,g^3\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,g^3\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2\right)}\right)-\frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A+3\,B\,n\right)\,1\mathrm{i}}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b
*g^3*x)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(
2*(a*d - b*c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b*g^3
+ 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(a
^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) + (B^2*d^2*((b*g^3*n*(a*d - b*c)*(2
*a*d - b*c))/(2*d^2) + (b^2*g^3*n*x*(a*d - b*c))/d + (a*b*g^3*n*(a*d - b*c)
)/(2*d)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b*g^3 + b^3*g^3*x^2 +
 2*a*b^2*g^3*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2*g^3 - 2*a^2*b*d^2*
g^3)/(2*b*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*g^3*(a*d
- b*c)^2)
```

3.17 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 448

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =& -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bBdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bd(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{b^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}\end{aligned}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(
-a*d+b*c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+
a)^3-2*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)+b*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)^2-2/9*b^2*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4
```

```
/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4/(
b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g
^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2549, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx=-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{2Bd^2n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(a+bx)(bc-ad)^3}+\frac{bd(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3}+\frac{bBdn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{2b^2B^2n^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}-\frac{2B^2d^2n^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{bB^2dn^2(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*n^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*n^2*(c
+ d*x)^2)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(
27*(b*c - a*d)^3*g^4*(a + b*x)^3) - (2*B*d^2*n*(c + d*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B*d*n*(c + d*x)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (
2*b^2*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d
)^3*g^4*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/((b*c - a*d)^3*g^4*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{x^4}-\frac{2bd(A+B\log(ex^n))^2}{x^3}+\frac{d^2(A+B\log(ex^n))^2}{x^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}-\frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&\quad+\frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}
\end{aligned}
$$

$$= -\frac{d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3} + \frac{(2b^2Bn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^4} - \frac{(2bBdn)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4} + \frac{(2Bd^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)} + \frac{bBdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.37

$$\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(12A(bc-ad)^3+4B(bc-ad)^3n-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2n(a+bx)+36Ad^2(bc-ad)(a\cdots\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d)
^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a
*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)
*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3
*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*
Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c
+ d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^
n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x
] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] -
 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^
3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b
```

```
*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(440) = 880.

Time = 16.00 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.56

method	result	size
parallelrisch	Expression too large to display	1146

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/54*(-108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n-108*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^6*c*d^3*n^2-108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*
b^5*d^4*n-108*A*B*x*a*b^6*c*d^3*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b
^5*c*d^3*n+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n-108*A*B*a^2*b^
5*c*d^3*n^2+54*A*B*a*b^6*c^2*d^2*n^2-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b
^7*d^4*n-54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*d^4*n-162*B^2*x^2*ln(
e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n^2-36*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b
^7*c*d^3*n^2+36*A*B*x^2*a*b^6*d^4*n^2-36*A*B*x^2*b^7*c*d^3*n^2-54*B^2*x*ln(
e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^4*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*
a^2*b^5*d^4*n^2+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^2*n^2-162*B^2*
x*a*b^6*c*d^3*n^3+90*A*B*x*a^2*b^5*d^4*n^2+18*A*B*x*b^7*c^2*d^2*n^2-54*B^2*
ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^3*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a*b^6*c^2*d^2*n-108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^3*n^2+54*
B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n^2-36*A*B*ln(e*((b*x+a)/(d*x+c
))^n)*b^7*c^3*d*n-108*B^2*a^2*b^5*c*d^3*n^3+27*B^2*a*b^6*c^2*d^2*n^3+66*A*B
*a^3*b^4*d^4*n^2-12*A*B*b^7*c^3*d*n^2-54*A^2*a^2*b^5*c*d^3*n+54*A^2*a*b^6*c
^2*d^2*n+85*B^2*a^3*b^4*d^4*n^3-4*B^2*b^7*c^3*d*n^3+18*A^2*a^3*b^4*d^4*n-18
*A^2*b^7*c^3*d*n-18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*d^4*n-66*B^2*x^
3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^4*n^2+66*B^2*x^2*a*b^6*d^4*n^3-66*B^2*x^2
*b^7*c*d^3*n^3+147*B^2*x*a^2*b^5*d^4*n^3+15*B^2*x*b^7*c^2*d^2*n^3-18*B^2*ln
(e*((b*x+a)/(d*x+c))^n)^2*b^7*c^3*d*n-12*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*
c^3*d*n^2)/g^4/(b*x+a)^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/b^5/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. $2(440) = 880$.

Time = 0.32 (sec) , antiderivative size = 1164, normalized size of antiderivative = 2.60

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{18\,A^2b^3c^3 - 54\,A^2ab^2c^2d + 54\,A^2a^2bcd^2 - 18\,A^2a^3d^3 + (4\,B^2b^3c^3 - 27\,B^2ab^2c^2d + 108\,B^2a^2bcd^2 - 85\,B^2}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3
*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2
*a^2*b*c*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*
b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d -
54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2
*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18
*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2
- 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n*x + (2*B^2*b^3*
c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3)*n + 6*(B^2*b
^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e)
 + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a
*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*
d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*
B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^
2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*
c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c
^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*
c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2
*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$=\frac{\int \frac{A^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx}{g^4}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

```
[Out] (Integral(A**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4
 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2
*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. $2(440) = 880$.

Time = 0.29 (sec) , antiderivative size = 1432, normalized size of antiderivative = 3.20

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*
d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6
*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*
g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3)*g^4)) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*
d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4
*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2
 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*
b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (4*b^
3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b
^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*
log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^
```

```
3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*
(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d
^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x +
 c))*n^2/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6
*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b
^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*
g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a
^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 1/3*B^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*
b*g^4) - 2/3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*
b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3
*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 1.32 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{54}\left(\frac{18\left(B^2b^2n^2-\frac{3(bx+a)B^2bdn^2}{dx+c}+\frac{3(bx+a)^2B^2d^2n^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(2B^2b^2n^2-\frac{9(bx+a)B^2bdn^2}{dx+c}+\frac{18(bx+a)^2}{(dx+c}\right.}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] -1/54*(18*(B^2*b^2*n^2 - 3*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 3*(b*x + a)^2*
B^2*d^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*b^2*c^2*g^
4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2
*g^4/(d*x + c)^3) + 6*(2*B^2*b^2*n^2 - 9*(b*x + a)*B^2*b*d*n^2/(d*x + c) +
18*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 6*B^2*b^2*n*log(e) - 18*(b*x + a)*
B^2*b*d*n*log(e)/(d*x + c) + 18*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c)^2 +
6*A*B*b^2*n - 18*(b*x + a)*A*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*A*B*d^2*n/(
d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 -
 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^
3) + (4*B^2*b^2*n^2 - 27*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 108*(b*x + a)^2*
B^2*d^2*n^2/(d*x + c)^2 + 12*B^2*b^2*n*log(e) - 54*(b*x + a)*B^2*b*d*n*log(
e)/(d*x + c) + 108*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c)^2 + 18*B^2*b^2*lo
g(e)^2 - 54*(b*x + a)*B^2*b*d*log(e)^2/(d*x + c) + 54*(b*x + a)^2*B^2*d^2*l
og(e)^2/(d*x + c)^2 + 12*A*B*b^2*n - 54*(b*x + a)*A*B*b*d*n/(d*x + c) + 108
*(b*x + a)^2*A*B*d^2*n/(d*x + c)^2 + 36*A*B*b^2*log(e) - 108*(b*x + a)*A*B*
b*d*log(e)/(d*x + c) + 108*(b*x + a)^2*A*B*d^2*log(e)/(d*x + c)^2 + 18*A^2*
```

```
b^2 - 54*(b*x + a)*A^2*b*d/(d*x + c) + 54*(b*x + a)^2*A^2*d^2/(d*x + c)^2)/
((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^
3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a
*d)^2)
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.32

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2\,n-42\,A\,B\,a\,b\,c\,d\,n+12\,A\,B\,b^2\,c^2\,n+85\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+4\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}+\frac{x\,(-5\,c}{}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\right.}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,A\,B}{3\,a^3\,b\,g^4+9\,a^2\,b^2\,g^4\,x+9\,a\,b^3\,g^4\,x^2+3\,b^4\,g^4\,x^3}\right.$$

$$+\frac{2\,B^2\,d^3\left(x\left(b\left(\frac{b\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{a\,b\,g^4\,n\,(a\,d-b\,c)}{d}\right)+\frac{2\,a\,b^2\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^2\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{d^2}\right)+a\left(\frac{b\,g^4\,n}{}\right.\right.}{3\,b\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)\left(3\,a^3\,b\,g^4+\right.}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{3\,b\,\left(a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3\right)}-\frac{B^2\,d^3}{3\,b\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$

$$-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{B\,d^3\,n\,(6\,A+11\,B\,n)\left(\frac{a^3\,b\,d^3\,g^4-a^2\,b^2\,c\,d^2\,g^4-a\,b^3\,c^2\,d\,g^4+b^4\,c^3\,g^4}{a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4}+2\,b\,d\,x\right)\left(a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4\right)\,1\mathrm{i}}{b\,g^4\,\left(11\,B^2\,d^3\,n^2+6\,A\,B\,d^3\,n\right)\,(a\,d-b\,c)^3}\right)\,(6\,A+}{9\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*
b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9
*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a^4*b*d*g^4) - log(e*((a + b*x)/(c + d*
x))^n)*((2*A*B)/(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2*b^2*g^4*x + 9*a*b^3*g^
4*x^2) + (2*B^2*d^3*(x*(b*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a
*b*g^4*n*(a*d - b*c))/d) + (2*a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d
- b*c)*(3*a*d - b*c))/d^2) + a*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)
 + (a*b*g^4*n*(a*d - b*c))/d) + (b*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d))/d^3 + (3*b^3*g^4*n*x^2*(a*d - b*c))/d))/(3*b*g^4*(a^3*d^3 - b^
```

```
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2
*b^2*g^4*x + 9*a*b^3*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*
b*(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x)) - (B^2*d^3)/(3
*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*ata
n((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 -
 a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d
*x)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(11*B^2*d^3*
n^2 + 6*A*B*d^3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*g^4*(a*d - b*c)^
3)
```

3.18 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 615

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx &= \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} \\
&+ \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} \\
&+ \frac{2Bd^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(a+bx)} \\
&- \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} \\
&+ \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} \\
&- \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} \\
&+ \frac{d^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)} \\
&- \frac{3bd^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} \\
&+ \frac{b^2d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} \\
&- \frac{b^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}
\end{aligned}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/
(-a*d+b*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*
```

```
x+a)^3-1/32*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*n*
(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^
2*B*d*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^
3-1/8*b^3*B*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b
*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/(b*x
+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/
(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^
5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
4/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4} + \frac{2b^2Bdn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{d^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4} + \frac{2Bd^3n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^5(a+bx)(bc-ad)^4} - \frac{3bd^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4} - \frac{3bBd^2n(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4} - \frac{b^3B^2n^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4} + \frac{2b^2B^2dn^2(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4} + \frac{2B^2d^3n^2(c+dx)}{g^5(a+bx)(bc-ad)^4} - \frac{3bB^2d^2n^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*n^2*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*n^2*
(c + d*x)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)
```

```
^3)/(9*(b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*n^2*(c + d*x)^4)/(32*(b*c
- a*d)^4*g^5*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*n*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*
b^2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d
)^4*g^5*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(8*(b*c - a*d)^4*g^5*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^4*g^5*(a + b*
x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$=\frac{\text{Subst}\left(\int\left(\frac{b^3(A+B\log(ex^n))^2}{x^5}-\frac{3b^2d(A+B\log(ex^n))^2}{x^4}+\frac{3bd^2(A+B\log(ex^n))^2}{x^3}-\frac{d^3(A+B\log(ex^n))^2}{x^2}\right)dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$=\frac{b^3\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^5}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}-\frac{(3b^2d)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^4}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}+\frac{(3bd^2)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^3}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}-\frac{d^3\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^2}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$=\frac{d^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)}-\frac{3bd^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2}+\frac{b^2d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3}-\frac{b^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}+\frac{(b^3Bn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^5}dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^5}-\frac{(2b^2Bdn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^4}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}+\frac{(3bBd^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^3}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}-\frac{(2Bd^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$=\frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2}+\frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4}+\frac{2Bd^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(a+bx)}-\frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(a+bx)^2}+\frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(a+bx)^3}-\frac{b^3Bn(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(a+bx)^4}+\frac{d^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)}-\frac{3bd^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2}+\frac{b^2d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3}-\frac{b^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(36A(bc-ad)^4+9B(bc-ad)^4n+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3n(a+bx)+72Ad^2(bc-\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*d
)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b
*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b
*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^
3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B
*d^4*n*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^4*Log[a + b*x]^2 + 3
6*B*(b*c - a*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B*d*(-(b*c) + a*d)^3*
(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)
^2*Log[e*((a + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Lo
g[e*((a + b*x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[e*((a
 + b*x)/(c + d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*n*(a
 + b*x)^4*Log[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]
*Log[c + d*x] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*n*(a + b*
x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*n*(a + b*x)^4*
PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*(a + b*x)^4*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(599) = 1198.

Time = 42.23 (sec) , antiderivative size = 2326, normalized size of antiderivative = 3.78

method	result	size
parallelrisch	Expression too large to display	2326

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/288*(144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n+576*A*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^
```

n)*a^8*b*c*d^4*n+1512*B^2*x^2*a^8*b*c*d^4*n^3-2400*B^2*x^2*a^7*b^2*c^2*d^3*
n^3+1218*B^2*x^2*a^6*b^3*c^3*d^2*n^3-384*B^2*x^2*a^5*b^4*c^4*d*n^3+288*A^2*
x^3*a^7*b^2*c*d^4*n-1152*A^2*x^3*a^6*b^3*c^2*d^3*n+1728*A^2*x^3*a^5*b^4*c^3
*d^2*n-1152*A^2*x^3*a^4*b^5*c^4*d*n+216*A*B*x^2*a^4*b^5*c^5*n^2+288*B^2*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^9*c*d^4*n+576*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*
a^9*c*d^4*n^2-1008*B^2*x*a^8*b*c^2*d^3*n^3+624*B^2*x*a^7*b^2*c^3*d^2*n^3-22
8*B^2*x*a^6*b^3*c^4*d*n^3+432*A^2*x^2*a^8*b*c*d^4*n-1728*A^2*x^2*a^7*b^2*c^
2*d^3*n+2592*A^2*x^2*a^6*b^3*c^3*d^2*n-1728*A^2*x^2*a^5*b^4*c^4*d*n+576*A*B
*x*a^9*c*d^4*n^2+144*A*B*x*a^5*b^4*c^5*n^2-432*B^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a^8*b*c^3*d^2*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c^4*d*n-432
*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^2*n^2+192*B^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^7*b^2*c^4*d*n^2-1152*A^2*x*a^8*b*c^2*d^3*n+1728*A^2*x*a^7*b^2*c^3
*d^2*n-1152*A^2*x*a^6*b^3*c^4*d*n+36*B^2*x*a^5*b^4*c^5*n^3+432*A^2*x^2*a^4*
b^5*c^5*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^9*c^2*d^3*n-72*B^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^6*b^3*c^5*n+576*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2
*d^3*n^2-36*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^5*n^2+288*A^2*x*a^9*c*d
^4*n+288*A^2*x*a^5*b^4*c^5*n+72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3
*c*d^4*n+300*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n^2+300*A*B*x^
4*a^6*b^3*c*d^4*n^2-576*A*B*x^4*a^5*b^4*c^2*d^3*n^2+432*A*B*x^4*a^4*b^5*c^3
*d^2*n^2-192*A*B*x^4*a^3*b^6*c^4*d*n^2+288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n
)^2*a^7*b^2*c*d^4*n+1056*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n^
2+144*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^2*d^3*n^2+1056*A*B*x^3*a^
7*b^2*c*d^4*n^2-2160*A*B*x^3*a^6*b^3*c^2*d^3*n^2+1728*A*B*x^3*a^5*b^4*c^3*d
^2*n^2-768*A*B*x^3*a^4*b^5*c^4*d*n^2+432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^
2*a^8*b*c*d^4*n+1296*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c*d^4*n^2+576*
A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2*d^3*n-144*A*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^6*b^3*c^5*n+576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^2*d^3*n^2-
72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^3*d^2*n^2+1296*A*B*x^2*a^8*b
*c*d^4*n^2-2880*A*B*x^2*a^7*b^2*c^2*d^3*n^2+2520*A*B*x^2*a^6*b^3*c^3*d^2*n^
2-1152*A*B*x^2*a^5*b^4*c^4*d*n^2+864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*
c^2*d^3*n^2-288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^3*d^2*n^2+48*B^2*
x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^4*d*n^2+576*A*B*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^9*c*d^4*n-1440*A*B*x*a^8*b*c^2*d^3*n^2+1440*A*B*x*a^7*b^2*c^3*d^2*
n^2-720*A*B*x*a^6*b^3*c^4*d*n^2-864*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3
*d^2*n+576*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d*n+415*B^2*x^4*a^6*b^
3*c*d^4*n^3-576*B^2*x^4*a^5*b^4*c^2*d^3*n^3+216*B^2*x^4*a^4*b^5*c^3*d^2*n^3
-64*B^2*x^4*a^3*b^6*c^4*d*n^3+36*A*B*x^4*a^2*b^7*c^5*n^2+1360*B^2*x^3*a^7*b
^2*c*d^4*n^3-2004*B^2*x^3*a^6*b^3*c^2*d^3*n^3+864*B^2*x^3*a^5*b^4*c^3*d^2*n
^3-256*B^2*x^3*a^4*b^5*c^4*d*n^3+72*A^2*x^4*a^6*b^3*c*d^4*n-288*A^2*x^4*a^5
*b^4*c^2*d^3*n+432*A^2*x^4*a^4*b^5*c^3*d^2*n-288*A^2*x^4*a^3*b^6*c^4*d*n+14
4*A*B*x^3*a^3*b^6*c^5*n^2+9*B^2*x^4*a^2*b^7*c^5*n^3+36*B^2*x^3*a^3*b^6*c^5*
n^3+72*A^2*x^4*a^2*b^7*c^5*n+54*B^2*x^2*a^4*b^5*c^5*n^3+288*A^2*x^3*a^3*b^6
*c^5*n+576*B^2*x*a^9*c*d^4*n^3)/g^5/(b*x+a)^4/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*
d+b^2*c^2)/c/n/a^6

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. 2(599) = 1198.

Time = 0.33 (sec) , antiderivative size = 1762, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*
a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d
^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^
4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^
2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3
*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^
4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^
4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log
((b*x + a)/(d*x + c))^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a
^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3
*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2
+ 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a
^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c
^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*
n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 13*
B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c
^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4
*B^2*a*b^3*d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2
*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n
)*log((b*x + a)/(d*x + c)))*log(e) - 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d
^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^
2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*
B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a
*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B*b^4*c^4 - 4*A*B*a*b^3*c
^3*d + 6*A*B*a^2*b^2*c^2*d^2 - 4*A*B*a^3*b*c*d^3)*n + 4*(12*A*B*a^3*b*d^4*n
 + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3
*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^
2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 +
6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^
6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
```

```
 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*
a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2136 vs. $2(599) = 1198$.

Time = 0.34 (sec) , antiderivative size = 2136, normalized size of antiderivative = 3.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3
*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*
a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^
5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5
*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^
5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g
^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) +
 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d
 + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(e*(b*x/(d*x +
```

```
c) + a/(d*x + c))^n) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 -
 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^
4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*
b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^
4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d
^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^
3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^
4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(
b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/4*B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 +
4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a
^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*
a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(599) = 1198$.

Time = 1.69 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.96

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] -1/288*(72*(B^2*b^3*n^2 - 4*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 6*(b*x + a)
^2*B^2*b*d^2*n^2/(d*x + c)^2 - 4*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3)*log((
b*x + a)/(d*x + c))^2/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*
a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (
b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(3*B^2*b^3*n^2 - 16*(b*x + a)*B^2*
b^2*d*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 48*(b*x +
a)^3*B^2*d^3*n^2/(d*x + c)^3 + 12*B^2*b^3*n*log(e) - 48*(b*x + a)*B^2*b^2*d
*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2*b*d^2*n*log(e)/(d*x + c)^2 - 48*(b
*x + a)^3*B^2*d^3*n*log(e)/(d*x + c)^3 + 12*A*B*b^3*n - 48*(b*x + a)*A*B*b^
```

```
2*d*n/(d*x + c) + 72*(b*x + a)^2*A*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*A
*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^3*c^3*g^5/(d*
x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*
c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + (9*B^2*b^3*n
^2 - 64*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 216*(b*x + a)^2*B^2*b*d^2*n^2/(
d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3 + 36*B^2*b^3*n*log(e)
- 192*(b*x + a)*B^2*b^2*d*n*log(e)/(d*x + c) + 432*(b*x + a)^2*B^2*b*d^2*n*
log(e)/(d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n*log(e)/(d*x + c)^3 + 72*B^2*
b^3*log(e)^2 - 288*(b*x + a)*B^2*b^2*d*log(e)^2/(d*x + c) + 432*(b*x + a)^2
*B^2*b*d^2*log(e)^2/(d*x + c)^2 - 288*(b*x + a)^3*B^2*d^3*log(e)^2/(d*x + c
)^3 + 36*A*B*b^3*n - 192*(b*x + a)*A*B*b^2*d*n/(d*x + c) + 432*(b*x + a)^2*
A*B*b*d^2*n/(d*x + c)^2 - 576*(b*x + a)^3*A*B*d^3*n/(d*x + c)^3 + 144*A*B*b
^3*log(e) - 576*(b*x + a)*A*B*b^2*d*log(e)/(d*x + c) + 864*(b*x + a)^2*A*B*
b*d^2*log(e)/(d*x + c)^2 - 576*(b*x + a)^3*A*B*d^3*log(e)/(d*x + c)^3 + 72*
A^2*b^3 - 288*(b*x + a)*A^2*b^2*d/(d*x + c) + 432*(b*x + a)^2*A^2*b*d^2/(d*
x + c)^2 - 288*(b*x + a)^3*A^2*d^3/(d*x + c)^3)/((b*x + a)^4*b^3*c^3*g^5/(d
*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b
*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4))*(b*c/(b*c -
a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 1769, normalized size of antiderivative = 2.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 -
 48*a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(25*B^2*d^4*n^2 +
 12*A*B*d^4*n)*(a*d - b*c)^4) + (B*d^5*n*x*(12*A + 25*B*n)*(b^4*c^3*g^5 - a
^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(25*B^2*d^
4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b
^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n
- 36*A*B*b^3*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156
*A*B*a*b^2*c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^
2*a*b^2*d^3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*
d^2*n))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 -
 53*B^2*a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B
*a*b^2*c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*
d^2*n))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 - 192*a^4*
b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2*b^5*c*d
*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5) + x^2*
```

```
(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) + 24*a^6*
b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2
*g^5*x^2 + 4*a^3*b*g^5*x)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x)
)^n)*((A*B)/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^
3 + 12*a^2*b^3*g^5*x^2) + (B^2*d^4*(x*(b*(a*((b*g^5*n*(a*d - b*c)*(4*a*d -
b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d - b*c)*(6*a^
2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + a*(b*((b*g^5*n*(a*d - b*c)*(4*a*d
- b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^5*n*(a*d - b*c)
)/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (b^2*g^5*n*(a*d - b*
c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + a*(a*((b*g^5*n*(a*d - b*c)
*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d -
b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + x^2*(b*(b*((b*g^5*n*(a*d
 - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^
5*n*(a*d - b*c))/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a*
b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2
)) + (2*b^4*g^5*n*x^3*(a*d - b*c))/d + (b*g^5*n*(a*d - b*c)*(4*a^3*d^3 - b^
3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4)))/(2*b*g^5*(2*a^4*b*g^5 + 2
*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
```

3.19 $\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log(e*((b*x + a)/(d*x + c
))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 34.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{2abx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(b**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*a*b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 27.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.20 $\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\operatorname{Int}\left(\frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
[Out] integral((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 16.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{bx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] g*(Integral(a/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
[Out] integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.21 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))
^n)), x)
```

Sympy [N/A]

Not integrable

Time = 16.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*b
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.22 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[Out] exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*
x+c))^n))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2549, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-((
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a + b*
x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= \frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2n(a+bx)} \\
&= \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx
= \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-((
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a + b*
x))
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Babx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n) + 2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.23 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 197

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)^2} - \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)}$$

```
[Out] b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2-d*exp(A/B/n)*(e*((b*x
+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-
a*d+b*c)^2/g^3/n/(b*x+a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used

= {2549, 2395, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(a+bx)^2(bc-ad)^2}$$
$$-\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(a+bx)(bc-ad)^2}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegra
lEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B*(b*c - a*d)^2*g^
3*n*(a + b*x)^2) - (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c
 - a*d)^2*g^3*n*(a + b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
```

```
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex^n))}-\frac{d}{x^2(A+B\log(ex^n))}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}-\frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= -\frac{\left(d\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3n(a+bx)}\\
&\quad+\frac{\left(b\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3n(a+bx)^2}\\
&= \frac{be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)^2}\\
&\quad-\frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)-d(a+bx)\right.}{B(bc-ad)^2g^3n(a+bx)^2}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*(b*E^(A/(B*n))*(e
*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(B*n)] - d*(a + b*x)*ExpIntegralEi[-((A + B*Log[e
*((a + b*x)/(c + d*x))^n])/(B*n))]))/(B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)
```

Maple [F]

$$\int \frac{1}{\left(bgx + ag\right)^{3} \left(A + B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)

[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = -\frac{de^{\left(\frac{B\log(e)+A}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right) - be^{\left(\frac{2\,(B\log(e)+A)}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2\,(B\log(e)+A)}{Bn}\right)}}{b^2x^2+2\,abx+a^2}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")

[Out] -(d*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)) - b*e^(2*(B*log(e) + A)/(B*n))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.24 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 44.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\begin{aligned}
&\int \frac{(ag + bgx)^2}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx \\
&= g^2\left(\int \frac{a^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\,dx\right. \\
&\quad + \int \frac{b^2x^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\,dx \\
&\quad \left.+ \int \frac{2abx}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)
\end{aligned}$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(a**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*a*b*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(ag + bgx)^2}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx + ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*
a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^
2 + a^2*b*d*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)
*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))
*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.25 $\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
[Out] integral((b*g*x + a*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*
((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 58.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g\left(\int \frac{a}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{bx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(a/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c*n
 - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*
c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.26 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [N/A]

Not integrable

Time = 130.89 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{\int \frac{1}{A^2a+A^2bx+2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral(1/(A**2*a + A**2*b*x + 2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n) + 2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*a*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*b*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
[Out] d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) -
 (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*
g*n*log(e) - a*d*g*n*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.27 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 153

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(a+bx)} - \frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] -exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d
*x+c))^n))/B/n)/B^2/(-a*d+b*c)/g^2/n^2/(b*x+a)+(-d*x-c)/B/(-a*d+b*c)/g^2/n/
(b*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2549, 2343, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(a+bx)(bc-ad)} - \frac{c+dx}{Bg^2n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -((E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-
((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*(b*c - a*d)*g^2*n^2*(
a + b*x))) - (c + d*x)/(B*(b*c - a*d)*g^2*n*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}$$

$$= -\frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)g^2n}$$

$$= -\frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)g^2n^2(a+bx)}$$

$$= -\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(a+bx)} - \frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{(c+dx)\left(Bn+e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)g^2n^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((c + d*x)*(B*n + E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpInteg
ralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*g^2*n^2*(a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])))
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.79

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{Bdnx+Bcn+(Abx+Aa+(Bbx+Ba)\log(e)+(Bbnx+Ban)\log\left(\frac{bx+a}{dx+c}\right)}{(AB^2b^2c-AB^2abd)g^2n^2x+(AB^2abc-AB^2a^2d)g^2n^2+((B^3b^2c-B^3abd)g^2n^2x+(B^3abc-B^3a^2d)g}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -(B*d*n*x + B*c*n + (A*b*x + A*a + (B*b*x + B*a)*log(e) + (B*b*n*x + B*a*n)
*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*
e^(-(B*log(e) + A)/(B*n))/(b*x + a)))/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*n^2*
x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2*n^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*n^2*
x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^2)*log(e) + ((B^3*b^2*c - B^3*a*b*d)*g^2*
n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n*log(e) - a^2*d*g
^2*n*log(e))*B^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n*log(e) -
 a*b*d*g^2*n*log(e))*B^2)*x + ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g
^2*n - a^2*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n)*B^
2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*log((d*x + c)^n)) + integrate(-1/(B^
2*a^2*g^2*n*log(e) + A*B*a^2*g^2*n + (B^2*b^2*g^2*n*log(e) + A*B*b^2*g^2*n)
*x^2 + 2*(B^2*a*b*g^2*n*log(e) + A*B*a*b*g^2*n)*x + (B^2*b^2*g^2*n*x^2 + 2*
B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((b*x + a)^n) - (B^2*b^2*g^2*n*x^2 + 2*
B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((d*x + c)^n)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.28 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 314

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2} + \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)} + \frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] -2*b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)^2+d*exp(A/B/n)*(
e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/
n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))-b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2395, 2343, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= -\frac{2be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(a+bx)^2(bc-ad)^2}$$
$$+\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(a+bx)(bc-ad)^2}$$
$$-\frac{b(c+dx)^2}{Bg^3n(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$
$$+\frac{d(c+dx)}{Bg^3n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] (-2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpInte
gralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*(b*c - a*d)
^2*g^3*n^2*(a + b*x)^2) + (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)
*(c + d*x)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/
(B^2*(b*c - a*d)^2*g^3*n^2*(a + b*x)) + (d*(c + d*x))/(B*(b*c - a*d)^2*g^3*
n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])) - (b*(c + d*x)^2)/(B*(b
*c - a*d)^2*g^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex^n))^2}-\frac{d}{x^2(A+B\log(ex^n))^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}-\frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} \\
&\quad -\frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} \\
&\quad -\frac{(2b)\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}+\frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}
\end{aligned}$$

$$= \frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left(d\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int\frac{e^{-\frac{x}{n}}}{A+Bx}dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2g^3n^2(a+bx)} - \frac{\left(2b\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\right)\text{Subst}\left(\int\frac{e^{-\frac{2x}{n}}}{A+Bx}dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2g^3n^2(a+bx)^2}$$

$$= -\frac{2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2} + \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)} + \frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.81

$$\int\frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}dx$$

$$= \frac{(c+dx)\left(B(-bc+ad)n-2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\left(A+B\right.\right.}{B^2(bc-ad)^2g^3n^2(a+b}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x
))^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a + b*x
)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a + b*x
)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c
 - a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(312) = 624.

Time = 0.29 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.40

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbcd-Bad^2)nx-\left(Ab^2dx^2+2\,Aabdx+Aa^2d+(Bb^2dx^2+2\,Babdx+Ba^2\right.}{(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2)g^3n^2x^2+2\,(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2)g^3n^2x+(AB^2}$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*c*d - B*a*d^2)*n*x - (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d + (B*b^2*d
*x^2 + 2*B*a*b*d*x + B*a^2*d)*log(e) + (B*b^2*d*n*x^2 + 2*B*a*b*d*n*x + B*a
^2*d*n)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*
x + c)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)) + 2*(A*b^3*x^2 + 2*A*a*b^2*x +
A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log(e) + (B*b^3*n*x^2 + 2*B*a
*b^2*n*x + B*a^2*b*n)*log((b*x + a)/(d*x + c)))*e^(2*(B*log(e) + A)/(B*n))*
log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2
 + 2*a*b*x + a^2)) + (B*b*c^2 - B*a*c*d)*n)/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3
*c*d + A*B^2*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^
2*c*d + A*B^2*a^3*b*d^2)*g^3*n^2*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d
 + A*B^2*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d
^2)*g^3*n^2*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3
*n^2*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^2)*log(e)
+ ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*x^2 + 2*(B^3*a
*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B^3*a^2*b^2*c^2
- 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n*log(e) - a^3
*d*g^3*n*log(e))*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n*lo
g(e) - a*b^2*d*g^3*n*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*n)*
A*B + (a*b^2*c*g^3*n*log(e) - a^2*b*d*g^3*n*log(e))*B^2)*x + ((b^3*c*g^3*n
- a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b
*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^3*c*g^3*n - a*b^2*d*g^3
*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3*n - a^
3*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x + 2*b*c - a*d)/(((b^4*
c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*n*log(e) - a*b^3*d*g^3*n*log(e))*
B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*g^3*n*log(e) - a^4*
d*g^3*n*log(e))*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B + (a*b^3*c*g
^3*n*log(e) - a^2*b^2*d*g^3*n*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3*n - a^3*
b*d*g^3*n)*A*B + (a^2*b^2*c*g^3*n*log(e) - a^3*b*d*g^3*n*log(e))*B^2)*x + (
(b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)
*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4
*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 +
3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*
g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.29 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= -\frac{B(bc-ad)^4g^4nx}{5b^4}-\frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} \\ &\quad -\frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d}-\frac{B(bc-ad)g^4n(c+dx)^4}{20bd} \\ &\quad -\frac{B(bc-ad)^5g^4n\log(a+bx)}{5b^5d}+\frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d}\end{aligned}$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*n*x/b^4-1/10*B*(-a*d+b*c)^3*g^4*n*(d*x+c)^2/b^3/d-1
/15*B*(-a*d+b*c)^2*g^4*n*(d*x+c)^3/b^2/d-1/20*B*(-a*d+b*c)*g^4*n*(d*x+c)^4/
b/d-1/5*B*(-a*d+b*c)^5*g^4*n*ln(b*x+a)/b^5/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\begin{aligned}\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)^5\log(a+bx)}{5b^5d}-\frac{Bg^4nx(bc-ad)^4}{5b^4} \\ &\quad -\frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d}-\frac{Bg^4n(c+dx)^3(bc-ad)^2}{15b^2d}-\frac{Bg^4n(c+dx)^4(bc-ad)}{20bd}\end{aligned}$$

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/5*(B*(b*c - a*d)^4*g^4*n*x)/b^4 - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2)/(1
0*b^3*d) - (B*(b*c - a*d)^2*g^4*n*(c + d*x)^3)/(15*b^2*d) - (B*(b*c - a*d)*
g^4*n*(c + d*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*g^4*n*Log[a + b*x])/(5*b^5*d
) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(B(bc-ad)n)\int\frac{(cg+dgx)^5}{(a+bx)(c+dx)}\,dx}{5dg}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(B(bc-ad)g^4n)\int\frac{(c+dx)^4}{a+bx}\,dx}{5d}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(B(bc-ad)g^4n)\int\left(\frac{d(bc-ad)^3}{b^4}+\frac{(bc-ad)^4}{b^4(a+bx)}+\frac{d(bc-ad)^2(c+dx)}{b^3}+\frac{d(bc-ad)(c+dx)^2}{b^2}+\frac{d(c+dx)^3}{b}\right)dx}{5d}$$

$$= -\frac{B(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} - \frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d} - \frac{B(bc-ad)g^4n(c+dx)^4}{20bd} - \frac{B(bc-ad)^5g^4n\log(a+bx)}{5b^5d} + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g^4\left(-\frac{B(bc-ad)n(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+dx)^2+4b^3(bc-ad)(c+dx)^3+3b^4(c+dx)^4+12(bc-ad)^4\log(a+bx))}{12b^5}+(c+dx)^5\left(A+\right.\right.}{5d}$$

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*(-1/12*(B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*
(c + d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c -
 a*d)^4*Log[a + b*x]))/b^5 + (c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])))/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. 2(176) = 352.

Time = 17.54 (sec) , antiderivative size = 864, normalized size of antiderivative = 4.60

method	result
parallelrisch	$\frac{60Bx^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5cd^4g^4n-180Aab^4c^4dg^4n-54Ba^4bcd^4g^4n^2+90Ba^3b^2c^2d^3g^4n^2-60Ba^2b^3c^3d^2g^4n^2-36Bab^4c^4dg}{}$

```
[In] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^4*n-180*A*a*b^4*c^4*d*
g^4*n-54*B*a^4*b*c*d^4*g^4*n^2+90*B*a^3*b^2*c^2*d^3*g^4*n^2-60*B*a^2*b^3*c^
3*d^2*g^4*n^2-36*B*a*b^4*c^4*d*g^4*n^2-48*B*x*b^5*c^4*d*g^4*n^2+12*B*x^5*ln
(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^4*n+3*B*x^4*a*b^4*d^5*g^4*n^2-3*B*x^4*b^5
*c*d^4*g^4*n^2-4*B*x^3*a^2*b^3*d^5*g^4*n^2-16*B*x^3*b^5*c^2*d^3*g^4*n^2+6*B
*x^2*a^3*b^2*d^5*g^4*n^2-36*B*x^2*b^5*c^3*d^2*g^4*n^2-12*B*x*a^4*b*d^5*g^4*
n^2+60*A*x^4*b^5*c*d^4*g^4*n+120*A*x^3*b^5*c^2*d^3*g^4*n+120*A*x^2*b^5*c^3*
d^2*g^4*n+60*A*x*b^5*c^4*d*g^4*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^
3*d^2*g^4*n+60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^4*d*g^4*n+60*B*x*a^3*b^2
*c*d^4*g^4*n^2-120*B*x*a^2*b^3*c^2*d^3*g^4*n^2+120*B*x*a*b^4*c^3*d^2*g^4*n^
```

```
2-60*B*ln(b*x+a)*a^4*b*c*d^4*g^4*n^2+120*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^4*n^
2-120*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^4*n^2+60*B*ln(b*x+a)*a*b^4*c^4*d*g^4*n^
2+20*B*x^3*a*b^4*c*d^4*g^4*n^2-30*B*x^2*a^2*b^3*c*d^4*g^4*n^2+60*B*x^2*a*b^
4*c^2*d^3*g^4*n^2+12*B*a^5*d^5*g^4*n^2+48*B*b^5*c^5*g^4*n^2+12*A*x^5*b^5*d^
5*g^4*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+12*B*ln(b*x+a)*a^5*d^5
*g^4*n^2-12*B*ln(b*x+a)*b^5*c^5*g^4*n^2+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)
*b^5*c^2*d^3*g^4*n-60*A*b^5*c^5*g^4*n)/n/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.04

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 - 12\,Bb^5c^5g^4n\log(dx+c) + 12\,(5\,Bab^4c^4d - 10\,Ba^2b^3c^3d^2 + 10\,Ba^3b^2c^2d^3 - 5\,Ba^4bcd^4 + B}{}$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*b^5*c^5*g^4*n*log(d*x + c) + 12*(5*B*a*b^
4*c^4*d - 10*B*a^2*b^3*c^3*d^2 + 10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 + B
*a^5*d^5)*g^4*n*log(b*x + a) + 3*(20*A*b^5*c*d^4*g^4 - (B*b^5*c*d^4 - B*a*b
^4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*c^2*d^3*g^4 - (4*B*b^5*c^2*d^3 - 5*B*a*b^4
*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*b^5*c^3*d^2*g^4 - (6*B*b^5*c^3
*d^2 - 10*B*a*b^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^4*n)*x^2 +
 12*(5*A*b^5*c^4*d*g^4 - (4*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3
*c^2*d^3 - 5*B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^5*g^4*x^
5 + 5*B*b^5*c*d^4*g^4*x^4 + 10*B*b^5*c^2*d^3*g^4*x^3 + 10*B*b^5*c^3*d^2*g^4
*x^2 + 5*B*b^5*c^4*d*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*c*d^
4*g^4*n*x^4 + 10*B*b^5*c^2*d^3*g^4*n*x^3 + 10*B*b^5*c^3*d^2*g^4*n*x^2 + 5*B
*b^5*c^4*d*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(176) = 352$.

Time = 0.21 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{1}{5}\,Bd^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{5}\,Ad^4g^4x^5+Bcd^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Acd^3g^4x^4+2\,Bc^2d^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+2\,Ac^2d^2g^4x^3+2\,Bc^3dg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Ac^3dg^4x^2$$
$$+\frac{1}{60}\,Bd^4g^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b}{b^4d^4}\right.$$
$$-\frac{1}{6}\,Bcd^3g^4n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3}{b^3d^3}\right.$$
$$+Bc^2d^2g^4n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2\,Bc^3dg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^4g^4x$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*d^4*g^4*x^
5 + B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^3*g^4*x^
4 + 2*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*c^2*d^
2*g^4*x^3 + 2*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*
c^3*d*g^4*x^2 + 1/60*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*c^2*d^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*c^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^4*g
^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(176) = 352$.

Time = 1.09 (sec) , antiderivative size = 1876, normalized size of antiderivative = 9.98

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^
4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c
*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d - 5*(b*x + a)
*b^4*d^2/(d*x + c) + 10*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - 10*(b*x + a)^3*b^
2*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b*d^5/(d*x + c)^4 - (b*x + a)^5*d^6/(d*x
+ c)^5) - (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 77*(b*x + a)*B*b
^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 462*(b*x + a)*B*a*
b^8*c^5*d^2*g^4*n/(d*x + c) + 94*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^
2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1155*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d
*x + c) - 564*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 54*(b*x + a)^
3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 1540*(b*x
 + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 1410*(b*x + a)^2*B*a^2*b^6*c^4*d^
4*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 1
2*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n -
 1155*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 1880*(b*x + a)^2*B*a^3*
b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*
x + c)^3 - 72*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*
d^6*g^4*n + 462*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 1410*(b*x + a)^
2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 1080*(b*x + a)^3*B*a^3*b^4*c^3*d^6*
g^4*n/(d*x + c)^3 + 180*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 7
7*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 564*(b*x + a)^2*B*a^5*b^3*c*d^7
*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 -
240*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 94*(b*x + a)^2*B*a^6*
b^2*d^8*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)
^3 + 180*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 54*(b*x + a)^3*B
*a^6*b*d^9*g^4*n/(d*x + c)^3 - 72*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)
^4 + 12*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 - 12*B*b^10*c^6*g^4*log(e)
 + 72*B*a*b^9*c^5*d*g^4*log(e) - 180*B*a^2*b^8*c^4*d^2*g^4*log(e) + 240*B*a
^3*b^7*c^3*d^3*g^4*log(e) - 180*B*a^4*b^6*c^2*d^4*g^4*log(e) + 72*B*a^5*b^5
*c*d^5*g^4*log(e) - 12*B*a^6*b^4*d^6*g^4*log(e) - 12*A*b^10*c^6*g^4 + 72*A*
a*b^9*c^5*d*g^4 - 180*A*a^2*b^8*c^4*d^2*g^4 + 240*A*a^3*b^7*c^3*d^3*g^4 - 1
80*A*a^4*b^6*c^2*d^4*g^4 + 72*A*a^5*b^5*c*d^5*g^4 - 12*A*a^6*b^4*d^6*g^4)/(
b^9*d - 5*(b*x + a)*b^8*d^2/(d*x + c) + 10*(b*x + a)^2*b^7*d^3/(d*x + c)^2
- 10*(b*x + a)^3*b^6*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b^5*d^5/(d*x + c)^4 -
```

```
(b*x + a)^5*b^4*d^6/(d*x + c)^5) + 12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^
4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^
2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log(b - (b*x + a
)*d/(d*x + c))/(b^5*d) - 12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B
*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*
g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(
b^5*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 1045, normalized size of antiderivative = 5.56

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{b}\right.}{10\,b\,d}\right.$$

$$-\frac{a\,c\,\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)}{2\,b\,d}$$

$$\left.+\frac{c^2\,d\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}\right)$$

$$-x^3\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}+\frac{A\,a\,c\,d^3\,g^4}{3\,b}\right)$$

$$+x^4\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{20\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{20\,b}\right)$$

$$+\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,c^4\,g^4\,x+2\,B\,c^3\,d\,g^4\,x^2+2\,B\,c^2\,d^2\,g^4\,x^3+B\,c\,d^3\,g^4\,x^4\right.$$

$$\left.+\frac{B\,d^4\,g^4\,x^5}{5}\right)+x\left(\frac{c^3\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{b}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b}{b}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{\ldots}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{\ldots}-\frac{A\,a\,c\,d^3\,g^4}{\ldots}\right)$$

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)

[Out] x^2*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))
/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d
^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(1
0*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (
A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(2*b*d) + (c^2*d*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/b) - x^3*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n -
B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(15*b
*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/(3*b) + (A*a*c*d
^3*g^4)/(3*b)) + x^4*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(2
0*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(20*b)) + log(e*((a + b*x)/(c + d*x))^n)
*((B*d^4*g^4*x^5)/5 + B*c^4*g^4*x + 2*B*c^3*d*g^4*x^2 + B*c*d^3*g^4*x^4 + 2
*B*c^2*d^2*g^4*x^3) + x*((c^3*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d*n - 2*B*b*c
*n))/b - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c
 + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d +
5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b +
(A*a*c*d^3*g^4)/b))/(5*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n
- B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(b*d) + (2*c^2*d*g^
4*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/b))/(5*b*d) + (a*c*((((d^3*g^4*(
5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c)
)/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*
n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(b*d)) + (log(a + b*x)*((B*a^5*d^4*g^
4*n)/5 + B*a*b^4*c^4*g^4*n - B*a^4*b*c*d^3*g^4*n - 2*B*a^2*b^3*c^3*d*g^4*n
+ 2*B*a^3*b^2*c^2*d^2*g^4*n))/b^5 + (A*d^4*g^4*x^5)/5 - (B*c^5*g^4*n*log(c
+ d*x))/(5*d)
```

3.30 $\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c+dx)^3}{12bd}-\frac{B(bc-ad)^4g^3n\log(a+bx)}{4b^4d}+\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2/b^2/d-1/
12*B*(-a*d+b*c)*g^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*n*ln(b*x+a)/b^4/
d+1/4*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)^4\log(a+bx)}{4b^4d}-\frac{Bg^3nx(bc-ad)^3}{4b^3}-\frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d}-\frac{Bg^3n(c+dx)^3(bc-ad)}{12bd}$$

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*n*x)/b^3 - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2)/(8
*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3
```

```
*n*Log[a + b*x])/(4*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}-\frac{(B(bc-ad)n)\int\frac{(cg+dgx)^4}{(a+bx)(c+dx)}\,dx}{4dg}\\
&= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}-\frac{\left(B(bc-ad)g^3n\right)\int\frac{(c+dx)^3}{a+bx}\,dx}{4d}\\
&= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}\\
&\quad-\frac{\left(B(bc-ad)g^3n\right)\int\left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}\\
&= -\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c+dx)^3}{12bd}\\
&\quad-\frac{B(bc-ad)^4g^3n\log(a+bx)}{4b^4d}+\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^3\left(-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{4d}$$

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(146) = 292.

Time = 7.14 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

method	result
parallelrisch	$\frac{24B\,x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^3g^3n+6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^3n-6B\ln(bx+a)a^4d^4g^3n^2-6B\ln(bx+a)b^4c^4g^3n^2+24Ax\,b^4c^3d\,g^3n}{}$

```
[In] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g^3*n+6*B*ln(e*((b*x+a)/
(d*x+c))^n)*b^4*c^4*g^3*n-6*B*ln(b*x+a)*a^4*d^4*g^3*n^2-6*B*ln(b*x+a)*b^4*c
^4*g^3*n^2+24*A*x*b^4*c^3*d*g^3*n-24*A*b^4*c^4*g^3*n+21*B*a^3*b*c*d^3*g^3*n
^2-24*B*a^2*b^2*c^2*d^2*g^3*n^2-9*B*a*b^3*c^3*d*g^3*n^2+6*B*x^4*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^
3*n^2-3*B*x^2*a^2*b^2*d^4*g^3*n^2-9*B*x^2*b^4*c^2*d^2*g^3*n^2+6*B*x*a^3*b*d
^4*g^3*n^2-18*B*x*b^4*c^3*d*g^3*n^2+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*
c^2*d^2*g^3*n+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^3*d*g^3*n-60*A*a*b^3*c
^3*d*g^3*n+24*A*x^3*b^4*c*d^3*g^3*n+36*A*x^2*b^4*c^2*d^2*g^3*n+24*B*ln(b*x+
a)*a^3*b*c*d^3*g^3*n^2-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^3*n^2+24*B*ln(b*x+a
)*a*b^3*c^3*d*g^3*n^2+12*B*x^2*a*b^3*c*d^3*g^3*n^2-24*B*x*a^2*b^2*c*d^3*g^3
*n^2+36*B*x*a*b^3*c^2*d^2*g^3*n^2-6*B*a^4*d^4*g^3*n^2+18*B*b^4*c^4*g^3*n^2+
6*A*x^4*b^4*d^4*g^3*n)/b^4/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time = 0.30 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.75

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4g^3x^4 - 6\,Bb^4c^4g^3n\log(dx+c) + 6\,(4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4)g^3n\log(bx+ }{}$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*b^4*c^4*g^3*n*log(d*x + c) + 6*(4*B*a*b^3*c
^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^3*n*log(b*x + a
) + 2*(12*A*b^4*c*d^3*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*
A*b^4*c^2*d^2*g^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*g^3
*n)*x^2 + 6*(4*A*b^4*c^3*d*g^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 4*B*a
^2*b^2*c*d^3 - B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*c*d^3
*g^3*x^3 + 6*B*b^4*c^2*d^2*g^3*x^2 + 4*B*b^4*c^3*d*g^3*x)*log(e) + 6*(B*b^4
*d^4*g^3*n*x^4 + 4*B*b^4*c*d^3*g^3*n*x^3 + 6*B*b^4*c^2*d^2*g^3*n*x^2 + 4*B*
b^4*c^3*d*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.20 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{4}\,Bd^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}\,Ad^3g^3x^4+Bcd^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Acd^2g^3x^3+\frac{3}{2}\,Bc^2dg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}\,Ac^2dg^3x^2$$
$$-\frac{1}{24}\,Bd^3g^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-}{b^3d^3}\right.$$
$$+\frac{1}{2}\,Bcd^2g^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}\,Bc^2dg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^3g^3x$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*g^3*x^
4 + B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*g^3*x^
3 + 3/2*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c^2*
d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(146) = 292.

Time = 0.84 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/24*(6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3
*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*
log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x +
a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^4*d^
5/(d*x + c)^4) - (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 26*(b*x + a
)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 130*(b*x + a)
*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x
+ c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*
n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 6*(b*x +
a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 260*(b*x
+ a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b^4*c^3*d^4*
g^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B
*a^5*b^3*d^5*g^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 210*(b
*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 60*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*g^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 105*
(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*b^2*c^
2*d^6*g^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 30
*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*g^3*
n/(d*x + c)^3 - 6*B*b^8*c^5*g^3*log(e) + 30*B*a*b^7*c^4*d*g^3*log(e) - 60*B
*a^2*b^6*c^3*d^2*g^3*log(e) + 60*B*a^3*b^5*c^2*d^3*g^3*log(e) - 30*B*a^4*b^
4*c*d^4*g^3*log(e) + 6*B*a^5*b^3*d^5*g^3*log(e) - 6*A*b^8*c^5*g^3 + 30*A*a*
b^7*c^4*d*g^3 - 60*A*a^2*b^6*c^3*d^2*g^3 + 60*A*a^3*b^5*c^2*d^3*g^3 - 30*A*
a^4*b^4*c*d^4*g^3 + 6*A*a^5*b^3*d^5*g^3)/(b^7*d - 4*(b*x + a)*b^6*d^2/(d*x
+ c) + 6*(b*x + a)^2*b^5*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b^4*d^4/(d*x + c)^
3 + (b*x + a)^4*b^3*d^5/(d*x + c)^4) + 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d
*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*
b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^4*d) -
6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 1
0*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log((b
*x + a)/(d*x + c))/(b^4*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^3\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$

$$-x^2\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)\,(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}+\frac{A\,a\,c\,d^2\,g^3}{2\,b}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^3\,g^3\,x+\frac{3\,B\,c^2\,d\,g^3\,x^2}{2}+B\,c\,d^2\,g^3\,x^3+\frac{B\,d^3\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}\right.}{4\,b\,d}\right.$$

$$+\frac{c^2\,g^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,b}$$

$$\left.-\frac{a\,c\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d}\right)$$

$$-\frac{\ln(a+b\,x)\,(B\,n\,a^4\,d^3\,g^3-4\,B\,n\,a^3\,b\,c\,d^2\,g^3+6\,B\,n\,a^2\,b^2\,c^2\,d\,g^3-4\,B\,n\,a\,b^3\,c^3\,g^3)}{4\,b^4}$$

$$+\frac{A\,d^3\,g^3\,x^4}{4}-\frac{B\,c^4\,g^3\,n\,\ln(c+d\,x)}{4\,d}$$

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*g^3
*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n -
 B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b
*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*d^2*
g^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*g^3*x^4)/4 + B*c^3*g^3
*x + (3*B*c^2*d*g^3*x^2)/2 + B*c*d^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((d^2
*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d +
4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a
```

```
*d*n - B*b*c*n))/b + (A*a*c*d^2*g^3)/b))/(4*b*d) + (c^2*g^3*(12*A*a*d + 8*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*g^3*(4*A*a*d + 16*A*b*c +
 B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b)))/(b*d)) - (
log(a + b*x)*(B*a^4*d^3*g^3*n - 4*B*a*b^3*c^3*g^3*n - 4*B*a^3*b*c*d^2*g^3*n
 + 6*B*a^2*b^2*c^2*d*g^3*n))/(4*b^4) + (A*d^3*g^3*x^4)/4 - (B*c^4*g^3*n*log
(c + d*x))/(4*d)
```

3.31 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$
\begin{aligned}
\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx
&= -\frac{B(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2}{6bd} \\
&\quad - \frac{B(bc-ad)^3g^2n\log(a+bx)}{3b^3d} + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}
\end{aligned}
$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*n*x/b^2-1/6*B*(-a*d+b*c)*g^2*n*(d*x+c)^2/b/d-1/3*B*
(-a*d+b*c)^3*g^2*n*ln(b*x+a)/b^3/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$
\begin{aligned}
\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx
&= \frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d} - \frac{Bg^2n(bc-ad)^3\log(a+bx)}{3b^3d} \\
&\quad - \frac{Bg^2nx(bc-ad)^2}{3b^2} - \frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}
\end{aligned}
$$

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*g^2*n*x)/b^2 - (B*(b*c - a*d)*g^2*n*(c + d*x)^2)/(6*b
*d) - (B*(b*c - a*d)^3*g^2*n*Log[a + b*x])/(3*b^3*d) + (g^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)n)\int\frac{(cg+dgx)^3}{(a+bx)(c+dx)}\,dx}{3dg}\\
&= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{\left(B(bc-ad)g^2n\right)\int\frac{(c+dx)^2}{a+bx}\,dx}{3d}\\
&= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{\left(B(bc-ad)g^2n\right)\int\left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx}{3d}\\
&= -\frac{B(bc-ad)^2g^2nx}{3b^2}-\frac{B(bc-ad)g^2n(c+dx)^2}{6bd}\\
&\quad-\frac{B(bc-ad)^3g^2n\log(a+bx)}{3b^3d}+\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2\left(-\frac{B(bc-ad)n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3}+(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}$$

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])))/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(116) = 232.

Time = 2.92 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

method	result
parallelrisch	$\frac{6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c^2d\,g^2n-6B\ln(bx+a)a^2bc\,d^2g^2n^2-6A\,b^3c^3g^2n+6Bxa\,b^2c\,d^2g^2n^2+6B\ln(bx+a)a\,b^2c^2d\,g^2n^2+2B\,a^3d^3g^2}{}$

```
[In] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*g^2*n-6*B*ln(b*x+a)*a^2*b*c*
d^2*g^2*n^2-6*A*b^3*c^3*g^2*n+6*B*x*a*b^2*c*d^2*g^2*n^2+6*B*ln(b*x+a)*a*b^2
*c^2*d*g^2*n^2+2*B*a^3*d^3*g^2*n^2+4*B*b^3*c^3*g^2*n^2+6*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*c*d^2*g^2*n-12*A*a*b^2*c^2*d*g^2*n+6*A*x^2*b^3*c*d^2*g^2*
n+6*A*x*b^3*c^2*d*g^2*n+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g^2*n+B*x
^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2-2*B*x*a^2*b*d^3*g^2*n^2-4*B*x*
b^3*c^2*d*g^2*n^2-5*B*a^2*b*c*d^2*g^2*n^2-B*a*b^2*c^2*d*g^2*n^2+2*A*x^3*b^3
*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+2*B*ln(b*x+a)*a^3*d^
3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b^3/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 - 2\,Bb^3c^3g^2n\log(dx+c) + 2\,(3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3)g^2n\log(bx+a) + (6\,Ab^3cd^2g}{}$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*b^3*c^3*g^2*n*log(d*x + c) + 2*(3*B*a*b^2*c^
2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*log(b*x + a) + (6*A*b^3*c*d^2*g^2
- (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*b^3*c^2*d*g^2 - (2*B*b^3*
c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*
B*b^3*c*d^2*g^2*x^2 + 3*B*b^3*c^2*d*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3
+ 3*B*b^3*c*d^2*g^2*n*x^2 + 3*B*b^3*c^2*d*g^2*n*x)*log((b*x + a)/(d*x + c))
)/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.19 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{3}\,Bd^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ad^2g^2x^3$$
$$+Bcdg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdg^2x^2$$
$$+\frac{1}{6}\,Bd^2g^2n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-Bcdg^2n\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^2g^2n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)$$
$$+Bc^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2g^2x$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*g^2*x^
3 + B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*g^2*x^2 +
1/6*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*g^2*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*c^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(116) = 232.

Time = 0.63 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{6}\left(\frac{2\,(Bb^4c^4g^2n-4\,Bab^3c^3dg^2n+6\,Ba^2b^2c^2d^2g^2n-4\,Ba^3bcd^3g^2n+Ba^4d^4g^2n)\log\left(\frac{bx+a}{dx+c}\right)}{b^3d-\frac{3\,(bx+a)b^2d^2}{dx+c}+\frac{3\,(bx+a)^2bd^3}{(dx+c)^2}-\frac{(bx+a)^3d^4}{(dx+c)^3}}-\frac{3\,Bb^6c^4g^2n}{\cdots}\right.$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n
 - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d
 - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*x +
 a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 5*(b
*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 20*(b*x
+ a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d
*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n
/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*
d^4*g^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 12*(b*x + a)^2*B
*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c
) - 8*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6
*g^2*n/(d*x + c)^2 - 2*B*b^6*c^4*g^2*log(e) + 8*B*a*b^5*c^3*d*g^2*log(e) -
12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 8*B*a^3*b^3*c*d^3*g^2*log(e) - 2*B*a^4*b^
2*d^4*g^2*log(e) - 2*A*b^6*c^4*g^2 + 8*A*a*b^5*c^3*d*g^2 - 12*A*a^2*b^4*c^2
*d^2*g^2 + 8*A*a^3*b^3*c*d^3*g^2 - 2*A*a^4*b^2*d^4*g^2)/(b^5*d - 3*(b*x + a
)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (b*x + a)^3*b^2*d
^4/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*
c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*
d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2
*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)
/(d*x + c))/(b^3*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,c^2\,g^2\,x+B\,c\,d\,g^2\,x^2+\frac{B\,d^2\,g^2\,x^3}{3}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d} - \frac{c\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d\,g^2}{b}\right) + x^2\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right) + \frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,d^2\,g^2-3\,B\,n\,a^2\,b\,c\,d\,g^2+3\,B\,n\,a\,b^2\,c^2\,g^2\right)}{3\,b^3} + \frac{A\,d^2\,g^2\,x^3}{3}-\frac{B\,c^3\,g^2\,n\,\ln(c+d\,x)}{3\,d}$$

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*g^2*x^3)/3 + B*c^2*g^2*x + B*c*d*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((d*g^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*n)
)/(3*b) - (A*d*g^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*g^2)/b) + x^2*((d*g^2*(3*A*a*d + 9*
A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*g^2*(3*a*d + 3*b*c))/(6*b)) + (log
(a + b*x)*(B*a^3*d^2*g^2*n + 3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*n))/(3
*b^3) + (A*d^2*g^2*x^3)/3 - (B*c^3*g^2*n*log(c + d*x))/(3*d)
```

3.32 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b-1/2*B*(-a*d+b*c)^2*g*n*ln(b*x+a)/b^2/d+1/2*g*(d*x
+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2547, 21, 45}

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Bgnx(bc-ad)}{2b}$$

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/b - (B*(b*c - a*d)^2*g*n*Log[a + b*x])/(2*b^2*d)
 + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(B(bc-ad)n)\int\frac{(cg+dgx)^2}{(a+bx)(c+dx)}\,dx}{2dg}\\
&= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(B(bc-ad)gn)\int\frac{c+dx}{a+bx}\,dx}{2d}\\
&= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(B(bc-ad)gn)\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}\\
&= -\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (g*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(80) = 160.

Time = 1.07 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{B x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^2 d^2 g n + A x^2 b^2 d^2 g n - B\ln(bx+a) a^2 d^2 g n^2 + 2B\ln(bx+a) abcdg n^2 - B\ln(bx+a) b^2 c^2 g n^2 + 2Bx\ln\left(e\left(\frac{bx+c}{dx+c}\right.\right.}{2}$

```
[In] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n-B*ln(b*x
+a)*a^2*d^2*g*n^2+2*B*ln(b*x+a)*a*b*c*d*g*n^2-B*ln(b*x+a)*b^2*c^2*g*n^2+2*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g*n^
2+2*A*x*b^2*c*d*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g*n^2
+B*b^2*c^2*g*n^2-3*A*a*b*c*d*g*n-2*A*b^2*c^2*g*n)/b^2/n/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{Ab^2d^2gx^2 - Bb^2c^2gn\log(dx+c) + (2\,Babcd - Ba^2d^2)gn\log(bx+a) + (2\,Ab^2cdg - (Bb^2cd - Babd^2)gn}{2\,b^2d}$$

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - B*b^2*c^2*g*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d^
2)*g*n*log(b*x + a) + (2*A*b^2*c*d*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (B*
b^2*d^2*g*x^2 + 2*B*b^2*c*d*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*c*d*
g*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(73) = 146.

Time = 55.24 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} cgx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acgx+\frac{Adgx^2}{2}+\frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcgnx}{2}+Bcgx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdgnx^2}{4}+\frac{Bdgx^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ cg\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Acgx+\frac{Adgx^2}{2}-\frac{Ba^2dgn\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{Ba^2dg\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2}+\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Bacg\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}+\frac{Bad}{2} \end{cases}$$

[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)

```
[Out] Piecewise((c*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*g*x +
A*d*g*x**2/2 + B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d) + B*c*g*n*x/2 + B*c*g
*x*log(e*(a/(c + d*x))**n) + B*d*g*n*x**2/4 + B*d*g*x**2*log(e*(a/(c + d*x)
)**n)/2, Eq(b, 0)), (c*g*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x
*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*g*x + A*d*g*x**2/2 - B*a**2*d*g*
n*log(c/d + x)/(2*b**2) - B*a**2*d*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(2*b**2) + B*a*c*g*n*log(c/d + x)/b + B*a*c*g*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/b + B*a*d*g*n*x/(2*b) - B*c**2*g*n*log(c/d + x)/(2*d) - B*c*g*
n*x/2 + B*c*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*g*x**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,Bdgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Adgx^2$$
$$-\frac{1}{2}\,Bdgn\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bcgn\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bcgx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acgx$$

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*g*x^2 - 1/2*
B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*g*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(80) = 160$.

Time = 0.44 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{2}\left(\frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d - \frac{2\,(bx+a)bd^2}{dx+c} + \frac{(bx+a)^2d^3}{(dx+c)^2}} - \frac{Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{(bx+a)B}{dx}\cdots}{\cdots}\right.$$

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3
*g*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*x
+ a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a)*
B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2
*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x +
 c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - B*b^4*c^3*g*log(e) + 3*B*a*b^3*c^
2*d*g*log(e) - 3*B*a^2*b^2*c*d^2*g*log(e) + B*a^3*b*d^3*g*log(e) - A*b^4*c^
3*g + 3*A*a*b^3*c^2*d*g - 3*A*a^2*b^2*c*d^2*g + A*a^3*b*d^3*g)/(b^3*d - 2*(
b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*g*
n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b*
x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^
2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d))*(b*c/(b*c
- a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x\left(\frac{g\,(2\,A\,a\,d + 4\,A\,b\,c + B\,a\,d\,n - B\,b\,c\,n)}{2\,b} - \frac{A\,g\,(2\,a\,d + 2\,b\,c)}{2\,b}\right)$$
$$+ \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,g\,x^2}{2} + B\,c\,g\,x\right)$$
$$- \frac{\ln(a+b\,x)\,(B\,a^2\,d\,g\,n - 2\,B\,a\,b\,c\,g\,n)}{2\,b^2} + \frac{A\,d\,g\,x^2}{2} - \frac{B\,c^2\,g\,n\,\ln(c+d\,x)}{2\,d}$$

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*g*(2*a*d + 2*b*c)
)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*g*x^2)/2 + B*c*g*x) - (log(
a + b*x)*(B*a^2*d*g*n - 2*B*a*b*c*g*n))/(2*b^2) + (A*d*g*x^2)/2 - (B*c^2*g*
n*log(c + d*x))/(2*d)
```

3.33 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}-\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/g-B*n*polylog(2
,d*(b*x+a)/b/(d*x+c))/d/g
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2543, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(
d*g)) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2543

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d)
/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c*
g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}+\frac{(B(bc-ad)n)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{dg}$$

$$=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}+\frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx,x,c+dx\right)}{d^2g}$$

$$=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}-\frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx,x,\frac{1}{c+dx}\right)}{d^2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{(B(bc-ad)n)\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}dx, x, \frac{1}{c+dx}\right)}{d^2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{Bn\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$$
$$= \frac{\log(g(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(g(c+dx))\right)-2Bn\,\text{PolyLog}\left(2, \frac{b}{b}\right.}{2dg}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] (Log[g*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[
e*((a + b*x)/(c + d*x))^n] + B*n*Log[g*(c + d*x)]) - 2*B*n*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(2*d*g)
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{dgx+cg}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dgx+cg}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*g*x + c*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\int \frac{A}{c+dx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g),x)
```

```
[Out] (Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(c + d*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dgx+cg}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="maxima
")
```

```
[Out] -1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)*
log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*g) - 2*integrate((n*
log(b*x + a) + log(e))/(d*g*x + c*g), x)) + A*log(d*g*x + c*g)/(d*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(79) = 158$.

Time = 57.49 (sec) , antiderivative size = 566, normalized size of antiderivative = 7.08

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$$
$$= \frac{1}{2}\left(\frac{(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n)\log\left(\frac{bx+a}{dx+c}\right)}{b^2dg - \frac{2\,(bx+a)bd^2g}{dx+c} + \frac{(bx+a)^2d^3g}{(dx+c)^2}} - \frac{Bb^4c^3n - 3\,Bab^3c^2dn - \frac{(bx+a)Bb^3c^3dn}{dx+c} + 3\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*lo
g((b*x + a)/(d*x + c))/(b^2*d*g - 2*(b*x + a)*b*d^2*g/(d*x + c) + (b*x + a)
^2*d^3*g/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b^3*
c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n/(d*
x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x + a)*
B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3*B*a^2
```

```
*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d - 3*A*
a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*g - 2*(b*x + a)*b^2*d^2*g/(d*x + c) + (
b*x + a)^2*b*d^3*g/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^
2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d*g) - (B*b
^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log((b*x +
a)/(d*x + c))/(b^2*d*g))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{c\,g+d\,g\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x), x)
```

3.34 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx=\frac{A(a+bx)}{(bc-ad)g^2(c+dx)}-\frac{Bn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}$$

```
[Out] A*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+B*(b*x+
a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2551, 2332}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx=\frac{A(a+bx)}{g^2(c+dx)(bc-ad)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)}-\frac{Bn(a+bx)}{g^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] (A*(a + b*x))/((b*c - a*d)*g^2*(c + d*x)) - (B*n*(a + b*x))/((b*c - a*d)*g^
2*(c + d*x)) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/((b*c - a*d)*g^
2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\ &= \frac{A(a+bx)}{(bc-ad)g^2(c+dx)} + \frac{B\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\ &= \frac{A(a+bx)}{(bc-ad)g^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)g^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg(cg+dgx)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}\right)}{dg^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g*(c*g + d*g*x))) + (B*(b*c - a
*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[
c + d*x])/(b*c - a*d)^2))/(d*g^2)
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{A}{g^{2}(dx+c)d}-\frac{B\left(\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)(bx+a)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{g^{2}(ad-cb)}$	81
parts	$-\frac{A}{g^{2}(dx+c)d}-\frac{B\left(\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)(bx+a)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{g^{2}(ad-cb)}$	81
parallelrisch	$-\frac{-Bab\,d^{3}n^{2}+B\,b^{2}c\,d^{2}n^{2}+Aab\,d^{3}n-A\,b^{2}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{3}n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{3}n}{g^{2}(dx+c)b\,d^{3}n(ad-cb)}$	129

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x,method=_RETURNVERBOSE)
[Out] -1/g^2*A/(d*x+c)/d-1/g^2*B/(a*d-b*c)*(ln(e*((b*x+a)/(d*x+c))^n)*(b*x+a)/(d*
x+c)-n*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(cg+dgx)^{2}}\,dx = -\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log(e)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^{2}-ad^{3})g^{2}x+(bc^{2}d-acd^{2})g^{2}}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="fric
as")
[Out] -(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x +
 B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d - a
*c*d^2)*g^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(cg+dgx)^{2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = Bn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log(bx+a)}{(bcd-ad^2)g^2}-\frac{b\log(dx+c)}{(bcd-ad^2)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2g^2x+cdg^2} - \frac{A}{d^2g^2x+cdg^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="maxi
ma")
```

```
[Out] B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}-\frac{(Bn-B\log(e)-A)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="giac
")
```

```
[Out] ((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) - (B*n - B*log(e) -
 A)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A-B\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^2,x)
```

```
[Out] (B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*g^2*(a*d - b*c)) -
 (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*g^2 + d*g^2*x)) - (A - B*n)/(d^2*
g^2*x + c*d*g^2)
```

3.35 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2g^3}$$

```
[Out] 1/4*B*n/d/g^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/g^3/(d*x+c)+1/2*b^2*B*n*ln(b
*x+a)/d/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^3/(d*x+c)
^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/g^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}+\frac{b^2Bn\log(a+bx)}{2dg^3(bc-ad)^2}-\frac{b^2Bn\log(c+dx)}{2dg^3(bc-ad)^2}+\frac{bBn}{2dg^3(c+dx)(bc-ad)}+\frac{Bn}{4dg^3(c+dx)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

```
[Out] (B*n)/(4*d*g^3*(c + d*x)^2) + (b*B*n)/(2*d*(b*c - a*d)*g^3*(c + d*x)) + (b^
2*B*n*Log[a + b*x])/(2*d*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(2*d*g^3*(c + d*x)^2) - (b^2*B*n*Log[c + d*x])/(2*d*(b*c - a*d)^2
*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(cg+dgx)^2}\,dx}{2dg}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^3}\,dx}{2dg^3}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx)}\right)dx}{2dg^3}\\
&= \frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3}\\
&\quad-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2g^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4dg^3(c+dx)^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*d
 + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 7.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2A\,a^2b\,d^5n+2A\,b^3c^2d^3n-B\,a^2b\,d^5n^2-3B\,b^3c^2d^3n^2+4Ba\,b^2c\,d^4n^2-4Aa\,b^2c\,d^4n-2B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^5n+2Bxa\,b^2d^5}{4g^3(dx+c)^2(a^2d^2-2abcd+b}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3*n-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^2
+4*B*a*b^2*c*d^4*n^2-4*A*a*b^2*c*d^4*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^
3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b*d^5*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((b*x+a
)/(d*x+c))^n)*a*b^2*c*d^4*n)/g^3/(d*x+c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^
4/n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n+2\,(Bb^2c^2-2}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)g^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d
 + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c*d
- B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2
*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(133) = 266$.

Time = 127.35 (sec) , antiderivative size = 2103, normalized size of antiderivative = 13.93

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**3,x)
```

```
[Out] Piecewise((-A/(2*c**2*d*g**3 + 4*c*d**2*g**3*x + 2*d**3*g**3*x**2) - B*log(
e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*c**2*d*g**3 + 4*c*d**2*g**3*x
 + 2*d**3*g**3*x**2), Eq(a, b*c/d)), ((A*x + B*a*log(e*(a/c + b*x/c)**n)/b
- B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**3*g**3), Eq(d, 0)), (-2*A*a**2*d
**2/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x**2 -
 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x**2 +
4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g**3*x**2)
+ 4*A*a*b*c*d/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g
**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g*
*3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g
**3*x**2) - 2*A*b**2*c**2/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4
*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a
*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2
*c**2*d**3*g**3*x**2) + B*a**2*d**2*n/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**
4*g**3*x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*
g**3*x - 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**
3*x + 4*b**2*c**2*d**3*g**3*x**2) - 2*B*a**2*d**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*
g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g
**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*
g**3*x**2) - 4*B*a*b*c*d*n/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x +
4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*
a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**
2*c**2*d**3*g**3*x**2) + 4*B*a*b*c*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x**2 - 8
*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x**2 + 4*
b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g**3*x**2) -
```

```
2*B*a*b*d**2*n*x/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**
5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4
*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**
3*g**3*x**2) + 3*B*b**2*c**2*n/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*
x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x
- 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4
*b**2*c**2*d**3*g**3*x**2) + 2*B*b**2*c*d*n*x/(4*a**2*c**2*d**3*g**3 + 8*a*
*2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c*
*2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*
d**2*g**3*x + 4*b**2*c**2*d**3*g**3*x**2) + 4*B*b**2*c*d*x*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a
**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b
*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c
**2*d**3*g**3*x**2) + 2*B*b**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n)/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x**2
 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x**2
+ 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g**3*x**2
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\,Bn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log\left(bx+a\right)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{2}{(b^2c^2d}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}-\frac{A}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*
g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^3*x +
 c^2*d*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.93 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Bd\log{(e)}-2\,Ad)(bx}{(bcg^3-adg^3)(dx+c)^2}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="giac
")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B*b*n/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2*B*d
*n/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n - 2
*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) - 4*(B*b
*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c
 - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,(a\,d-b\,c)^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,g^3\,(a\,d-b\,c)^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^3,x)
```

```
[Out] (B*b^2*n*atanh((2*a^2*d^3*g^3 - 2*b^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)^2) +
(2*b*d*x)/(a*d - b*c)))/(d*g^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c + d*
x))^n))/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d*g^3*x)) - ((2*A*a*d - 2*A*b*c -
 B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2*d*g
^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x)
```

3.36 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2}+\frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)}+\frac{b^3Bn\log(a+bx)}{3d(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}-\frac{b^3Bn\log(c+dx)}{3d(bc-ad)^3g^4}$$

```
[Out] 1/9*B*n/d/g^4/(d*x+c)^3+1/6*b*B*n/d/(-a*d+b*c)/g^4/(d*x+c)^2+1/3*b^2*B*n/d/
(-a*d+b*c)^2/g^4/(d*x+c)+1/3*b^3*B*n*ln(b*x+a)/d/(-a*d+b*c)^3/g^4+1/3*(-A-B
*ln(e*((b*x+a)/(d*x+c))^n))/d/g^4/(d*x+c)^3-1/3*b^3*B*n*ln(d*x+c)/d/(-a*d+b
*c)^3/g^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}+\frac{b^3Bn\log(a+bx)}{3dg^4(bc-ad)^3}-\frac{b^3Bn\log(c+dx)}{3dg^4(bc-ad)^3}+\frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2}+\frac{bBn}{6dg^4(c+dx)^2(bc-ad)}+\frac{Bn}{9dg^4(c+dx)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (B*n)/(9*d*g^4*(c + d*x)^3) + (b*B*n)/(6*d*(b*c - a*d)*g^4*(c + d*x)^2) + (
b^2*B*n)/(3*d*(b*c - a*d)^2*g^4*(c + d*x)) + (b^3*B*n*Log[a + b*x])/(3*d*(b
*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d*g^4*(c + d*x
)^3) - (b^3*B*n*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(cg+dgx)^3}\,dx}{3dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^4}\,dx}{3dg^4}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3} + \frac{(B(bc-ad)n)\int\left(\frac{b^4}{(bc-ad)^4(a+bx)}-\frac{d}{(bc-ad)(c+dx)^4}-\frac{bd}{(bc-ad)^2(c+dx)^3}-\frac{b^2d}{(bc-ad)^3(c+dx)^2}-\frac{b^3d}{(bc-ad)^4(c+dx)}\right)}{3dg^4}$$

$$= \frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2}+\frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)} + \frac{b^3Bn\log(a+bx)}{3d(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}-\frac{b^3Bn\log(c+dx)}{3d(bc-ad)^3g^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$= \frac{-6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(2a^2d^2-abd(7c+3dx)+b^2\left(11c^2+15cdx+6d^2x^2\right)\right)+6b^3(c+dx)^3\log(a+bx)-6b^3(c+dx)^3\log(}{(bc-ad)^3}}{18dg^4(c+dx)^3}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (-6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(2*a^2*d^2 -
 a*b*d*(7*c + 3*d*x) + b^2*(11*c^2 + 15*c*d*x + 6*d^2*x^2)) + 6*b^3*(c + d*
x)^3*Log[a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(18*d*g
^4*(c + d*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(174) = 348.

Time = 16.01 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{9B\,a^2b^2c\,d^6n^2-18Ba\,b^3c^2d^5n^2-18A\,a^2b^2c\,d^6n+18Aa\,b^3c^2d^5n+18B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^6n+18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^2}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(9*B*a^2*b^2*c*d^6*n^2-18*B*a*b^3*c^2*d^5*n^2-18*A*a^2*b^2*c*d^6*n+18
*A*a*b^3*c^2*d^5*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n+18*B*x*ln
(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n-18*B*x*a*b^3*c*d^6*n^2-18*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d
^5*n-2*B*a^3*b*d^7*n^2+11*B*b^4*c^3*d^4*n^2+6*A*a^3*b*d^7*n-6*A*b^4*c^3*d^4
*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d^7*n+6*B*x^3*ln(e*((b*x+a)/(d*x+c))
^n)*b^4*d^7*n-6*B*x^2*a*b^3*d^7*n^2+6*B*x^2*b^4*c*d^6*n^2+3*B*x*a^2*b^2*d^7
*n^2+15*B*x*b^4*c^2*d^5*n^2)/g^4/(d*x+c)^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/n/b/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(171) = 342.

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.64

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = -\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(5\,Bb^3c^2d - 6\,Bab^2cd^2 + B}{18\,((b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 - 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + B*
a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2 - 2*B*a
^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*l
og(e) - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^2 + 3*B*b^3*c^2*d*n*x + (3*B
*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*log((b*x + a)/(d*x + c)))/((
b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c
^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^
5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d
 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(171) = 342.

Time = 0.21 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$=\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2+11\,b^2c^2-7\,abcd+2\,a^2d^2+3\,(5\,b^2cd-abd^2)x}{(b^2c^2d^4-2\,abcd^5+a^2d^6)g^4x^3+3\,(b^2c^3d^3-2\,abc^2d^4+a^2cd^5)g^4x^2+3\,(b^2c^4d^2-2\,abc^3d^3+a^2c^2d\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(d^4g^4x^3+3\,cd^3g^4x^2+3\,c^2d^2g^4x+c^3dg^4)}$$
$$-\frac{A}{3\,(d^4g^4x^3+3\,cd^3g^4x^2+3\,c^2d^2g^4x+c^3dg^4)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c*
d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^3
*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
+ a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*b
^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^
4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*x^3 + 3
*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3 + 3*c*d^
3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(171) = 342.

Time = 0.68 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.21

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$=\frac{1}{18}\left(6\left(\frac{3\,(bx+a)Bb^2n}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)}-\frac{3\,(bx+a)^2Bbdn}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{(bx}{(b^2c^2g^4-2\,abc}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="giac
")
```

```
[Out] 1/18*(6*(3*(b*x + a)*B*b^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(
d*x + c)) - 3*(b*x + a)^2*B*b*d*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)^2) + (b*x + a)^3*B*d^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*
d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c)) - 2*(B*d^2*n - 3*B*d^2*log(
e) - 3*A*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x
 + c)^3) + 9*(B*b*d*n - 2*B*b*d*log(e) - 2*A*b*d)*(b*x + a)^2/((b^2*c^2*g^4
```

```
 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) - 18*(B*b^2*n - B*b^2*log(e) -
 A*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*
(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{B\,a^2\,d\,n}{9\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{A\,a^2\,d}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{A\,b^2\,c^2}{3\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,d\,g^4\,(c+d\,x)^3} + \frac{2\,A\,a\,b\,c}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{B\,b^2\,d\,n\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{7\,B\,a\,b\,c\,n}{18\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{11\,B\,b^2\,c^2\,n}{18\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{5\,B\,b^2\,c\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{B\,a\,b\,d\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,d\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^4,x)
```

```
[Out] (B*a^2*d*n)/(9*g^4*(a*d - b*c)^2*(c + d*x)^3) - (A*a^2*d)/(3*g^4*(a*d - b*c
)^2*(c + d*x)^3) - (A*b^2*c^2)/(3*d*g^4*(a*d - b*c)^2*(c + d*x)^3) - (B*log
(e*((a + b*x)/(c + d*x))^n))/(3*d*g^4*(c + d*x)^3) + (B*b^3*n*atan((a*d*1i
+ b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*d*g^4*(a*d - b*c)^3) + (2*A*a*b*c)
/(3*g^4*(a*d - b*c)^2*(c + d*x)^3) + (B*b^2*d*n*x^2)/(3*g^4*(a*d - b*c)^2*(
c + d*x)^3) - (7*B*a*b*c*n)/(18*g^4*(a*d - b*c)^2*(c + d*x)^3) + (11*B*b^2*
c^2*n)/(18*d*g^4*(a*d - b*c)^2*(c + d*x)^3) + (5*B*b^2*c*n*x)/(6*g^4*(a*d -
 b*c)^2*(c + d*x)^3) - (B*a*b*d*n*x)/(6*g^4*(a*d - b*c)^2*(c + d*x)^3)
```

3.37 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \frac{Bn}{16dg^5(c+dx)^4} + \frac{bBn}{12d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2} + \frac{b^3Bn}{4d(bc-ad)^3g^5(c+dx)} + \frac{b^4Bn\log(a+bx)}{4d(bc-ad)^4g^5} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} - \frac{b^4Bn\log(c+dx)}{4d(bc-ad)^4g^5}$$

```
[Out] 1/16*B*n/d/g^5/(d*x+c)^4+1/12*b*B*n/d/(-a*d+b*c)/g^5/(d*x+c)^3+1/8*b^2*B*n/
d/(-a*d+b*c)^2/g^5/(d*x+c)^2+1/4*b^3*B*n/d/(-a*d+b*c)^3/g^5/(d*x+c)+1/4*b^4
*B*n*ln(b*x+a)/d/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^
5/(d*x+c)^4-1/4*b^4*B*n*ln(d*x+c)/d/(-a*d+b*c)^4/g^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used

$= \{2547, 21, 46\}$

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4} + \frac{b^4Bn\log(a+bx)}{4dg^5(bc-ad)^4} - \frac{b^4Bn\log(c+dx)}{4dg^5(bc-ad)^4} + \frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3} + \frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2} + \frac{bBn}{12dg^5(c+dx)^3(bc-ad)} + \frac{Bn}{16dg^5(c+dx)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (B*n)/(16*d*g^5*(c + d*x)^4) + (b*B*n)/(12*d*(b*c - a*d)*g^5*(c + d*x)^3) +
 (b^2*B*n)/(8*d*(b*c - a*d)^2*g^5*(c + d*x)^2) + (b^3*B*n)/(4*d*(b*c - a*d)
^3*g^5*(c + d*x)) + (b^4*B*n*Log[a + b*x])/(4*d*(b*c - a*d)^4*g^5) - (A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(4*d*g^5*(c + d*x)^4) - (b^4*B*n*Log[c + d
*x])/(4*d*(b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(cg+dgx)^4}\,dx}{4dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^5}\,dx}{4dg^5}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5}{(bc-ad)^5(a+bx)} - \frac{d}{(bc-ad)(c+dx)^5} - \frac{bd}{(bc-ad)^2(c+dx)^4} - \frac{b^2d}{(bc-ad)^3(c+dx)^3} - \frac{b^3d}{(bc-ad)^4(c+dx)^2}\right.}{4dg^5}$$

$$= \frac{Bn}{16dg^5(c+dx)^4} + \frac{bBn}{12d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2} + \frac{b^3Bn}{4d(bc-ad)^3g^5(c+dx)} + \frac{b^4Bn\log(a+bx)}{4d(bc-ad)^4g^5} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} - \frac{b^4Bn\log(c+dx)}{4d(bc-ad)^4g^5}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4} + \frac{Bn\left(\frac{3(bc-ad)^4}{(c+dx)^4} + \frac{4b(bc-ad)^3}{(c+dx)^3} + \frac{6b^2(bc-ad)^2}{(c+dx)^2} + \frac{12b^3(bc-ad)}{c+dx} + 12b^4\log(a+bx) - 12b^4\log(c+dx)\right)}{12(bc-ad)^4}}{4dg^5}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)^4) + (B*n*((3*(b*c - a*
d)^4)/(c + d*x)^4 + (4*b*(b*c - a*d)^3)/(c + d*x)^3 + (6*b^2*(b*c - a*d)^2)
/(c + d*x)^2 + (12*b^3*(b*c - a*d))/(c + d*x) + 12*b^4*Log[a + b*x] - 12*b^
4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*d*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. 2(204) = 408.

Time = 42.25 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(96*B*x^2*a^4*b*c^5*d^4*n^2-210*B*x^2*a^3*b^2*c^6*d^3*n^2+240*B*x^2*a^
2*b^3*c^7*d^2*n^2-108*B*x^2*a*b^4*c^8*d*n^2-288*A*x^2*a^4*b*c^5*d^4*n+432*A
*x^2*a^3*b^2*c^6*d^3*n-288*A*x^2*a^2*b^3*c^7*d^2*n+72*A*x^2*a*b^4*c^8*d*n+4
8*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^9*n+60*B*x*a^4*b*c^6*d^3*n^2-120*B*
x*a^3*b^2*c^7*d^2*n^2+120*B*x*a^2*b^3*c^8*d*n^2-192*A*x*a^4*b*c^6*d^3*n+288
*A*x*a^3*b^2*c^7*d^2*n-192*A*x*a^2*b^3*c^8*d*n+48*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^4*b*c^7*d^2*n-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^8*d*n+16*B*x^4*
a^4*b*c^3*d^6*n^2-36*B*x^4*a^3*b^2*c^4*d^5*n^2+48*B*x^4*a^2*b^3*c^5*d^4*n^2
-25*B*x^4*a*b^4*c^6*d^3*n^2-48*A*x^4*a^4*b*c^3*d^6*n+72*A*x^4*a^3*b^2*c^4*d
^5*n-48*A*x^4*a^2*b^3*c^5*d^4*n+12*A*x^4*a*b^4*c^6*d^3*n+64*B*x^3*a^4*b*c^4
*d^5*n^2-144*B*x^3*a^3*b^2*c^5*d^4*n^2+180*B*x^3*a^2*b^3*c^6*d^3*n^2-88*B*x
^3*a*b^4*c^7*d^2*n^2-192*A*x^3*a^4*b*c^4*d^5*n+288*A*x^3*a^3*b^2*c^5*d^4*n-
192*A*x^3*a^2*b^3*c^6*d^3*n+48*A*x^3*a*b^4*c^7*d^2*n-3*B*x^4*a^5*c^2*d^7*n^
2+12*A*x^4*a^5*c^2*d^7*n-12*B*x^3*a^5*c^3*d^6*n^2+48*A*x^3*a^5*c^3*d^6*n-18
*B*x^2*a^5*c^4*d^5*n^2+72*A*x^2*a^5*c^4*d^5*n-12*B*x*a^5*c^5*d^4*n^2-48*B*x
*a*b^4*c^9*n^2+48*A*x*a^5*c^5*d^4*n+48*A*x*a*b^4*c^9*n-12*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^5*c^6*d^3*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n+12*B*
x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^6*d^3*n+48*B*x^3*ln(e*((b*x+a)/(d*x+c
))^n)*a*b^4*c^7*d^2*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^8*d*n)/g^5
/(d*x+c)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/
n/a/c^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(201) = 402.

Time = 0.31 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.42

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{12\,Ab^4c^4-48\,Aab^3c^3d+72\,Aa^2b^2c^2d^2-48\,Aa^3bcd^3+12\,Aa^4d^4-12\,(Bb^4cd^3-Bab^3d^4)nx^3-6\,(7\,Bb^4c}{48\,((b^4c^4d^5-4\,ab^3c^3d^6+6}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 - 6*(7*B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 - 4*(13*B*b^4*c^3*d - 18*B*a
*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n*x - (25*B*b^4*c^4 - 48*B*
a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + 3*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*b^4*c*d^3*n*x^3 + 6*B*b^4*c^2*d^
2*n*x^2 + 4*B*b^4*c^3*d*n*x + (4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*
a^3*b*c*d^3 - B*a^4*d^4)*n)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b
^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*
c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8
)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*
c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b
^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^
7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(201) = 402$.

Time = 0.21 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.03

$$\begin{aligned}&\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx\\&=\frac{1}{48}Bn\left(\frac{12\,b^3d^3x^3+25\,b^3c^3-23\,ab^2c^2d+\cdots}{(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8)g^5x^4+4\,(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d^6-a^3cd^7)g^5x^3+6\,(\cdots}\right.\\&\quad-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}\\&\quad-\frac{A}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}\end{aligned}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^
8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7
)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^
6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g
^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^
3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d
^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4 + 4*c*d^4*g^
5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(201) = 402$.

Time = 0.85 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.18

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{1}{48}\left(12\left(\frac{4\,(bx+a)Bb^3n}{(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3d^3g^5)(dx+c)} - \frac{6\,(bx+a)^2Bb^2dn}{(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3d^3g\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="giac
")
```

```
[Out] 1/48*(12*(4*(b*x + a)*B*b^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c
*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B*b^2*d*n/((b^3*c^3*g^5
- 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 4*(b*
x + a)^3*B*b*d^2*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 -
a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B*d^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2
*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*log((b*x + a)/(d*x
+ c)) + 3*(B*d^3*n - 4*B*d^3*log(e) - 4*A*d^3)*(b*x + a)^4/((b^3*c^3*g^5 -
3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) - 16*(B*b
*d^2*n - 3*B*b*d^2*log(e) - 3*A*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*
c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B*b^2*d*n -
 2*B*b^2*d*log(e) - 2*A*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^
5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B*b^3*n - B*b^3*log
(e) - A*b^3)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^
5 - a^3*d^3*g^5)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$=\frac{B\,b^4\,n\,\mathrm{atanh}\left(\frac{4\,a^4\,d^5\,g^5-8\,a^3\,b\,c\,d^4\,g^5+8\,a\,b^3\,c^3\,d^2\,g^5-4\,b^4\,c^4\,d\,g^5}{4\,d\,g^5\,(a\,d-b\,c)^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{2\,d\,g^5\,(a\,d-b\,c)^4}$$
$$-\frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{4\,d\,\left(c^4\,g^5+4\,c^3\,d\,g^5\,x+6\,c^2\,d^2\,g^5\,x^2+4\,c\,d^3\,g^5\,x^3+d^4\,g^5\,x^4\right)}$$
$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-3\,B\,a^3\,d^3\,n+25\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-23\,B\,a\,b^2\,c^2\,d\,n+13\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,x\,\left(B\,n\,a^2\,d^3-5\,B\,n\,a\,b\,c\,d\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\right.}}{4\,c^4\,d\,g^5+16\,c^3\,d^2\,g^5\,x+24\,c^2\,d^3\,g^5\,x^2+16\,c\,d^4}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atanh((4*a^4*d^5*g^5 - 4*b^4*c^4*d*g^5 - 8*a^3*b*c*d^4*g^5 + 8*a*b
^3*c^3*d^2*g^5)/(4*d*g^5*(a*d - b*c)^4) + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*d*g^5*(a*d - b*c)^4) - (B*l
og(e*((a + b*x)/(c + d*x))^n))/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^
3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 3
*B*a^3*d^3*n + 25*B*b^3*c^3*n + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 23*B*
a*b^2*c^2*d*n + 13*B*a^2*b*c*d^2*n)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) + (b*x*(B*a^2*d^3*n + 13*B*b^2*c^2*d*n - 5*B*a*b*c*d^2*n)
)/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (b^2*x^2*(B*a*d
^3*n - 7*B*b*c*d^2*n))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)/(4*c^4*d*g^5 + 4*d^5*g^5*x^4 + 16*c^3*d^2*g^5*x + 16*c*d^4*g^5*x^3 + 24*c
^2*d^3*g^5*x^2)
```

3.38 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 544

$$\begin{aligned}
&\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d} \\
&\quad + \frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d} - \frac{2B(bc-ad)^4g^4n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^5} \\
&\quad - \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} \\
&\quad - \frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} \\
&\quad - \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&\quad + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} \\
&\quad + \frac{13B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d} + \frac{5B^2(bc-ad)^5g^4n^2\log(c+dx)}{6b^5d} \\
&\quad + \frac{2B(bc-ad)^5g^4n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} \\
&\quad - \frac{2B^2(bc-ad)^5g^4n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}
\end{aligned}$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*n^2*x/b^4+7/60*B^2*(-a*d+b*c)^3*g^4*n^2*(d*x+c)^
2/b^3/d+1/30*B^2*(-a*d+b*c)^2*g^4*n^2*(d*x+c)^3/b^2/d-2/5*B*(-a*d+b*c)^4*g^
4*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^5-1/5*B*(-a*d+b*c)^3*g^4*n*(d
*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-2/15*B*(-a*d+b*c)^2*g^4*n*(d*
```

```
x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*c)*g^4*n*(d*x+c
)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/d+13/30*B^2*(-a*d+b*c)^5*g^4*n^2*ln((b*x+a)/(d*x+c))/b^5/d
+5/6*B^2*(-a*d+b*c)^5*g^4*n^2*ln(d*x+c)/b^5/d+2/5*B*(-a*d+b*c)^5*g^4*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^5/d-2/5*B^2*(-a*d+b
*c)^5*g^4*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^5/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{2Bg^4n(bc-ad)^5\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5d} - \frac{2Bg^4n(a+bx)(bc-ad)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5} - \frac{Bg^4n(c+dx)^2(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^3d} - \frac{2Bg^4n(c+dx)^3(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{15b^2d} - \frac{Bg^4n(c+dx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10bd} + \frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d} - \frac{2B^2g^4n^2(bc-ad)^5\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} + \frac{13B^2g^4n^2(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d} + \frac{5B^2g^4n^2(bc-ad)^5\log(c+dx)}{6b^5d} + \frac{13B^2g^4n^2x(bc-ad)^4}{30b^4} + \frac{7B^2g^4n^2(c+dx)^2(bc-ad)^3}{60b^3d} + \frac{B^2g^4n^2(c+dx)^3(bc-ad)^2}{30b^2d}$$

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(30*b^4) + (7*B^2*(b*c - a*d)^3*g^4*n^2*(c
 + d*x)^2)/(60*b^3*d) + (B^2*(b*c - a*d)^2*g^4*n^2*(c + d*x)^3)/(30*b^2*d)
- (2*B*(b*c - a*d)^4*g^4*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(5*b^5) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(5*b^3*d) - (2*B*(b*c - a*d)^2*g^4*n*(c + d*x)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) + (g^4*(c + d*x)^5*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(5*d) + (13*B^2*(b*c - a*d)^5*g^4*n^2*Log[(
a + b*x)/(c + d*x)])/(30*b^5*d) + (5*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x]
```

```
)/(6*b^5*d) + (2*B*(b*c - a*d)^5*g^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g
^4*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)^5g^4n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5d}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)^5g^4n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b} - \frac{(2B(bc-ad)^5g^4n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5bd}$$

$$= -\frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)^5g^4n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2} - \frac{(2B(bc-ad)^5g^4n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2d} + \frac{(B^2(bc-ad)^5g^4n^2)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10bd}$$

$$= -\frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} - \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3} - \frac{(2B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3d} + \frac{(2B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\frac{1}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{15b^2d} + \frac{(B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^4x}+\frac{d}{b(b-dx)^4}+\frac{d}{b^2(b-dx)^3}+\frac{d}{b^3(b-dx)^2}+\frac{d}{b^4(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{10bd}$$

$$= \frac{B^2(bc-ad)^4g^4n^2x}{10b^4} + \frac{B^2(bc-ad)^3g^4n^2(c+dx)^2}{20b^3d} + \frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d} - \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} - \frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} - \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} + \frac{B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{10b^5d} + \frac{B^2(bc-ad)^5g^4n^2\log(c+dx)}{10b^5d} - \frac{(2B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^4} - \frac{(2B(bc-ad)^5g^4n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^4d} + \frac{(B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3d} + \frac{(2B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{15b^2d}$$

$$
\begin{aligned}
&= \frac{7B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d} + \frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d} \\
&\quad - \frac{2B(bc-ad)^4g^4n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^5} \\
&\quad - \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} \\
&\quad - \frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} \\
&\quad - \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&\quad + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} \\
&\quad + \frac{7B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d} + \frac{7B^2(bc-ad)^5g^4n^2\log(c+dx)}{30b^5d} \\
&\quad + \frac{2B(bc-ad)^5g^4n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} \\
&\quad + \frac{(2B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^5} \\
&\quad - \frac{(2B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^5d} \\
&\quad + \frac{(B^2(bc-ad)^5g^4n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d} \\
&+ \frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d} - \frac{2B(bc-ad)^4g^4n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^5} \\
&- \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} \\
&- \frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} \\
&- \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&+ \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} \\
&+ \frac{13B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d} + \frac{5B^2(bc-ad)^5g^4n^2\log(c+dx)}{6b^5d} \\
&+ \frac{2B(bc-ad)^5g^4n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} \\
&- \frac{2B^2(bc-ad)^5g^4n^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.98

$$
\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{g^4\left((c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(24Abd(bc-ad)^3x-12B(bc-ad)^3n(bdx+(bc-ad)\log(a+bx))-4B(bc-ad)^2n(\right.}{}\right.}{}
$$

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^4*((c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 +
2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*
b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b
*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b
^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3
*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c
+ d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a +
b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c +
d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(12*b^5)))
/(5*d)
```

Maple [F]

$$\int (dgx + cg)^4 \left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^2 dx$$

```
[In] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg + dgx)^4 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx$$
$$= \int (dgx + cg)^4 \left(B\log\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) + A\right)^2 dx$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4*
A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 + 6*
B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2*g^4
*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (cg + dgx)^4 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2880 vs. 2(519) = 1038.

Time = 0.72 (sec) , antiderivative size = 2880, normalized size of antiderivative = 5.29

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*d^4*g^
4*x^5 + 2*A*B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d^3*g^4*x^4 + 4*A*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A^2*c^2*d^2*g^4*x^3 + 4*A*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 2*A^2*c^3*d*g^4*x^2 + 1/30*A*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*c^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2*b^2*c^3*d^2*g^
4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (25*g^4*n^2 - 12*
g^4*n*log(e))*b^4*c^5)*B^2*log(d*x + c)/(b^4*d) - 2/5*(b^5*c^5*g^4*n^2 - 5*
a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n
^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^5*d) + 1/60
*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 24*B^2*b^5*c^5*g^4*n^2*log(b*x + a)*log
(d*x + c) - 12*B^2*b^5*c^5*g^4*n^2*log(d*x + c)^2 + 6*(a*b^4*d^5*g^4*n*log(
e) - (g^4*n*log(e) - 10*g^4*log(e)^2)*b^5*c*d^4)*B^2*x^4 + 2*((g^4*n^2 - 16
*g^4*n*log(e) + 60*g^4*log(e)^2)*b^5*c^2*d^3 - 2*(g^4*n^2 - 10*g^4*n*log(e)
)*a*b^4*c*d^4 + (g^4*n^2 - 4*g^4*n*log(e))*a^2*b^3*d^5)*B^2*x^3 + ((13*g^4*
n^2 - 72*g^4*n*log(e) + 120*g^4*log(e)^2)*b^5*c^3*d^2 - 3*(11*g^4*n^2 - 40*
g^4*n*log(e))*a*b^4*c^2*d^3 + 3*(9*g^4*n^2 - 20*g^4*n*log(e))*a^2*b^3*c*d^4
 - (7*g^4*n^2 - 12*g^4*n*log(e))*a^3*b^2*d^5)*B^2*x^2 - 12*(5*a*b^4*c^4*d*g
^4*n^2 - 10*a^2*b^3*c^3*d^2*g^4*n^2 + 10*a^3*b^2*c^2*d^3*g^4*n^2 - 5*a^4*b*
c*d^4*g^4*n^2 + a^5*d^5*g^4*n^2)*B^2*log(b*x + a)^2 + 2*((23*g^4*n^2 - 48*g
^4*n*log(e) + 30*g^4*log(e)^2)*b^5*c^4*d - (79*g^4*n^2 - 120*g^4*n*log(e))*
a*b^4*c^3*d^2 + 6*(17*g^4*n^2 - 20*g^4*n*log(e))*a^2*b^3*c^2*d^3 - (59*g^4*
n^2 - 60*g^4*n*log(e))*a^3*b^2*c*d^4 + (13*g^4*n^2 - 12*g^4*n*log(e))*a^4*b
*d^5)*B^2*x - 2*(12*(4*g^4*n^2 - 5*g^4*n*log(e))*a*b^4*c^4*d - 12*(13*g^4*n
```

```
^2 - 10*g^4*n*log(e))*a^2*b^3*c^3*d^2 + 4*(49*g^4*n^2 - 30*g^4*n*log(e))*a^
3*b^2*c^2*d^3 - (113*g^4*n^2 - 60*g^4*n*log(e))*a^4*b*c*d^4 + (25*g^4*n^2 -
 12*g^4*n*log(e))*a^5*d^5)*B^2*log(b*x + a) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B
^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2*g^4*
x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n)^2 + 12*(B^2*b^5*d^5*g^4*x^5 +
 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2*
g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((d*x + c)^n)^2 + 2*(12*B^2*b^5*d^5*g^4
*x^5*log(e) - 12*B^2*b^5*c^5*g^4*n*log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4
*n - 20*g^4*log(e))*b^5*c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d
^5*g^4*n - 2*(2*g^4*n - 15*g^4*log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c
^2*d^3*g^4*n - 5*a^2*b^3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10*
g^4*log(e))*b^5*c^3*d^2)*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3*
c^2*d^3*g^4*n + 5*a^3*b^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4*
log(e))*b^5*c^4*d)*B^2*x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4
*n + 10*a^3*b^2*c^2*d^3*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*lo
g(b*x + a))*log((b*x + a)^n) - 2*(12*B^2*b^5*d^5*g^4*x^5*log(e) - 12*B^2*b^
5*c^5*g^4*n*log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4*n - 20*g^4*log(e))*b^5
*c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d^5*g^4*n - 2*(2*g^4*n -
 15*g^4*log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c^2*d^3*g^4*n - 5*a^2*b^
3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10*g^4*log(e))*b^5*c^3*d^2
)*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3*c^2*d^3*g^4*n + 5*a^3*b
^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4*log(e))*b^5*c^4*d)*B^2*
x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4*n + 10*a^3*b^2*c^2*d^3
*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*log(b*x + a) + 12*(B^2*b^
5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B
^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^5*d)
```

Giac [F]

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)^4\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)^4\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.39 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 454

$$\begin{aligned}
&\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d} \\
&\quad - \frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} \\
&\quad - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\
&\quad + \frac{5B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2(bc-ad)^4g^3n^2\log(c+dx)}{12b^4d} \\
&\quad + \frac{B(bc-ad)^4g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad - \frac{B^2(bc-ad)^4g^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}$$

```
[Out] 5/12*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2
/b^2/d-1/2*B*(-a*d+b*c)^3*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4
-1/4*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1
/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g^3
*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*g^3*n^
2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*g^3*n^2*ln(d*x+c)/b^4/d+
```

```
1/2*B*(-a*d+b*c)^4*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b
^4/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{Bg^3n(bc-ad)^4\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4d} - \frac{Bg^3n(a+bx)(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4} - \frac{Bg^3n(c+dx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b^2d} - \frac{Bg^3n(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6bd} + \frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d} - \frac{B^2g^3n^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} + \frac{5B^2g^3n^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2g^3n^2(bc-ad)^4\log(c+dx)}{12b^4d} + \frac{5B^2g^3n^2x(bc-ad)^3}{12b^3} + \frac{B^2g^3n^2(c+dx)^2(bc-ad)^2}{12b^2d}$$

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*b^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c +
d*x)^2)/(12*b^2*d) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(2*b^4) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (g^3*(c + d*x)^4*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (5*B^2*(b*c - a*d)^4*g^3*n^2*Log[(
a + b*x)/(c + d*x)])/(12*b^4*d) + (11*B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x
])/(12*b^4*d) + (B*(b*c - a*d)^4*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3
*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
```

```
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 g^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd}$$

$$= -\frac{B(bc-ad)g^3 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2} - \frac{\left(B(bc-ad)^4 g^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 d} + \frac{\left(B^2(bc-ad)^4 g^3 n^2\right)\text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd}$$

$$= -\frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(B(bc-ad)^4g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^3} - \frac{(B(bc-ad)^4g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^3d} + \frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4b^2d} + \frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{6bd}$$

$$= \frac{B^2(bc-ad)^3g^3n^2x}{6b^3} + \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d} - \frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} + \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d} + \frac{B^2(bc-ad)^4g^3n^2\log(c+dx)}{6b^4d} + \frac{B(bc-ad)^4g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} + \frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^4} - \frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^4d} + \frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{4b^2d}$$

$$= \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d} - \frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} + \frac{5B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2(bc-ad)^4g^3n^2\log(c+dx)}{12b^4d} + \frac{B(bc-ad)^4g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} - \frac{B^2(bc-ad)^4g^3n^2\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$= \frac{g^3\left((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd(b\right.}{}}{}$$

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c
 - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c -
a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])))/(3*b^4)))/(4*d)
```

Maple [F]

$$\int (dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dgx+cg)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^3*g^3*x^3 + 3*A^2*c*d^2*g^3*x^2 + 3*A^2*c^2*d*g^3*x + A^2*c^
3*g^3 + (B^2*d^3*g^3*x^3 + 3*B^2*c*d^2*g^3*x^2 + 3*B^2*c^2*d*g^3*x + B^2*c^
3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*g^3*x^3 + 3*A*B*c*d^2*
g^3*x^2 + 3*A*B*c^2*d*g^3*x + A*B*c^3*g^3)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(433) = 866$.

Time = 0.71 (sec) , antiderivative size = 2129, normalized size of antiderivative = 4.69

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*g^
3*x^4 + 2*A*B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d^2*g^3*x^3 + 3*A*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/2*A^2*c^2*d*g^3*x^2 - 1/12*A*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A^2*c^3*g^3*x - 1/12*(26*a*b^2*c^3*d*g^3*n^2 - 21*a^2*b*c^2*d^2*g^3*n^
2 + 6*a^3*c*d^3*g^3*n^2 - (11*g^3*n^2 - 6*g^3*n*log(e))*b^3*c^4)*B^2*log(d*
x + c)/(b^3*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c
^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b
^4*d) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*B^2*b^4*c^4*g^3*n^2*log(b*
x + a)*log(d*x + c) - 3*B^2*b^4*c^4*g^3*n^2*log(d*x + c)^2 + 2*(a*b^3*d^4*g
^3*n*log(e) - (g^3*n*log(e) - 6*g^3*log(e)^2)*b^4*c*d^3)*B^2*x^3 + ((g^3*n^
2 - 9*g^3*n*log(e) + 18*g^3*log(e)^2)*b^4*c^2*d^2 - 2*(g^3*n^2 - 6*g^3*n*lo
g(e))*a*b^3*c*d^3 + (g^3*n^2 - 3*g^3*n*log(e))*a^2*b^2*d^4)*B^2*x^2 - 3*(4*
a*b^3*c^3*d*g^3*n^2 - 6*a^2*b^2*c^2*d^2*g^3*n^2 + 4*a^3*b*c*d^3*g^3*n^2 - a
^4*d^4*g^3*n^2)*B^2*log(b*x + a)^2 + ((7*g^3*n^2 - 18*g^3*n*log(e) + 12*g^3
*log(e)^2)*b^4*c^3*d - (19*g^3*n^2 - 36*g^3*n*log(e))*a*b^3*c^2*d^2 + (17*g
^3*n^2 - 24*g^3*n*log(e))*a^2*b^2*c*d^3 - (5*g^3*n^2 - 6*g^3*n*log(e))*a^3*
b*d^4)*B^2*x - (6*(3*g^3*n^2 - 4*g^3*n*log(e))*a*b^3*c^3*d - 9*(5*g^3*n^2 -
 4*g^3*n*log(e))*a^2*b^2*c^2*d^2 + 2*(19*g^3*n^2 - 12*g^3*n*log(e))*a^3*b*c
*d^3 - (11*g^3*n^2 - 6*g^3*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4
*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*
b^4*c^3*d*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*
d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*log((d*x +
 c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) - 6*B^2*b^4*c^4*g^3*n*log(d*x + c)
 + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*log(e))*b^4*c*d^3)*B^2*x^3 + 3*(4*a
*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*log(e))*b^4*c^2*d^2
)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n + a^3*b*d^4*g^
```

```
3*n - (3*g^3*n - 4*g^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n -
6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*log(b*x
+ a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) - 6*B^2*b^4*c^4*g^3*
n*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*log(e))*b^4*c*d^3)*B^
2*x^3 + 3*(4*a*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*log(e
))*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n
+ a^3*b*d^4*g^3*n - (3*g^3*n - 4*g^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*
c^3*d*g^3*n - 6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n
)*B^2*log(b*x + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B
^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^4*d)
```

Giac [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.40 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
&\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} - \frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3g^2n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*g^2*n*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)^3*g
^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{2Bg^2n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d}-\frac{2Bg^2n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3}-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd}+\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d}-\frac{2B^2g^2n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}+\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{B^2g^2n^2(bc-ad)^3\log(c+dx)}{b^3d}+\frac{B^2g^2n^2x(bc-ad)^2}{3b^2}$$

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b^2) - (2*B*(b*c - a*d)^2*g^2*n*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3) - (B*(b*c - a*d)*g^2*n*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*d) + (g^2*(c + d*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (B^2*(b*c - a*d)^3*g^2*n^
2*Log[(a + b*x)/(c + d*x)])/(3*b^3*d) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[c +
d*x])/(b^3*d) + (2*B*(b*c - a*d)^3*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3
*g^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$=\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d}$$

$$=\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b}-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$=-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd}+\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2}-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2d}+\frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$=-\frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3}-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd}+\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d}+\frac{2B(bc-ad)^3g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}+\frac{(2B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3}-\frac{(2B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} - \frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{b^3d} + \frac{2B(bc-ad)^3g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} - \frac{2B^2(bc-ad)^3g^2n^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$= \frac{g^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\,\mathrm{l}\right.}{}\right.}{}$$

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a + b*
x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x
] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

Maple [F]

$$\int (dgx+cg)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dgx + cg)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^2*g^2*x^2 + 2*A^2*c*d*g^2*x + A^2*c^2*g^2 + (B^2*d^2*g^2*x^2
 + 2*B^2*c*d*g^2*x + B^2*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*d^2*g^2*x^2 + 2*A*B*c*d*g^2*x + A*B*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(346) = 692$.

Time = 0.71 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.08

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*g^
2*x^3 + 2*A*B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*d*
g^2*x^2 + 1/3*A*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
c*d*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*c^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^2*g^2*
```

```
x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*g^2*x - 1/3*(5*a*b*c^2*d
*g^2*n^2 - 2*a^2*c*d^2*g^2*n^2 - (3*g^2*n^2 - 2*g^2*n*log(e))*b^2*c^3)*B^2*
log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2
*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*d
^3*g^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*g^2*n^2*log(b*x + a)*log(d*x + c) - B^2
*b^3*c^3*g^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (g^2*n*log(e) -
 3*g^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*g^2*n^2 - 3*a^2*b*c*d^
2*g^2*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2 - 4*g^2*n*log(e
) + 3*g^2*log(e)^2)*b^3*c^2*d - 2*(g^2*n^2 - 3*g^2*n*log(e))*a*b^2*c*d^2 +
(g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*g^2*n^2 - 3*g^2*n*log(e
))*a*b^2*c^2*d - (7*g^2*n^2 - 6*g^2*n*log(e))*a^2*b*c*d^2 + (3*g^2*n^2 - 2*
g^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*
c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^
2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((d*x + c)^n)^2
 + (2*B^2*b^3*d^3*g^2*x^3*log(e) - 2*B^2*b^3*c^3*g^2*n*log(d*x + c) + (a*b^
2*d^3*g^2*n - (g^2*n - 6*g^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*
g^2*n - a^2*b*d^3*g^2*n - (2*g^2*n - 3*g^2*log(e))*b^3*c^2*d)*B^2*x + 2*(3*
a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*
log((b*x + a)^n) - (2*B^2*b^3*d^3*g^2*x^3*log(e) - 2*B^2*b^3*c^3*g^2*n*log(
d*x + c) + (a*b^2*d^3*g^2*n - (g^2*n - 6*g^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2
*(3*a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n - (2*g^2*n - 3*g^2*log(e))*b^3*c^2*
d)*B^2*x + 2*(3*a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^
2*log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b
^3*c^2*d*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d)
```

Giac [F]

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dgx+cg)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (c\,g+d\,g\,x)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.41 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\begin{aligned}\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx &= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}\\ &\quad+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d}\\ &\quad+\frac{B(bc-ad)^2gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}\\ &\quad-\frac{B^2(bc-ad)^2gn^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*g*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*g*n^2*ln(d*x+c)/b^2
/d+B*(-a*d+b*c)^2*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x
+a))/b^2/d-B^2*(-a*d+b*c)^2*g*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31}

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{Bgn(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d} - \frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2} + \frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d} - \frac{B^2gn^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} + \frac{B^2gn^2(bc-ad)^2\log(c+dx)}{b^2d}$$

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2)
 + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d) + (B^2*(b
*c - a*d)^2*g*n^2*Log[c + d*x])/(b^2*d) + (B*(b*c - a*d)^2*g*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*d) -
(B^2*(b*c - a*d)^2*g*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^2 g\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{\left(B(bc-ad)^2 gn\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d}$$

$$= \frac{g(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{\left(B(bc-ad)^2 gn\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b} - \frac{\left(B(bc-ad)^2 gn\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} \\
&\quad+\frac{B(bc-ad)^2gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad+\frac{(B^2(bc-ad)^2gn^2)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2} \\
&\quad-\frac{(B^2(bc-ad)^2gn^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d} \\
&= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d} \\
&\quad+\frac{B(bc-ad)^2gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad-\frac{B^2(bc-ad)^2gn^2\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$
\int(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
=\frac{g\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)\cdots\right.\right.}{\cdots}\right.}{2d}
$$

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*n
*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c -
 a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b
^2))/(2*d)
```

Maple [F]

$$\int (dgx + cg)\left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right)\right)^{2} dx$$

```
[In] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^{n}\right)\right)^{2} dx = \int (dgx + cg)\left(B\log\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right)^{2} dx$$

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^{n}\right)\right)^{2} dx = \text{Timed out}$$

```
[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(217) = 434.

Time = 0.68 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = ABdgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2dgx^2 - ABdgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) + 2ABcgn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) + 2ABcgx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + A^2cgx - \frac{(acdgn^2-(gn^2-gn\log(e))bc^2)B^2\log(dx+c)}{bd} - \frac{(b^2c^2gn^2-2abcdgn^2+a^2d^2gn^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} + \frac{2B^2b^2c^2gn^2\log(bx+a)\log(dx+c) - B^2b^2c^2gn^2\log(dx+c)^2 + B^2b^2d^2gx^2\log(e)^2 - (2abcdgn^2 - a^2}{}$$

[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")

[Out] A*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*g*x^2 - A*B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*g*x*log(e*(b*x /(d*x + c) + a/(d*x + c))^n) + A^2*c*g*x - (a*c*d*g*n^2 - (g*n^2 - g*n*log(e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2* d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*log(b*x + a)*l og(d*x + c) - B^2*b^2*c^2*g*n^2*log(d*x + c)^2 + B^2*b^2*d^2*g*x^2*log(e)^2 - (2*a*b*c*d*g*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*lo g(e) - (g*n*log(e) - g*log(e)^2)*b^2*c*d)*B^2*x - 2*((g*n^2 - 2*g*n*log(e)) *a*b*c*d - (g*n^2 - g*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2*b^2*d^2*g* x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^ 2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - B^2*b^2*c^2*g *n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b *c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2 *g*x^2*log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*lo g(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a) + (B^ 2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b ^2*d)

Giac [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.42 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/g-2*B*n*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/g+2*B^2*n^2*pol
ylog(3,d*(b*x+a)/b/(d*x+c))/d/g
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2551, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = -\frac{2Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{dg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])
/(d*g)) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/(d*g) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))])/(d*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} + \frac{(2Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg} + \frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg} + \frac{2B^2n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}dx$$
$$= \frac{A^2\log(c+dx)+2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] (A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*
d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/(b
*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c +
b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a + b*x)
/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*
x))])/(d*g)
```

Maple [F]

$$\int\frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dgx+cg}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(d*g*x + c*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = \frac{\int \frac{A^2}{c+dx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g),x)
```

```
[Out] (Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(c + d*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*g) + A^2*log(d*g*x + c*g)/(d*g) - in
tegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log
(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^n)
+ B^2*log(e) + A*B)*log((d*x + c)^n))/(d*g*x + c*g), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="giac
")

[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*g*x + c*g), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{c\,g+d\,g\,x}\,dx$$

[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x),x)

[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x), x)

3.43 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}$$

```
[Out] -2*A*B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/g^2/(d
*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)+(b*x
+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2551, 2333, 2332}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(c+dx)(bc-ad)} - \frac{2ABn(a+bx)}{g^2(c+dx)(bc-ad)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)} + \frac{2B^2n^2(a+bx)}{g^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-2*A*B*n*(a + b*x))/((b*c - a*d)*g^2*(c + d*x)) + (2*B^2*n^2*(a + b*x))/((
b*c - a*d)*g^2*(c + d*x)) - (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n])/((b*c - a*d)*g^2*(c + d*x)) + ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/((b*c - a*d)*g^2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}-\frac{(2Bn)\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}\\
&\quad -\frac{\left(2B^2n\right)\text{Subst}\left(\int \log\left(ex^n\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)}\\
&\quad -\frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}+\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$
$$= \frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B\,\mathrm{l}\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*
Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*n*
(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*(c + d*x
))
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2B^2ab\,d^3n^3-2B^2b^2c\,d^2n^3+A^2ab\,d^3n-A^2b^2c\,d^2n+2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n+2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n-2ABab\,d^3n^2+}{g^2(dx+c}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -(2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+2*A*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n+2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*
d^3*n-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n^2+B^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a*b*d^3*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n^2)/g^2/
(d*x+c)/b/d^3/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{A^2bc - A^2ad + 2\,(B^2bc - B^2ad)n^2 + (B^2bc - B^2ad)\log\left(e\right)^2 - (B^2bdn^2x + B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^2 - 2\,(A$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="fr
icas")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(e
)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b*c
 - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*n*x
 + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a*d*n
 + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3
)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{g^2}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**2,x)
```

```
[Out] (Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x**2)
, x))/g**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 0.20 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = 2\,ABn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2}-\frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right) + \left(2\,n\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2}-\frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)-\frac{((bdx+bc)\log}{} \right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{d^2g^2x+cdg^2}-\frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2g^2x+cdg^2}-\frac{A^2}{d^2g^2x+cdg^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="ma
xima")
```

```
[Out] 2*A*B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b
*log(d*x + c)/((b*c*d - a*d^2)*g^2)) + (2*n*(1/(d^2*g^2*x + c*d*g^2) + b*lo
g(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log(d*x + c)/((b*c*d - a*d^2)*g^2))*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d
*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2
*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*d*g^2
- a*c*d^2*g^2 + (b*c*d^2*g^2 - a*d^3*g^2)*x))*B^2 - B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(d^2*g^2*x + c*d*g^2) - 2*A*B*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)/(d^2*g^2*x + c*d*g^2) - A^2/(d^2*g^2*x + c*d*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.99 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)g^2}-\frac{2\,(B^2n^2-B^2n\log\left(e\right)-ABn)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}+\frac{(2\,B^2n^2-2\,B^2n\log\left(e\right)}{} \right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="gi
ac")
```

```
[Out] ((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*g^2) - 2*(B^2*n^2
- B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*g^2)
+ (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) + A^2
)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{2\,A\,B}{x\,d^2\,g^2+c\,d\,g^2}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,g^2+d\,g^2\,x\right)} + \frac{B^2\,b}{d\,g^2\,\left(a\,d-b\,c\right)}\right) - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,g^2+c\,d\,g^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,g^2+b\,c\,d\,g^2}{d\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^2,x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*g^2*x + c*d*g^2) - (2*A*B)/(
d^2*g^2*x + c*d*g^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*g^2 + d
*g^2*x)) + (B^2*b)/(d*g^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d^2*
g^2*x + c*d*g^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*g^2 + b*c*d*g^2)/(d*g^2))
*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*g^2*(a*d - b*c))
```

3.44 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)} + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2g^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)}$$

```
[Out] -1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/g^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*d
+b*c)^2/g^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/g^3/(d*x+c)-2*b*B^2*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g^3/(d*x+c)+1/2*B*d*n*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^3/(d*x+c)^2-1/2*d*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)^2+b*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2551, 2367, 2333, 2332, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = \frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(c+dx)^2(bc-ad)^2} + \frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(c+dx)(bc-ad)^2} - \frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(c+dx)^2(bc-ad)^2} - \frac{2AbBn(a+bx)}{g^3(c+dx)(bc-ad)^2} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^3(c+dx)(bc-ad)^2} + \frac{2bB^2n^2(a+bx)}{g^3(c+dx)(bc-ad)^2} - \frac{B^2dn^2(a+bx)^2}{4g^3(c+dx)^2(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] -1/4*(B^2*d*n^2*(a + b*x)^2)/((b*c - a*d)^2*g^3*(c + d*x)^2) - (2*A*b*B*n*(
a + b*x))/((b*c - a*d)^2*g^3*(c + d*x)) + (2*b*B^2*n^2*(a + b*x))/((b*c - a
*d)^2*g^3*(c + d*x)) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
/((b*c - a*d)^2*g^3*(c + d*x)) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^3*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^2*g^3*(c + d*x)^2) + (b*(
a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g^3*(c +
d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (b-dx)\left(A+B\log\left(ex^n\right)\right)^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{\text{Subst}\left(\int \left(b(A+B\log\left(ex^n\right))^2-dx(A+B\log\left(ex^n\right))^2\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{b\text{Subst}\left(\int (A+B\log\left(ex^n\right))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}-\frac{d\text{Subst}\left(\int x(A+B\log\left(ex^n\right))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= -\frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2}+\frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)}\\
&\quad -\frac{(2bBn)\text{Subst}\left(\int (A+B\log\left(ex^n\right))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&\quad +\frac{(Bdn)\text{Subst}\left(\int x(A+B\log\left(ex^n\right))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}
\end{aligned}
$$

$$= -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)} - \frac{(2bB^2n)\,\text{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)} + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2g^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$

$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x
] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d
*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*
b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x
)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^
2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(311) = 622.

Time = 7.41 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

method	result
parallelrisch	$-\frac{-8B^2ab^2cd^4n^3-2AB\,a^2bd^5n^2-6AB\,b^3c^2d^3n^2-4A^2ab^2cd^4n+2A^2b^3c^2d^3n+2A^2a^2bd^5n+7B^2b^3c^2d^3n^3+B^2a^2bd^5n^3-4A}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4*(-8*B^2*a*b^2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A^
2*a*b^2*c*d^4*n+2*A^2*b^3*c^2*d^3*n+2*A^2*a^2*b*d^5*n+7*B^2*b^3*c^2*d^3*n^3
+B^2*a^2*b*d^5*n^3-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n-4*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^4*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^2*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n^2+4*A*B*x*a*b^2*d^
5*n^2-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*c*d^4*n
+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n^2+4*A*B*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^5*n+8*A*B*a*b^2*c*d^4*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b
^3*c*d^4*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n-2*B^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n
^2-6*B^2*x*a*b^2*d^5*n^3+6*B^2*x*b^3*c*d^4*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n^2)/g^3/(d*x+c
)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(311) = 622.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.06

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{2A^2b^2c^2-4A^2abcd+2A^2a^2d^2+(7B^2b^2c^2-8B^2abcd+B^2a^2d^2)n^2+2\left(B^2b^2c^2-2B^2abcd+B^2a^2d^2\right)}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B^
2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2
)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c*d
- B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A*B*a
*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*B*b^2
```

```
*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2
 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*d + B^2
*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b*c*d - B
^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*d - B^2*a
^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*B*a*b*c*d
- A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b*d^2)*n^2)*
x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2
 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c
^3*d^2 + a^2*c^2*d^3)*g^3)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{g^3}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**3,x)
```

```
[Out] (Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(311) = 622$.

Time = 0.24 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{2}ABn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{}{(b^2c}\right.$$
$$+\frac{1}{4}\left(2\,n\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{}{(b}\right.\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}-\frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^3g^3x^2+2\,cd^2g^3x+c^2dg^3}$$
$$-\frac{A^2}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^
2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*g^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*
d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) +
 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d
*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3))*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*l
og(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*
g^3 - 2*a*b*c^3*d^2*g^3 + a^2*c^2*d^3*g^3 + (b^2*c^2*d^3*g^3 - 2*a*b*c*d^4*
g^3 + a^2*d^5*g^3)*x^2 + 2*(b^2*c^3*d^2*g^3 - 2*a*b*c^2*d^3*g^3 + a^2*c*d^4
*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(d^3*g^3*x
^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A^2/(d^3*g^3*x^2 + 2*c*d^
2*g^3*x + c^2*d*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.97 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log(e)-}{(bcg^3-adg^3)(a}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2
*B^2*d*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2 +
2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*g^3 - a*d*g
^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/((b*c
*g^3 - a*d*g^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*B^2*d
*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*(b*x +
 a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n*log(e)
 + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/((b*c*g^3
 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)} - \frac{B^2\,b^2}{2\,d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$- \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)} - \frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$
$$- \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2} + \frac{B^2\,b^2\left(\frac{d^2\,g^3\,n\,x\,(a\,d-b\,c)}{b} - \frac{d\,g^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2} + \frac{c\,d\,g^3\,n\,(a\,d-b\,c)}{2\,b}\right)}{d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2\right)}\right)$$
$$- \frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A-3\,B\,n)\,1\mathrm{i}}{d\,g^3\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d
*g^3*x)) - (B^2*b^2)/(2*d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n)/(
2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*g^3
+ 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(c
^2*d*g^3 + d^3*g^3*x^2 + 2*c*d^2*g^3*x) + (B^2*b^2*((d^2*g^3*n*x*(a*d - b*c
))/b - (d*g^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*g^3*n*(a*d - b*c)
)/(2*b)))/(d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*g^3 + d^3*g^3*x^2 +
 2*c*d^2*g^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*g^3 - 2*b^2*c^2*d*
g^3)/(2*d*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d*g^3*(a*d
- b*c)^2)
```

3.45 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 429

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx &= \frac{2B^2d^2n^2(a+bx)^3}{27(bc-ad)^3g^4(c+dx)^3} \\ &- \frac{bB^2dn^2(a+bx)^2}{2(bc-ad)^3g^4(c+dx)^2} + \frac{2b^2B^2n^2(a+bx)}{(bc-ad)^3g^4(c+dx)} \\ &- \frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} \\ &+ \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} \\ &- \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} \\ &- \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} \\ &+ \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} \\ &- \frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}\end{aligned}$$

```
[Out] 2/27*B^2*d^2*n^2*(b*x+a)^3/(-a*d+b*c)^3/g^4/(d*x+c)^3-1/2*b*B^2*d*n^2*(b*x+
a)^2/(-a*d+b*c)^3/g^4/(d*x+c)^2+2*b^2*B^2*n^2*(b*x+a)/(-a*d+b*c)^3/g^4/(d*x
+c)-2/9*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/
(d*x+c)^3+b*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
4/(d*x+c)^2-2*b^2*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/
g^4/(d*x+c)-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^4/(d*x+c)^3+2/3*b^3*B
```

```
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^3/g^4-1
/3*b^3*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^3/g^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2551, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \frac{2b^3Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3dg^4(bc-ad)^3} - \frac{2b^2Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(c+dx)(bc-ad)^3} - \frac{2Bd^2n(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(c+dx)^3(bc-ad)^3} + \frac{bBdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(c+dx)^2(bc-ad)^3} - \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3dg^4(c+dx)^3} - \frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3dg^4(bc-ad)^3} + \frac{2b^2B^2n^2(a+bx)}{g^4(c+dx)(bc-ad)^3} + \frac{2B^2d^2n^2(a+bx)^3}{27g^4(c+dx)^3(bc-ad)^3} - \frac{bB^2dn^2(a+bx)^2}{2g^4(c+dx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

```
[Out] (2*B^2*d^2*n^2*(a + b*x)^3)/(27*(b*c - a*d)^3*g^4*(c + d*x)^3) - (b*B^2*d*n
^2*(a + b*x)^2)/(2*(b*c - a*d)^3*g^4*(c + d*x)^2) + (2*b^2*B^2*n^2*(a + b*x
))/((b*c - a*d)^3*g^4*(c + d*x)) - (2*B*d^2*n*(a + b*x)^3*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^3*g^4*(c + d*x)^3) + (b*B*d*n*(a + b*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(c + d*x)^2
) - (2*b^2*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*
d)^3*g^4*(c + d*x)) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*g^4*(c
+ d*x)^3) + (2*b^3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)
/(c + d*x)])/(3*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[(a + b*x)/(c + d*x)
]^2)/(3*d*(b*c - a*d)^3*g^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (b-dx)^2 \left(A+B\log\left(ex^n\right)\right)^2\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^4}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2Bn)\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))}{x}\, dx, x, \frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}$$

$$= -\frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} + \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} - \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} - \frac{(2B^2n^2)\,\text{Subst}\left(\int\left(-\frac{1}{6}d(18b^2-9bdx+2d^2x^2)+\frac{b^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}$$

$$= -\frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} + \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} - \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} + \frac{(B^2n^2)\,\text{Subst}\left(\int(18b^2-9bdx+2d^2x^2)\,dx, x, \frac{a+bx}{c+dx}\right)}{9(bc-ad)^3g^4} - \frac{(2b^3B^2n^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}$$

$$= \frac{2B^2d^2n^2(a+bx)^3}{27(bc-ad)^3g^4(c+dx)^3} - \frac{bB^2dn^2(a+bx)^2}{2(bc-ad)^3g^4(c+dx)^2} + \frac{2b^2B^2n^2(a+bx)}{(bc-ad)^3g^4(c+dx)} - \frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} + \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} - \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} - \frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$= \frac{-18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(12A(bc-ad)^3-4B(bc-ad)^3n+18Ab(bc-ad)^2(c+dx)-15bB(bc-ad)^2n(c+dx)+36Ab^2(bc-ad)(c+\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

```
[Out] (-18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d)^3 -
4*B*(b*c - a*d)^3*n + 18*A*b*(b*c - a*d)^2*(c + d*x) - 15*b*B*(b*c - a*d)^2
*n*(c + d*x) + 36*A*b^2*(b*c - a*d)*(c + d*x)^2 - 66*b^2*B*(b*c - a*d)*n*(c
 + d*x)^2 + 36*A*b^3*(c + d*x)^3*Log[a + b*x] - 66*b^3*B*n*(c + d*x)^3*Log[
a + b*x] - 18*b^3*B*n*(c + d*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e
*((a + b*x)/(c + d*x))^n] + 18*b*B*(b*c - a*d)^2*(c + d*x)*Log[e*((a + b*x)
/(c + d*x))^n] + 36*b^2*B*(b*c - a*d)*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x
))^n] + 36*b^3*B*(c + d*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] -
36*A*b^3*(c + d*x)^3*Log[c + d*x] + 66*b^3*B*n*(c + d*x)^3*Log[c + d*x] + 3
6*b^3*B*n*(c + d*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*b
^3*B*(c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*b^3*B*n*(
c + d*x)^3*Log[c + d*x]^2 + 36*b^3*B*n*(c + d*x)^3*Log[a + b*x]*Log[(b*(c +
 d*x))/(b*c - a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
/(b*c - a*d)^3)/(54*d*g^4*(c + d*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(417) = 834.

Time = 16.05 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.67

method	result	size
parallelrisch	Expression too large to display	1146

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/54*(54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^6*n-36*B^2*x^2*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^3*d^7*n^2-162*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4
*c*d^6*n^2-36*A*B*x^2*a*b^3*d^7*n^2+36*A*B*x^2*b^4*c*d^6*n^2+54*B^2*x*ln(e*
((b*x+a)/(d*x+c))^n)^2*b^4*c^2*d^5*n+36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b
```

```
^4*d^7*n+54*A*B*a^2*b^2*c*d^6*n^2-108*A*B*a*b^3*c^2*d^5*n^2+108*A*B*x^2*ln(
e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^
3*c*d^6*n^2+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n-108*A*B*x*a*b
^3*c*d^6*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n+108*A*B*ln(e
*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^2*d^7*n^2-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n^2+162*B^2*x
*a*b^3*c*d^6*n^3+18*A*B*x*a^2*b^2*d^7*n^2+90*A*B*x*b^4*c^2*d^5*n^2-54*B^2*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c*d^6*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)
^2*a*b^3*c^2*d^5*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n^2-108*B
^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n^2+36*A*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a^3*b*d^7*n-27*B^2*a^2*b^2*c*d^6*n^3+108*B^2*a*b^3*c^2*d^5*n^3-12*A*B*
a^3*b*d^7*n^2+66*A*B*b^4*c^3*d^4*n^2-54*A^2*a^2*b^2*c*d^6*n+54*A^2*a*b^3*c^
2*d^5*n+4*B^2*a^3*b*d^7*n^3-85*B^2*b^4*c^3*d^4*n^3+18*A^2*a^3*b*d^7*n-18*A^
2*b^4*c^3*d^4*n+18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^7*n-66*B^2*x^3
*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^7*n^2+66*B^2*x^2*a*b^3*d^7*n^3-66*B^2*x^2*
b^4*c*d^6*n^3-15*B^2*x*a^2*b^2*d^7*n^3-147*B^2*x*b^4*c^2*d^5*n^3+18*B^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*a^3*b*d^7*n-12*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
*d^7*n^2)/g^4/(d*x+c)^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/d^5/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(417) = 834$.

Time = 0.29 (sec) , antiderivative size = 1167, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = -\frac{18\,A^2b^3c^3-54\,A^2ab^2c^2d+54\,A^2a^2bcd^2-18\,A^2a^3d^3+(85\,B^2b^3c^3-108\,B^2ab^2c^2d+27\,B^2a^2bcd^2-4\,B}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (85*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2 - B^2*a^3*d^3)*log(e)^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*c*d^2
*n^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 +
 B^2*a^3*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 - 6*(11*A*B*b^3*c^3 - 18*A*B*
a*b^2*c^2*d + 9*A*B*a^2*b*c*d^2 - 2*A*B*a^3*d^3)*n + 3*((49*B^2*b^3*c^2*d -
 54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 - 6*(5*A*B*b^3*c^2*d - 6*A*B*a*b
^2*c*d^2 + A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18
*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2
- 3*(5*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x - (11*B^2*b^3
```

```
*c^3 - 18*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 - 2*B^2*a^3*d^3)*n - 6*(B^2*b
^3*d^3*n*x^3 + 3*B^2*b^3*c*d^2*n*x^2 + 3*B^2*b^3*c^2*d*n*x + (3*B^2*a*b^2*c
^2*d - 3*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n)*log((b*x + a)/(d*x + c)))*log(e)
 + 6*((11*B^2*b^3*d^3*n^2 - 6*A*B*b^3*d^3*n)*x^3 + (18*B^2*a*b^2*c^2*d - 9*
B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*A*B*b^3*c*d^2*n - (9*B^2*b^3*c*
d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*A*B*a*b^2*c^2*d - 3*A*B*a^2*b*c*d^2
+ A*B*a^3*d^3)*n - 3*(6*A*B*b^3*c^2*d*n - (6*B^2*b^3*c^2*d + 6*B^2*a*b^2*c*
d^2 - B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a*
b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c
^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^
4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2
 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$= \frac{\int \frac{A^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{g^4}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**4,x)
```

```
[Out] (Integral(A**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**4
 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**4 + 4*c**3*d*x + 6*c**2
*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(417) = 834.

Time = 0.28 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="ma
xima")
```

```
[Out] 1/9*A*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c
*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^
3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
```

```
 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*
b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g
^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^
3*d^4)*g^4)) + 1/54*(6*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d
^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*
x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2
- 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3
*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (85*b^
3*c^3 - 108*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 4*a^3*d^3 + 66*(b^3*c*d^2 - a*b^
2*d^3)*x^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*l
og(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3
)*log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 66*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a) - 6*(
11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*b^3*c^3 + 6*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a))*log(d*x +
c))*n^2/(b^3*c^6*d*g^4 - 3*a*b^2*c^5*d^2*g^4 + 3*a^2*b*c^4*d^3*g^4 - a^3*c^
3*d^4*g^4 + (b^3*c^3*d^4*g^4 - 3*a*b^2*c^2*d^5*g^4 + 3*a^2*b*c*d^6*g^4 - a^
3*d^7*g^4)*x^3 + 3*(b^3*c^4*d^3*g^4 - 3*a*b^2*c^3*d^4*g^4 + 3*a^2*b*c^2*d^5
*g^4 - a^3*c*d^6*g^4)*x^2 + 3*(b^3*c^5*d^2*g^4 - 3*a*b^2*c^4*d^3*g^4 + 3*a^
2*b*c^3*d^4*g^4 - a^3*c^2*d^5*g^4)*x))*B^2 - 1/3*B^2*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)^2/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d
*g^4) - 2/3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*x^3 + 3*c*d
^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A^2/(d^4*g^4*x^3 + 3*c*d^3*
g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

Giac [A] (verification not implemented)

none

Time = 1.31 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.81

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$= \frac{1}{54}\left(18\left(\frac{3\,(bx+a)B^2b^2n^2}{(b^2c^2g^4 - 2\,abcdg^4 + a^2d^2g^4)(dx+c)} - \frac{3\,(bx+a)^2B^2bdn^2}{(b^2c^2g^4 - 2\,abcdg^4 + a^2d^2g^4)(dx+c)^2} + \frac{(bx}{(b^2c^2g^4 - 2\,ab}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="gi
ac")
```

```
[Out] 1/54*(18*(3*(b*x + a)*B^2*b^2*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)) - 3*(b*x + a)^2*B^2*b*d*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 +
a^2*d^2*g^4)*(d*x + c)^2) + (b*x + a)^3*B^2*d^2*n^2/((b^2*c^2*g^4 - 2*a*b*c
```

```
*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c))^2 - 6*(2*(B^2*
d^2*n^2 - 3*B^2*d^2*n*log(e) - 3*A*B*d^2*n)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a
*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 9*(B^2*b*d*n^2 - 2*B^2*b*d*n*log(e
) - 2*A*B*b*d*n)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(
d*x + c)^2) + 18*(B^2*b^2*n^2 - B^2*b^2*n*log(e) - A*B*b^2*n)*(b*x + a)/((b
^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*log((b*x + a)/(d*x +
c)) + 2*(2*B^2*d^2*n^2 - 6*B^2*d^2*n*log(e) + 9*B^2*d^2*log(e)^2 - 6*A*B*d^
2*n + 18*A*B*d^2*log(e) + 9*A^2*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*
g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 27*(B^2*b*d*n^2 - 2*B^2*b*d*n*log(e) + 2*
B^2*b*d*log(e)^2 - 2*A*B*b*d*n + 4*A*B*b*d*log(e) + 2*A^2*b*d)*(b*x + a)^2/
((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) + 54*(2*B^2*b^2*n
^2 - 2*B^2*b^2*n*log(e) + B^2*b^2*log(e)^2 - 2*A*B*b^2*n + 2*A*B*b^2*log(e)
 + A^2*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c
)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{3\,d\,\left(c^3\,g^4+3\,c^2\,d\,g^4\,x+3\,c\,d^2\,g^4\,x^2+d^3\,g^4\,x^3\right)}+\frac{B^2\,b^3}{3\,d\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$
$$-\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2-12\,A\,B\,a^2\,d^2\,n+42\,A\,B\,a\,b\,c\,d\,n-66\,A\,B\,b^2\,c^2\,n+4\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+85\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}-\frac{x\,(-4\cdots}{\cdots}}{x\,\left(27\,a\,c^2\,d^3\,g^4-27\,b\,c^3\,d^2\,g^4\right)-x^2\,\left(27\,b\,c^2\,d^3\,g^4-27\,a\,c\,d^4\,g^4\right)+x^3\,\left(9\,a\,d^5\cdots\right.}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,A\,B}{3\,c^3\,d\,g^4+9\,c^2\,d^2\,g^4\,x+9\,c\,d^3\,g^4\,x^2+3\,d^4\,g^4\,x^3}\right.$$
$$+\frac{2\,B^2\,b^3\,\left(x\,\left(d\,\left(\frac{d\,g^4\,n\,(a\,d-b\,c)\,(a\,d-3\,b\,c)}{2\,b^2}-\frac{c\,d\,g^4\,n\,(a\,d-b\,c)}{b}\right)-\frac{2\,c\,d^2\,g^4\,n\,(a\,d-b\,c)}{b}+\frac{d^2\,g^4\,n\,(a\,d-b\,c)\,(a\,d-3\,b\,c)}{b^2}\right)+c\,\left(\frac{d\,g^4\,n\,(\cdots}{\cdots}\right.\right.}{3\,d\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)\,\left(3\,c^3\,d\,g^4+9\cdots\right.}$$
$$-\frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{B\,b^3\,n\,(6\,A-11\,B\,n)\,\left(\frac{a^3\,d^4\,g^4-a^2\,b\,c\,d^3\,g^4-a\,b^2\,c^2\,d^2\,g^4+b^3\,c^3\,d\,g^4}{a^2\,d^3\,g^4-2\,a\,b\,c\,d^2\,g^4+b^2\,c^2\,d\,g^4}+2\,b\,d\,x\right)\,\left(a^2\,d^3\,g^4-2\,a\,b\,c\,d^2\,g^4+b^2\,c^2\,d\,g^4\right)\,1{}\mathrm{i}}{d\,g^4\,\left(11\,B^2\,b^3\,n^2-6\,A\,B\,b^3\,n\right)\,{(a\,d-b\,c)}^3}\right)\,(6\,A-1\cdots}{9\,d\,g^4\,{(a\,d-b\,c)}^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^4,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*d*(c^3*g^4 + d^3*g^4*x^3 + 3*c*d
^2*g^4*x^2 + 3*c^2*d*g^4*x)) + (B^2*b^3)/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2))) - ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 4*B^2*a^
```

2*d^2*n^2 + 85*B^2*b^2*c^2*n^2 - 36*A^2*a*b*c*d - 12*A*B*a^2*d^2*n - 66*A*B
*b^2*c^2*n - 23*B^2*a*b*c*d*n^2 + 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) - (x*(5
*B^2*a*b*d^2*n^2 - 49*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n + 30*A*B*b^2*c*d*n)
)/(2*(a*d - b*c)) + (b*x^2*(11*B^2*b*d^2*n^2 - 6*A*B*b*d^2*n))/(a*d - b*c))
/(x*(27*a*c^2*d^3*g^4 - 27*b*c^3*d^2*g^4) - x^2*(27*b*c^2*d^3*g^4 - 27*a*c*
d^4*g^4) + x^3*(9*a*d^5*g^4 - 9*b*c*d^4*g^4) + 9*a*c^3*d^2*g^4 - 9*b*c^4*d*
g^4) - log(e*((a + b*x)/(c + d*x))^n)*((2*A*B)/(3*c^3*d*g^4 + 3*d^4*g^4*x^3
 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2) + (2*B^2*b^3*(x*(d*((d*g^4*n*(a*d - b
c)(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (2*c*d^2*g^4*n*(a
*d - b*c))/b + (d^2*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/b^2) + c*((d*g^4*n*(a*
d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (d*g^4*n*(a*
d - b*c)*(a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/b^3 - (3*d^3*g^4*n*x^2*(a*d - b
*c))/b))/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*c^
3*d*g^4 + 3*d^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2))) - (B*b^3*n*a
tan((B*b^3*n*(6*A - 11*B*n)*((a^3*d^4*g^4 + b^3*c^3*d*g^4 - a^2*b*c*d^3*g^4
 - a*b^2*c^2*d^2*g^4)/(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4) + 2*b
*d*x)*(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4)*1i)/(d*g^4*(11*B^2*b^
3*n^2 - 6*A*B*b^3*n)*(a*d - b*c)^3))*(6*A - 11*B*n)*2i)/(9*d*g^4*(a*d - b*c
)^3)

3.46 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 536

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx =& -\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3} \\
&-\frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2}+\frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)} \\
&+\frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4} \\
&-\frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3} \\
&+\frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2} \\
&-\frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)} \\
&-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} \\
&+\frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5} \\
&-\frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4d(bc-ad)^4g^5}
\end{aligned}
$$

```
[Out] -1/32*B^2*d^3*n^2*(b*x+a)^4/(-a*d+b*c)^4/g^5/(d*x+c)^4+2/9*b*B^2*d^2*n^2*(b
*x+a)^3/(-a*d+b*c)^4/g^5/(d*x+c)^3-3/4*b^2*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^4
/g^5/(d*x+c)^2+2*b^3*B^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^5/(d*x+c)+1/8*B*d^3*n*(
b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^4-2/3*b*B
```

```
*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^3
+3/2*b^2*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(
d*x+c)^2-2*b^3*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5
/(d*x+c)-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^5/(d*x+c)^4+1/2*b^4*B*n*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^4/g^5-1/4*
b^4*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^4/g^5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2551, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}dx=\frac{b^4Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2dg^5(bc-ad)^4}-\frac{2b^3Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^5(c+dx)(bc-ad)^4}+\frac{3b^2Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(c+dx)^2(bc-ad)^4}+\frac{Bd^3n(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(c+dx)^4(bc-ad)^4}-\frac{2bBd^2n(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(c+dx)^3(bc-ad)^4}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4dg^5(c+dx)^4}-\frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4dg^5(bc-ad)^4}+\frac{2b^3B^2n^2(a+bx)}{g^5(c+dx)(bc-ad)^4}-\frac{3b^2B^2dn^2(a+bx)^2}{4g^5(c+dx)^2(bc-ad)^4}-\frac{B^2d^3n^2(a+bx)^4}{32g^5(c+dx)^4(bc-ad)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9g^5(c+dx)^3(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] -1/32*(B^2*d^3*n^2*(a + b*x)^4)/((b*c - a*d)^4*g^5*(c + d*x)^4) + (2*b*B^2*
d^2*n^2*(a + b*x)^3)/(9*(b*c - a*d)^4*g^5*(c + d*x)^3) - (3*b^2*B^2*d*n^2*(
a + b*x)^2)/(4*(b*c - a*d)^4*g^5*(c + d*x)^2) + (2*b^3*B^2*n^2*(a + b*x))/(
(b*c - a*d)^4*g^5*(c + d*x)) + (B*d^3*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(8*(b*c - a*d)^4*g^5*(c + d*x)^4) - (2*b*B*d^2*n*(a + b*x)
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^4*g^5*(c + d*x)^3
) + (3*b^2*B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*
c - a*d)^4*g^5*(c + d*x)^2) - (2*b^3*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/((b*c - a*d)^4*g^5*(c + d*x)) - (A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2/(4*d*g^5*(c + d*x)^4) + (b^4*B*n*(A + B*Log[e*((a + b*x)/(c +
```

```
 d*x))^n])*Log[(a + b*x)/(c + d*x)])/(2*d*(b*c - a*d)^4*g^5) - (b^4*B^2*n^2
*Log[(a + b*x)/(c + d*x)]^2)/(4*d*(b*c - a*d)^4*g^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (b-dx)^3 \left(A+B\log\left(ex^n\right)\right)^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4 g^5}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(Bn)\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5}$$

$$= \frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4} - \frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3} + \frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2} - \frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5} - \frac{(B^2n^2)\,\text{Subst}\left(\int\left(-4b^3d+3b^2d^2x-\frac{4}{3}bd^3x^2+\frac{d^4x^3}{4}+\frac{b^4\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5}$$

$$= -\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4} + \frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3} - \frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2} + \frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)} + \frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4} - \frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3} + \frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2} - \frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5} - \frac{(b^4B^2n^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5}$$

$$= -\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4} + \frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3} - \frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2} + \frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)} + \frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4} - \frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3} + \frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2} - \frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5} - \frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4d(bc-ad)^4g^5}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.31

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$$
$$= \frac{-72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(36A(bc-ad)^4-9B(bc-ad)^4n+48Ab(bc-ad)^3(c+dx)-28bB(bc-ad)^3n(c+dx)+72Ab^2(bc-ad)^2(c+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*d)^4 -
9*B*(b*c - a*d)^4*n + 48*A*b*(b*c - a*d)^3*(c + d*x) - 28*b*B*(b*c - a*d)^3
*n*(c + d*x) + 72*A*b^2*(b*c - a*d)^2*(c + d*x)^2 - 78*b^2*B*(b*c - a*d)^2*
n*(c + d*x)^2 + 144*A*b^3*(b*c - a*d)*(c + d*x)^3 - 300*b^3*B*(b*c - a*d)*n
*(c + d*x)^3 + 144*A*b^4*(c + d*x)^4*Log[a + b*x] - 300*b^4*B*n*(c + d*x)^4
*Log[a + b*x] - 72*b^4*B*n*(c + d*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a*d)^4*
Log[e*((a + b*x)/(c + d*x))^n] + 48*b*B*(b*c - a*d)^3*(c + d*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + 72*b^2*B*(b*c - a*d)^2*(c + d*x)^2*Log[e*((a + b*x)/(
c + d*x))^n] + 144*b^3*B*(b*c - a*d)*(c + d*x)^3*Log[e*((a + b*x)/(c + d*x)
)^n] + 144*b^4*B*(c + d*x)^4*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] -
144*A*b^4*(c + d*x)^4*Log[c + d*x] + 300*b^4*B*n*(c + d*x)^4*Log[c + d*x] +
 144*b^4*B*n*(c + d*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 1
44*b^4*B*(c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 72*b^4*B
*n*(c + d*x)^4*Log[c + d*x]^2 + 144*b^4*B*n*(c + d*x)^4*Log[a + b*x]*Log[(b
*(c + d*x))/(b*c - a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/(b*c - a*d)^4)/(288*d*g^5*(c + d*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(518) = 1036.

Time = 42.18 (sec) , antiderivative size = 2326, normalized size of antiderivative = 4.34

method	result	size
parallelrisch	Expression too large to display	2326

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/288*(-1056*A*B*x^3*a*b^4*c^7*d^2*n^2+432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a*b^4*c^8*d*n+72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^6*d^3*n^2-
```

```
576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^7*d^2*n^2+2160*A*B*x^3*a^2*
b^3*c^6*d^3*n^2+9*B^2*x^4*a^5*c^2*d^7*n^3+36*B^2*x^3*a^5*c^3*d^6*n^3+72*A^2
*x^4*a^5*c^2*d^7*n+54*B^2*x^2*a^5*c^4*d^5*n^3+288*A^2*x^3*a^5*c^3*d^6*n+36*
B^2*x*a^5*c^5*d^4*n^3+576*B^2*x*a*b^4*c^9*n^3+432*A^2*x^2*a^5*c^4*d^5*n-72*
B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*c^6*d^3*n+288*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^2*b^3*c^9*n+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^6*d^3*n^2-576*B
^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n^2+288*A^2*x*a^5*c^5*d^4*n+288*A^
2*x*a*b^4*c^9*n-64*B^2*x^4*a^4*b*c^3*d^6*n^3+216*B^2*x^4*a^3*b^2*c^4*d^5*n^
3-576*B^2*x^4*a^2*b^3*c^5*d^4*n^3+415*B^2*x^4*a*b^4*c^6*d^3*n^3-36*A*B*x^4*
a^5*c^2*d^7*n^2-256*B^2*x^3*a^4*b*c^4*d^5*n^3+864*B^2*x^3*a^3*b^2*c^5*d^4*n
^3-2004*B^2*x^3*a^2*b^3*c^6*d^3*n^3+1360*B^2*x^3*a*b^4*c^7*d^2*n^3-1152*A^2
*x*a^2*b^3*c^8*d*n-144*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^6*d^3*n+576*A*B*
ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n+144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^4*c^6*d^3*n+576*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^7*d^2*n+86
4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^8*d*n-288*A^2*x^4*a^4*b*c^3*d^6
*n+432*A^2*x^4*a^3*b^2*c^4*d^5*n-288*A^2*x^4*a^2*b^3*c^5*d^4*n+72*A^2*x^4*a
*b^4*c^6*d^3*n-144*A*B*x^3*a^5*c^3*d^6*n^2-384*B^2*x^2*a^4*b*c^5*d^4*n^3+12
18*B^2*x^2*a^3*b^2*c^6*d^3*n^3-2400*B^2*x^2*a^2*b^3*c^7*d^2*n^3+1512*B^2*x^
2*a*b^4*c^8*d*n^3-1152*A^2*x^3*a^4*b*c^4*d^5*n+1728*A^2*x^3*a^3*b^2*c^5*d^4
*n-1152*A^2*x^3*a^2*b^3*c^6*d^3*n+288*A^2*x^3*a*b^4*c^7*d^2*n-216*A*B*x^2*a
^5*c^4*d^5*n^2+288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^9*n-576*B^2*x*
ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^9*n^2-228*B^2*x*a^4*b*c^6*d^3*n^3+624*B^2
*x*a^3*b^2*c^7*d^2*n^3-1008*B^2*x*a^2*b^3*c^8*d*n^3-1728*A^2*x^2*a^4*b*c^5*
d^4*n+2592*A^2*x^2*a^3*b^2*c^6*d^3*n-1728*A^2*x^2*a^2*b^3*c^7*d^2*n+432*A^2
*x^2*a*b^4*c^8*d*n-144*A*B*x*a^5*c^5*d^4*n^2-576*A*B*x*a*b^4*c^9*n^2+288*B^
2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^7*d^2*n-432*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^3*b^2*c^8*d*n-192*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^7*d^2*n^2+
432*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^8*d*n^2-1152*A^2*x*a^4*b*c^6*d^
3*n+1728*A^2*x*a^3*b^2*c^7*d^2*n-1296*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^4*c^8*d*n^2+1152*A*B*x^2*a^4*b*c^5*d^4*n^2-2520*A*B*x^2*a^3*b^2*c^6*d^3*n^
2+2880*A*B*x^2*a^2*b^3*c^7*d^2*n^2-1296*A*B*x^2*a*b^4*c^8*d*n^2-48*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^6*d^3*n^2+288*B^2*x*ln(e*((b*x+a)/(d*x+c))^
n)*a^3*b^2*c^7*d^2*n^2-864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^8*d*n^
2+576*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^9*n+720*A*B*x*a^4*b*c^6*d^3*n
^2-1440*A*B*x*a^3*b^2*c^7*d^2*n^2+1440*A*B*x*a^2*b^3*c^8*d*n^2+576*A*B*ln(e
*((b*x+a)/(d*x+c))^n)*a^4*b*c^7*d^2*n-864*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3
*b^2*c^8*d*n+72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^6*d^3*n-300*B^2
*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^6*d^3*n^2+192*A*B*x^4*a^4*b*c^3*d^6*
n^2-432*A*B*x^4*a^3*b^2*c^4*d^5*n^2+576*A*B*x^4*a^2*b^3*c^5*d^4*n^2-300*A*B
*x^4*a*b^4*c^6*d^3*n^2+288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^7*d^
2*n-144*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^6*d^3*n^2-1056*B^2*x^3*
ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^7*d^2*n^2+768*A*B*x^3*a^4*b*c^4*d^5*n^2-1
728*A*B*x^3*a^3*b^2*c^5*d^4*n^2)/g^5/(d*x+c)^4/(a*d-b*c)^2/(a^2*d^2-2*a*b*c
*d+b^2*c^2)/n/c^6/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(518) = 1036$.

Time = 0.32 (sec) , antiderivative size = 1768, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 - 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (415*B^2*b^4*c^4 - 576*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*b*c*d^3 + 9*B^2*a^4*d
^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 13*B^2*a^2*b^2*d^
4)*n^2 - 12*(7*A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^
2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3
*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*b^4*c*d^
3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x + (4*B^2*a*b^
3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2)*log
((b*x + a)/(d*x + c))^2 - 12*(25*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 36*A*B*
a^2*b^2*c^2*d^2 - 16*A*B*a^3*b*c*d^3 + 3*A*B*a^4*d^4)*n + 4*((271*B^2*b^4*c
^3*d - 324*B^2*a*b^3*c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^3*b*d^4)*n^2
- 12*(13*A*B*b^4*c^3*d - 18*A*B*a*b^3*c^2*d^2 + 6*A*B*a^2*b^2*c*d^3 - A*B*a
^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c
^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n*x^3 - 6*(7*B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*
n*x^2 - 4*(13*B^2*b^4*c^3*d - 18*B^2*a*b^3*c^2*d^2 + 6*B^2*a^2*b^2*c*d^3 -
B^2*a^3*b*d^4)*n*x - (25*B^2*b^4*c^4 - 48*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*
c^2*d^2 - 16*B^2*a^3*b*c*d^3 + 3*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4
*B^2*b^4*c*d^3*n*x^3 + 6*B^2*b^4*c^2*d^2*n*x^2 + 4*B^2*b^4*c^3*d*n*x + (4*B
^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n
)*log((b*x + a)/(d*x + c)))*log(e) + 12*((25*B^2*b^4*d^4*n^2 - 12*A*B*b^4*d
^4*n)*x^4 - 4*(12*A*B*b^4*c*d^3*n - (22*B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*n^
2)*x^3 + (48*B^2*a*b^3*c^3*d - 36*B^2*a^2*b^2*c^2*d^2 + 16*B^2*a^3*b*c*d^3
- 3*B^2*a^4*d^4)*n^2 - 6*(12*A*B*b^4*c^2*d^2*n - (18*B^2*b^4*c^2*d^2 + 8*B^
2*a*b^3*c*d^3 - B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(4*A*B*a*b^3*c^3*d - 6*A*B*a
^2*b^2*c^2*d^2 + 4*A*B*a^3*b*c*d^3 - A*B*a^4*d^4)*n - 4*(12*A*B*b^4*c^3*d*n
 - (12*B^2*b^4*c^3*d + 18*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 + B^2*a^3
*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 +
 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*c^5*d^4 - 4*
a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*g^5*x^3 +
6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^
4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 -
```

```
 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^
2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2138 vs. $2(518) = 1036$.

Time = 0.33 (sec) , antiderivative size = 2138, normalized size of antiderivative = 3.99

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="ma
xima")
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2
- 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*
d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d
^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*
d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3
*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)
*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d
^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 - 3*a^3
*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*g^
5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*g^5
*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*g^
5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*g
^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g^5) +
 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*
a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))*log(e*(b*x/(d*x +
```

```
c) + a/(d*x + c))^n) - (415*b^4*c^4 - 576*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^
2 - 64*a^3*b*c*d^3 + 9*a^4*d^4 + 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(163*b
^4*c^2*d^2 - 176*a*b^3*c*d^3 + 13*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4)*log(d*x + c)^2 + 4*(271*b^4*c^3*d - 324*a*b^3*c^2*d^2 + 60*a^2*b^2*c
*d^3 - 7*a^3*b*d^4)*x + 300*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*
x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a) - 12*(25*b^4*d^4*x^4 + 100*b^4*
c*d^3*x^3 + 150*b^4*c^2*d^2*x^2 + 100*b^4*c^3*d*x + 25*b^4*c^4 + 12*(b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(
b*x + a))*log(d*x + c))*n^2/(b^4*c^8*d*g^5 - 4*a*b^3*c^7*d^2*g^5 + 6*a^2*b^
2*c^6*d^3*g^5 - 4*a^3*b*c^5*d^4*g^5 + a^4*c^4*d^5*g^5 + (b^4*c^4*d^5*g^5 -
4*a*b^3*c^3*d^6*g^5 + 6*a^2*b^2*c^2*d^7*g^5 - 4*a^3*b*c*d^8*g^5 + a^4*d^9*g
^5)*x^4 + 4*(b^4*c^5*d^4*g^5 - 4*a*b^3*c^4*d^5*g^5 + 6*a^2*b^2*c^3*d^6*g^5
- 4*a^3*b*c^2*d^7*g^5 + a^4*c*d^8*g^5)*x^3 + 6*(b^4*c^6*d^3*g^5 - 4*a*b^3*c
^5*d^4*g^5 + 6*a^2*b^2*c^4*d^5*g^5 - 4*a^3*b*c^3*d^6*g^5 + a^4*c^2*d^7*g^5)
*x^2 + 4*(b^4*c^7*d^2*g^5 - 4*a*b^3*c^6*d^3*g^5 + 6*a^2*b^2*c^5*d^4*g^5 - 4
*a^3*b*c^4*d^5*g^5 + a^4*c^3*d^6*g^5)*x))*B^2 - 1/4*B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 +
4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c
^4*d*g^5) - 1/4*A^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*
c^3*d^2*g^5*x + c^4*d*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(518) = 1036$.

Time = 1.69 (sec) , antiderivative size = 1265, normalized size of antiderivative = 2.36

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/288*(72*(4*(b*x + a)*B^2*b^3*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^
2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B^2*b^2*d*n^2/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2
) + 4*(b*x + a)^3*B^2*b*d^2*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b
*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B^2*d^3*n^2/((b^3*c^3*
g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*lo
g((b*x + a)/(d*x + c))^2 + 12*(3*(B^2*d^3*n^2 - 4*B^2*d^3*n*log(e) - 4*A*B*
d^3*n)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 -
a^3*d^3*g^5)*(d*x + c)^4) - 16*(B^2*b*d^2*n^2 - 3*B^2*b*d^2*n*log(e) - 3*A*
```

```
B*b*d^2*n)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^
5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B^2*b^2*d*n^2 - 2*B^2*b^2*d*n*log(e) -
2*A*B*b^2*d*n)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^
2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B^2*b^3*n^2 - B^2*b^3*n*log(e) - A*
B*b^3*n)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 -
a^3*d^3*g^5)*(d*x + c)))*log((b*x + a)/(d*x + c)) - 9*(B^2*d^3*n^2 - 4*B^2*
d^3*n*log(e) + 8*B^2*d^3*log(e)^2 - 4*A*B*d^3*n + 16*A*B*d^3*log(e) + 8*A^2
*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a
^3*d^3*g^5)*(d*x + c)^4) + 32*(2*B^2*b*d^2*n^2 - 6*B^2*b*d^2*n*log(e) + 9*B
^2*b*d^2*log(e)^2 - 6*A*B*b*d^2*n + 18*A*B*b*d^2*log(e) + 9*A^2*b*d^2)*(b*x
 + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5
)*(d*x + c)^3) - 216*(B^2*b^2*d*n^2 - 2*B^2*b^2*d*n*log(e) + 2*B^2*b^2*d*lo
g(e)^2 - 2*A*B*b^2*d*n + 4*A*B*b^2*d*log(e) + 2*A^2*b^2*d)*(b*x + a)^2/((b^
3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^
2) + 288*(2*B^2*b^3*n^2 - 2*B^2*b^3*n*log(e) + B^2*b^3*log(e)^2 - 2*A*B*b^3
*n + 2*A*B*b^3*log(e) + A^2*b^3)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^
5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(
b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 1765, normalized size of antiderivative = 3.29

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atan((B*b^4*n*(12*A - 25*B*n)*(24*a^4*d^5*g^5 - 24*b^4*c^4*d*g^5 -
 48*a^3*b*c*d^4*g^5 + 48*a*b^3*c^3*d^2*g^5)*1i)/(24*d*g^5*(25*B^2*b^4*n^2 -
 12*A*B*b^4*n)*(a*d - b*c)^4) + (B*b^5*n*x*(12*A - 25*B*n)*(a^3*d^4*g^5 - b
^3*c^3*d*g^5 - 3*a^2*b*c*d^3*g^5 + 3*a*b^2*c^2*d^2*g^5)*2i)/(g^5*(25*B^2*b^
4*n^2 - 12*A*B*b^4*n)*(a*d - b*c)^4))*(12*A - 25*B*n)*1i)/(12*d*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 9*B^2*a^3*d^3*n^2 - 415*B^2*b
^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 - 36*A*B*a^3*d^3*n +
 300*A*B*b^3*c^3*n + 161*B^2*a*b^2*c^2*d*n^2 - 55*B^2*a^2*b*c*d^2*n^2 - 276
*A*B*a*b^2*c^2*d*n + 156*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(13*B^2
*a*b^2*d^3*n^2 - 163*B^2*b^3*c*d^2*n^2 - 12*A*B*a*b^2*d^3*n + 84*A*B*b^3*c*
d^2*n))/(2*(a*d - b*c)) - (x*(7*B^2*a^2*b*d^3*n^2 + 271*B^2*b^3*c^2*d*n^2 -
 53*B^2*a*b^2*c*d^2*n^2 - 12*A*B*a^2*b*d^3*n - 156*A*B*b^3*c^2*d*n + 60*A*B
*a*b^2*c*d^2*n))/(3*(a*d - b*c)) - (b*x^3*(25*B^2*b^2*d^3*n^2 - 12*A*B*b^2*
d^3*n))/(a*d - b*c))/(x*(96*a^2*c^3*d^4*g^5 + 96*b^2*c^5*d^2*g^5 - 192*a*b*
c^4*d^3*g^5) + x^3*(96*a^2*c*d^6*g^5 + 96*b^2*c^3*d^4*g^5 - 192*a*b*c^2*d^5
*g^5) + x^4*(24*a^2*d^7*g^5 + 24*b^2*c^2*d^5*g^5 - 48*a*b*c*d^6*g^5) + x^2*
(144*a^2*c^2*d^5*g^5 + 144*b^2*c^4*d^3*g^5 - 288*a*b*c^3*d^4*g^5) + 24*b^2*
```

```
c^6*d*g^5 + 24*a^2*c^4*d^3*g^5 - 48*a*b*c^5*d^2*g^5) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^3 + 6*c^2*d^2
*g^5*x^2 + 4*c^3*d*g^5*x)) - (B^2*b^4)/(4*d*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x)
)^n)*((A*B)/(2*c^4*d*g^5 + 2*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^
3 + 12*c^2*d^3*g^5*x^2) - (B^2*b^4*(x*(d*(c*((d*g^5*n*(a*d - b*c)*(a*d - 4*
b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d - b*c)*(a^2*
d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + c*(d*((d*g^5*n*(a*d - b*c)*(a*d -
4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^5*n*(a*d - b*c)
)/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (d^2*g^5*n*(a*d - b*
c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(2*b^3)) + c*(c*((d*g^5*n*(a*d - b*c)
*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d -
b*c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + x^2*(d*(d*((d*g^5*n*(a*d
 - b*c)*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^
5*n*(a*d - b*c))/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (3*c*
d^3*g^5*n*(a*d - b*c))/(2*b) + (d^3*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(2*b^2
)) - (2*d^4*g^5*n*x^3*(a*d - b*c))/b + (d*g^5*n*(a*d - b*c)*(a^3*d^3 - 4*b^
3*c^3 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2))/(2*b^4)))/(2*d*g^5*(2*c^4*d*g^5 + 2
*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^3 + 12*c^2*d^3*g^5*x^2)*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
```

3.47 $\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Int}\left(\frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Defer[Int][(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx+cg)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B*log(e*((b*x + a)/(d*x + c
))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 24.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g^2\left(\int \frac{c^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{d^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{2cdx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(c**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(d**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*c*d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 27.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\left(c\,g+d\,g\,x\right)^2}{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.48 $\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Defer[Int][(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx+cg}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
[Out] integral((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g\left(\int \frac{c}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{dx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] g*(Integral(c/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
[Out] integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 15.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{c\,g+d\,g\,x}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.49 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

```
[In] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
[Out] integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(e*((b*x + a)/(d*x + c))
^n)), x)
```

Sympy [N/A]

Not integrable

Time = 18.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Ac+Adx+Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c + A*d*x + B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 9.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.50 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 96

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/exp(A/B/n)/g^2
/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2551, 2337, 2209}

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(c+dx)(bc-ad)}$$

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B*
(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\ &= \frac{\left((a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2n(c+dx)}\\ &= \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B*
(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Maple [F]

$$\int \frac{1}{\left(dgx+cg\right)^{2}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

```
[In] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(-\frac{B\log(e)+A}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)}{(Bbc-Bad)g^2n}$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] e^(-(B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))/(
d*x + c))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F]

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Ac^2+2Acdx+Ad^2x^2+Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Bcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c**2 + 2*A*c*d*x + A*d**2*x**2 + B*c**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n) + 2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(c\,g+d\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.51 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)} - \frac{de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)^2}$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(A/B/n)
/g^3/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-d*(b*x+a)^2*Ei(2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n/((e*((b*x+a)/(d
*x+c))^n)^(2/n))/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{2551, 2367, 2337, 2209, 2347\}$

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(c+dx)(bc-ad)^2}$$
$$- \frac{d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B*(b*c - a*d)^2*E^(A/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)) - (d*(a + b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(B*n)])/(B*(b*c - a*d)^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))
^n)^(2/n)*(c + d*x)^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b-dx}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{A+B\log(ex^n)} - \frac{dx}{A+B\log(ex^n)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{b\text{Subst}\left(\int \frac{1}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{x}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= -\frac{\left(d(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3n(c+dx)^2} \\
&\quad + \frac{\left(b(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3n(c+dx)} \\
&= \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)} \\
&\quad - \frac{de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$
\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx
$$
$$
= \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right) - d(a+bx)\right.}{B(bc-ad)^2g^3n(c+dx)^2}
$$

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*(b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpI
ntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)] - d*(a + b*x)*ExpIn
tegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]))/(B*(b*c - a*d)
^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Maple [F]

$$\int \frac{1}{(dgx+cg)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

```
[In] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\left(be^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)-d\operatorname{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\left(\frac{2\,(B\log(e)+A)}{Bn}\right)}}{d^2x^2+2\,cdx+c^2}\right)\right)e^{\left(-\frac{2\,(B\log}{B}\right.}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] (b*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))
/(d*x + c)) - d*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(e) + A)/
(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((B*b^2*c^2
- 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
[Out] integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
[Out] integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.52 $\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Defer[Int][(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx+cg)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B^2*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 45.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g^2\left(\int \frac{c^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{d^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{2cdx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(c**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*c*d*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b*d^3*g^2*x^4 + a*c^3*g^2 + (3*b*c*d^2*g^2 + a*d^3*g^2)*x^3 + 3*(b*c^2*d*
g^2 + a*c*d^2*g^2)*x^2 + (b*c^3*g^2 + 3*a*c^2*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b*d^3*g^2*x^3 + b*
c^3*g^2 + 3*a*c^2*d*g^2 + 3*(3*b*c*d^2*g^2 + a*d^3*g^2)*x^2 + 6*(b*c^2*d*g^
2 + a*c*d^2*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)
*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))
*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(c\,g+d\,g\,x)^2}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.53 $\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx+cg}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
[Out] integral((d*g*x + c*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*
((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 58.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g\left(\int \frac{c}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{dx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(c/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d^2*g*x^3 + a*c^2*g + (2*b*c*d*g + a*d^2*g)*x^2 + (b*c^2*g + 2*a*c*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b*d^2*g*x^2 + b*c^2*g + 2*a*c*d*g + 2*(2*b*c*d*g + a*d^2*g)*x)/((b*c*n
 - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*
c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{dgx + cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
[Out] integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{c\,g + d\,g\,x}{\left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.54 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
[Out] integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [N/A]

Not integrable

Time = 139.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{\int \frac{1}{A^2c+A^2dx+2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}}{g}$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral(1/(A**2*c + A**2*d*x + 2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n) + 2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*c*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) -
 (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*
g*n*log(e) - a*d*g*n*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.55 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/exp(A/B/n)/g
^2/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)+(-b*x-a)/B/(-a*d+b*c)/g^2/n/
(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2551, 2334, 2337, 2209}

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(c+dx)(bc-ad)} - \frac{a+bx}{Bg^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B^
2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)) - (a + b*x)/(B*(b*c - a*d)*g^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}$$

$$= -\frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)g^2n}$$

$$= -\frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)g^2n^2(c+dx)}$$

$$= \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\mathrm{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\left(Be^{\frac{A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} - \mathrm{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\right.\right.\right.\right.}{B^2(bc-ad)g^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((a + b*x)*(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1) - ExpInteg
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d
*x))^n)^n^(-1)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))
```

Maple [F]

$$\int \frac{1}{(dgx+cg)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{\left((Bbnx+Ban)e^{\left(\frac{B\log(e)+A}{Bn}\right)} - \left(Adx+Ac+(Bdx+Bc)\log(e)+(Bdnx+Bcn)\log\left(\right.\right.\right.}{(AB^2bcd-AB^2ad^2)g^2n^2x+(AB^2bc^2-AB^2acd)g^2n^2+((B^3bcd-B^3ad^2)g^2n^2x+(B^3bc^2-B^3acd)g^2n}$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*n*x + B*a*n)*e^((B*log(e) + A)/(B*n)) - (A*d*x + A*c + (B*d*x + B*c)
*log(e) + (B*d*n*x + B*c*n)*log((b*x + a)/(d*x + c)))*log_integral((b*x + a
)*e^((B*log(e) + A)/(B*n))/(d*x + c)))*e^(-(B*log(e) + A)/(B*n))/((A*B^2*b*
c*d - A*B^2*a*d^2)*g^2*n^2*x + (A*B^2*b*c^2 - A*B^2*a*c*d)*g^2*n^2 + ((B^3*
b*c*d - B^3*a*d^2)*g^2*n^2*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^2)*log(e) + ((
B^3*b*c*d - B^3*a*d^2)*g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^3)*log((b*
x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(b*x + a)/((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n*log(e) - a*c*d*g
^2*n*log(e))*B^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n*log(e) -
 a*d^2*g^2*n*log(e))*B^2)*x + ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g
^2*n - a*c*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n)*B^
2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*log((d*x + c)^n)) - integrate(-1/(B^
2*c^2*g^2*n*log(e) + A*B*c^2*g^2*n + (B^2*d^2*g^2*n*log(e) + A*B*d^2*g^2*n)
*x^2 + 2*(B^2*c*d*g^2*n*log(e) + A*B*c*d*g^2*n)*x + (B^2*d^2*g^2*n*x^2 + 2*
B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((b*x + a)^n) - (B^2*d^2*g^2*n*x^2 + 2*
B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((d*x + c)^n)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\left(\frac{bx+a}{\left(B^2g^2n^2\log\left(\frac{bx+a}{dx+c}\right)+B^2g^2n\log(e)+ABg^2n\right)(dx+c)}-\frac{\mathrm{Ei}\left(\frac{\log(e)}{n}+\frac{A}{Bn}+\right.}{B^2e}\right.$$

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] -(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*((b*x + a)/((B^2*g^2*n^2*log((b*x
+ a)/(d*x + c)) + B^2*g^2*n*log(e) + A*B*g^2*n)*(d*x + c)) - Ei(log(e)/n +
A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/(B^2*e^(1/n)*g^2*n^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{\left(c\,g+d\,g\,x\right)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.56 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} - \frac{2de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)^2} - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(A/B/
n)/g^3/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-2*d*(b*x+a)^2*Ei(2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n^2/((e*(
(b*x+a)/(d*x+c))^n)^(2/n))/(d*x+c)^2+(-b*x-a)/B/(-a*d+b*c)/g^3/n/(d*x+c)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2551, 2357, 2367, 2337, 2209, 2347}

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= -\frac{2d(a+bx)^2 e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(c+dx)^2(bc-ad)^2}$$
$$+\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(c+dx)(bc-ad)^2}$$
$$-\frac{a+bx}{Bg^3n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] (b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B^2*(b*c - a*d)^2*E^(A/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c
 + d*x)) - (2*d*(a + b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)])/(B^2*(b*c - a*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/
(c + d*x))^n)^(2/n)*(c + d*x)^2) - (a + b*x)/(B*(b*c - a*d)*g^3*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b-dx}{(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= -\frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{b-dx}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}-\frac{b\text{Subst}\left(\int \frac{1}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}\\
&= -\frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\\
&\quad+\frac{2\text{Subst}\left(\int \left(\frac{b}{A+B\log(ex^n)}-\frac{dx}{A+B\log(ex^n)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}\\
&\quad-\frac{\left(b(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2g^3n^2(c+dx)}
\end{aligned}$$

$$= -\frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{(2b)\text{Subst}\left(\int\frac{1}{A+B\log(ex^n)}\,dx,x,\frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n} - \frac{(2d)\text{Subst}\left(\int\frac{x}{A+B\log(ex^n)}\,dx,x,\frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3n}$$

$$= -\frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{\left(2d(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{A+Bx}\,dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2g^3n^2(c+dx)^2} + \frac{\left(2b(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{A+Bx}\,dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2g^3n^2(c+dx)}$$

$$= \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} - \frac{2de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)^2} - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.12

$$\int\frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

$$= \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(-B(bc-ad)e^{\frac{2A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}+be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralE}\right.}{B^2(bc-ad)^2g^3n^2}$$

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((a + b*x)*(-(B*(b*c - a*d)*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(
2/n)) + b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpInteg
```

```
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c - a
*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Maple [F]

$$\int \frac{1}{\left(dgx+cg\right)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

```
[In] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(256) = 512.

Time = 0.28 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.01

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{\Big((Abd^2x^2+2\,Abcdx+Abc^2+(Bbd^2x^2+2\,Bbcdx+Bbc^2)\log(e)+(Bbd^2nx^2+2\,Bbc\cdots}{(AB^2b^2c^2d^2-2\,AB^2abcd^3+AB^2a^2d^4)g^3n^2x^2+2\,(AB^2b^2c^3d-2\,AB^2abc^2d^2+AB^2a^2cd^3)g^3n^2x+(AB^2\cdots}$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] ((A*b*d^2*x^2 + 2*A*b*c*d*x + A*b*c^2 + (B*b*d^2*x^2 + 2*B*b*c*d*x + B*b*c^
2)*log(e) + (B*b*d^2*n*x^2 + 2*B*b*c*d*n*x + B*b*c^2*n)*log((b*x + a)/(d*x
+ c)))*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B
*n))/(d*x + c)) - ((B*b^2*c - B*a*b*d)*n*x + (B*a*b*c - B*a^2*d)*n)*e^(2*(B
*log(e) + A)/(B*n)) - 2*(A*d^3*x^2 + 2*A*c*d^2*x + A*c^2*d + (B*d^3*x^2 + 2
*B*c*d^2*x + B*c^2*d)*log(e) + (B*d^3*n*x^2 + 2*B*c*d^2*n*x + B*c^2*d*n)*lo
g((b*x + a)/(d*x + c)))*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(
e) + A)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((A*
B^2*b^2*c^2*d^2 - 2*A*B^2*a*b*c*d^3 + A*B^2*a^2*d^4)*g^3*n^2*x^2 + 2*(A*B^2
*b^2*c^3*d - 2*A*B^2*a*b*c^2*d^2 + A*B^2*a^2*c*d^3)*g^3*n^2*x + (A*B^2*b^2*
c^4 - 2*A*B^2*a*b*c^3*d + A*B^2*a^2*c^2*d^2)*g^3*n^2 + ((B^3*b^2*c^2*d^2 -
2*B^3*a*b*c*d^3 + B^3*a^2*d^4)*g^3*n^2*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c
^2*d^2 + B^3*a^2*c*d^3)*g^3*n^2*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^
2*c^2*d^2)*g^3*n^2)*log(e) + ((B^3*b^2*c^2*d^2 - 2*B^3*a*b*c*d^3 + B^3*a^2*
d^4)*g^3*n^3*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c^2*d^2 + B^3*a^2*c*d^3)*g^
```

```
3*n^3*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^2*c^2*d^2)*g^3*n^3)*log((b
*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
[Out] -(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n*log(e) - a*c^2
*d*g^3*n*log(e))*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*n*
log(e) - a*d^3*g^3*n*log(e))*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*n)*
A*B + (b*c^2*d*g^3*n*log(e) - a*c*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^2*g^3*
n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3
*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^2*g^3*n - a*d^3*g^3
*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n - a*c^
2*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x - b*c + 2*a*d)/(((b*c*
d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^3*n*log(e) - a*d^4*g^3*n*log(e))*
B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^3*n*log(e) - a*c^3*
d*g^3*n*log(e))*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B + (b*c^2*d^2
*g^3*n*log(e) - a*c*d^3*g^3*n*log(e))*B^2)*x^2 + 3*((b*c^3*d*g^3*n - a*c^2*
d^2*g^3*n)*A*B + (b*c^3*d*g^3*n*log(e) - a*c^2*d^2*g^3*n*log(e))*B^2)*x + (
(b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)
*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3
*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 +
3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*
g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.27

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \left(\frac{b\text{Ei}\left(\frac{\log(e)}{n}+\frac{A}{Bn}+\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\left(\frac{1}{n}\right)}} - \frac{2\,d\text{Ei}\left(\frac{2\,\log(e)}{n}+\frac{2\,A}{Bn}+2\,\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{2\,A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\frac{2}{n}}} - \frac{}{B^2bcg^3n^2\log\left(\frac{bx}{dx}\right.}\right.$$

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] (b*Ei(log(e)/n + A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/((B^2*b*c
*g^3*n^2 - B^2*a*d*g^3*n^2)*e^(1/n)) - 2*d*Ei(2*log(e)/n + 2*A/(B*n) + 2*lo
g((b*x + a)/(d*x + c)))*e^(-2*A/(B*n))/((B^2*b*c*g^3*n^2 - B^2*a*d*g^3*n^2)
*e^(2/n)) - ((b*x + a)*b/(d*x + c) - (b*x + a)^2*d/(d*x + c)^2)/(B^2*b*c*g^
3*n^2*log((b*x + a)/(d*x + c)) - B^2*a*d*g^3*n^2*log((b*x + a)/(d*x + c)) +
 B^2*b*c*g^3*n*log(e) - B^2*a*d*g^3*n*log(e) + A*B*b*c*g^3*n - A*B*a*d*g^3*
n))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.57 $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 364

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2df}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,nx^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)nx^3}{15b^2d^2}-\frac{B(bc-ad)g^4nx^4}{20bd}-\frac{B(bf-ag)^5n\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}+\frac{B(df-cg)^5n\log(c+dx)}{5d^5g}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b
^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*
d*g-b*c*g+5*b*d*f)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*n*x^4/b/d-1/5*B*(-a*
g+b*f)^5*n*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+
1/5*B*(-c*g+d*f)^5*n*ln(d*x+c)/d^5/g
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2547, 84}

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= -\frac{Bg^2nx^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{10b^3d^3}$$
$$+\frac{Bgnx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-\right.}{5b^4d^4}$$
$$+\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$-\frac{Bg^3nx^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4nx^4(bc-ad)}{20bd}+\frac{Bn(df-cg)^5\log(c+dx)}{5d^5g}$$

```
[In] Int[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10
*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*
d*f*g^2 - c^3*g^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*
b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*n*x^2)/(10*b^
3*d^3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*n*x^3)/(15*b^2*d^2) -
 (B*(b*c - a*d)*g^4*n*x^4)/(20*b*d) - (B*(b*f - a*g)^5*n*Log[a + b*x])/(5*b
^5*g) + ((f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*g) + (B*(d*
f - c*g)^5*n*Log[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g} - \frac{(B(bc-ad)n)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$= \frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}$$

$$-\frac{(B(bc-ad)n)\int\left(\frac{g^2\left(-a^3d^3g^3+a^2bd^2g^2(5df-cg)-ab^2dg\left(10d^2f^2-5cdfg+c^2g^2\right)+b^3\left(10d^3f^3-10cd^2f^2g+5c^2dfg^2-c^3g^3\right)\right)}{b^4d^4}+\right.}{}$$

$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g}{5b^4d^4}$$

$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,nx^2}{10b^3d^3}$$

$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)nx^3}{15b^2d^2}$$

$$-\frac{B(bc-ad)g^4nx^4}{20bd}-\frac{B(bf-ag)^5n\log(a+bx)}{5b^5g}$$

$$+\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}+\frac{B(df-cg)^5n\log(c+dx)}{5d^5g}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.78

$$\int(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{\frac{B(-bc+ad)g^2nx\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+\right.\right.}{12b^4d^4}}{}$$

```
[In] Integrate[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*n*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c
*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c
*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g
^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*n*Log[a + b*x])/b^5
+ (f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(d*f - c*g)^5*n*L
og[c + d*x])/d^5)/(5*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. 2(350) = 700.

Time = 5.46 (sec) , antiderivative size = 1160, normalized size of antiderivative = 3.19

method	result	size
parallelrisch	Expression too large to display	1160

```
[In] int((g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*g^4*n+60*A*x^4*a*b^5*c
*d^5*f*g^3*n+12*A*x^5*a*b^5*c*d^5*g^4*n-120*B*ln(b*x+a)*a^3*b^3*c*d^5*f^3*g
*n^2+60*B*ln(b*x+a)*a*b^5*c^5*d*f*g^3*n^2-120*B*ln(b*x+a)*a*b^5*c^4*d^2*f^2
*g^2*n^2+120*B*ln(b*x+a)*a*b^5*c^3*d^3*f^3*g*n^2+60*B*x^4*ln(e*((b*x+a)/(d*
x+c))^n)*a*b^5*c*d^5*f*g^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^
5*f^2*g^2*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*f^3*g*n+20*B*x^
3*a^2*b^4*c*d^5*f*g^3*n^2-20*B*x^3*a*b^5*c^2*d^4*f*g^3*n^2+120*A*x^3*a*b^5*
c*d^5*f^2*g^2*n-30*B*x^2*a^3*b^3*c*d^5*f*g^3*n^2+60*B*x^2*a^2*b^4*c*d^5*f^2
*g^2*n^2+30*B*x^2*a*b^5*c^3*d^3*f*g^3*n^2-60*B*x^2*a*b^5*c^2*d^4*f^2*g^2*n^
2+120*A*x^2*a*b^5*c*d^5*f^3*g*n+60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^
5*f^4*n+60*B*x*a^4*b^2*c*d^5*f*g^3*n^2-120*B*x*a^3*b^3*c*d^5*f^2*g^2*n^2+12
0*B*x*a^2*b^4*c*d^5*f^3*g*n^2-60*B*x*a*b^5*c^4*d^2*f*g^3*n^2+120*B*x*a*b^5*
c^3*d^3*f^2*g^2*n^2-120*B*x*a*b^5*c^2*d^4*f^3*g*n^2-60*B*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^5*c^5*d*f*g^3*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^4*d^2*f
^2*g^2*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^3*d^3*f^3*g*n-60*B*ln(b*x+
a)*a^5*b*c*d^5*f*g^3*n^2+120*B*ln(b*x+a)*a^4*b^2*c*d^5*f^2*g^2*n^2+12*B*ln(
e*((b*x+a)/(d*x+c))^n)*a*b^5*c^6*g^4*n+12*B*ln(b*x+a)*a^6*c*d^5*g^4*n^2-12*
B*ln(b*x+a)*a*b^5*c^6*g^4*n^2+60*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^4*
f^4*n+60*B*ln(b*x+a)*a^2*b^4*c*d^5*f^4*n^2-60*B*ln(b*x+a)*a*b^5*c^2*d^4*f^4
*n^2+3*B*x^4*a^2*b^4*c*d^5*g^4*n^2-3*B*x^4*a*b^5*c^2*d^4*g^4*n^2-4*B*x^3*a^
3*b^3*c*d^5*g^4*n^2+4*B*x^3*a*b^5*c^3*d^3*g^4*n^2+6*B*x^2*a^4*b^2*c*d^5*g^4
*n^2-6*B*x^2*a*b^5*c^4*d^2*g^4*n^2-12*B*x*a^5*b*c*d^5*g^4*n^2+12*B*x*a*b^5*
c^5*d*g^4*n^2+60*A*x*a*b^5*c*d^5*f^4*n)/a/c/d^5/n/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(350) = 700.

Time = 0.66 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.02

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{12\,Ab^5d^5g^4x^5+3\,(20\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4n)x^4+4\,(30\,Ab^5d^5f^2g^2-(5\,(Bb^5cd^4-Bab^4d^5)fg}{}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)
*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*(20*A*b^5*d^5*f^3*
g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*
d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n)*x^2 + 12*(5*B*a*b^4*d^
5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f
*g^3 + B*a^5*d^5*g^4)*n*log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2
*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4
)*n*log(d*x + c) + 12*(5*A*b^5*d^5*f^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^
3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 + 5*(B*b^5*c^3*d^2 - B*a^3
*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*n)*x + 12*(B*b^5*d^5*g^4
*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*
g*x^2 + 5*B*b^5*d^5*f^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*d^5*f
*g^3*n*x^4 + 10*B*b^5*d^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*n*x^2 + 5*B*b^
5*d^5*f^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.73

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{5}\,Bg^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{5}\,Ag^4x^5+Bfg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Afg^3x^4+2\,Bf^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+2\,Af^2g^2x^3+2\,Bf^3gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Af^3gx^2$$
$$+\frac{1}{60}\,Bg^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b^4c}{b^4d^4}\right.$$
$$-\frac{1}{6}\,Bfg^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-}{b^3d^3}\right.$$
$$+Bf^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2\,Bf^3gn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bf^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^4x$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/5*B*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*g^4*x^5 + B*f*
g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^3*x^4 + 2*B*f^2*g^2*
x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g*x^
2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^3*g*x^2 + 1/60*B*g^4*n*(12
*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4
)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 -
 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*B*f*g^3*n*(6*a^4*log(b*x + a)/b
^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*f^2*g^2*n*(2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*B*f^3*g*n*(a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*f^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11996 vs. 2(350) = 700.

Time = 1.67 (sec) , antiderivative size = 11996, normalized size of antiderivative = 32.96

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] 1/60*(12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n - 20*(b*x + a)*B*b
^5*c^2*d^5*f^4*n/(d*x + c) + 5*B*a^2*b^4*d^6*f^4*n + 40*(b*x + a)*B*a*b^4*c
*d^6*f^4*n/(d*x + c) + 30*(b*x + a)^2*B*b^4*c^2*d^6*f^4*n/(d*x + c)^2 - 20*
(b*x + a)*B*a^2*b^3*d^7*f^4*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^3*c*d^7*f^4*
n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^3*c^2*d^7*f^4*n/(d*x + c)^3 + 30*(b*x +
a)^2*B*a^2*b^2*d^8*f^4*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a*b^2*c*d^8*f^4*n/(
d*x + c)^3 + 5*(b*x + a)^4*B*b^2*c^2*d^8*f^4*n/(d*x + c)^4 - 20*(b*x + a)^3
*B*a^2*b*d^9*f^4*n/(d*x + c)^3 - 10*(b*x + a)^4*B*a*b*c*d^9*f^4*n/(d*x + c)
^4 + 5*(b*x + a)^4*B*a^2*d^10*f^4*n/(d*x + c)^4 - 10*B*b^6*c^3*d^3*f^3*g*n
+ 10*B*a*b^5*c^2*d^4*f^3*g*n + 50*(b*x + a)*B*b^5*c^3*d^4*f^3*g*n/(d*x + c)
 + 10*B*a^2*b^4*c*d^5*f^3*g*n - 70*(b*x + a)*B*a*b^4*c^2*d^5*f^3*g*n/(d*x +
 c) - 90*(b*x + a)^2*B*b^4*c^3*d^5*f^3*g*n/(d*x + c)^2 - 10*B*a^3*b^3*d^6*f
^3*g*n - 10*(b*x + a)*B*a^2*b^3*c*d^6*f^3*g*n/(d*x + c) + 150*(b*x + a)^2*B
*a*b^3*c^2*d^6*f^3*g*n/(d*x + c)^2 + 70*(b*x + a)^3*B*b^3*c^3*d^6*f^3*g*n/(
d*x + c)^3 + 30*(b*x + a)*B*a^3*b^2*d^7*f^3*g*n/(d*x + c) - 30*(b*x + a)^2*
B*a^2*b^2*c*d^7*f^3*g*n/(d*x + c)^2 - 130*(b*x + a)^3*B*a*b^2*c^2*d^7*f^3*g
*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^3*d^7*f^3*g*n/(d*x + c)^4 - 30*(b*x
 + a)^2*B*a^3*b*d^8*f^3*g*n/(d*x + c)^2 + 50*(b*x + a)^3*B*a^2*b*c*d^8*f^3*
g*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^2*d^8*f^3*g*n/(d*x + c)^4 + 10*(b*
x + a)^3*B*a^3*d^9*f^3*g*n/(d*x + c)^3 - 20*(b*x + a)^4*B*a^2*c*d^9*f^3*g*n
/(d*x + c)^4 + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^3*f^2*g^2*n -
50*(b*x + a)*B*b^5*c^4*d^3*f^2*g^2*n/(d*x + c) + 50*(b*x + a)*B*a*b^4*c^3*d
^4*f^2*g^2*n/(d*x + c) + 100*(b*x + a)^2*B*b^4*c^4*d^4*f^2*g^2*n/(d*x + c)^
2 - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 30*(b*x + a)*B*a^2*b^3*c^2*d^5*f^2*g^2*n
/(d*x + c) - 130*(b*x + a)^2*B*a*b^3*c^3*d^5*f^2*g^2*n/(d*x + c)^2 - 90*(b*
x + a)^3*B*b^3*c^4*d^5*f^2*g^2*n/(d*x + c)^3 + 10*B*a^4*b^2*d^6*f^2*g^2*n -
 10*(b*x + a)*B*a^3*b^2*c*d^6*f^2*g^2*n/(d*x + c) - 30*(b*x + a)^2*B*a^2*b^
2*c^2*d^6*f^2*g^2*n/(d*x + c)^2 + 150*(b*x + a)^3*B*a*b^2*c^3*d^6*f^2*g^2*n
/(d*x + c)^3 + 30*(b*x + a)^4*B*b^2*c^4*d^6*f^2*g^2*n/(d*x + c)^4 - 20*(b*x
 + a)*B*a^4*b*d^7*f^2*g^2*n/(d*x + c) + 50*(b*x + a)^2*B*a^3*b*c*d^7*f^2*g^
2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^2*b*c^2*d^7*f^2*g^2*n/(d*x + c)^3 - 60
*(b*x + a)^4*B*a*b*c^3*d^7*f^2*g^2*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^4*d^8
*f^2*g^2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^3*c*d^8*f^2*g^2*n/(d*x + c)^3 +
 30*(b*x + a)^4*B*a^2*c^2*d^8*f^2*g^2*n/(d*x + c)^4 - 5*B*b^6*c^5*d*f*g^3*n
 + 5*B*a*b^5*c^4*d^2*f*g^3*n + 25*(b*x + a)*B*b^5*c^5*d^2*f*g^3*n/(d*x + c)
```

- 25*(b*x + a)*B*a*b^4*c^4*d^3*f*g^3*n/(d*x + c) - 50*(b*x + a)^2*B*b^4*c^
5*d^3*f*g^3*n/(d*x + c)^2 + 50*(b*x + a)^2*B*a*b^3*c^4*d^4*f*g^3*n/(d*x + c
)^2 + 50*(b*x + a)^3*B*b^3*c^5*d^4*f*g^3*n/(d*x + c)^3 + 5*B*a^4*b^2*c*d^5*
f*g^3*n - 20*(b*x + a)*B*a^3*b^2*c^2*d^5*f*g^3*n/(d*x + c) + 30*(b*x + a)^2
*B*a^2*b^2*c^3*d^5*f*g^3*n/(d*x + c)^2 - 70*(b*x + a)^3*B*a*b^2*c^4*d^5*f*g
^3*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^5*d^5*f*g^3*n/(d*x + c)^4 - 5*B*a
^5*b*d^6*f*g^3*n + 15*(b*x + a)*B*a^4*b*c*d^6*f*g^3*n/(d*x + c) - 10*(b*x +
a)^2*B*a^3*b*c^2*d^6*f*g^3*n/(d*x + c)^2 - 10*(b*x + a)^3*B*a^2*b*c^3*d^6*
f*g^3*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^4*d^6*f*g^3*n/(d*x + c)^4 + 5*
(b*x + a)*B*a^5*d^7*f*g^3*n/(d*x + c) - 20*(b*x + a)^2*B*a^4*c*d^7*f*g^3*n/
(d*x + c)^2 + 30*(b*x + a)^3*B*a^3*c^2*d^7*f*g^3*n/(d*x + c)^3 - 20*(b*x +
a)^4*B*a^2*c^3*d^7*f*g^3*n/(d*x + c)^4 + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^
4*n - 5*(b*x + a)*B*b^5*c^6*d*g^4*n/(d*x + c) + 5*(b*x + a)*B*a*b^4*c^5*d^2
*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^4*c^6*d^2*g^4*n/(d*x + c)^2 - 10*(b*x
+ a)^2*B*a*b^3*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^3*c^6*d^3*g^
4*n/(d*x + c)^3 + 10*(b*x + a)^3*B*a*b^2*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x
+ a)^4*B*b^2*c^6*d^4*g^4*n/(d*x + c)^4 - B*a^5*b*c*d^5*g^4*n + 5*(b*x + a)
*B*a^4*b*c^2*d^5*g^4*n/(d*x + c) - 10*(b*x + a)^2*B*a^3*b*c^3*d^5*g^4*n/(d*
x + c)^2 + 10*(b*x + a)^3*B*a^2*b*c^4*d^5*g^4*n/(d*x + c)^3 - 10*(b*x + a)^
4*B*a*b*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*d^6*g^4*n - 5*(b*x + a)*B*a^5*c*d
^6*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*a^4*c^2*d^6*g^4*n/(d*x + c)^2 - 10*(b
*x + a)^3*B*a^3*c^3*d^6*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*a^2*c^4*d^6*g^4
*n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*
x + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x +
c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) - (
120*B*b^10*c^3*d^3*f^3*g*n - 360*B*a*b^9*c^2*d^4*f^3*g*n - 480*(b*x + a)*B*
b^9*c^3*d^4*f^3*g*n/(d*x + c) + 360*B*a^2*b^8*c*d^5*f^3*g*n + 1440*(b*x + a
)*B*a*b^8*c^2*d^5*f^3*g*n/(d*x + c) + 720*(b*x + a)^2*B*b^8*c^3*d^5*f^3*g*n
/(d*x + c)^2 - 120*B*a^3*b^7*d^6*f^3*g*n - 1440*(b*x + a)*B*a^2*b^7*c*d^6*f
^3*g*n/(d*x + c) - 2160*(b*x + a)^2*B*a*b^7*c^2*d^6*f^3*g*n/(d*x + c)^2 - 4
80*(b*x + a)^3*B*b^7*c^3*d^6*f^3*g*n/(d*x + c)^3 + 480*(b*x + a)*B*a^3*b^6*
d^7*f^3*g*n/(d*x + c) + 2160*(b*x + a)^2*B*a^2*b^6*c*d^7*f^3*g*n/(d*x + c)^
2 + 1440*(b*x + a)^3*B*a*b^6*c^2*d^7*f^3*g*n/(d*x + c)^3 + 120*(b*x + a)^4*
B*b^6*c^3*d^7*f^3*g*n/(d*x + c)^4 - 720*(b*x + a)^2*B*a^3*b^5*d^8*f^3*g*n/(
d*x + c)^2 - 1440*(b*x + a)^3*B*a^2*b^5*c*d^8*f^3*g*n/(d*x + c)^3 - 360*(b*
x + a)^4*B*a*b^5*c^2*d^8*f^3*g*n/(d*x + c)^4 + 480*(b*x + a)^3*B*a^3*b^4*d^
9*f^3*g*n/(d*x + c)^3 + 360*(b*x + a)^4*B*a^2*b^4*c*d^9*f^3*g*n/(d*x + c)^4
- 120*(b*x + a)^4*B*a^3*b^3*d^10*f^3*g*n/(d*x + c)^4 - 180*B*b^10*c^4*d^2*
f^2*g^2*n + 360*B*a*b^9*c^3*d^3*f^2*g^2*n + 780*(b*x + a)*B*b^9*c^4*d^3*f^2
*g^2*n/(d*x + c) - 1680*(b*x + a)*B*a*b^8*c^3*d^4*f^2*g^2*n/(d*x + c) - 126
0*(b*x + a)^2*B*b^8*c^4*d^4*f^2*g^2*n/(d*x + c)^2 - 360*B*a^3*b^7*c*d^5*f^2
*g^2*n + 360*(b*x + a)*B*a^2*b^7*c^2*d^5*f^2*g^2*n/(d*x + c) + 2880*(b*x +
a)^2*B*a*b^7*c^3*d^5*f^2*g^2*n/(d*x + c)^2 + 900*(b*x + a)^3*B*b^7*c^4*d^5*
f^2*g^2*n/(d*x + c)^3 + 180*B*a^4*b^6*d^6*f^2*g^2*n + 1200*(b*x + a)*B*a^3*
b^6*c*d^6*f^2*g^2*n/(d*x + c) - 1080*(b*x + a)^2*B*a^2*b^6*c^2*d^6*f^2*g^2*

n/(d*x + c)^2 - 2160*(b*x + a)^3*B*a*b^6*c^3*d^6*f^2*g^2*n/(d*x + c)^3 - 24
0*(b*x + a)^4*B*b^6*c^4*d^6*f^2*g^2*n/(d*x + c)^4 - 660*(b*x + a)*B*a^4*b^5
*d^7*f^2*g^2*n/(d*x + c) - 1440*(b*x + a)^2*B*a^3*b^5*c*d^7*f^2*g^2*n/(d*x
+ c)^2 + 1080*(b*x + a)^3*B*a^2*b^5*c^2*d^7*f^2*g^2*n/(d*x + c)^3 + 600*(b*
x + a)^4*B*a*b^5*c^3*d^7*f^2*g^2*n/(d*x + c)^4 + 900*(b*x + a)^2*B*a^4*b^4*
d^8*f^2*g^2*n/(d*x + c)^2 + 720*(b*x + a)^3*B*a^3*b^4*c*d^8*f^2*g^2*n/(d*x
+ c)^3 - 360*(b*x + a)^4*B*a^2*b^4*c^2*d^8*f^2*g^2*n/(d*x + c)^4 - 540*(b*x
 + a)^3*B*a^4*b^3*d^9*f^2*g^2*n/(d*x + c)^3 - 120*(b*x + a)^4*B*a^3*b^3*c*d
^9*f^2*g^2*n/(d*x + c)^4 + 120*(b*x + a)^4*B*a^4*b^2*d^10*f^2*g^2*n/(d*x +
c)^4 + 110*B*b^10*c^5*d*f*g^3*n - 190*B*a*b^9*c^4*d^2*f*g^3*n - 490*(b*x +
a)*B*b^9*c^5*d^2*f*g^3*n/(d*x + c) + 20*B*a^2*b^8*c^3*d^3*f*g^3*n + 890*(b*
x + a)*B*a*b^8*c^4*d^3*f*g^3*n/(d*x + c) + 830*(b*x + a)^2*B*b^8*c^5*d^3*f*
g^3*n/(d*x + c)^2 - 20*B*a^3*b^7*c^2*d^4*f*g^3*n - 100*(b*x + a)*B*a^2*b^7*
c^3*d^4*f*g^3*n/(d*x + c) - 1630*(b*x + a)^2*B*a*b^7*c^4*d^4*f*g^3*n/(d*x +
 c)^2 - 630*(b*x + a)^3*B*b^7*c^5*d^4*f*g^3*n/(d*x + c)^3 + 190*B*a^4*b^6*c
*d^5*f*g^3*n - 140*(b*x + a)*B*a^3*b^6*c^2*d^5*f*g^3*n/(d*x + c) + 380*(b*x
 + a)^2*B*a^2*b^6*c^3*d^5*f*g^3*n/(d*x + c)^2 + 1350*(b*x + a)^3*B*a*b^6*c^
4*d^5*f*g^3*n/(d*x + c)^3 + 180*(b*x + a)^4*B*b^6*c^5*d^5*f*g^3*n/(d*x + c)
^4 - 110*B*a^5*b^5*d^6*f*g^3*n - 530*(b*x + a)*B*a^4*b^5*c*d^6*f*g^3*n/(d*x
 + c) + 340*(b*x + a)^2*B*a^3*b^5*c^2*d^6*f*g^3*n/(d*x + c)^2 - 540*(b*x +
a)^3*B*a^2*b^5*c^3*d^6*f*g^3*n/(d*x + c)^3 - 420*(b*x + a)^4*B*a*b^5*c^4*d^
6*f*g^3*n/(d*x + c)^4 + 370*(b*x + a)*B*a^5*b^4*d^7*f*g^3*n/(d*x + c) + 550
*(b*x + a)^2*B*a^4*b^4*c*d^7*f*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a^3*b^
4*c^2*d^7*f*g^3*n/(d*x + c)^3 + 240*(b*x + a)^4*B*a^2*b^4*c^3*d^7*f*g^3*n/(
d*x + c)^4 - 470*(b*x + a)^2*B*a^5*b^3*d^8*f*g^3*n/(d*x + c)^2 - 270*(b*x +
 a)^3*B*a^4*b^3*c*d^8*f*g^3*n/(d*x + c)^3 + 270*(b*x + a)^3*B*a^5*b^2*d^9*f
*g^3*n/(d*x + c)^3 + 60*(b*x + a)^4*B*a^4*b^2*c*d^9*f*g^3*n/(d*x + c)^4 - 6
0*(b*x + a)^4*B*a^5*b*d^10*f*g^3*n/(d*x + c)^4 - 25*B*b^10*c^6*g^4*n + 40*B
*a*b^9*c^5*d*g^4*n + 113*(b*x + a)*B*b^9*c^6*d*g^4*n/(d*x + c) - 5*B*a^2*b^
8*c^4*d^2*g^4*n - 188*(b*x + a)*B*a*b^8*c^5*d^2*g^4*n/(d*x + c) - 196*(b*x
+ a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 + 25*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4
*n/(d*x + c) + 346*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 + 156*(b*x
 + a)^3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 5*B*a^4*b^6*c^2*d^4*g^4*n - 50*(b
*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c)^2 - 306*(b*x + a)^3*B*a*b^6*c^5
*d^4*g^4*n/(d*x + c)^3 - 48*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 4
0*B*a^5*b^5*c*d^5*g^4*n + 35*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) -
60*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x + c)^2 + 90*(b*x + a)^3*B*a^2*b
^5*c^4*d^5*g^4*n/(d*x + c)^3 + 108*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x +
 c)^4 + 25*B*a^6*b^4*d^6*g^4*n + 92*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x +
c) - 40*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 60*(b*x + a)^3*B*
a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 - 60*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/
(d*x + c)^4 - 77*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 94*(b*x + a)^2*B
*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 + 94*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x
+ c)^2 + 54*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 - 54*(b*x + a)^3*
B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 12*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c

)^4 + 12*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 - 60*B*b^10*c^2*d^4*f^4*l
og(e) + 120*B*a*b^9*c*d^5*f^4*log(e) + 240*(b*x + a)*B*b^9*c^2*d^5*f^4*log(
e)/(d*x + c) - 60*B*a^2*b^8*d^6*f^4*log(e) - 480*(b*x + a)*B*a*b^8*c*d^6*f^
4*log(e)/(d*x + c) - 360*(b*x + a)^2*B*b^8*c^2*d^6*f^4*log(e)/(d*x + c)^2 +
 240*(b*x + a)*B*a^2*b^7*d^7*f^4*log(e)/(d*x + c) + 720*(b*x + a)^2*B*a*b^7
*c*d^7*f^4*log(e)/(d*x + c)^2 + 240*(b*x + a)^3*B*b^7*c^2*d^7*f^4*log(e)/(d
x + c)^3 - 360(b*x + a)^2*B*a^2*b^6*d^8*f^4*log(e)/(d*x + c)^2 - 480*(b*x
 + a)^3*B*a*b^6*c*d^8*f^4*log(e)/(d*x + c)^3 - 60*(b*x + a)^4*B*b^6*c^2*d^8
*f^4*log(e)/(d*x + c)^4 + 240*(b*x + a)^3*B*a^2*b^5*d^9*f^4*log(e)/(d*x + c
)^3 + 120*(b*x + a)^4*B*a*b^5*c*d^9*f^4*log(e)/(d*x + c)^4 - 60*(b*x + a)^4
*B*a^2*b^4*d^10*f^4*log(e)/(d*x + c)^4 + 120*B*b^10*c^3*d^3*f^3*g*log(e) -
120*B*a*b^9*c^2*d^4*f^3*g*log(e) - 600*(b*x + a)*B*b^9*c^3*d^4*f^3*g*log(e)
/(d*x + c) - 120*B*a^2*b^8*c*d^5*f^3*g*log(e) + 840*(b*x + a)*B*a*b^8*c^2*d
^5*f^3*g*log(e)/(d*x + c) + 1080*(b*x + a)^2*B*b^8*c^3*d^5*f^3*g*log(e)/(d*
x + c)^2 + 120*B*a^3*b^7*d^6*f^3*g*log(e) + 120*(b*x + a)*B*a^2*b^7*c*d^6*f
^3*g*log(e)/(d*x + c) - 1800*(b*x + a)^2*B*a*b^7*c^2*d^6*f^3*g*log(e)/(d*x
+ c)^2 - 840*(b*x + a)^3*B*b^7*c^3*d^6*f^3*g*log(e)/(d*x + c)^3 - 360*(b*x
+ a)*B*a^3*b^6*d^7*f^3*g*log(e)/(d*x + c) + 360*(b*x + a)^2*B*a^2*b^6*c*d^7
*f^3*g*log(e)/(d*x + c)^2 + 1560*(b*x + a)^3*B*a*b^6*c^2*d^7*f^3*g*log(e)/(
d*x + c)^3 + 240*(b*x + a)^4*B*b^6*c^3*d^7*f^3*g*log(e)/(d*x + c)^4 + 360*(
b*x + a)^2*B*a^3*b^5*d^8*f^3*g*log(e)/(d*x + c)^2 - 600*(b*x + a)^3*B*a^2*b
^5*c*d^8*f^3*g*log(e)/(d*x + c)^3 - 480*(b*x + a)^4*B*a*b^5*c^2*d^8*f^3*g*l
og(e)/(d*x + c)^4 - 120*(b*x + a)^3*B*a^3*b^4*d^9*f^3*g*log(e)/(d*x + c)^3
+ 240*(b*x + a)^4*B*a^2*b^4*c*d^9*f^3*g*log(e)/(d*x + c)^4 - 120*B*b^10*c^4
*d^2*f^2*g^2*log(e) + 120*B*a*b^9*c^3*d^3*f^2*g^2*log(e) + 600*(b*x + a)*B*
b^9*c^4*d^3*f^2*g^2*log(e)/(d*x + c) - 600*(b*x + a)*B*a*b^8*c^3*d^4*f^2*g^
2*log(e)/(d*x + c) - 1200*(b*x + a)^2*B*b^8*c^4*d^4*f^2*g^2*log(e)/(d*x + c
)^2 + 120*B*a^3*b^7*c*d^5*f^2*g^2*log(e) - 360*(b*x + a)*B*a^2*b^7*c^2*d^5*
f^2*g^2*log(e)/(d*x + c) + 1560*(b*x + a)^2*B*a*b^7*c^3*d^5*f^2*g^2*log(e)/
(d*x + c)^2 + 1080*(b*x + a)^3*B*b^7*c^4*d^5*f^2*g^2*log(e)/(d*x + c)^3 - 1
20*B*a^4*b^6*d^6*f^2*g^2*log(e) + 120*(b*x + a)*B*a^3*b^6*c*d^6*f^2*g^2*log
(e)/(d*x + c) + 360*(b*x + a)^2*B*a^2*b^6*c^2*d^6*f^2*g^2*log(e)/(d*x + c)^
2 - 1800*(b*x + a)^3*B*a*b^6*c^3*d^6*f^2*g^2*log(e)/(d*x + c)^3 - 360*(b*x
+ a)^4*B*b^6*c^4*d^6*f^2*g^2*log(e)/(d*x + c)^4 + 240*(b*x + a)*B*a^4*b^5*d
^7*f^2*g^2*log(e)/(d*x + c) - 600*(b*x + a)^2*B*a^3*b^5*c*d^7*f^2*g^2*log(e
)/(d*x + c)^2 + 360*(b*x + a)^3*B*a^2*b^5*c^2*d^7*f^2*g^2*log(e)/(d*x + c)^
3 + 720*(b*x + a)^4*B*a*b^5*c^3*d^7*f^2*g^2*log(e)/(d*x + c)^4 - 120*(b*x +
 a)^2*B*a^4*b^4*d^8*f^2*g^2*log(e)/(d*x + c)^2 + 360*(b*x + a)^3*B*a^3*b^4*
c*d^8*f^2*g^2*log(e)/(d*x + c)^3 - 360*(b*x + a)^4*B*a^2*b^4*c^2*d^8*f^2*g^
2*log(e)/(d*x + c)^4 + 60*B*b^10*c^5*d*f*g^3*log(e) - 60*B*a*b^9*c^4*d^2*f*
g^3*log(e) - 300*(b*x + a)*B*b^9*c^5*d^2*f*g^3*log(e)/(d*x + c) + 300*(b*x
+ a)*B*a*b^8*c^4*d^3*f*g^3*log(e)/(d*x + c) + 600*(b*x + a)^2*B*b^8*c^5*d^3
*f*g^3*log(e)/(d*x + c)^2 - 600*(b*x + a)^2*B*a*b^7*c^4*d^4*f*g^3*log(e)/(d
x + c)^2 - 600(b*x + a)^3*B*b^7*c^5*d^4*f*g^3*log(e)/(d*x + c)^3 - 60*B*a
^4*b^6*c*d^5*f*g^3*log(e) + 240*(b*x + a)*B*a^3*b^6*c^2*d^5*f*g^3*log(e)/(d

x + c) - 360(b*x + a)^2*B*a^2*b^6*c^3*d^5*f*g^3*log(e)/(d*x + c)^2 + 840*
(b*x + a)^3*B*a*b^6*c^4*d^5*f*g^3*log(e)/(d*x + c)^3 + 240*(b*x + a)^4*B*b^
6*c^5*d^5*f*g^3*log(e)/(d*x + c)^4 + 60*B*a^5*b^5*d^6*f*g^3*log(e) - 180*(b
*x + a)*B*a^4*b^5*c*d^6*f*g^3*log(e)/(d*x + c) + 120*(b*x + a)^2*B*a^3*b^5*
c^2*d^6*f*g^3*log(e)/(d*x + c)^2 + 120*(b*x + a)^3*B*a^2*b^5*c^3*d^6*f*g^3*
log(e)/(d*x + c)^3 - 480*(b*x + a)^4*B*a*b^5*c^4*d^6*f*g^3*log(e)/(d*x + c)
^4 - 60*(b*x + a)*B*a^5*b^4*d^7*f*g^3*log(e)/(d*x + c) + 240*(b*x + a)^2*B*
a^4*b^4*c*d^7*f*g^3*log(e)/(d*x + c)^2 - 360*(b*x + a)^3*B*a^3*b^4*c^2*d^7*
f*g^3*log(e)/(d*x + c)^3 + 240*(b*x + a)^4*B*a^2*b^4*c^3*d^7*f*g^3*log(e)/(
d*x + c)^4 - 12*B*b^10*c^6*g^4*log(e) + 12*B*a*b^9*c^5*d*g^4*log(e) + 60*(b
*x + a)*B*b^9*c^6*d*g^4*log(e)/(d*x + c) - 60*(b*x + a)*B*a*b^8*c^5*d^2*g^4
*log(e)/(d*x + c) - 120*(b*x + a)^2*B*b^8*c^6*d^2*g^4*log(e)/(d*x + c)^2 +
120*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*log(e)/(d*x + c)^2 + 120*(b*x + a)^3*B*
b^7*c^6*d^3*g^4*log(e)/(d*x + c)^3 - 120*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*lo
g(e)/(d*x + c)^3 - 60*(b*x + a)^4*B*b^6*c^6*d^4*g^4*log(e)/(d*x + c)^4 + 12
*B*a^5*b^5*c*d^5*g^4*log(e) - 60*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*log(e)/(d*
x + c) + 120*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*log(e)/(d*x + c)^2 - 120*(b*
x + a)^3*B*a^2*b^5*c^4*d^5*g^4*log(e)/(d*x + c)^3 + 120*(b*x + a)^4*B*a*b^5
*c^5*d^5*g^4*log(e)/(d*x + c)^4 - 12*B*a^6*b^4*d^6*g^4*log(e) + 60*(b*x + a
)*B*a^5*b^4*c*d^6*g^4*log(e)/(d*x + c) - 120*(b*x + a)^2*B*a^4*b^4*c^2*d^6*
g^4*log(e)/(d*x + c)^2 + 120*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*log(e)/(d*x
+ c)^3 - 60*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*log(e)/(d*x + c)^4 - 60*A*b^1
0*c^2*d^4*f^4 + 120*A*a*b^9*c*d^5*f^4 + 240*(b*x + a)*A*b^9*c^2*d^5*f^4/(d*
x + c) - 60*A*a^2*b^8*d^6*f^4 - 480*(b*x + a)*A*a*b^8*c*d^6*f^4/(d*x + c) -
360*(b*x + a)^2*A*b^8*c^2*d^6*f^4/(d*x + c)^2 + 240*(b*x + a)*A*a^2*b^7*d^
7*f^4/(d*x + c) + 720*(b*x + a)^2*A*a*b^7*c*d^7*f^4/(d*x + c)^2 + 240*(b*x
+ a)^3*A*b^7*c^2*d^7*f^4/(d*x + c)^3 - 360*(b*x + a)^2*A*a^2*b^6*d^8*f^4/(d
x + c)^2 - 480(b*x + a)^3*A*a*b^6*c*d^8*f^4/(d*x + c)^3 - 60*(b*x + a)^4*
A*b^6*c^2*d^8*f^4/(d*x + c)^4 + 240*(b*x + a)^3*A*a^2*b^5*d^9*f^4/(d*x + c)
^3 + 120*(b*x + a)^4*A*a*b^5*c*d^9*f^4/(d*x + c)^4 - 60*(b*x + a)^4*A*a^2*b
^4*d^10*f^4/(d*x + c)^4 + 120*A*b^10*c^3*d^3*f^3*g - 120*A*a*b^9*c^2*d^4*f^
3*g - 600*(b*x + a)*A*b^9*c^3*d^4*f^3*g/(d*x + c) - 120*A*a^2*b^8*c*d^5*f^3
g + 840(b*x + a)*A*a*b^8*c^2*d^5*f^3*g/(d*x + c) + 1080*(b*x + a)^2*A*b^8
*c^3*d^5*f^3*g/(d*x + c)^2 + 120*A*a^3*b^7*d^6*f^3*g + 120*(b*x + a)*A*a^2*
b^7*c*d^6*f^3*g/(d*x + c) - 1800*(b*x + a)^2*A*a*b^7*c^2*d^6*f^3*g/(d*x + c
)^2 - 840*(b*x + a)^3*A*b^7*c^3*d^6*f^3*g/(d*x + c)^3 - 360*(b*x + a)*A*a^3
*b^6*d^7*f^3*g/(d*x + c) + 360*(b*x + a)^2*A*a^2*b^6*c*d^7*f^3*g/(d*x + c)^
2 + 1560*(b*x + a)^3*A*a*b^6*c^2*d^7*f^3*g/(d*x + c)^3 + 240*(b*x + a)^4*A*
b^6*c^3*d^7*f^3*g/(d*x + c)^4 + 360*(b*x + a)^2*A*a^3*b^5*d^8*f^3*g/(d*x +
c)^2 - 600*(b*x + a)^3*A*a^2*b^5*c*d^8*f^3*g/(d*x + c)^3 - 480*(b*x + a)^4*
A*a*b^5*c^2*d^8*f^3*g/(d*x + c)^4 - 120*(b*x + a)^3*A*a^3*b^4*d^9*f^3*g/(d*
x + c)^3 + 240*(b*x + a)^4*A*a^2*b^4*c*d^9*f^3*g/(d*x + c)^4 - 120*A*b^10*c
^4*d^2*f^2*g^2 + 120*A*a*b^9*c^3*d^3*f^2*g^2 + 600*(b*x + a)*A*b^9*c^4*d^3*
f^2*g^2/(d*x + c) - 600*(b*x + a)*A*a*b^8*c^3*d^4*f^2*g^2/(d*x + c) - 1200*
(b*x + a)^2*A*b^8*c^4*d^4*f^2*g^2/(d*x + c)^2 + 120*A*a^3*b^7*c*d^5*f^2*g^2

- 360*(b*x + a)*A*a^2*b^7*c^2*d^5*f^2*g^2/(d*x + c) + 1560*(b*x + a)^2*A*a
*b^7*c^3*d^5*f^2*g^2/(d*x + c)^2 + 1080*(b*x + a)^3*A*b^7*c^4*d^5*f^2*g^2/(
d*x + c)^3 - 120*A*a^4*b^6*d^6*f^2*g^2 + 120*(b*x + a)*A*a^3*b^6*c*d^6*f^2*
g^2/(d*x + c) + 360*(b*x + a)^2*A*a^2*b^6*c^2*d^6*f^2*g^2/(d*x + c)^2 - 180
0*(b*x + a)^3*A*a*b^6*c^3*d^6*f^2*g^2/(d*x + c)^3 - 360*(b*x + a)^4*A*b^6*c
^4*d^6*f^2*g^2/(d*x + c)^4 + 240*(b*x + a)*A*a^4*b^5*d^7*f^2*g^2/(d*x + c)
- 600*(b*x + a)^2*A*a^3*b^5*c*d^7*f^2*g^2/(d*x + c)^2 + 360*(b*x + a)^3*A*a
^2*b^5*c^2*d^7*f^2*g^2/(d*x + c)^3 + 720*(b*x + a)^4*A*a*b^5*c^3*d^7*f^2*g^
2/(d*x + c)^4 - 120*(b*x + a)^2*A*a^4*b^4*d^8*f^2*g^2/(d*x + c)^2 + 360*(b*
x + a)^3*A*a^3*b^4*c*d^8*f^2*g^2/(d*x + c)^3 - 360*(b*x + a)^4*A*a^2*b^4*c^
2*d^8*f^2*g^2/(d*x + c)^4 + 60*A*b^10*c^5*d*f*g^3 - 60*A*a*b^9*c^4*d^2*f*g^
3 - 300*(b*x + a)*A*b^9*c^5*d^2*f*g^3/(d*x + c) + 300*(b*x + a)*A*a*b^8*c^4
*d^3*f*g^3/(d*x + c) + 600*(b*x + a)^2*A*b^8*c^5*d^3*f*g^3/(d*x + c)^2 - 60
0*(b*x + a)^2*A*a*b^7*c^4*d^4*f*g^3/(d*x + c)^2 - 600*(b*x + a)^3*A*b^7*c^5
*d^4*f*g^3/(d*x + c)^3 - 60*A*a^4*b^6*c*d^5*f*g^3 + 240*(b*x + a)*A*a^3*b^6
*c^2*d^5*f*g^3/(d*x + c) - 360*(b*x + a)^2*A*a^2*b^6*c^3*d^5*f*g^3/(d*x + c
)^2 + 840*(b*x + a)^3*A*a*b^6*c^4*d^5*f*g^3/(d*x + c)^3 + 240*(b*x + a)^4*A
*b^6*c^5*d^5*f*g^3/(d*x + c)^4 + 60*A*a^5*b^5*d^6*f*g^3 - 180*(b*x + a)*A*a
^4*b^5*c*d^6*f*g^3/(d*x + c) + 120*(b*x + a)^2*A*a^3*b^5*c^2*d^6*f*g^3/(d*x
 + c)^2 + 120*(b*x + a)^3*A*a^2*b^5*c^3*d^6*f*g^3/(d*x + c)^3 - 480*(b*x +
a)^4*A*a*b^5*c^4*d^6*f*g^3/(d*x + c)^4 - 60*(b*x + a)*A*a^5*b^4*d^7*f*g^3/(
d*x + c) + 240*(b*x + a)^2*A*a^4*b^4*c*d^7*f*g^3/(d*x + c)^2 - 360*(b*x + a
)^3*A*a^3*b^4*c^2*d^7*f*g^3/(d*x + c)^3 + 240*(b*x + a)^4*A*a^2*b^4*c^3*d^7
*f*g^3/(d*x + c)^4 - 12*A*b^10*c^6*g^4 + 12*A*a*b^9*c^5*d*g^4 + 60*(b*x + a
)*A*b^9*c^6*d*g^4/(d*x + c) - 60*(b*x + a)*A*a*b^8*c^5*d^2*g^4/(d*x + c) -
120*(b*x + a)^2*A*b^8*c^6*d^2*g^4/(d*x + c)^2 + 120*(b*x + a)^2*A*a*b^7*c^5
*d^3*g^4/(d*x + c)^2 + 120*(b*x + a)^3*A*b^7*c^6*d^3*g^4/(d*x + c)^3 - 120*
(b*x + a)^3*A*a*b^6*c^5*d^4*g^4/(d*x + c)^3 - 60*(b*x + a)^4*A*b^6*c^6*d^4*
g^4/(d*x + c)^4 + 12*A*a^5*b^5*c*d^5*g^4 - 60*(b*x + a)*A*a^4*b^5*c^2*d^5*g
^4/(d*x + c) + 120*(b*x + a)^2*A*a^3*b^5*c^3*d^5*g^4/(d*x + c)^2 - 120*(b*x
 + a)^3*A*a^2*b^5*c^4*d^5*g^4/(d*x + c)^3 + 120*(b*x + a)^4*A*a*b^5*c^5*d^5
*g^4/(d*x + c)^4 - 12*A*a^6*b^4*d^6*g^4 + 60*(b*x + a)*A*a^5*b^4*c*d^6*g^4/
(d*x + c) - 120*(b*x + a)^2*A*a^4*b^4*c^2*d^6*g^4/(d*x + c)^2 + 120*(b*x +
a)^3*A*a^3*b^4*c^3*d^6*g^4/(d*x + c)^3 - 60*(b*x + a)^4*A*a^2*b^4*c^4*d^6*g
^4/(d*x + c)^4)/(b^9*d^5 - 5*(b*x + a)*b^8*d^6/(d*x + c) + 10*(b*x + a)^2*b
^7*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^6*d^8/(d*x + c)^3 + 5*(b*x + a)^4*b^5
*d^9/(d*x + c)^4 - (b*x + a)^5*b^4*d^10/(d*x + c)^5) + 12*(5*B*b^6*c^2*d^4*
f^4*n - 10*B*a*b^5*c*d^5*f^4*n + 5*B*a^2*b^4*d^6*f^4*n - 10*B*b^6*c^3*d^3*f
^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 10*B*a^2*b^4*c*d^5*f^3*g*n - 10*B*a^3
*b^3*d^6*f^3*g*n + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^3*f^2*g^2*
n - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 10*B*a^4*b^2*d^6*f^2*g^2*n - 5*B*b^6*c^5
*d*f*g^3*n + 5*B*a*b^5*c^4*d^2*f*g^3*n + 5*B*a^4*b^2*c*d^5*f*g^3*n - 5*B*a^
5*b*d^6*f*g^3*n + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^4*n - B*a^5*b*c*d^5*g^4
*n + B*a^6*d^6*g^4*n)*log(b - (b*x + a)*d/(d*x + c))/(b^5*d^5) - 12*(5*B*b^
6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n + 5*B*a^2*b^4*d^6*f^4*n - 10*B*b^6

```
*c^3*d^3*f^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 10*B*a^2*b^4*c*d^5*f^3*g*n
- 10*B*a^3*b^3*d^6*f^3*g*n + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^
3*f^2*g^2*n - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 10*B*a^4*b^2*d^6*f^2*g^2*n - 5
*B*b^6*c^5*d*f*g^3*n + 5*B*a*b^5*c^4*d^2*f*g^3*n + 5*B*a^4*b^2*c*d^5*f*g^3*
n - 5*B*a^5*b*d^6*f*g^3*n + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^4*n - B*a^5*b
*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d^5))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 1433, normalized size of antiderivative = 3.94

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((f + g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*
n)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x^2*((20*A*a*c*f*g^3 + 20
*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2*n - 1
0*B*b*c*f^2*g^2*n)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^
4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5
*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 +
20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b
*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*
A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/
(5*b*d)))/(2*b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B
*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d
 + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A
*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(15*b*d) + (A*a*c*g^4)/(3
*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g + 30*A*a*c*f^2*g
^2 + 10*B*a*d*f^3*g*n - 10*B*b*c*f^3*g*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A
*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*
d*f^2*g^2*n - 10*B*b*c*f^2*g^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^
4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*
g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*
A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c
*f*g^3*n)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a
*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g^4
 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*
b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 2
0*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b*
d) + (A*a*c*g^4)/(b*d)))/(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^4*x^
5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + (A*g^4*x^
```

```
5)/5 + (log(a + b*x)*((B*a^5*g^4*n)/5 + B*a*b^4*f^4*n + 2*B*a^3*b^2*f^2*g^2
*n - B*a^4*b*f*g^3*n - 2*B*a^2*b^3*f^3*g*n))/b^5 - (log(c + d*x)*(B*c^5*g^4
*n + 5*B*c*d^4*f^4*n + 10*B*c^3*d^2*f^2*g^2*n - 5*B*c^4*d*f*g^3*n - 10*B*c^
2*d^3*f^3*g*n))/(5*d^5)
```

3.58 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,nx}{4b^3d^3} \\ &\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)nx^2}{8b^2d^2}-\frac{B(bc-ad)g^3nx^3}{12bd}-\frac{B(bf-ag)^4n\log(a+bx)}{4b^4g} \\ &\quad +\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g}+\frac{B(df-cg)^4n\log(c+dx)}{4d^4g}\end{aligned}$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*g^3*n*x^3/b/d-1/4*B*(-a*g+b*f)^4*n*ln(b*x+a)/b^4
/g+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/4*B*(-c*g+d*f)^4*n*ln(
d*x+c)/d^4/g
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{2547, 84\}$

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= -\frac{Bgnx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{4b^3d^3} + \frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g} - \frac{Bn(bf-ag)^4\log(a+bx)}{4b^4g} - \frac{Bg^2nx^2(bc-ad)(-adg-bcg+4bdf)}{8b^2d^2} - \frac{Bg^3nx^3(bc-ad)}{12bd} + \frac{Bn(df-cg)^4\log(c+dx)}{4d^4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*n*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c
*g - a*d*g)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*n*x^3)/(12*b*d) - (B*(b
*f - a*g)^4*n*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(4*g) + (B*(d*f - c*g)^4*n*Log[c + d*x])/(4*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int \frac{(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$
$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int \left(\frac{g^2\left(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)\right)}{b^3d^3} + \frac{g^3(4bdf-bcg-adg)x}{b^2d^2} + \frac{g^4x^2}{bd} + \frac{(bf-ag)}{b^3(bc-ad)(a}\right.}{4g}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))\, nx}{4b^3d^3} - \frac{B(bc-ad)g^2(4bdf - bcg - adg)nx^2}{8b^2d^2} - \frac{B(bc-ad)g^3nx^3}{12bd} - \frac{B(bf-ag)^4 n \log(a+bx)}{4b^4g} + \frac{(f+gx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} + \frac{B(df-cg)^4 n \log(c+dx)}{4d^4g}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int (f+gx)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{(f+gx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{Bn(6bd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+3b^2d^2(bc-ad)g^3(4bdf}{6b^4d^4}}{4g}$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(223) = 446.

Time = 2.96 (sec) , antiderivative size = 976, normalized size of antiderivative = 4.15

method	result
parallelrisch	$\frac{-24B\ln(bx+a)b^4c^3dfg^2n^2-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^3n-6B\ln(bx+a)a^4d^4g^3n^2+6B\ln(bx+a)b^4c^4g^3n^2-3Ba^3bcd^3g^3n^2+3Ba}{}$

```
[In] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-24*B*ln(b*x+a)*b^4*c^3*d*f*g^2*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4
*c^4*g^3*n-6*B*ln(b*x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+a)*b^4*c^4*g^3*n^2-3*B*
a^3*b*c*d^3*g^3*n^2+3*B*a*b^3*c^3*d*g^3*n^2+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2+12*B*a
^2*b^2*c*d^3*f*g^2*n^2-12*B*a*b^3*c^2*d^2*f*g^2*n^2-36*A*a*b^3*c*d^3*f^2*g*
n-3*B*x^2*a^2*b^2*d^4*g^3*n^2+3*B*x^2*b^4*c^2*d^2*g^3*n^2+6*B*x*a^3*b*d^4*g
^3*n^2-6*B*x*b^4*c^3*d*g^3*n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*f^2*g*n^2+24*B*x^
```

```
3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f*g^2*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*b^4*d^4*f^2*g*n+12*B*x^2*a*b^3*d^4*f*g^2*n^2-12*B*x^2*b^4*c*d^3*f*g^2*n
^2-24*B*x*a^2*b^2*d^4*f*g^2*n^2+36*B*x*a*b^3*d^4*f^2*g*n^2+24*B*x*b^4*c^2*d
^2*f*g^2*n^2-36*B*x*b^4*c*d^3*f^2*g*n^2+24*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*
c^3*d*f*g^2*n-36*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*f^2*g*n+24*B*ln(b*
x+a)*a^3*b*d^4*f*g^2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*f^2*g*n^2+24*B*ln(b*x+a
)*a*b^3*d^4*f^3*n^2+24*A*x^3*b^4*d^4*f*g^2*n+36*A*x^2*b^4*d^4*f^2*g*n+24*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^3*n+24*B*ln(e*((b*x+a)/(d*x+c))^n)*b^
4*c*d^3*f^3*n-24*B*ln(b*x+a)*b^4*c*d^3*f^3*n^2+24*B*a^3*b*d^4*f*g^2*n^2-36*
B*a^2*b^2*d^4*f^2*g*n^2-24*B*b^4*c^3*d*f*g^2*n^2+36*B*b^4*c^2*d^2*f^2*g*n^2
+24*A*x*b^4*d^4*f^3*n-24*A*a*b^3*d^4*f^3*n-24*A*b^4*c*d^3*f^3*n-6*B*a^4*d^4
*g^3*n^2+6*B*b^4*c^4*g^3*n^2+6*A*x^4*b^4*d^4*g^3*n)/b^4/d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(223) = 446$.

Time = 0.41 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.22

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4g^3x^4+2\,(12\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3n)x^3+3\,(12\,Ab^4d^4f^2g-(4\,(Bb^4cd^3-Bab^3d^4)fg^2}{}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3*n)*x^3 + 3*(12*A*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*
g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*B*a*b^3*d^4*f^3 -
6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*n*log(b*x + a)
 - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b
^4*c^4*g^3)*n*log(d*x + c) + 6*(4*A*b^4*d^4*f^3 - (6*(B*b^4*c*d^3 - B*a*b^3
*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 + (B*b^4*c^3*d - B*a^
3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4
*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^4 + 4*B*b
^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*x)*log((b*
x + a)/(d*x + c)))/(b^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.89

$$\begin{aligned}
&\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{4}Bg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Ag^3x^4+Bfg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad + Afg^2x^3+\frac{3}{2}Bf^2gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Af^2gx^2 \\
&\quad - \frac{1}{24}Bg^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-a}{b^3d^3}\right. \\
&\quad + \frac{1}{2}Bfg^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad - \frac{3}{2}Bf^2gn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad + Bf^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^3x
\end{aligned}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] 1/4*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*g^3*x^4 + B*f*
g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^2*x^3 + 3/2*B*f^2*g*
x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*f^2*g*x^2 - 1/24*B*g^3*n
*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^2*g*n*(
a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^3*
n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6772 vs. 2(223) = 446.

Time = 1.05 (sec) , antiderivative size = 6772, normalized size of antiderivative = 28.82

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4
*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d
^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b
*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/
(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^
2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)
^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2*g*n +
6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x + c) +
6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(d*x + c)
- 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*d^5*f^2*g*
n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*b
^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g*n/(d*x + c)
 - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a*b*
c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n/(d*x + c)^2 +
 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^4*d*f*g^2*n - 4
*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2*n/(d*x + c) + 1
6*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*b^3*c^4*d^
3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b*x + a)*B*a^2*b^2*
c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d^4*f*g^2*n/(d*x + c
)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)^3 + 4*B*a^4*b*d^5*f*g^
2*n - 8*(b*x + a)*B*a^3*b*c*d^5*f*g^2*n/(d*x + c) + 24*(b*x + a)^3*B*a*b*c^
3*d^5*f*g^2*n/(d*x + c)^3 - 4*(b*x + a)*B*a^4*d^6*f*g^2*n/(d*x + c) + 12*(b
*x + a)^2*B*a^3*c*d^6*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*a^2*c^2*d^6*f*
g^2*n/(d*x + c)^3 - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + 4*(b*x + a)*B*b
^4*c^5*d*g^3*n/(d*x + c) - 4*(b*x + a)*B*a*b^3*c^4*d^2*g^3*n/(d*x + c) - 6*
(b*x + a)^2*B*b^3*c^5*d^2*g^3*n/(d*x + c)^2 + 6*(b*x + a)^2*B*a*b^2*c^4*d^3
*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*b^2*c^5*d^3*g^3*n/(d*x + c)^3 + B*a^4*
b*c*d^4*g^3*n - 4*(b*x + a)*B*a^3*b*c^2*d^4*g^3*n/(d*x + c) + 6*(b*x + a)^2
*B*a^2*b*c^3*d^4*g^3*n/(d*x + c)^2 - 8*(b*x + a)^3*B*a*b*c^4*d^4*g^3*n/(d*x
 + c)^3 - B*a^5*d^5*g^3*n + 4*(b*x + a)*B*a^4*c*d^5*g^3*n/(d*x + c) - 6*(b*
x + a)^2*B*a^3*c^2*d^5*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*a^2*c^3*d^5*g^3*
n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x
 + c) + 6*(b*x + a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3
 + (b*x + a)^4*d^8/(d*x + c)^4) - (36*B*b^8*c^3*d^2*f^2*g*n - 108*B*a*b^7*c
^2*d^3*f^2*g*n - 108*(b*x + a)*B*b^7*c^3*d^3*f^2*g*n/(d*x + c) + 108*B*a^2*
```

b^6*c*d^4*f^2*g*n + 324*(b*x + a)*B*a*b^6*c^2*d^4*f^2*g*n/(d*x + c) + 108*(
b*x + a)^2*B*b^6*c^3*d^4*f^2*g*n/(d*x + c)^2 - 36*B*a^3*b^5*d^5*f^2*g*n - 3
24*(b*x + a)*B*a^2*b^5*c*d^5*f^2*g*n/(d*x + c) - 324*(b*x + a)^2*B*a*b^5*c^
2*d^5*f^2*g*n/(d*x + c)^2 - 36*(b*x + a)^3*B*b^5*c^3*d^5*f^2*g*n/(d*x + c)^
3 + 108*(b*x + a)*B*a^3*b^4*d^6*f^2*g*n/(d*x + c) + 324*(b*x + a)^2*B*a^2*b
^4*c*d^6*f^2*g*n/(d*x + c)^2 + 108*(b*x + a)^3*B*a*b^4*c^2*d^6*f^2*g*n/(d*x
 + c)^3 - 108*(b*x + a)^2*B*a^3*b^3*d^7*f^2*g*n/(d*x + c)^2 - 108*(b*x + a)
^3*B*a^2*b^3*c*d^7*f^2*g*n/(d*x + c)^3 + 36*(b*x + a)^3*B*a^3*b^2*d^8*f^2*g
*n/(d*x + c)^3 - 36*B*b^8*c^4*d*f*g^2*n + 72*B*a*b^7*c^3*d^2*f*g^2*n + 120*
(b*x + a)*B*b^7*c^4*d^2*f*g^2*n/(d*x + c) - 264*(b*x + a)*B*a*b^6*c^3*d^3*f
*g^2*n/(d*x + c) - 132*(b*x + a)^2*B*b^6*c^4*d^3*f*g^2*n/(d*x + c)^2 - 72*B
*a^3*b^5*c*d^4*f*g^2*n + 72*(b*x + a)*B*a^2*b^5*c^2*d^4*f*g^2*n/(d*x + c) +
 312*(b*x + a)^2*B*a*b^5*c^3*d^4*f*g^2*n/(d*x + c)^2 + 48*(b*x + a)^3*B*b^5
*c^4*d^4*f*g^2*n/(d*x + c)^3 + 36*B*a^4*b^4*d^5*f*g^2*n + 168*(b*x + a)*B*a
^3*b^4*c*d^5*f*g^2*n/(d*x + c) - 144*(b*x + a)^2*B*a^2*b^4*c^2*d^5*f*g^2*n/
(d*x + c)^2 - 120*(b*x + a)^3*B*a*b^4*c^3*d^5*f*g^2*n/(d*x + c)^3 - 96*(b*x
 + a)*B*a^4*b^3*d^6*f*g^2*n/(d*x + c) - 120*(b*x + a)^2*B*a^3*b^3*c*d^6*f*g
^2*n/(d*x + c)^2 + 72*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f*g^2*n/(d*x + c)^3 + 8
4*(b*x + a)^2*B*a^4*b^2*d^7*f*g^2*n/(d*x + c)^2 + 24*(b*x + a)^3*B*a^3*b^2*
c*d^7*f*g^2*n/(d*x + c)^3 - 24*(b*x + a)^3*B*a^4*b*d^8*f*g^2*n/(d*x + c)^3
+ 11*B*b^8*c^5*g^3*n - 19*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d*g^
3*n/(d*x + c) + 2*B*a^2*b^6*c^3*d^2*g^3*n + 70*(b*x + a)*B*a*b^6*c^4*d^2*g^
3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 2*B*a^3*b^
5*c^2*d^3*g^3*n - 8*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 93*(b*x +
 a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c^5*d^3*g^3*
n/(d*x + c)^3 + 19*B*a^4*b^4*c*d^4*g^3*n - 16*(b*x + a)*B*a^3*b^4*c^2*d^4*g
^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 42*(b
*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5*g^3*n - 34*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) + 18*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 2
6*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*a^4*b^2*c*d^6*
g^3*n/(d*x + c)^2 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 6*(b*x +
 a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x
+ c)^3 - 24*B*b^8*c^2*d^3*f^3*log(e) + 48*B*a*b^7*c*d^4*f^3*log(e) + 72*(b*
x + a)*B*b^7*c^2*d^4*f^3*log(e)/(d*x + c) - 24*B*a^2*b^6*d^5*f^3*log(e) - 1
44*(b*x + a)*B*a*b^6*c*d^5*f^3*log(e)/(d*x + c) - 72*(b*x + a)^2*B*b^6*c^2*
d^5*f^3*log(e)/(d*x + c)^2 + 72*(b*x + a)*B*a^2*b^5*d^6*f^3*log(e)/(d*x + c
) + 144*(b*x + a)^2*B*a*b^5*c*d^6*f^3*log(e)/(d*x + c)^2 + 24*(b*x + a)^3*B
*b^5*c^2*d^6*f^3*log(e)/(d*x + c)^3 - 72*(b*x + a)^2*B*a^2*b^4*d^7*f^3*log(
e)/(d*x + c)^2 - 48*(b*x + a)^3*B*a*b^4*c*d^7*f^3*log(e)/(d*x + c)^3 + 24*(
b*x + a)^3*B*a^2*b^3*d^8*f^3*log(e)/(d*x + c)^3 + 36*B*b^8*c^3*d^2*f^2*g*lo
g(e) - 36*B*a*b^7*c^2*d^3*f^2*g*log(e) - 144*(b*x + a)*B*b^7*c^3*d^3*f^2*g*
log(e)/(d*x + c) - 36*B*a^2*b^6*c*d^4*f^2*g*log(e) + 216*(b*x + a)*B*a*b^6*
c^2*d^4*f^2*g*log(e)/(d*x + c) + 180*(b*x + a)^2*B*b^6*c^3*d^4*f^2*g*log(e)
/(d*x + c)^2 + 36*B*a^3*b^5*d^5*f^2*g*log(e) - 324*(b*x + a)^2*B*a*b^5*c^2*

d^5*f^2*g*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*b^5*c^3*d^5*f^2*g*log(e)/(d*x + c)^3 - 72*(b*x + a)*B*a^3*b^4*d^6*f^2*g*log(e)/(d*x + c) + 108*(b*x + a)^2*B*a^2*b^4*c*d^6*f^2*g*log(e)/(d*x + c)^2 + 144*(b*x + a)^3*B*a*b^4*c^2*d^6*f^2*g*log(e)/(d*x + c)^3 + 36*(b*x + a)^2*B*a^3*b^3*d^7*f^2*g*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a^2*b^3*c*d^7*f^2*g*log(e)/(d*x + c)^3 - 24*B*b^8*c^4*d*f*g^2*log(e) + 24*B*a*b^7*c^3*d^2*f*g^2*log(e) + 96*(b*x + a)*B*b^7*c^4*d^2*f*g^2*log(e)/(d*x + c) - 96*(b*x + a)*B*a*b^6*c^3*d^3*f*g^2*log(e)/(d*x + c) - 144*(b*x + a)^2*B*b^6*c^4*d^3*f*g^2*log(e)/(d*x + c)^2 + 24*B*a^3*b^5*c*d^4*f*g^2*log(e) - 72*(b*x + a)*B*a^2*b^5*c^2*d^4*f*g^2*log(e)/(d*x + c) + 216*(b*x + a)^2*B*a*b^5*c^3*d^4*f*g^2*log(e)/(d*x + c)^2 + 72*(b*x + a)^3*B*b^5*c^4*d^4*f*g^2*log(e)/(d*x + c)^3 - 24*B*a^4*b^4*d^5*f*g^2*log(e) + 48*(b*x + a)*B*a^3*b^4*c*d^5*f*g^2*log(e)/(d*x + c) - 144*(b*x + a)^3*B*a*b^4*c^3*d^5*f*g^2*log(e)/(d*x + c)^3 + 24*(b*x + a)*B*a^4*b^3*d^6*f*g^2*log(e)/(d*x + c) - 72*(b*x + a)^2*B*a^3*b^3*c*d^6*f*g^2*log(e)/(d*x + c)^2 + 72*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f*g^2*log(e)/(d*x + c)^3 + 6*B*b^8*c^5*g^3*log(e) - 6*B*a*b^7*c^4*d*g^3*log(e) - 24*(b*x + a)*B*b^7*c^5*d*g^3*log(e)/(d*x + c) + 24*(b*x + a)*B*a*b^6*c^4*d^2*g^3*log(e)/(d*x + c) + 36*(b*x + a)^2*B*b^6*c^5*d^2*g^3*log(e)/(d*x + c)^2 - 36*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*log(e)/(d*x + c)^2 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^3*log(e)/(d*x + c)^3 - 6*B*a^4*b^4*c*d^4*g^3*log(e) + 24*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3*log(e)/(d*x + c) - 36*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*log(e)/(d*x + c)^2 + 48*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*log(e)/(d*x + c)^3 + 6*B*a^5*b^3*d^5*g^3*log(e) - 24*(b*x + a)*B*a^4*b^3*c*d^5*g^3*log(e)/(d*x + c) + 36*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*log(e)/(d*x + c)^2 - 24*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*log(e)/(d*x + c)^3 - 24*A*b^8*c^2*d^3*f^3 + 48*A*a*b^7*c*d^4*f^3 + 72*(b*x + a)*A*b^7*c^2*d^4*f^3/(d*x + c) - 24*A*a^2*b^6*d^5*f^3 - 144*(b*x + a)*A*a*b^6*c*d^5*f^3/(d*x + c) - 72*(b*x + a)^2*A*b^6*c^2*d^5*f^3/(d*x + c)^2 + 72*(b*x + a)*A*a^2*b^5*d^6*f^3/(d*x + c) + 144*(b*x + a)^2*A*a*b^5*c*d^6*f^3/(d*x + c)^2 + 24*(b*x + a)^3*A*b^5*c^2*d^6*f^3/(d*x + c)^3 - 72*(b*x + a)^2*A*a^2*b^4*d^7*f^3/(d*x + c)^2 - 48*(b*x + a)^3*A*a*b^4*c*d^7*f^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^2*b^3*d^8*f^3/(d*x + c)^3 + 36*A*b^8*c^3*d^2*f^2*g - 36*A*a*b^7*c^2*d^3*f^2*g - 144*(b*x + a)*A*b^7*c^3*d^3*f^2*g/(d*x + c) - 36*A*a^2*b^6*c*d^4*f^2*g + 216*(b*x + a)*A*a*b^6*c^2*d^4*f^2*g/(d*x + c) + 180*(b*x + a)^2*A*b^6*c^3*d^4*f^2*g/(d*x + c)^2 + 36*A*a^3*b^5*d^5*f^2*g - 324*(b*x + a)^2*A*a*b^5*c^2*d^5*f^2*g/(d*x + c)^2 - 72*(b*x + a)^3*A*b^5*c^3*d^5*f^2*g/(d*x + c)^3 - 72*(b*x + a)*A*a^3*b^4*d^6*f^2*g/(d*x + c) + 108*(b*x + a)^2*A*a^2*b^4*c*d^6*f^2*g/(d*x + c)^2 + 144*(b*x + a)^3*A*a*b^4*c^2*d^6*f^2*g/(d*x + c)^3 + 36*(b*x + a)^2*A*a^3*b^3*d^7*f^2*g/(d*x + c)^2 - 72*(b*x + a)^3*A*a^2*b^3*c*d^7*f^2*g/(d*x + c)^3 - 24*A*b^8*c^4*d*f*g^2 + 24*A*a*b^7*c^3*d^2*f*g^2 + 96*(b*x + a)*A*b^7*c^4*d^2*f*g^2/(d*x + c) - 96*(b*x + a)*A*a*b^6*c^3*d^3*f*g^2/(d*x + c) - 144*(b*x + a)^2*A*b^6*c^4*d^3*f*g^2/(d*x + c)^2 + 24*A*a^3*b^5*c*d^4*f*g^2 - 72*(b*x + a)*A*a^2*b^5*c^2*d^4*f*g^2/(d*x + c) + 216*(b*x + a)^2*A*a*b^5*c^3*d^4*f*g^2/(d*x + c)^2 + 72*(b*x + a)^3*A*b^5*c^4*d^4*f*g^2/(d*x + c)^3 - 24*A*a^4*b^4*d^5*f*g^2 + 48*(b*x + a)*A*a^3*b^4*c*d^5*f*g^2/(d*x + c) - 144*(b*x + a)^3*A*a*b^

```
4*c^3*d^5*f*g^2/(d*x + c)^3 + 24*(b*x + a)*A*a^4*b^3*d^6*f*g^2/(d*x + c) -
72*(b*x + a)^2*A*a^3*b^3*c*d^6*f*g^2/(d*x + c)^2 + 72*(b*x + a)^3*A*a^2*b^3
*c^2*d^6*f*g^2/(d*x + c)^3 + 6*A*b^8*c^5*g^3 - 6*A*a*b^7*c^4*d*g^3 - 24*(b*
x + a)*A*b^7*c^5*d*g^3/(d*x + c) + 24*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x +
c) + 36*(b*x + a)^2*A*b^6*c^5*d^2*g^3/(d*x + c)^2 - 36*(b*x + a)^2*A*a*b^5*
c^4*d^3*g^3/(d*x + c)^2 - 24*(b*x + a)^3*A*b^5*c^5*d^3*g^3/(d*x + c)^3 - 6*
A*a^4*b^4*c*d^4*g^3 + 24*(b*x + a)*A*a^3*b^4*c^2*d^4*g^3/(d*x + c) - 36*(b*
x + a)^2*A*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 + 48*(b*x + a)^3*A*a*b^4*c^4*d^4
*g^3/(d*x + c)^3 + 6*A*a^5*b^3*d^5*g^3 - 24*(b*x + a)*A*a^4*b^3*c*d^5*g^3/(
d*x + c) + 36*(b*x + a)^2*A*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 24*(b*x + a)^
3*A*a^2*b^3*c^3*d^5*g^3/(d*x + c)^3)/(b^7*d^4 - 4*(b*x + a)*b^6*d^5/(d*x +
c) + 6*(b*x + a)^2*b^5*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b^4*d^7/(d*x + c)^3
+ (b*x + a)^4*b^3*d^8/(d*x + c)^4) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c
*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^
2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b
^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n +
4*B*a^4*b*d^5*f*g^2*n - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d
^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^4*d^4) - 6*(
4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^3*n - 6*B
*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d^4*f^2*g*
n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g
^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b^5*c^5*g^3*n
+ B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a
)/(d*x + c))/(b^4*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.26

$$\int (f+gx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g\,n-6\,B\,b\,c\,f^2\,g\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)\,(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A \cdots}{\cdots}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)\,(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2\,n-4\,B\,b\,c\,f\,g^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{12\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)+\frac{A\,g^3\,x^4}{4}$$

$$-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,g^3-4\,B\,n\,a^3\,b\,f\,g^2+6\,B\,n\,a^2\,b^2\,f^2\,g-4\,B\,n\,a\,b^3\,f^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,g^3-4\,B\,n\,c^3\,d\,f\,g^2+6\,B\,n\,c^2\,d^2\,f^2\,g-4\,B\,n\,c\,d^3\,f^3\right)}{4\,d^4}$$

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g*n - 6*B*b*c*f^2*g*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4
*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d
```

```
*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*
n)/(4*b*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3
 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2
 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4
*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 1
2*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*n)/(8*b*d) + (A*a*c*g^3)/(2
*b*d)) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B
*b*c*g^3*n)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + log(e*((a + b*x)
/(c + d*x))^n)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3)
+ (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^3*n - 4*B*a*b^3*f^3*n - 4*B*a^3*b*
f*g^2*n + 6*B*a^2*b^2*f^2*g*n))/(4*b^4) + (log(c + d*x)*(B*c^4*g^3*n - 4*B*
c*d^3*f^3*n - 4*B*c^3*d*f*g^2*n + 6*B*c^2*d^2*f^2*g*n))/(4*d^4)
```

3.59 $\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 157

$$\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2}-\frac{B(bc-ad)g^2nx^2}{6bd}-\frac{B(bf-ag)^3n\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g}+\frac{B(df-cg)^3n\log(c+dx)}{3d^3g}$$

```
[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2
*n*x^2/b/d-1/3*B*(-a*g+b*f)^3*n*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/g+1/3*B*(-c*g+d*f)^3*n*ln(d*x+c)/d^3/g
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2547, 84}

$$\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bgnx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2nx^2(bc-ad)}{6bd}+\frac{Bn(df-cg)^3\log(c+dx)}{3d^3g}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(b^2*d^2) - (B*(b*c -
a*d)*g^2*n*x^2)/(6*b*d) - (B*(b*f - a*g)^3*n*Log[a + b*x])/(3*b^3*g) + ((f
+ g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) + (B*(d*f - c*g)^3*n
*Log[c + d*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g}-\frac{(B(bc-ad)n)\int\frac{(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}\\
&= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g}\\
&\quad -\frac{(B(bc-ad)n)\int\left(\frac{g^2(3bdf-bcg-adg)}{b^2d^2}+\frac{g^3x}{bd}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(-bc+ad)(c+dx)}\right)dx}{3g}\\
&= -\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2}\\
&\quad -\frac{B(bc-ad)g^2nx^2}{6bd}-\frac{B(bf-ag)^3n\log(a+bx)}{3b^3g}\\
&\quad +\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g}+\frac{B(df-cg)^3n\log(c+dx)}{3d^3g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{Bn(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg}{2b^3d^3}}{3g}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/
(3*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(147) = 294$.

Time = 1.63 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.93

method	result
parallelrisch	$\frac{-6Aab^2cd^2fgn+6Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^3fgn+6Bxab^2d^3fgn^2-6Bxb^3cd^2fgn^2-6B\ln(bx+a)a^2bd^3fgn^2+6B\ln(bx+a)}{}$

```
[In] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-6*A*a*b^2*c*d^2*f*g*n+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*f*g*n
+6*B*x*a*b^2*d^3*f*g*n^2-6*B*x*b^3*c*d^2*f*g*n^2-6*B*ln(b*x+a)*a^2*b*d^3*f*
g*n^2+6*B*ln(b*x+a)*b^3*c^2*d*f*g*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2
*d*f*g*n-6*B*a^2*b*d^3*f*g*n^2+6*B*b^3*c^2*d*f*g*n^2+6*B*ln(e*((b*x+a)/(d*x
+c))^n)*b^3*c*d^2*f^2*n-6*A*a*b^2*d^3*f^2*n-6*A*b^3*c*d^2*f^2*n+6*A*x*b^3*d
^3*f^2*n+2*B*a^3*d^3*g^2*n^2-2*B*b^3*c^3*g^2*n^2+6*A*x^2*b^3*d^3*f*g*n+6*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*f^2*n+6*B*ln(b*x+a)*a*b^2*d^3*f^2*n^2-6
*B*ln(b*x+a)*b^3*c*d^2*f^2*n^2+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g^
2*n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2-2*B*x*a^2*b*d^3*g^2*n^2
+2*B*x*b^3*c^2*d*g^2*n^2+B*a^2*b*c*d^2*g^2*n^2-B*a*b^2*c^2*d*g^2*n^2+2*A*x^
3*b^3*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+2*B*ln(b*x+a)*a
^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b^3/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(147) = 294.

Time = 0.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.13

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{2\,Ab^3d^3g^2x^3 + (6\,Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2\,(3\,Bab^2d^3f^2 - 3\,Ba^2bd^3fg + Ba^3d^3g^2)n\log\left(bx\right.}{}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g
^2*n)*x^2 + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*n*log
(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*n*log
(d*x + c) + 2*(3*A*b^3*d^3*f^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g - (B*b^
3*c^2*d - B*a^2*b*d^3)*g^2)*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x
^2 + 3*B*b^3*d^3*f^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3 + 3*B*b^3*d^3*f*g*n
*x^2 + 3*B*b^3*d^3*f^2*n*x)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.80

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{3}\,Bg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ag^2x^3$$
$$+ Bfgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afgx^2$$
$$+ \frac{1}{6}\,Bg^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$- Bfgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+ Bf^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^2x$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*g^2*x^3 + B*f*
g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g*x^2 + 1/6*B*g^2*n*(2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*f*g*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^2*n*(a*log(b*x + a)/b - c*log(d
*x + c)/d) + B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. $2(147) = 294$.

Time = 0.72 (sec) , antiderivative size = 3408, normalized size of antiderivative = 21.71

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c
^2*d^3*f^2*n/(d*x + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4
*f^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x +
 a)*B*a^2*b*d^5*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)
^2 + 3*(b*x + a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*
a*b^3*c^2*d^2*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b
^2*c*d^3*f*g*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)
^2*B*b^2*c^3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^
2*b*c*d^4*f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2
```

+ 3*(b*x + a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(
d*x + c)^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*
d*g^2*n/(d*x + c) + 3*(b*x + a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x +
a)^2*B*b^2*c^4*d^2*g^2*n/(d*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*
a^2*b*c^2*d^3*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)
^2 + B*a^4*d^4*g^2*n - 3*(b*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a
)^2*B*a^2*c^2*d^4*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3 - 3*
(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3
*d^6/(d*x + c)^3) - (6*B*b^6*c^3*d*f*g*n - 18*B*a*b^5*c^2*d^2*f*g*n - 12*(b
*x + a)*B*b^5*c^3*d^2*f*g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f*g*n + 36*(b*x
+ a)*B*a*b^4*c^2*d^3*f*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f*g*n/(d
*x + c)^2 - 6*B*a^3*b^3*d^4*f*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f*g*n/(d*x
 + c) - 18*(b*x + a)^2*B*a*b^3*c^2*d^4*f*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a
^3*b^2*d^5*f*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g*n/(d*x + c)
^2 - 6*(b*x + a)^2*B*a^3*b*d^6*f*g*n/(d*x + c)^2 - 3*B*b^6*c^4*g^2*n + 6*B*
a*b^5*c^3*d*g^2*n + 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) - 16*(b*x + a)*
B*a*b^4*c^3*d^2*g^2*n/(d*x + c) - 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x +
c)^2 - 6*B*a^3*b^3*c*d^3*g^2*n + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x +
 c) + 10*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^
2*n + 8*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2
*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 2*(b
*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6*g^2*n/(
d*x + c)^2 - 6*B*b^6*c^2*d^2*f^2*log(e) + 12*B*a*b^5*c*d^3*f^2*log(e) + 12*
(b*x + a)*B*b^5*c^2*d^3*f^2*log(e)/(d*x + c) - 6*B*a^2*b^4*d^4*f^2*log(e) -
 24*(b*x + a)*B*a*b^4*c*d^4*f^2*log(e)/(d*x + c) - 6*(b*x + a)^2*B*b^4*c^2*
d^4*f^2*log(e)/(d*x + c)^2 + 12*(b*x + a)*B*a^2*b^3*d^5*f^2*log(e)/(d*x + c
) + 12*(b*x + a)^2*B*a*b^3*c*d^5*f^2*log(e)/(d*x + c)^2 - 6*(b*x + a)^2*B*a
^2*b^2*d^6*f^2*log(e)/(d*x + c)^2 + 6*B*b^6*c^3*d*f*g*log(e) - 6*B*a*b^5*c^
2*d^2*f*g*log(e) - 18*(b*x + a)*B*b^5*c^3*d^2*f*g*log(e)/(d*x + c) - 6*B*a^
2*b^4*c*d^3*f*g*log(e) + 30*(b*x + a)*B*a*b^4*c^2*d^3*f*g*log(e)/(d*x + c)
+ 12*(b*x + a)^2*B*b^4*c^3*d^3*f*g*log(e)/(d*x + c)^2 + 6*B*a^3*b^3*d^4*f*g
log(e) - 6(b*x + a)*B*a^2*b^3*c*d^4*f*g*log(e)/(d*x + c) - 24*(b*x + a)^2
*B*a*b^3*c^2*d^4*f*g*log(e)/(d*x + c)^2 - 6*(b*x + a)*B*a^3*b^2*d^5*f*g*log
(e)/(d*x + c) + 12*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g*log(e)/(d*x + c)^2 - 2*B
*b^6*c^4*g^2*log(e) + 2*B*a*b^5*c^3*d*g^2*log(e) + 6*(b*x + a)*B*b^5*c^4*d*
g^2*log(e)/(d*x + c) - 6*(b*x + a)*B*a*b^4*c^3*d^2*g^2*log(e)/(d*x + c) - 6
*(b*x + a)^2*B*b^4*c^4*d^2*g^2*log(e)/(d*x + c)^2 + 2*B*a^3*b^3*c*d^3*g^2*l
og(e) - 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*log(e)/(d*x + c) + 12*(b*x + a)^2
*B*a*b^3*c^3*d^3*g^2*log(e)/(d*x + c)^2 - 2*B*a^4*b^2*d^4*g^2*log(e) + 6*(b
*x + a)*B*a^3*b^2*c*d^4*g^2*log(e)/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2*c^2*
d^4*g^2*log(e)/(d*x + c)^2 - 6*A*b^6*c^2*d^2*f^2 + 12*A*a*b^5*c*d^3*f^2 + 1
2*(b*x + a)*A*b^5*c^2*d^3*f^2/(d*x + c) - 6*A*a^2*b^4*d^4*f^2 - 24*(b*x + a
)*A*a*b^4*c*d^4*f^2/(d*x + c) - 6*(b*x + a)^2*A*b^4*c^2*d^4*f^2/(d*x + c)^2
 + 12*(b*x + a)*A*a^2*b^3*d^5*f^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c*d^5*
f^2/(d*x + c)^2 - 6*(b*x + a)^2*A*a^2*b^2*d^6*f^2/(d*x + c)^2 + 6*A*b^6*c^3

```
*d*f*g - 6*A*a*b^5*c^2*d^2*f*g - 18*(b*x + a)*A*b^5*c^3*d^2*f*g/(d*x + c) -
 6*A*a^2*b^4*c*d^3*f*g + 30*(b*x + a)*A*a*b^4*c^2*d^3*f*g/(d*x + c) + 12*(b
*x + a)^2*A*b^4*c^3*d^3*f*g/(d*x + c)^2 + 6*A*a^3*b^3*d^4*f*g - 6*(b*x + a)
*A*a^2*b^3*c*d^4*f*g/(d*x + c) - 24*(b*x + a)^2*A*a*b^3*c^2*d^4*f*g/(d*x +
c)^2 - 6*(b*x + a)*A*a^3*b^2*d^5*f*g/(d*x + c) + 12*(b*x + a)^2*A*a^2*b^2*c
*d^5*f*g/(d*x + c)^2 - 2*A*b^6*c^4*g^2 + 2*A*a*b^5*c^3*d*g^2 + 6*(b*x + a)*
A*b^5*c^4*d*g^2/(d*x + c) - 6*(b*x + a)*A*a*b^4*c^3*d^2*g^2/(d*x + c) - 6*(
b*x + a)^2*A*b^4*c^4*d^2*g^2/(d*x + c)^2 + 2*A*a^3*b^3*c*d^3*g^2 - 6*(b*x +
 a)*A*a^2*b^3*c^2*d^3*g^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^3*d^3*g^2/(d
*x + c)^2 - 2*A*a^4*b^2*d^4*g^2 + 6*(b*x + a)*A*a^3*b^2*c*d^4*g^2/(d*x + c)
 - 6*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2)/(b^5*d^3 - 3*(b*x + a)*
b^4*d^4/(d*x + c) + 3*(b*x + a)^2*b^3*d^5/(d*x + c)^2 - (b*x + a)^3*b^2*d^6
/(d*x + c)^3) + 2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*
b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2
*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n
- B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b^
3*d^3) - 2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4
*f^2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*
f*g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3
*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.36

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d}\right.$$
$$-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g\,n-3\,B\,b\,c\,f\,g\,n}{3\,b\,d}$$
$$\left.+\frac{A\,a\,c\,g^2}{b\,d}\right)+\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$+\frac{A\,g^2\,x^3}{3}+\frac{\ln\left(a+b\,x\right)\,\left(B\,n\,a^3\,g^2-3\,B\,n\,a^2\,b\,f\,g+3\,B\,n\,a\,b^2\,f^2\right)}{3\,b^3}$$
$$-\frac{\ln\left(c+d\,x\right)\,\left(B\,n\,c^3\,g^2-3\,B\,n\,c^2\,d\,f\,g+3\,B\,n\,c\,d^2\,f^2\right)}{3\,d^3}$$

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/
(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*(((3*a*d + 3*b*c)*((3*A*a*d*
g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/(3*b*d) - (A*g
^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*
d*f*g + 6*A*b*c*f*g + 3*B*a*d*f*g*n - 3*B*b*c*f*g*n)/(3*b*d) + (A*a*c*g^2)/
(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^
2) + (A*g^2*x^3)/3 + (log(a + b*x)*(B*a^3*g^2*n + 3*B*a*b^2*f^2*n - 3*B*a^2
*b*f*g*n))/(3*b^3) - (log(c + d*x)*(B*c^3*g^2*n + 3*B*c*d^2*f^2*n - 3*B*c^2
*d*f*g*n))/(3*d^3)
```

3.60 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 115

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=-\frac{B(bc-ad)gnx}{2bd}-\frac{B(bf-ag)^2n\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}+\frac{B(df-cg)^2n\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b/d-1/2*B*(-a*g+b*f)^2*n*ln(b*x+a)/b^2/g+1/2*(g*x+f
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/2*B*(-c*g+d*f)^2*n*ln(d*x+c)/d^2/g
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2547, 84}

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgnx(bc-ad)}{2bd}+\frac{Bn(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/(b*d) - (B*(b*f - a*g)^2*n*Log[a + b*x])/(2*b^2*
g) + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) + (B*(d*f -
 c*g)^2*n*Log[c + d*x])/(2*d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}-\frac{(B(bc-ad)n)\int\frac{(f+gx)^2}{(a+bx)(c+dx)}\,dx}{2g}\\
&=\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}-\frac{(B(bc-ad)n)\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(-bc+ad)(c+dx)}\right)dx}{2g}\\
&=-\frac{B(bc-ad)gnx}{2bd}-\frac{B(bf-ag)^2n\log(a+bx)}{2b^2g}\\
&\quad+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}+\frac{B(df-cg)^2n\log(c+dx)}{2d^2g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04

$$\int(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{-Bd^2(bf-ag)^2n\log(a+bx)+b\Big(d\big(B(-bc+ad)g^2nx+Abd(f+gx)^2\big)+bBd^2(f+gx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\cdots}{2b^2d^2g}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*n*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*n*x + A
*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + b*
B*(d*f - c*g)^2*n*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(107) = 214$.

Time = 0.81 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.18

method	result
parallelrisch	$\frac{-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{2}f-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}cdf-B\ln(bx+a)a^{2}d^{2}gn+B\ln(dx+c)b^{2}c^{2}gn+B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{2}g-B}{}$

```
[In] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*f-2*B*ln(e*((b*x+a)/(d*x+c))^n)
*b^2*c*d*f-B*ln(b*x+a)*a^2*d^2*g*n+B*ln(d*x+c)*b^2*c^2*g*n+B*x^2*ln(e*((b*x
+a)/(d*x+c))^n)*b^2*d^2*g-B*a^2*d^2*g*n+B*b^2*c^2*g*n-4*B*ln(d*x+c)*b^2*c*d
*f*n+B*x*a*b*d^2*g*n-B*x*b^2*c*d*g*n-B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*g+
4*B*ln(b*x+a)*a*b*d^2*f*n+2*B*ln(b*x+a)*b^2*c*d*f*n-2*B*ln(d*x+c)*a*b*d^2*f
*n+B*ln(b*x+a)*a*b*c*d*g*n-B*ln(d*x+c)*a*b*c*d*g*n-A*a*b*c*d*g-2*A*a*b*d^2*
f-2*A*b^2*c*d*f+2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*f+A*x^2*b^2*d^2*g+2
*A*x*b^2*d^2*f)/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)dx$$
$$=\frac{Ab^{2}d^{2}gx^{2}+(2\,Babd^{2}f-Ba^{2}d^{2}g)n\log(bx+a)-(2\,Bb^{2}cdf-Bb^{2}c^{2}g)n\log(dx+c)+(2\,Ab^{2}d^{2}f-(Bb^{2}}{2\,b^{2}d^{2}}$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + (2*B*a*b*d^2*f - B*a^2*d^2*g)*n*log(b*x + a) - (2*B*
b^2*c*d*f - B*b^2*c^2*g)*n*log(d*x + c) + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a
*b*d^2)*g*n)*x + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log(e) + (B*b^2*d^2*g*
n*x^2 + 2*B*b^2*d^2*f*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(97) = 194$.

Time = 59.23 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.29

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases}\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right)\left(fx+\frac{gx^2}{2}\right)\\ Afx+\frac{Agx^2}{2}-\frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d^2}+\frac{Bcf\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}-\frac{Bcgnx}{2d}+Bfnx+Bfx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bgnx^2}{4}+\frac{Bg\cdots}{} \\ Afx+\frac{Agx^2}{2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b^2}+\frac{Baf\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}+\frac{Bagnx}{2b}-Bfnx+Bfx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bgnx^2}{4}+\cdots \\ Afx+\frac{Agx^2}{2}-\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2}+\frac{Bafn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Baf\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}+\frac{Bagnx}{2b}+\cdots\end{cases}$$

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise(((A + B*log(e*(a/c)**n))*(f*x + g*x**2/2), Eq(b, 0) & Eq(d, 0)),
(A*f*x + A*g*x**2/2 - B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d**2) + B*c*f*log
(e*(a/(c + d*x))**n)/d - B*c*g*n*x/(2*d) + B*f*n*x + B*f*x*log(e*(a/(c + d*
x))**n) + B*g*n*x**2/4 + B*g*x**2*log(e*(a/(c + d*x))**n)/2, Eq(b, 0)), (A*
f*x + A*g*x**2/2 - B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b**2) + B*a*f*log(e*
(a/c + b*x/c)**n)/b + B*a*g*n*x/(2*b) - B*f*n*x + B*f*x*log(e*(a/c + b*x/c)
**n) - B*g*n*x**2/4 + B*g*x**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)), (A*f*x
 + A*g*x**2/2 - B*a**2*g*n*log(c/d + x)/(2*b**2) - B*a**2*g*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(2*b**2) + B*a*f*n*log(c/d + x)/b + B*a*f*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*g*n*x/(2*b) + B*c**2*g*n*log(c/d +
x)/(2*d**2) - B*c*f*n*log(c/d + x)/d - B*c*g*n*x/(2*d) + B*f*x*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n) + B*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,Bgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Agx^2$$
$$-\frac{1}{2}\,Bgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afx$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/2*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*g*x^2 - 1/2*B*g*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f
*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A*f*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(107) = 214$.

Time = 0.57 (sec) , antiderivative size = 1215, normalized size of antiderivative = 10.57

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f
*n/(d*x + c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) -
2*(b*x + a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2
*(b*x + a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*
b*c^2*d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x
+ c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d^3/(d*x + c) + (b*
x + a)^2*d^4/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a
)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c
^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x
 + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - 2*B*b^4*c^2*d*f*log(e) + 4*B*a*
b^3*c*d^2*f*log(e) + 2*(b*x + a)*B*b^3*c^2*d^2*f*log(e)/(d*x + c) - 2*B*a^2
*b^2*d^3*f*log(e) - 4*(b*x + a)*B*a*b^2*c*d^3*f*log(e)/(d*x + c) + 2*(b*x +
 a)*B*a^2*b*d^4*f*log(e)/(d*x + c) + B*b^4*c^3*g*log(e) - B*a*b^3*c^2*d*g*l
og(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(e)/(d*x + c) - B*a^2*b^2*c*d^2*g*log(
e) + 4*(b*x + a)*B*a*b^2*c^2*d^2*g*log(e)/(d*x + c) + B*a^3*b*d^3*g*log(e)
- 2*(b*x + a)*B*a^2*b*c*d^3*g*log(e)/(d*x + c) - 2*A*b^4*c^2*d*f + 4*A*a*b^
3*c*d^2*f + 2*(b*x + a)*A*b^3*c^2*d^2*f/(d*x + c) - 2*A*a^2*b^2*d^3*f - 4*(
b*x + a)*A*a*b^2*c*d^3*f/(d*x + c) + 2*(b*x + a)*A*a^2*b*d^4*f/(d*x + c) +
A*b^4*c^3*g - A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) - A*a^2
*b^2*c*d^2*g + 4*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) + A*a^3*b*d^3*g - 2*
(b*x + a)*A*a^2*b*c*d^3*g/(d*x + c))/(b^3*d^2 - 2*(b*x + a)*b^2*d^3/(d*x +
c) + (b*x + a)^2*b*d^4/(d*x + c)^2) + (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*
f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2
*g*n - B*a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d^2) - (2*B*b^3*
c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b
^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/
(b^2*d^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g\,n-B\,b\,c\,g\,n}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g\,n-2\,B\,a\,b\,f\,n\right)}{2\,b^2}$$
$$+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g\,n-2\,B\,c\,d\,f\,n\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((2*A*a*d*g + 2*A*b*c*g + 2*A*b*d*f + B*a*d*g*n - B*b*c*g*n)/(2*b*d) - (A
*g*(2*a*d + 2*b*c))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*f*x + (B*g
*x^2)/2) - (log(a + b*x)*(B*a^2*g*n - 2*B*a*b*f*n))/(2*b^2) + (log(c + d*x)
*(B*c^2*g*n - 2*B*c*d*f*n))/(2*d^2) + (A*g*x^2)/2
```

3.61 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2535, 31}

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[In] Int[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= Ax + B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\,dx\\ &= Ax + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{(B(bc-ad)n)\int \frac{1}{c+dx}\,dx}{b}\\ &= Ax + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)n\log(c+dx)}{bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Ax + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)n\log(c+dx)}{bd}$$

```
[In] Integrate[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

method	result	size
default	$Ax+B\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$	82
parts	$Ax+B\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$	82
parallelrisch	$\frac{B\left(\ln(bx+a)ad\,n^2-\ln(bx+a)bc\,n^2+x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)bdn+\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)bcn\right)}{bdn}+Ax$	87

```
[In] int(A+B*ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

```
[Out] A*x+B*(ln(e*((b*x+a)/(d*x+c))^n)*x+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)+a/
(a*d-b*c)/b*ln(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{Bbdnx\log\left(\frac{bx+a}{dx+c}\right)+Badn\log(bx+a)-Bbcn\log(dx+c)+Bbdx\log(e)+Abdx}{bd}$$

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

```
[Out] (B*b*d*n*x*log((b*x + a)/(d*x + c)) + B*a*d*n*log(b*x + a) - B*b*c*n*log(d*
x + c) + B*b*d*x*log(e) + A*b*d*x)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.68

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = Ax + B\left(\begin{cases} x\log\left(e\left(\frac{a}{c}\right)^n\right) & \text{for } b=0 \wedge d=0 \\ \frac{c\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+nx+x\log\left(e\left(\frac{a}{c+dx}\right)^n\right) & \text{for } b=0 \\ \frac{a\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-nx+x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right) & \text{for } d=0 \\ \frac{an\log(c+dx)}{b}+\frac{a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}-\frac{cn\log(c+dx)}{d}+x\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] A*x + B*Piecewise((x*log(e*(a/c)**n), Eq(b, 0) & Eq(d, 0)), (c*log(e*(a/(c
+ d*x))**n)/d + n*x + x*log(e*(a/(c + d*x))**n), Eq(b, 0)), (a*log(e*(a/c +
 b*x/c)**n)/b - n*x + x*log(e*(a/c + b*x/c)**n), Eq(d, 0)), (a*n*log(c + d*
x)/b + a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b - c*n*log(c + d*x)/d + x
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = Bn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) + Bx\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax$$

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

```
[Out] B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*x*log(e*((b*x + a)/(d*x + c))
^n) + A*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(56) = 112$.

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.34

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= B\left(\frac{(b^2c^2n-2\,abcdn+a^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{b^2c^2\log(e)-2\,abcd\log(e)+a^2d^2\log(e)}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(b^2c^2n-2\,abcdn+\right.$$
$$+Ax$$

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

```
[Out] B*((b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d - (b
*x + a)*d^2/(d*x + c)) + (b^2*c^2*log(e) - 2*a*b*c*d*log(e) + a^2*d^2*log(e
))/(b*d - (b*x + a)*d^2/(d*x + c)) + (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*
log(b - (b*x + a)*d/(d*x + c))/(b*d) - (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n
)*log((b*x + a)/(d*x + c))/(b*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2) +
 A*x
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = A\,x+B\,x\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)+\frac{B\,a\,n\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln\left(c+d\,x\right)}{d}$$

```
[In] int(A + B*log(e*((a + b*x)/(c + d*x))^n),x)
```

```
[Out] A*x + B*x*log(e*((a + b*x)/(c + d*x))^n) + (B*a*n*log(a + b*x))/b - (B*c*n*
log(c + d*x))/d
```

3.62 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

```
[Out] -B*n*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*((b*x+a)/(d*x+c))^n))*
ln(g*x+f)/g+B*n*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*n*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+B*n*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

= {2545, 2441, 2440, 2438}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g} - \frac{Bn\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} - \frac{Bn\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g} + \frac{Bn\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g} + \frac{Bn\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

```
[Out] -((B*n*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[f + g*x])/g + (B*n*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (B*n*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B
*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(bBn)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bdn)\int\frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + (Bn)\int\frac{\log\left(\frac{g(a+bx)}{-bf+ag}\right)}{f+gx}\,dx - (Bn)\int\frac{\log\left(\frac{g(c+dx)}{-df+cg}\right)}{f+gx}\,dx$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + \frac{(Bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bf+ag}\right)}{x}\,dx,x,f+gx\right)}{g} - \frac{(Bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{-df+cg}\right)}{x}\,dx,x,f+gx\right)}{g}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{Bn\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \frac{\left(A-Bn\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-Bn\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+ \cdots}{g}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

```
[Out] ((A - B*n*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[e*((a + b*x)/(c + d*x))
^n] + B*n*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*n*PolyLog[2,
(b*(f + g*x))/(b*f - a*g)] + B*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [F]

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx+f}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(g*x + f), x)
+ A*log(g*x + f)/g
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f+g\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x), x)
```

3.63 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)} + \frac{B(bc-ad)n\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*n*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2553, 2351, 31}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)} + \frac{Bn(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*g)*(f + g*x))
+ (B*(b*c - a*d)*n*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)} - \frac{(B(bc-ad)n)\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bf-ag}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)} + \frac{B(bc-ad)n\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} + \frac{Bn(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)) + (B*n*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(91) = 182.

Time = 3.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.00

method	result
parallelrisch	$\frac{Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)abcd\,f^{2}n-Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}cdfgn+B\ln(bx+a)x\,a^{2}cdfg\,n^{2}-B\ln(bx+a)xab\,c^{2}fg\,n^{2}-B\ln(gx+f)x\,a^{2}c}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*f^2*n-B*x*ln(e*((b*x+a)/(d*x+c))^n)*
a^2*c*d*f*g*n+B*ln(b*x+a)*x*a^2*c*d*f*g*n^2-B*ln(b*x+a)*x*a*b*c^2*f*g*n^2-B
*ln(g*x+f)*x*a^2*c*d*f*g*n^2+B*ln(g*x+f)*x*a*b*c^2*f*g*n^2+A*x*a^2*c^2*g^2*
n-A*x*a^2*c*d*f*g*n-A*x*a*b*c^2*f*g*n+A*x*a*b*c*d*f^2*n-B*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*c^2*f*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c^2*f^2*n+B*ln(b*x+a
)*a^2*c*d*f^2*n^2-B*ln(b*x+a)*a*b*c^2*f^2*n^2-B*ln(g*x+f)*a^2*c*d*f^2*n^2+B
*ln(g*x+f)*a*b*c^2*f^2*n^2)/(a*g-b*f)/(g*x+f)/n/(c*g-d*f)/a/c/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(91) = 182.

Time = 3.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.23

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(f+gx)^{2}}\,dx =$$
$$-\frac{Abdf^{2}+Aacg^{2}-(Abc+Aad)fg+(Bbdf^{2}+Bacg^{2}-(Bbc+Bad)fg)n\log\left(\frac{bx+a}{dx+c}\right)-((Bbdfg-Bbc}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g + (B*b*d*f^2 + B*a*c*g^2 - (B
*b*c + B*a*d)*f*g)*n*log((b*x + a)/(d*x + c)) - ((B*b*d*f*g - B*b*c*g^2)*n*
x + (B*b*d*f^2 - B*b*c*f*g)*n)*log(b*x + a) + ((B*b*d*f*g - B*a*d*g^2)*n*x
+ (B*b*d*f^2 - B*a*d*f*g)*n)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*n*x + (B*b
*c - B*a*d)*f*g*n)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*
f*g)*log(e))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 +
a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = Bn\left(\frac{b\log(bx+a)}{bfg-ag^2}-\frac{d\log(dx+c)}{dfg-cg^2}+\frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{g^2x+fg}-\frac{A}{g^2x+fg}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="maxima")
[Out] B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c
 - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(g^2*x + f*g) - A/(g^2*x + f*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(91) = 182.

Time = 0.55 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.07

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \left(\frac{(Bb^2c^2n-2\,Babcdn+Ba^2d^2n)\log\left(-bf+\frac{(bx+a)df}{dx+c}+ag-\frac{(bx+a)cg}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{(Bb^2c^2n-2\,Babcdn}{bdf^2-\frac{(bx+a)d^2f^2}{dx+c}-bcfg-adfg}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="giac")
[Out] ((B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log(-b*f + (b*x + a)*d*f/(d*x
+ c) + a*g - (b*x + a)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^
```

```
2) + (B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(
b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d
*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*n - 2*
B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*
d*f*g + a*c*g^2) + (B*b^2*c^2*log(e) - 2*B*a*b*c*d*log(e) + B*a^2*d^2*log(e
) + A*b^2*c^2 - 2*A*a*b*c*d + A*a^2*d^2)/(b*d*f^2 - (b*x + a)*d^2*f^2/(d*x
+ c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d*f*g/(d*x + c) + a*c*g^2 - (b*x +
 a)*c^2*g^2/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{\ln\left(f+g\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g} - \frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g\,\left(f+g\,x\right)}-\frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^2,x)
```

```
[Out] (B*d*n*log(c + d*x))/(c*g^2 - d*f*g) - (log(f + g*x)*(B*a*d*n - B*b*c*n))/(
a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (B*log(e*((a + b*x)/(c + d*x))^n))
/(g*(f + g*x)) - (B*b*n*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x)
```

3.64 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 190

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}-\frac{Bd^2n\log(c+dx)}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)n\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*n*ln(b*x+a)/g/(
-a*g+b*f)^2+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^2-1/2*B*d^2*n*ln
(d*x+c)/g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*ln(g*x+f)/
(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2547, 84}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2} - \frac{Bn(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2} - \frac{Bd^2n\log(c+dx)}{2g(df-cg)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log[a
 + b*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*g*
(f + g*x)^2) - (B*d^2*n*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*
(2*b*d*f - b*c*g - a*d*g)*n*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}-\frac{g^2(-2bdf+\ldots}{(bf-ag)^2(df-\ldots}\right.}{2g}$$

$$= -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}-\frac{Bd^2n\log(c+dx)}{2g(df-cg)^2}+\frac{B(bc-ad)(2bdf-bcg-adg)n\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}+B(bc-ad)n\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2) + B*(b*c - a*d)*n*((
b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f -
 a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g
 + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. 2(183) = 366.

Time = 14.21 (sec) , antiderivative size = 1376, normalized size of antiderivative = 7.24

method	result	size
parallelrisch	Expression too large to display	1376

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*B*ln(g*x+f)*b^3*c*d^2*f^3*g^2*n-B*ln(b*x+a)*x^2*b^3*c^2*d*g^5*n-B*
x*a^2*b*c*d^2*g^5*n+B*x*a^2*b*d^3*f*g^4*n+B*x*a*b^2*c^2*d*g^5*n-B*x*a*b^2*d
^3*f^2*g^3*n-B*x*b^3*c^2*d*f*g^4*n+B*x*b^3*c*d^2*f^2*g^3*n-2*B*ln(e*((b*x+a
)/(d*x+c))^n)*a^2*b*c*d^2*f*g^4-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c^2*d*f
*g^4+4*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^2*f^2*g^3+A*a^2*b*c^2*d*g^5+A*
a^2*b*d^3*f^2*g^3-2*A*a*b^2*d^3*f^3*g^2+A*b^3*c^2*d*f^2*g^3-2*A*b^3*c*d^2*f
^3*g^2-2*B*ln(d*x+c)*x^2*a*b^2*d^3*f*g^4*n+2*B*ln(g*x+f)*x^2*a*b^2*d^3*f*g^
4*n-2*B*ln(g*x+f)*x^2*b^3*c*d^2*f*g^4*n-2*B*ln(b*x+a)*x*b^3*c^2*d*f*g^4*n+4
*B*ln(b*x+a)*x*b^3*c*d^2*f^2*g^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*f*g^4*n-4*B*ln
(d*x+c)*x*a*b^2*d^3*f^2*g^3*n-2*B*ln(g*x+f)*x*a^2*b*d^3*f*g^4*n+4*B*ln(g*x+
f)*x*a*b^2*d^3*f^2*g^3*n+2*B*ln(g*x+f)*x*b^3*c^2*d*f*g^4*n-4*B*ln(g*x+f)*x*
b^3*c*d^2*f^2*g^3*n+2*B*ln(b*x+a)*x^2*b^3*c*d^2*f*g^4*n-B*a^2*b*c*d^2*f*g^4
*n+B*a*b^2*c^2*d*f*g^4*n-B*ln(b*x+a)*x^2*b^3*d^3*f^2*g^3*n+B*ln(d*x+c)*x^2*
a^2*b*d^3*g^5*n+B*ln(d*x+c)*x^2*b^3*d^3*f^2*g^3*n-B*ln(g*x+f)*x^2*a^2*b*d^3
*g^5*n+B*ln(g*x+f)*x^2*b^3*c^2*d*g^5*n-2*B*ln(b*x+a)*x*b^3*d^3*f^3*g^2*n+2*
B*ln(d*x+c)*x*b^3*d^3*f^3*g^2*n-B*ln(b*x+a)*b^3*c^2*d*f^2*g^3*n+2*B*ln(b*x+
a)*b^3*c*d^2*f^3*g^2*n+B*ln(d*x+c)*a^2*b*d^3*f^2*g^3*n-2*B*ln(d*x+c)*a*b^2*
```

```
d^3*f^3*g^2*n-B*ln(g*x+f)*a^2*b*d^3*f^2*g^3*n+2*B*ln(g*x+f)*a*b^2*d^3*f^3*g
^2*n+B*ln(g*x+f)*b^3*c^2*d*f^2*g^3*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*f^
4*g+B*a^2*b*d^3*f^2*g^3*n-B*a*b^2*d^3*f^3*g^2*n-B*b^3*c^2*d*f^2*g^3*n+B*b^3
*c*d^2*f^3*g^2*n-2*A*a^2*b*c*d^2*f*g^4-2*A*a*b^2*c^2*d*f*g^4+4*A*a*b^2*c*d^
2*f^2*g^3+A*b^3*d^3*f^4*g-B*ln(b*x+a)*b^3*d^3*f^4*g*n+B*ln(d*x+c)*b^3*d^3*f
^4*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*c^2*d*g^5+B*ln(e*((b*x+a)/(d*x+c))
^n)*a^2*b*d^3*f^2*g^3-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*f^3*g^2+B*ln(
e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*f^2*g^3-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*
c*d^2*f^3*g^2)/(c^2*g^2-2*c*d*f*g+d^2*f^2)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(g*x
+f)^2/b/d/g^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(180) = 360$.

Time = 45.64 (sec) , antiderivative size = 1175, normalized size of antiderivative = 6.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas")
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*(A*b^2*c*d + A*a*b*d^2)*f^3*g + (A*
b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*f^2*g^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*f*g^
3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a
*b*c^2 - B*a^2*c*d)*g^4)*n*x + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*
d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B
*a*b*c^2 + B*a^2*c*d)*f*g^3)*n*log((b*x + a)/(d*x + c)) + ((B*b^2*c*d - B*a
*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2 + (B*a*b*c^2 - B*a^2*c*d)*f
*g^3)*n - ((B*b^2*d^2*f^2*g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*n*x^2 +
2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*n*x + (B*b^2*d^
2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2)*n)*log(b*x + a) + ((B*b^2*d^
2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g^4)*n*x^2 + 2*(B*b^2*d^2*f^3*g -
 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*n*x + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*
f^3*g + B*a^2*d^2*f^2*g^2)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*f*
g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2
*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*f^3*
g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2)*n)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a
^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B
*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log(e))/(b^2*d^2*f^6*g
 + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*
c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f
^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*
g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g
^4 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$$
$$= \frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g - 2\,abfg^2 + a^2g^3} - \frac{d^2\log(dx+c)}{d^2f^2g - 2\,cdfg^2 + c^2g^3} + \frac{(2\,(b^2cd - abd^2)f - (b^2c^2 - a^2}{b^2d^2f^4 + a^2c^2g^4 - 2\,(b^2cd + abd^2)f^3g + (b^2c^2 + 4\,}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(g^3x^2 + 2\,fg^2x + f^2g)} - \frac{A}{2\,(g^3x^2 + 2\,fg^2x + f^2g)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B*n - 1/2*B*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. 2(180) = 360.

Time = 0.86 (sec) , antiderivative size = 2994, normalized size of antiderivative = 15.76

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")
```

[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c
^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b*f +
(b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^2*d^2*f^4 - 2*b
^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*
d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*B*b^3*c
^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f*n/(d*x + c) +
2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) - 2*(b*x + a)*B*a
^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2*(b*x + a)*B*b^
2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*b*c^2*d^2*g*n/(
d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x + c))*log((b*x
+ a)/(d*x + c))/(b^2*d^2*f^4 - 2*(b*x + a)*b*d^3*f^4/(d*x + c) + (b*x + a)^
2*d^4*f^4/(d*x + c)^2 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + 6*(b*x + a)*b*c
*d^2*f^3*g/(d*x + c) + 2*(b*x + a)*a*d^3*f^3*g/(d*x + c) - 4*(b*x + a)^2*c*
d^3*f^3*g/(d*x + c)^2 + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 - 6*(b*x + a)*b
*c^2*d*f^2*g^2/(d*x + c) + a^2*d^2*f^2*g^2 - 6*(b*x + a)*a*c*d^2*f^2*g^2/(d
x + c) + 6(b*x + a)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 2*a*b*c^2*f*g^3 + 2*(
b*x + a)*b*c^3*f*g^3/(d*x + c) - 2*a^2*c*d*f*g^3 + 6*(b*x + a)*a*c^2*d*f*g^
3/(d*x + c) - 4*(b*x + a)^2*c^3*d*f*g^3/(d*x + c)^2 + a^2*c^2*g^4 - 2*(b*x
+ a)*a*c^3*g^4/(d*x + c) + (b*x + a)^2*c^4*g^4/(d*x + c)^2) - (2*B*b^3*c^2*
d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c
^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2
*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*
f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4
) + (B*b^4*c^3*f*g*n - 3*B*a*b^3*c^2*d*f*g*n - (b*x + a)*B*b^3*c^3*d*f*g*n/
(d*x + c) + 3*B*a^2*b^2*c*d^2*f*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*f*g*n/(d*
x + c) - B*a^3*b*d^3*f*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*f*g*n/(d*x + c) + (b
*x + a)*B*a^3*d^4*f*g*n/(d*x + c) - B*a*b^3*c^3*g^2*n + (b*x + a)*B*b^3*c^4
*g^2*n/(d*x + c) + 3*B*a^2*b^2*c^2*d*g^2*n - 3*(b*x + a)*B*a*b^2*c^3*d*g^2*
n/(d*x + c) - 3*B*a^3*b*c*d^2*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^2*g^2*n/(d*
x + c) + B*a^4*d^3*g^2*n - (b*x + a)*B*a^3*c*d^3*g^2*n/(d*x + c) + 2*B*b^4*
c^2*d*f^2*log(e) - 4*B*a*b^3*c*d^2*f^2*log(e) - 2*(b*x + a)*B*b^3*c^2*d^2*f
^2*log(e)/(d*x + c) + 2*B*a^2*b^2*d^3*f^2*log(e) + 4*(b*x + a)*B*a*b^2*c*d^
3*f^2*log(e)/(d*x + c) - 2*(b*x + a)*B*a^2*b*d^4*f^2*log(e)/(d*x + c) - B*b
^4*c^3*f*g*log(e) - B*a*b^3*c^2*d*f*g*log(e) + 2*(b*x + a)*B*b^3*c^3*d*f*g*
log(e)/(d*x + c) + 5*B*a^2*b^2*c*d^2*f*g*log(e) - 2*(b*x + a)*B*a*b^2*c^2*d
^2*f*g*log(e)/(d*x + c) - 3*B*a^3*b*d^3*f*g*log(e) - 2*(b*x + a)*B*a^2*b*c*
d^3*f*g*log(e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*f*g*log(e)/(d*x + c) + B*a
*b^3*c^3*g^2*log(e) - B*a^2*b^2*c^2*d*g^2*log(e) - 2*(b*x + a)*B*a*b^2*c^3*
d*g^2*log(e)/(d*x + c) - B*a^3*b*c*d^2*g^2*log(e) + 4*(b*x + a)*B*a^2*b*c^2
*d^2*g^2*log(e)/(d*x + c) + B*a^4*d^3*g^2*log(e) - 2*(b*x + a)*B*a^3*c*d^3*
g^2*log(e)/(d*x + c) + 2*A*b^4*c^2*d*f^2 - 4*A*a*b^3*c*d^2*f^2 - 2*(b*x + a
)*A*b^3*c^2*d^2*f^2/(d*x + c) + 2*A*a^2*b^2*d^3*f^2 + 4*(b*x + a)*A*a*b^2*c
*d^3*f^2/(d*x + c) - 2*(b*x + a)*A*a^2*b*d^4*f^2/(d*x + c) - A*b^4*c^3*f*g
- A*a*b^3*c^2*d*f*g + 2*(b*x + a)*A*b^3*c^3*d*f*g/(d*x + c) + 5*A*a^2*b^2*c
*d^2*f*g - 2*(b*x + a)*A*a*b^2*c^2*d^2*f*g/(d*x + c) - 3*A*a^3*b*d^3*f*g -

2*(b*x + a)*A*a^2*b*c*d^3*f*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*f*g/(d*x + c) + A*a*b^3*c^3*g^2 - A*a^2*b^2*c^2*d*g^2 - 2*(b*x + a)*A*a*b^2*c^3*d*g^2/(d*x + c) - A*a^3*b*c*d^2*g^2 + 4*(b*x + a)*A*a^2*b*c^2*d^2*g^2/(d*x + c) + A*a^4*d^3*g^2 - 2*(b*x + a)*A*a^3*c*d^3*g^2/(d*x + c))/(b^3*d^2*f^5 - 2*(b*x + a)*b^2*d^3*f^5/(d*x + c) + (b*x + a)^2*b*d^4*f^5/(d*x + c)^2 - 2*b^3*c*d*f^4*g - 3*a*b^2*d^2*f^4*g + 6*(b*x + a)*b^2*c*d^2*f^4*g/(d*x + c) + 4*(b*x + a)*a*b*d^3*f^4*g/(d*x + c) - 4*(b*x + a)^2*b*c*d^3*f^4*g/(d*x + c)^2 - (b*x + a)^2*a*d^4*f^4*g/(d*x + c)^2 + b^3*c^2*f^3*g^2 + 6*a*b^2*c*d*f^3*g^2 - 6*(b*x + a)*b^2*c^2*d*f^3*g^2/(d*x + c) + 3*a^2*b*d^2*f^3*g^2 - 12*(b*x + a)*a*b*c*d^2*f^3*g^2/(d*x + c) + 6*(b*x + a)^2*b*c^2*d^2*f^3*g^2/(d*x + c)^2 - 2*(b*x + a)*a^2*d^3*f^3*g^2/(d*x + c) + 4*(b*x + a)^2*a*c*d^3*f^3*g^2/(d*x + c)^2 - 3*a*b^2*c^2*f^2*g^3 + 2*(b*x + a)*b^2*c^3*f^2*g^3/(d*x + c) - 6*a^2*b*c*d*f^2*g^3 + 12*(b*x + a)*a*b*c^2*d*f^2*g^3/(d*x + c) - 4*(b*x + a)^2*b*c^3*d*f^2*g^3/(d*x + c)^2 - a^3*d^2*f^2*g^3 + 6*(b*x + a)*a^2*c*d^2*f^2*g^3/(d*x + c) - 6*(b*x + a)^2*a*c^2*d^2*f^2*g^3/(d*x + c)^2 + 3*a^2*b*c^2*f*g^4 - 4*(b*x + a)*a*b*c^3*f*g^4/(d*x + c) + (b*x + a)^2*b*c^4*f*g^4/(d*x + c)^2 + 2*a^3*c*d*f*g^4 - 6*(b*x + a)*a^2*c^2*d*f*g^4/(d*x + c) + 4*(b*x + a)^2*a*c^3*d*f*g^4/(d*x + c)^2 - a^3*c^2*g^5 + 2*(b*x + a)*a^2*c^3*g^5/(d*x + c) - (b*x + a)^2*a*c^4*g^5/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$$

$$= \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n)-2\,B\,a\,b\,d^2\,f\,n+2\,B\,b^2\,c\,d\,f\,n\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d\,f}$$

$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g\,n+B\,b\,c\,f\,g\,n}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,(B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$

$$-\frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,g\,(f^2+2\,f\,g\,x+g^2\,x^2)}+\frac{B\,b^2\,n\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,d^2\,n\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^3,x)

[Out] (log(f + g*x)*(g*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*f*n + 2*B*b^2*c*d*f*n))/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 - 4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d*f*g*n + B*b*c*f*g*n)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2*n - B*b*c*g^2*n))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^2 + 4*f*g^2*x) - (B*log(e*((a + b*x)/(c + d*x))^n))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g^2) - (B*d^2*n*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)

3.65 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx &= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} \\ &+ \frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} - \frac{Bd^3n\log(c+dx)}{3g(df-cg)^3} \\ &+ \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

```
[Out] -1/6*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*n*ln(b*x+a)/
g/(-a*g+b*f)^3+1/3*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^3-1/3*B*d^3*n
*ln(d*x+c)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f
)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{2547, 84\}$

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$$
$$= \frac{Bn(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}+\frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3}-\frac{Bn(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2}$$
$$-\frac{Bn(bc-ad)}{6(f+gx)^2(bf-ag)(df-cg)}-\frac{Bd^3n\log(c+dx)}{3g(df-cg)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*
d)*(2*b*d*f - b*c*g - a*d*g)*n)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) +
 (b^3*B*n*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(3*g*(f + g*x)^3) - (B*d^3*n*Log[c + d*x])/(3*g*(d*f - c*g)^3) +
(B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2))*n*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}$$
$$+\frac{(B(bc-ad)n)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^3}-\frac{g^2(-2bdf+bc}{(bf-ag)^2(df-cg}\right.}{3g}$$

$$= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} - \frac{Bd^3n\log(c+dx)}{3g(df-cg)^3} + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\log(f+gx)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3} + B(bc-ad)n\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2} + \frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3\log}{(bc-ad)}\right.}{3g}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3) + B*(b*c - a*d)*n*(-
1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g)
)/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)
*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a
^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)
)*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3256 vs. 2(274) = 548.

Time = 41.68 (sec) , antiderivative size = 3257, normalized size of antiderivative = 11.51

method	result	size
parallelrisch	Expression too large to display	3257

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*f^4*g^4*n+2*B*ln(g*x+f)*a^
3*b*d^4*f^3*g^5*n^2-6*B*ln(g*x+f)*a^2*b^2*d^4*f^4*g^4*n^2+6*B*ln(g*x+f)*a*b
^3*d^4*f^5*g^3*n^2-2*B*ln(g*x+f)*b^4*c^3*d*f^3*g^5*n^2+6*B*ln(g*x+f)*b^4*c^
2*d^2*f^4*g^4*n^2-6*B*ln(g*x+f)*b^4*c*d^3*f^5*g^3*n^2-2*B*ln(b*x+a)*x^3*a^3
*b*d^4*g^8*n^2+2*B*ln(b*x+a)*x^3*b^4*c^3*d*g^8*n^2+2*B*ln(g*x+f)*x^3*a^3*b*
d^4*g^8*n^2-2*B*ln(g*x+f)*x^3*b^4*c^3*d*g^8*n^2-3*B*a^3*b*d^4*f^3*g^5*n^2+8
*B*a^2*b^2*d^4*f^4*g^4*n^2-5*B*a*b^3*d^4*f^5*g^3*n^2+3*B*b^4*c^3*d*f^3*g^5*
n^2-8*B*b^4*c^2*d^2*f^4*g^4*n^2+5*B*b^4*c*d^3*f^5*g^3*n^2+2*A*a^3*b*c^3*d*g
```

^8*n-2*A*a^3*b*d^4*f^3*g^5*n+6*A*a^2*b^2*d^4*f^4*g^4*n-6*A*a*b^3*d^4*f^5*g^
3*n-2*A*b^4*c^3*d*f^3*g^5*n+6*A*b^4*c^2*d^2*f^4*g^4*n-6*A*b^4*c*d^3*f^5*g^3
*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^3*d*g^8*n-2*B*ln(e*((b*x+a)/(d*x+c
))^n)*b^4*c^3*d*f^3*g^5*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*f^4*g^4
*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*f^5*g^3*n-2*B*ln(b*x+a)*a^3*b*d^
4*f^3*g^5*n^2+6*B*ln(b*x+a)*a^2*b^2*d^4*f^4*g^4*n^2-6*B*ln(b*x+a)*a*b^3*d^4
*f^5*g^3*n^2+2*B*ln(b*x+a)*b^4*c^3*d*f^3*g^5*n^2-6*B*ln(b*x+a)*b^4*c^2*d^2*
f^4*g^4*n^2+6*B*ln(b*x+a)*b^4*c*d^3*f^5*g^3*n^2-6*B*ln(b*x+a)*x^2*a^3*b*d^4
*f*g^7*n^2+18*B*ln(b*x+a)*x^2*a^2*b^2*d^4*f^2*g^6*n^2-18*B*ln(b*x+a)*x^2*a*
b^3*d^4*f^3*g^5*n^2+6*B*ln(b*x+a)*x^2*b^4*c^3*d*f*g^7*n^2-18*B*ln(b*x+a)*x^
2*b^4*c^2*d^2*f^2*g^6*n^2+18*B*ln(b*x+a)*x^2*b^4*c*d^3*f^3*g^5*n^2+18*B*ln(
g*x+f)*x^2*b^4*c^2*d^2*f^2*g^6*n^2-18*B*ln(g*x+f)*x^2*b^4*c*d^3*f^3*g^5*n^2
-6*B*ln(b*x+a)*x*a^3*b*d^4*f^2*g^6*n^2+18*B*ln(b*x+a)*x*a^2*b^2*d^4*f^3*g^5
*n^2-18*B*ln(b*x+a)*x*a*b^3*d^4*f^4*g^4*n^2+6*B*ln(b*x+a)*x*b^4*c^3*d*f^2*g
^6*n^2-18*B*ln(b*x+a)*x*b^4*c^2*d^2*f^3*g^5*n^2+18*B*ln(b*x+a)*x*b^4*c*d^3*
f^4*g^4*n^2+6*B*ln(g*x+f)*x*a^3*b*d^4*f^2*g^6*n^2-18*B*ln(g*x+f)*x*a^2*b^2*
d^4*f^3*g^5*n^2+18*B*ln(g*x+f)*x*a*b^3*d^4*f^4*g^4*n^2-6*B*ln(g*x+f)*x*b^4*
c^3*d*f^2*g^6*n^2-6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^4*g^4*n+2*B*x
^2*a^3*b*c*d^3*g^8*n^2-2*B*x^2*a^3*b*d^4*f*g^7*n^2+6*B*x^2*a^2*b^2*d^4*f^2*
g^6*n^2-2*B*x^2*a*b^3*c^3*d*g^8*n^2-4*B*x^2*a*b^3*d^4*f^3*g^5*n^2+2*B*x^2*b
^4*c^3*d*f*g^7*n^2-6*B*x^2*b^4*c^2*d^2*f^2*g^6*n^2+4*B*x^2*b^4*c*d^3*f^3*g^
5*n^2-6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^5*g^3*n-B*x*a^3*b*c^2*d^2*g
^8*n^2-5*B*x*a^3*b*d^4*f^2*g^6*n^2+B*x*a^2*b^2*c^3*d*g^8*n^2+14*B*x*a^2*b^2
*d^4*f^3*g^5*n^2-9*B*x*a*b^3*d^4*f^4*g^4*n^2+5*B*x*b^4*c^3*d*f^2*g^6*n^2-14
*B*x*b^4*c^2*d^2*f^3*g^5*n^2+9*B*x*b^4*c*d^3*f^4*g^4*n^2+18*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b^2*c^2*d^2*f^2*g^6*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b
^2*c*d^3*f^3*g^5*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*f^2*g^6*n-18*B
ln(e((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^2*f^3*g^5*n+6*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*a^3*b*d^4*f*g^7*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4*
f^2*g^6*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*f^3*g^5*n-6*B*x^2*a^
2*b^2*c*d^3*f*g^7*n^2+6*B*x^2*a*b^3*c^2*d^2*f*g^7*n^2+6*B*x*ln(e*((b*x+a)/(
d*x+c))^n)*a^3*b*d^4*f^2*g^6*n-18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4
*f^3*g^5*n+18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*f^4*g^4*n+6*B*x*a^3*b
*c*d^3*f*g^7*n^2-15*B*x*a^2*b^2*c*d^3*f^2*g^6*n^2-6*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a^3*b*c^2*d^2*f*g^7*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c*d^3*f^2*g^
6*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^3*d*f*g^7*n+18*B*ln(g*x+f)*x*b^
4*c^2*d^2*f^3*g^5*n^2-18*B*ln(g*x+f)*x*b^4*c*d^3*f^4*g^4*n^2+6*B*ln(g*x+f)*
x^2*a^3*b*d^4*f*g^7*n^2-18*B*ln(g*x+f)*x^2*a^2*b^2*d^4*f^2*g^6*n^2+18*B*ln(
g*x+f)*x^2*a*b^3*d^4*f^3*g^5*n^2-6*B*ln(g*x+f)*x^2*b^4*c^3*d*f*g^7*n^2-6*B*
x*a*b^3*c^3*d*f*g^7*n^2+15*B*x*a*b^3*c^2*d^2*f^2*g^6*n^2-6*B*x^3*ln(e*((b*x
+a)/(d*x+c))^n)*a^2*b^2*d^4*f*g^7*n+6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3
*d^4*f^2*g^6*n-B*a^3*b*c^2*d^2*f*g^7*n^2-6*B*ln(b*x+a)*x^3*b^4*c^2*d^2*f*g^
7*n^2+6*B*ln(b*x+a)*x^3*b^4*c*d^3*f^2*g^6*n^2-6*B*ln(g*x+f)*x^3*a^2*b^2*d^4
*f*g^7*n^2+6*B*ln(g*x+f)*x^3*a*b^3*d^4*f^2*g^6*n^2+6*B*ln(g*x+f)*x^3*b^4*c^
2*d^2*f*g^7*n^2-6*B*ln(g*x+f)*x^3*b^4*c*d^3*f^2*g^6*n^2+6*B*ln(b*x+a)*x^3*a

```
^2*b^2*d^4*f*g^7*n^2-6*B*ln(b*x+a)*x^3*a*b^3*d^4*f^2*g^6*n^2+4*B*a^3*b*c*d^
3*f^2*g^6*n^2+B*a^2*b^2*c^3*d*f*g^7*n^2-9*B*a^2*b^2*c*d^3*f^3*g^5*n^2-4*B*a
*b^3*c^3*d*f^2*g^6*n^2+9*B*a*b^3*c^2*d^2*f^3*g^5*n^2-6*A*a^3*b*c^2*d^2*f*g^
7*n+6*A*a^3*b*c*d^3*f^2*g^6*n-6*A*a^2*b^2*c^3*d*f*g^7*n+18*A*a^2*b^2*c^2*d^
2*f^2*g^6*n-18*A*a^2*b^2*c*d^3*f^3*g^5*n+6*A*a*b^3*c^3*d*f^2*g^6*n-18*A*a*b
^3*c^2*d^2*f^3*g^5*n+18*A*a*b^3*c*d^3*f^4*g^4*n+2*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a^3*b*d^4*g^8*n-2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^3*g^5*n+
2*A*b^4*d^4*f^6*g^2*n)/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(g*x+f
)^3/(c*g-d*f)/(c^2*g^2-2*c*d*f*g+d^2*f^2)/n/b/d/g^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(271) = 542$.

Time = 0.24 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.01

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \frac{1}{6}\left(\frac{2\,b^3\log(bx+a)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4} - \frac{2\,d^3\log(dx+c)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4} + \frac{}{b^3d^3f^6+a^3c^3g^6-3\,(b\cdots}\right. - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)} - \frac{A}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*B*n - 1/3*B*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*A/(g
^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9692 vs. $2(271) = 542$.

Time = 0.85 (sec) , antiderivative size = 9692, normalized size of antiderivative = 34.25

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2
*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*
n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c
*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b
*x + a)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g
 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*
c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g
^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*
a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^2*d^2*f^2
*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c^2*d^3*f^2*n/(d*x + c) + 3*
B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4*f^2*n/(d*x + c) + 3*(b*x +
 a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x + a)*B*a^2*b*d^5*f^2*n/(d*x
+ c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^2*d^
6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 9*(b*
x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^3*f*g*n - 15*(b*x +
a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*b^2*c^3*d^3*f*g*n/(d*x
```

+ c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^2*b*c*d^4*f*g*n/(d*x + c) +
12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2 + 3*(b*x + a)*B*a^3*d^5*f*g
*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(d*x + c)^2 + B*b^4*c^4*g^2*
n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*d*g^2*n/(d*x + c) + 3*(b*x
+ a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^4*d^2*g^2*n/(d
*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^3*g^2*n/(d*x +
c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*d^4*g^2*n - 3*(b
*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*a^2*c^2*d^4*g^2*n/(d*
x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*(b*x + a)*b^2*d^4*f^6/(
d*x + c) + 3*(b*x + a)^2*b*d^5*f^6/(d*x + c)^2 - (b*x + a)^3*d^6*f^6/(d*x +
c)^3 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 12*(b*x + a)*b^2*c*d^3*f^5*
g/(d*x + c) + 6*(b*x + a)*a*b*d^4*f^5*g/(d*x + c) - 15*(b*x + a)^2*b*c*d^4*
f^5*g/(d*x + c)^2 - 3*(b*x + a)^2*a*d^5*f^5*g/(d*x + c)^2 + 6*(b*x + a)^3*c
*d^5*f^5*g/(d*x + c)^3 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 18*(
b*x + a)*b^2*c^2*d^2*f^4*g^2/(d*x + c) + 3*a^2*b*d^3*f^4*g^2 - 24*(b*x + a)
*a*b*c*d^3*f^4*g^2/(d*x + c) + 30*(b*x + a)^2*b*c^2*d^3*f^4*g^2/(d*x + c)^2
- 3*(b*x + a)*a^2*d^4*f^4*g^2/(d*x + c) + 15*(b*x + a)^2*a*c*d^4*f^4*g^2/(
d*x + c)^2 - 15*(b*x + a)^3*c^2*d^4*f^4*g^2/(d*x + c)^3 - b^3*c^3*f^3*g^3 -
9*a*b^2*c^2*d*f^3*g^3 + 12*(b*x + a)*b^2*c^3*d*f^3*g^3/(d*x + c) - 9*a^2*b
*c*d^2*f^3*g^3 + 36*(b*x + a)*a*b*c^2*d^2*f^3*g^3/(d*x + c) - 30*(b*x + a)^
2*b*c^3*d^2*f^3*g^3/(d*x + c)^2 - a^3*d^3*f^3*g^3 + 12*(b*x + a)*a^2*c*d^3*
f^3*g^3/(d*x + c) - 30*(b*x + a)^2*a*c^2*d^3*f^3*g^3/(d*x + c)^2 + 20*(b*x
+ a)^3*c^3*d^3*f^3*g^3/(d*x + c)^3 + 3*a*b^2*c^3*f^2*g^4 - 3*(b*x + a)*b^2*
c^4*f^2*g^4/(d*x + c) + 9*a^2*b*c^2*d*f^2*g^4 - 24*(b*x + a)*a*b*c^3*d*f^2*
g^4/(d*x + c) + 15*(b*x + a)^2*b*c^4*d*f^2*g^4/(d*x + c)^2 + 3*a^3*c*d^2*f^
2*g^4 - 18*(b*x + a)*a^2*c^2*d^2*f^2*g^4/(d*x + c) + 30*(b*x + a)^2*a*c^3*d
^2*f^2*g^4/(d*x + c)^2 - 15*(b*x + a)^3*c^4*d^2*f^2*g^4/(d*x + c)^3 - 3*a^2
*b*c^3*f*g^5 + 6*(b*x + a)*a*b*c^4*f*g^5/(d*x + c) - 3*(b*x + a)^2*b*c^5*f*
g^5/(d*x + c)^2 - 3*a^3*c^2*d*f*g^5 + 12*(b*x + a)*a^2*c^3*d*f*g^5/(d*x + c
) - 15*(b*x + a)^2*a*c^4*d*f*g^5/(d*x + c)^2 + 6*(b*x + a)^3*c^5*d*f*g^5/(d
*x + c)^3 + a^3*c^3*g^6 - 3*(b*x + a)*a^2*c^4*g^6/(d*x + c) + 3*(b*x + a)^2
*a*c^5*g^6/(d*x + c)^2 - (b*x + a)^3*c^6*g^6/(d*x + c)^3) - 2*(3*B*b^4*c^2*
d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f
*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*
g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c*d^3*g^2*n + B*a^4*d
^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b
^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^
4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a
^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*
f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + (6*B*b^6*c
^3*d*f^3*g*n - 18*B*a*b^5*c^2*d^2*f^3*g*n - 12*(b*x + a)*B*b^5*c^3*d^2*f^3*
g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f^3*g*n + 36*(b*x + a)*B*a*b^4*c^2*d^3*f
^3*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f^3*g*n/(d*x + c)^2 - 6*B*a^
3*b^3*d^4*f^3*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f^3*g*n/(d*x + c) - 18*(b*
x + a)^2*B*a*b^3*c^2*d^4*f^3*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a^3*b^2*d^5*f

^3*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f^3*g*n/(d*x + c)^2 - 6*(
b*x + a)^2*B*a^3*b*d^6*f^3*g*n/(d*x + c)^2 - 3*B*b^6*c^4*f^2*g^2*n - 6*B*a*
b^5*c^3*d*f^2*g^2*n + 17*(b*x + a)*B*b^5*c^4*d*f^2*g^2*n/(d*x + c) + 36*B*a
^2*b^4*c^2*d^2*f^2*g^2*n - 32*(b*x + a)*B*a*b^4*c^3*d^2*f^2*g^2*n/(d*x + c)
 - 14*(b*x + a)^2*B*b^4*c^4*d^2*f^2*g^2*n/(d*x + c)^2 - 42*B*a^3*b^3*c*d^3*
f^2*g^2*n - 6*(b*x + a)*B*a^2*b^3*c^2*d^3*f^2*g^2*n/(d*x + c) + 38*(b*x + a
)^2*B*a*b^3*c^3*d^3*f^2*g^2*n/(d*x + c)^2 + 15*B*a^4*b^2*d^4*f^2*g^2*n + 40
*(b*x + a)*B*a^3*b^2*c*d^4*f^2*g^2*n/(d*x + c) - 30*(b*x + a)^2*B*a^2*b^2*c
^2*d^4*f^2*g^2*n/(d*x + c)^2 - 19*(b*x + a)*B*a^4*b*d^5*f^2*g^2*n/(d*x + c)
 + 2*(b*x + a)^2*B*a^3*b*c*d^5*f^2*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*
d^6*f^2*g^2*n/(d*x + c)^2 + 6*B*a*b^5*c^4*f*g^3*n - 5*(b*x + a)*B*b^5*c^5*f
*g^3*n/(d*x + c) - 6*B*a^2*b^4*c^3*d*f*g^3*n - 9*(b*x + a)*B*a*b^4*c^4*d*f*
g^3*n/(d*x + c) + 10*(b*x + a)^2*B*b^4*c^5*d*f*g^3*n/(d*x + c)^2 - 18*B*a^3
*b^3*c^2*d^2*f*g^3*n + 50*(b*x + a)*B*a^2*b^3*c^3*d^2*f*g^3*n/(d*x + c) - 2
2*(b*x + a)^2*B*a*b^3*c^4*d^2*f*g^3*n/(d*x + c)^2 + 30*B*a^4*b^2*c*d^3*f*g^
3*n - 46*(b*x + a)*B*a^3*b^2*c^2*d^3*f*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*a^
2*b^2*c^3*d^3*f*g^3*n/(d*x + c)^2 - 12*B*a^5*b*d^4*f*g^3*n + 3*(b*x + a)*B*
a^4*b*c*d^4*f*g^3*n/(d*x + c) + 14*(b*x + a)^2*B*a^3*b*c^2*d^4*f*g^3*n/(d*x
 + c)^2 + 7*(b*x + a)*B*a^5*d^5*f*g^3*n/(d*x + c) - 8*(b*x + a)^2*B*a^4*c*d
^5*f*g^3*n/(d*x + c)^2 - 3*B*a^2*b^4*c^4*g^4*n + 5*(b*x + a)*B*a*b^4*c^5*g^
4*n/(d*x + c) - 2*(b*x + a)^2*B*b^4*c^6*g^4*n/(d*x + c)^2 + 6*B*a^3*b^3*c^3
*d*g^4*n - 8*(b*x + a)*B*a^2*b^3*c^4*d*g^4*n/(d*x + c) + 2*(b*x + a)^2*B*a*
b^3*c^5*d*g^4*n/(d*x + c)^2 - 6*(b*x + a)*B*a^3*b^2*c^3*d^2*g^4*n/(d*x + c)
 + 6*(b*x + a)^2*B*a^2*b^2*c^4*d^2*g^4*n/(d*x + c)^2 - 6*B*a^5*b*c*d^3*g^4*
n + 16*(b*x + a)*B*a^4*b*c^2*d^3*g^4*n/(d*x + c) - 10*(b*x + a)^2*B*a^3*b*c
^3*d^3*g^4*n/(d*x + c)^2 + 3*B*a^6*d^4*g^4*n - 7*(b*x + a)*B*a^5*c*d^4*g^4*
n/(d*x + c) + 4*(b*x + a)^2*B*a^4*c^2*d^4*g^4*n/(d*x + c)^2 + 6*B*b^6*c^2*d
^2*f^4*log(e) - 12*B*a*b^5*c*d^3*f^4*log(e) - 12*(b*x + a)*B*b^5*c^2*d^3*f^
4*log(e)/(d*x + c) + 6*B*a^2*b^4*d^4*f^4*log(e) + 24*(b*x + a)*B*a*b^4*c*d^
4*f^4*log(e)/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^2*d^4*f^4*log(e)/(d*x + c)^2
 - 12*(b*x + a)*B*a^2*b^3*d^5*f^4*log(e)/(d*x + c) - 12*(b*x + a)^2*B*a*b^3
*c*d^5*f^4*log(e)/(d*x + c)^2 + 6*(b*x + a)^2*B*a^2*b^2*d^6*f^4*log(e)/(d*x
 + c)^2 - 6*B*b^6*c^3*d*f^3*g*log(e) - 6*B*a*b^5*c^2*d^2*f^3*g*log(e) + 18*
(b*x + a)*B*b^5*c^3*d^2*f^3*g*log(e)/(d*x + c) + 30*B*a^2*b^4*c*d^3*f^3*g*l
og(e) - 6*(b*x + a)*B*a*b^4*c^2*d^3*f^3*g*log(e)/(d*x + c) - 12*(b*x + a)^2
*B*b^4*c^3*d^3*f^3*g*log(e)/(d*x + c)^2 - 18*B*a^3*b^3*d^4*f^3*g*log(e) - 4
2*(b*x + a)*B*a^2*b^3*c*d^4*f^3*g*log(e)/(d*x + c) + 12*(b*x + a)^2*B*a*b^3
*c^2*d^4*f^3*g*log(e)/(d*x + c)^2 + 30*(b*x + a)*B*a^3*b^2*d^5*f^3*g*log(e)
/(d*x + c) + 12*(b*x + a)^2*B*a^2*b^2*c*d^5*f^3*g*log(e)/(d*x + c)^2 - 12*(
b*x + a)^2*B*a^3*b*d^6*f^3*g*log(e)/(d*x + c)^2 + 2*B*b^6*c^4*f^2*g^2*log(e
) + 10*B*a*b^5*c^3*d*f^2*g^2*log(e) - 6*(b*x + a)*B*b^5*c^4*d*f^2*g^2*log(e
)/(d*x + c) - 6*B*a^2*b^4*c^2*d^2*f^2*g^2*log(e) - 30*(b*x + a)*B*a*b^4*c^3
*d^2*f^2*g^2*log(e)/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^4*d^2*f^2*g^2*log(e)/
(d*x + c)^2 - 26*B*a^3*b^3*c*d^3*f^2*g^2*log(e) + 54*(b*x + a)*B*a^2*b^3*c^
2*d^3*f^2*g^2*log(e)/(d*x + c) + 12*(b*x + a)^2*B*a*b^3*c^3*d^3*f^2*g^2*log

(e)/(d*x + c)^2 + 20*B*a^4*b^2*d^4*f^2*g^2*log(e) + 6*(b*x + a)*B*a^3*b^2*c
*d^4*f^2*g^2*log(e)/(d*x + c) - 36*(b*x + a)^2*B*a^2*b^2*c^2*d^4*f^2*g^2*lo
g(e)/(d*x + c)^2 - 24*(b*x + a)*B*a^4*b*d^5*f^2*g^2*log(e)/(d*x + c) + 12*(
b*x + a)^2*B*a^3*b*c*d^5*f^2*g^2*log(e)/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*d
^6*f^2*g^2*log(e)/(d*x + c)^2 - 4*B*a*b^5*c^4*f*g^3*log(e) - 2*B*a^2*b^4*c^
3*d*f*g^3*log(e) + 12*(b*x + a)*B*a*b^4*c^4*d*f*g^3*log(e)/(d*x + c) + 6*B*
a^3*b^3*c^2*d^2*f*g^3*log(e) + 6*(b*x + a)*B*a^2*b^3*c^3*d^2*f*g^3*log(e)/(
d*x + c) - 12*(b*x + a)^2*B*a*b^3*c^4*d^2*f*g^3*log(e)/(d*x + c)^2 + 10*B*a
^4*b^2*c*d^3*f*g^3*log(e) - 42*(b*x + a)*B*a^3*b^2*c^2*d^3*f*g^3*log(e)/(d*
x + c) + 12*(b*x + a)^2*B*a^2*b^2*c^3*d^3*f*g^3*log(e)/(d*x + c)^2 - 10*B*a
^5*b*d^4*f*g^3*log(e) + 18*(b*x + a)*B*a^4*b*c*d^4*f*g^3*log(e)/(d*x + c) +
 12*(b*x + a)^2*B*a^3*b*c^2*d^4*f*g^3*log(e)/(d*x + c)^2 + 6*(b*x + a)*B*a^
5*d^5*f*g^3*log(e)/(d*x + c) - 12*(b*x + a)^2*B*a^4*c*d^5*f*g^3*log(e)/(d*x
 + c)^2 + 2*B*a^2*b^4*c^4*g^4*log(e) - 2*B*a^3*b^3*c^3*d*g^4*log(e) - 6*(b*
x + a)*B*a^2*b^3*c^4*d*g^4*log(e)/(d*x + c) + 6*(b*x + a)*B*a^3*b^2*c^3*d^2
*g^4*log(e)/(d*x + c) + 6*(b*x + a)^2*B*a^2*b^2*c^4*d^2*g^4*log(e)/(d*x + c
)^2 - 2*B*a^5*b*c*d^3*g^4*log(e) + 6*(b*x + a)*B*a^4*b*c^2*d^3*g^4*log(e)/(
d*x + c) - 12*(b*x + a)^2*B*a^3*b*c^3*d^3*g^4*log(e)/(d*x + c)^2 + 2*B*a^6*
d^4*g^4*log(e) - 6*(b*x + a)*B*a^5*c*d^4*g^4*log(e)/(d*x + c) + 6*(b*x + a)
^2*B*a^4*c^2*d^4*g^4*log(e)/(d*x + c)^2 + 6*A*b^6*c^2*d^2*f^4 - 12*A*a*b^5*
c*d^3*f^4 - 12*(b*x + a)*A*b^5*c^2*d^3*f^4/(d*x + c) + 6*A*a^2*b^4*d^4*f^4
+ 24*(b*x + a)*A*a*b^4*c*d^4*f^4/(d*x + c) + 6*(b*x + a)^2*A*b^4*c^2*d^4*f^
4/(d*x + c)^2 - 12*(b*x + a)*A*a^2*b^3*d^5*f^4/(d*x + c) - 12*(b*x + a)^2*A
*a*b^3*c*d^5*f^4/(d*x + c)^2 + 6*(b*x + a)^2*A*a^2*b^2*d^6*f^4/(d*x + c)^2
- 6*A*b^6*c^3*d*f^3*g - 6*A*a*b^5*c^2*d^2*f^3*g + 18*(b*x + a)*A*b^5*c^3*d^
2*f^3*g/(d*x + c) + 30*A*a^2*b^4*c*d^3*f^3*g - 6*(b*x + a)*A*a*b^4*c^2*d^3*
f^3*g/(d*x + c) - 12*(b*x + a)^2*A*b^4*c^3*d^3*f^3*g/(d*x + c)^2 - 18*A*a^3
*b^3*d^4*f^3*g - 42*(b*x + a)*A*a^2*b^3*c*d^4*f^3*g/(d*x + c) + 12*(b*x + a
)^2*A*a*b^3*c^2*d^4*f^3*g/(d*x + c)^2 + 30*(b*x + a)*A*a^3*b^2*d^5*f^3*g/(d
x + c) + 12(b*x + a)^2*A*a^2*b^2*c*d^5*f^3*g/(d*x + c)^2 - 12*(b*x + a)^2
*A*a^3*b*d^6*f^3*g/(d*x + c)^2 + 2*A*b^6*c^4*f^2*g^2 + 10*A*a*b^5*c^3*d*f^2
g^2 - 6(b*x + a)*A*b^5*c^4*d*f^2*g^2/(d*x + c) - 6*A*a^2*b^4*c^2*d^2*f^2*
g^2 - 30*(b*x + a)*A*a*b^4*c^3*d^2*f^2*g^2/(d*x + c) + 6*(b*x + a)^2*A*b^4*
c^4*d^2*f^2*g^2/(d*x + c)^2 - 26*A*a^3*b^3*c*d^3*f^2*g^2 + 54*(b*x + a)*A*a
^2*b^3*c^2*d^3*f^2*g^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^3*d^3*f^2*g^2/(
d*x + c)^2 + 20*A*a^4*b^2*d^4*f^2*g^2 + 6*(b*x + a)*A*a^3*b^2*c*d^4*f^2*g^2
/(d*x + c) - 36*(b*x + a)^2*A*a^2*b^2*c^2*d^4*f^2*g^2/(d*x + c)^2 - 24*(b*x
 + a)*A*a^4*b*d^5*f^2*g^2/(d*x + c) + 12*(b*x + a)^2*A*a^3*b*c*d^5*f^2*g^2/
(d*x + c)^2 + 6*(b*x + a)^2*A*a^4*d^6*f^2*g^2/(d*x + c)^2 - 4*A*a*b^5*c^4*f
*g^3 - 2*A*a^2*b^4*c^3*d*f*g^3 + 12*(b*x + a)*A*a*b^4*c^4*d*f*g^3/(d*x + c)
 + 6*A*a^3*b^3*c^2*d^2*f*g^3 + 6*(b*x + a)*A*a^2*b^3*c^3*d^2*f*g^3/(d*x + c
) - 12*(b*x + a)^2*A*a*b^3*c^4*d^2*f*g^3/(d*x + c)^2 + 10*A*a^4*b^2*c*d^3*f
g^3 - 42(b*x + a)*A*a^3*b^2*c^2*d^3*f*g^3/(d*x + c) + 12*(b*x + a)^2*A*a^
2*b^2*c^3*d^3*f*g^3/(d*x + c)^2 - 10*A*a^5*b*d^4*f*g^3 + 18*(b*x + a)*A*a^4
*b*c*d^4*f*g^3/(d*x + c) + 12*(b*x + a)^2*A*a^3*b*c^2*d^4*f*g^3/(d*x + c)^2

+ 6*(b*x + a)*A*a^5*d^5*f*g^3/(d*x + c) - 12*(b*x + a)^2*A*a^4*c*d^5*f*g^3
/(d*x + c)^2 + 2*A*a^2*b^4*c^4*g^4 - 2*A*a^3*b^3*c^3*d*g^4 - 6*(b*x + a)*A*
a^2*b^3*c^4*d*g^4/(d*x + c) + 6*(b*x + a)*A*a^3*b^2*c^3*d^2*g^4/(d*x + c) +
6*(b*x + a)^2*A*a^2*b^2*c^4*d^2*g^4/(d*x + c)^2 - 2*A*a^5*b*c*d^3*g^4 + 6*
(b*x + a)*A*a^4*b*c^2*d^3*g^4/(d*x + c) - 12*(b*x + a)^2*A*a^3*b*c^3*d^3*g^
4/(d*x + c)^2 + 2*A*a^6*d^4*g^4 - 6*(b*x + a)*A*a^5*c*d^4*g^4/(d*x + c) + 6
*(b*x + a)^2*A*a^4*c^2*d^4*g^4/(d*x + c)^2)/(b^5*d^3*f^8 - 3*(b*x + a)*b^4*
d^4*f^8/(d*x + c) + 3*(b*x + a)^2*b^3*d^5*f^8/(d*x + c)^2 - (b*x + a)^3*b^2
*d^6*f^8/(d*x + c)^3 - 3*b^5*c*d^2*f^7*g - 5*a*b^4*d^3*f^7*g + 12*(b*x + a)
*b^4*c*d^3*f^7*g/(d*x + c) + 12*(b*x + a)*a*b^3*d^4*f^7*g/(d*x + c) - 15*(b
*x + a)^2*b^3*c*d^4*f^7*g/(d*x + c)^2 - 9*(b*x + a)^2*a*b^2*d^5*f^7*g/(d*x
+ c)^2 + 6*(b*x + a)^3*b^2*c*d^5*f^7*g/(d*x + c)^3 + 2*(b*x + a)^3*a*b*d^6*
f^7*g/(d*x + c)^3 + 3*b^5*c^2*d*f^6*g^2 + 15*a*b^4*c*d^2*f^6*g^2 - 18*(b*x
+ a)*b^4*c^2*d^2*f^6*g^2/(d*x + c) + 10*a^2*b^3*d^3*f^6*g^2 - 48*(b*x + a)*
a*b^3*c*d^3*f^6*g^2/(d*x + c) + 30*(b*x + a)^2*b^3*c^2*d^3*f^6*g^2/(d*x + c
)^2 - 18*(b*x + a)*a^2*b^2*d^4*f^6*g^2/(d*x + c) + 45*(b*x + a)^2*a*b^2*c*d
^4*f^6*g^2/(d*x + c)^2 - 15*(b*x + a)^3*b^2*c^2*d^4*f^6*g^2/(d*x + c)^3 + 9
*(b*x + a)^2*a^2*b*d^5*f^6*g^2/(d*x + c)^2 - 12*(b*x + a)^3*a*b*c*d^5*f^6*g
^2/(d*x + c)^3 - (b*x + a)^3*a^2*d^6*f^6*g^2/(d*x + c)^3 - b^5*c^3*f^5*g^3
- 15*a*b^4*c^2*d*f^5*g^3 + 12*(b*x + a)*b^4*c^3*d*f^5*g^3/(d*x + c) - 30*a^
2*b^3*c*d^2*f^5*g^3 + 72*(b*x + a)*a*b^3*c^2*d^2*f^5*g^3/(d*x + c) - 30*(b*
x + a)^2*b^3*c^3*d^2*f^5*g^3/(d*x + c)^2 - 10*a^3*b^2*d^3*f^5*g^3 + 72*(b*x
+ a)*a^2*b^2*c*d^3*f^5*g^3/(d*x + c) - 90*(b*x + a)^2*a*b^2*c^2*d^3*f^5*g^
3/(d*x + c)^2 + 20*(b*x + a)^3*b^2*c^3*d^3*f^5*g^3/(d*x + c)^3 + 12*(b*x +
a)*a^3*b*d^4*f^5*g^3/(d*x + c) - 45*(b*x + a)^2*a^2*b*c*d^4*f^5*g^3/(d*x +
c)^2 + 30*(b*x + a)^3*a*b*c^2*d^4*f^5*g^3/(d*x + c)^3 - 3*(b*x + a)^2*a^3*d
^5*f^5*g^3/(d*x + c)^2 + 6*(b*x + a)^3*a^2*c*d^5*f^5*g^3/(d*x + c)^3 + 5*a*
b^4*c^3*f^4*g^4 - 3*(b*x + a)*b^4*c^4*f^4*g^4/(d*x + c) + 30*a^2*b^3*c^2*d*
f^4*g^4 - 48*(b*x + a)*a*b^3*c^3*d*f^4*g^4/(d*x + c) + 15*(b*x + a)^2*b^3*c
^4*d*f^4*g^4/(d*x + c)^2 + 30*a^3*b^2*c*d^2*f^4*g^4 - 108*(b*x + a)*a^2*b^2
*c^2*d^2*f^4*g^4/(d*x + c) + 90*(b*x + a)^2*a*b^2*c^3*d^2*f^4*g^4/(d*x + c)
^2 - 15*(b*x + a)^3*b^2*c^4*d^2*f^4*g^4/(d*x + c)^3 + 5*a^4*b*d^3*f^4*g^4 -
48*(b*x + a)*a^3*b*c*d^3*f^4*g^4/(d*x + c) + 90*(b*x + a)^2*a^2*b*c^2*d^3*
f^4*g^4/(d*x + c)^2 - 40*(b*x + a)^3*a*b*c^3*d^3*f^4*g^4/(d*x + c)^3 - 3*(b
*x + a)*a^4*d^4*f^4*g^4/(d*x + c) + 15*(b*x + a)^2*a^3*c*d^4*f^4*g^4/(d*x +
c)^2 - 15*(b*x + a)^3*a^2*c^2*d^4*f^4*g^4/(d*x + c)^3 - 10*a^2*b^3*c^3*f^3
g^5 + 12(b*x + a)*a*b^3*c^4*f^3*g^5/(d*x + c) - 3*(b*x + a)^2*b^3*c^5*f^3
*g^5/(d*x + c)^2 - 30*a^3*b^2*c^2*d*f^3*g^5 + 72*(b*x + a)*a^2*b^2*c^3*d*f^
3*g^5/(d*x + c) - 45*(b*x + a)^2*a*b^2*c^4*d*f^3*g^5/(d*x + c)^2 + 6*(b*x +
a)^3*b^2*c^5*d*f^3*g^5/(d*x + c)^3 - 15*a^4*b*c*d^2*f^3*g^5 + 72*(b*x + a)
*a^3*b*c^2*d^2*f^3*g^5/(d*x + c) - 90*(b*x + a)^2*a^2*b*c^3*d^2*f^3*g^5/(d*
x + c)^2 + 30*(b*x + a)^3*a*b*c^4*d^2*f^3*g^5/(d*x + c)^3 - a^5*d^3*f^3*g^5
+ 12*(b*x + a)*a^4*c*d^3*f^3*g^5/(d*x + c) - 30*(b*x + a)^2*a^3*c^2*d^3*f^
3*g^5/(d*x + c)^2 + 20*(b*x + a)^3*a^2*c^3*d^3*f^3*g^5/(d*x + c)^3 + 10*a^3
*b^2*c^3*f^2*g^6 - 18*(b*x + a)*a^2*b^2*c^4*f^2*g^6/(d*x + c) + 9*(b*x + a)

^2*a*b^2*c^5*f^2*g^6/(d*x + c)^2 - (b*x + a)^3*b^2*c^6*f^2*g^6/(d*x + c)^3
+ 15*a^4*b*c^2*d*f^2*g^6 - 48*(b*x + a)*a^3*b*c^3*d*f^2*g^6/(d*x + c) + 45*
(b*x + a)^2*a^2*b*c^4*d*f^2*g^6/(d*x + c)^2 - 12*(b*x + a)^3*a*b*c^5*d*f^2*
g^6/(d*x + c)^3 + 3*a^5*c*d^2*f^2*g^6 - 18*(b*x + a)*a^4*c^2*d^2*f^2*g^6/(d
x + c) + 30(b*x + a)^2*a^3*c^3*d^2*f^2*g^6/(d*x + c)^2 - 15*(b*x + a)^3*a
^2*c^4*d^2*f^2*g^6/(d*x + c)^3 - 5*a^4*b*c^3*f*g^7 + 12*(b*x + a)*a^3*b*c^4
*f*g^7/(d*x + c) - 9*(b*x + a)^2*a^2*b*c^5*f*g^7/(d*x + c)^2 + 2*(b*x + a)^
3*a*b*c^6*f*g^7/(d*x + c)^3 - 3*a^5*c^2*d*f*g^7 + 12*(b*x + a)*a^4*c^3*d*f*
g^7/(d*x + c) - 15*(b*x + a)^2*a^3*c^4*d*f*g^7/(d*x + c)^2 + 6*(b*x + a)^3*
a^2*c^5*d*f*g^7/(d*x + c)^3 + a^5*c^3*g^8 - 3*(b*x + a)*a^4*c^4*g^8/(d*x +
c) + 3*(b*x + a)^2*a^3*c^5*g^8/(d*x + c)^2 - (b*x + a)^3*a^2*c^6*g^8/(d*x +
c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \frac{B\,d^3\,n\,\ln\left(c+d\,x\right)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g} - \frac{\ln\left(f+g\,x\right)\,\left(g^2\,\left(B\,a^3\,d^3\,n-B\,b^3\,c\right.\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f} - \frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,g\,\left(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3\right)} - \frac{B\,b^3\,n\,\ln\left(a+b\,x\right)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g} - \frac{\frac{2\,A\,a^2\,c^2\,g^4+2\,A\,b^2\,d^2\,f^4+2\,A\,a^2\,d^2\,f^2\,g^2+2\,A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2\,n-3\,B\,b^2\,c^2\,f^2\,g^2\,n-4\,A\,a\,b\,c^2\,f\,g^3-4\,A\,a\,b\,d^2\,f^3\,g-4\,A\,a^2\,c\,d\,f\,g^3-}{2\,\left(a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+\right.}}{}$$

[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^4,x)

[Out] (B*d^3*n*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*d
*f*g^3) - (log(f + g*x)*(g^2*(B*a^3*d^3*n - B*b^3*c^3*n) - g*(3*B*a^2*b*d^3
*f*n - 3*B*b^3*c^2*d*f*n) + 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c*d^2*f^2*n))/(3*
a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2
*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g +
9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2
*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d
^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - (B*log(e*((a + b*x)/(c + d*x))^n))/(
3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (B*b^3*n*log(a + b*x))/(3*
a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) - ((2*A*a^2*c^2*g^
4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2 + 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d
^2*f^2*g^2*n - 3*B*b^2*c^2*f^2*g^2*n - 4*A*a*b*c^2*f*g^3 - 4*A*a*b*d^2*f^3*
g - 4*A*a^2*c*d*f*g^3 - 4*A*b^2*c*d*f^3*g + 8*A*a*b*c*d*f^2*g^2 + B*a*b*c^2
*f*g^3*n - 5*B*a*b*d^2*f^3*g*n - B*a^2*c*d*f*g^3*n + 5*B*b^2*c*d*f^3*g*n)/(
2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^
2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f

```
^2*g^2)) + (x*(B*a*b*c^2*g^4*n - B*a^2*c*d*g^4*n + 5*B*a^2*d^2*f*g^3*n - 5*
B*b^2*c^2*f*g^3*n - 9*B*a*b*d^2*f^2*g^2*n + 9*B*b^2*c*d*f^2*g^2*n))/(2*(a^2
*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^
3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2
)) + (x^2*(B*a^2*d^2*g^4*n - B*b^2*c^2*g^4*n - 2*B*a*b*d^2*f*g^3*n + 2*B*b^
2*c*d*f*g^3*n))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2)
```

3.66 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 388

$$\begin{aligned}&\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx\\&=-\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2}\\&\quad-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n}{4(bf-ag)^3(df-cg)^3(f+gx)}\\&\quad+\frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4}-\frac{Bd^4n\log(c+dx)}{4g(df-cg)^4}\\&\quad-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\log(f+gx)}{4(bf-ag)^4(df-cg)^4}\end{aligned}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n/(-a*g+
b*f)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*n*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*
ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^4-1/4*B*d^4*n*ln(d*x+c)/g/(-c*g+d*f)^4
-1/4*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2547, 84}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$$
$$= -\frac{Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}$$
$$-\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+2d^2f^2))\right)}{4(bf-ag)^4(df-cg)^4}$$
$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}+\frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4}-\frac{Bn(bc-ad)(-adg-bcg+2bdf)}{8(f+gx)^2(bf-ag)^2(df-cg)^2}$$
$$-\frac{Bn(bc-ad)}{12(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4n\log(c+dx)}{4g(df-cg)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a
*d)*(2*b*d*f - b*c*g - a*d*g)*n)/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2
) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 -
3*c*d*f*g + c^2*g^2))*n)/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B
*n*Log[a + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(4*g*(f + g*x)^4) - (B*d^4*n*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2
*f^2 - 2*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)(f+gx)^4} - \frac{g^2(-2bdf+}{(bf-ag)^2(df-}\right.}{}$$

$$= -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n}{4(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} - \frac{Bd^4n\log(c+dx)}{4g(df-cg)^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\log(f+gx)}{4(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4} + B(bc-ad)n\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3} + \frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{g(a^2d^2g^2+abdg(-3df+cg)+}{(bf-ag)^3(df-}\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(-
1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g)
)/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(
-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f
 - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^
4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)
*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log
[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5205 vs. $2(377) = 754$.

Time = 256.76 (sec) , antiderivative size = 5206, normalized size of antiderivative = 13.42

method	result	size
parallelrisch	Expression too large to display	5206

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1761 vs. $2(374) = 748$.

Time = 0.31 (sec) , antiderivative size = 1761, normalized size of antiderivative = 4.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="maxima")
[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
```

```
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*B*n - 1/4*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*
x + f^4*g) - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f
^4*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21743 vs. $2(374) = 748$.

Time = 1.21 (sec) , antiderivative size = 21743, normalized size of antiderivative = 56.04

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")
```

[Out] 1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x + c) + 6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(d*x + c) - 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*d^5*f^2*g*n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*b^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g*n/(d*x + c) - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a*b*c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2*n/(d*x + c) + 16*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*b^3*c^4*d^3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b*x + a)*B*a^2*b^2*c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d^4*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)^3 + 4*B*a^4*b*d^5*f*g^2*n - 8*(b*x + a)*B*a^3*b*c*d^5*f*g^2*n/(d*x + c) + 24*(b*x + a)^3*B*a*b*c^3*d^5*f*g^2*n/(d*x + c)^3 - 4*(b*x + a)*B*a^4*d^6*f*g^2*n/(d*x + c) + 12*(b*x + a)^2*B*a^3*c*d^6*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*a^2*c^2*d^6*f*g^2*n/(d*x + c)^3 - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + 4*(b*x + a)*B*b^4*c^5*d*g^3*n/(d*x + c) - 4*(b*x + a)*B*a*b^3*c^4*d^2*g^3*n/(d*x + c) - 6*(b*x + a)^2*B*b^3*c^5*d^2*g^3*n/(d*x + c)^2 + 6*(b*x + a)^2*B*a*b^2*c^4*d^3*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*b^2*c^5*d^3*g^3*n/(d*x + c)^3 + B*a^4*b*c*d^4*g^3*n - 4*(b*x + a)*B*a^3*b*c^2*d^4*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*a^2*b*c^3*d^4*g^3*n/(d*x + c)^2 - 8*(b*x + a)^3*B*a*b*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*d^5*g^3*n + 4*(b*x + a)*B*a^4*c*d^5*g^3*n/(d*x + c) - 6*(b*x + a)^2*B*a^3*c^2*d^5*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*a^2*c^3*d^5*g^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^4*d^4*

f^8 - 4*(b*x + a)*b^3*d^5*f^8/(d*x + c) + 6*(b*x + a)^2*b^2*d^6*f^8/(d*x +
c)^2 - 4*(b*x + a)^3*b*d^7*f^8/(d*x + c)^3 + (b*x + a)^4*d^8*f^8/(d*x + c)^
4 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 20*(b*x + a)*b^3*c*d^4*f^7*g/(d
x + c) + 12(b*x + a)*a*b^2*d^5*f^7*g/(d*x + c) - 36*(b*x + a)^2*b^2*c*d^5
*f^7*g/(d*x + c)^2 - 12*(b*x + a)^2*a*b*d^6*f^7*g/(d*x + c)^2 + 28*(b*x + a
)^3*b*c*d^6*f^7*g/(d*x + c)^3 + 4*(b*x + a)^3*a*d^7*f^7*g/(d*x + c)^3 - 8*(
b*x + a)^4*c*d^7*f^7*g/(d*x + c)^4 + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3
*f^6*g^2 - 40*(b*x + a)*b^3*c^2*d^3*f^6*g^2/(d*x + c) + 6*a^2*b^2*d^4*f^6*g
^2 - 60*(b*x + a)*a*b^2*c*d^4*f^6*g^2/(d*x + c) + 90*(b*x + a)^2*b^2*c^2*d^
4*f^6*g^2/(d*x + c)^2 - 12*(b*x + a)*a^2*b*d^5*f^6*g^2/(d*x + c) + 72*(b*x
+ a)^2*a*b*c*d^5*f^6*g^2/(d*x + c)^2 - 84*(b*x + a)^3*b*c^2*d^5*f^6*g^2/(d*
x + c)^3 + 6*(b*x + a)^2*a^2*d^6*f^6*g^2/(d*x + c)^2 - 28*(b*x + a)^3*a*c*d
^6*f^6*g^2/(d*x + c)^3 + 28*(b*x + a)^4*c^2*d^6*f^6*g^2/(d*x + c)^4 - 4*b^4
*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 + 40*(b*x + a)*b^3*c^3*d^2*f^5*g^
3/(d*x + c) - 24*a^2*b^2*c*d^3*f^5*g^3 + 120*(b*x + a)*a*b^2*c^2*d^3*f^5*g^
3/(d*x + c) - 120*(b*x + a)^2*b^2*c^3*d^3*f^5*g^3/(d*x + c)^2 - 4*a^3*b*d^4
*f^5*g^3 + 60*(b*x + a)*a^2*b*c*d^4*f^5*g^3/(d*x + c) - 180*(b*x + a)^2*a*b
*c^2*d^4*f^5*g^3/(d*x + c)^2 + 140*(b*x + a)^3*b*c^3*d^4*f^5*g^3/(d*x + c)^
3 + 4*(b*x + a)*a^3*d^5*f^5*g^3/(d*x + c) - 36*(b*x + a)^2*a^2*c*d^5*f^5*g^
3/(d*x + c)^2 + 84*(b*x + a)^3*a*c^2*d^5*f^5*g^3/(d*x + c)^3 - 56*(b*x + a)
^4*c^3*d^5*f^5*g^3/(d*x + c)^4 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 -
20*(b*x + a)*b^3*c^4*d*f^4*g^4/(d*x + c) + 36*a^2*b^2*c^2*d^2*f^4*g^4 - 12
0*(b*x + a)*a*b^2*c^3*d^2*f^4*g^4/(d*x + c) + 90*(b*x + a)^2*b^2*c^4*d^2*f^
4*g^4/(d*x + c)^2 + 16*a^3*b*c*d^3*f^4*g^4 - 120*(b*x + a)*a^2*b*c^2*d^3*f^
4*g^4/(d*x + c) + 240*(b*x + a)^2*a*b*c^3*d^3*f^4*g^4/(d*x + c)^2 - 140*(b*
x + a)^3*b*c^4*d^3*f^4*g^4/(d*x + c)^3 + a^4*d^4*f^4*g^4 - 20*(b*x + a)*a^3
*c*d^4*f^4*g^4/(d*x + c) + 90*(b*x + a)^2*a^2*c^2*d^4*f^4*g^4/(d*x + c)^2 -
140*(b*x + a)^3*a*c^3*d^4*f^4*g^4/(d*x + c)^3 + 70*(b*x + a)^4*c^4*d^4*f^4
*g^4/(d*x + c)^4 - 4*a*b^3*c^4*f^3*g^5 + 4*(b*x + a)*b^3*c^5*f^3*g^5/(d*x +
c) - 24*a^2*b^2*c^3*d*f^3*g^5 + 60*(b*x + a)*a*b^2*c^4*d*f^3*g^5/(d*x + c)
- 36*(b*x + a)^2*b^2*c^5*d*f^3*g^5/(d*x + c)^2 - 24*a^3*b*c^2*d^2*f^3*g^5
+ 120*(b*x + a)*a^2*b*c^3*d^2*f^3*g^5/(d*x + c) - 180*(b*x + a)^2*a*b*c^4*d
^2*f^3*g^5/(d*x + c)^2 + 84*(b*x + a)^3*b*c^5*d^2*f^3*g^5/(d*x + c)^3 - 4*a
^4*c*d^3*f^3*g^5 + 40*(b*x + a)*a^3*c^2*d^3*f^3*g^5/(d*x + c) - 120*(b*x +
a)^2*a^2*c^3*d^3*f^3*g^5/(d*x + c)^2 + 140*(b*x + a)^3*a*c^4*d^3*f^3*g^5/(d
x + c)^3 - 56(b*x + a)^4*c^5*d^3*f^3*g^5/(d*x + c)^4 + 6*a^2*b^2*c^4*f^2*
g^6 - 12*(b*x + a)*a*b^2*c^5*f^2*g^6/(d*x + c) + 6*(b*x + a)^2*b^2*c^6*f^2*
g^6/(d*x + c)^2 + 16*a^3*b*c^3*d*f^2*g^6 - 60*(b*x + a)*a^2*b*c^4*d*f^2*g^6
/(d*x + c) + 72*(b*x + a)^2*a*b*c^5*d*f^2*g^6/(d*x + c)^2 - 28*(b*x + a)^3*
b*c^6*d*f^2*g^6/(d*x + c)^3 + 6*a^4*c^2*d^2*f^2*g^6 - 40*(b*x + a)*a^3*c^3*
d^2*f^2*g^6/(d*x + c) + 90*(b*x + a)^2*a^2*c^4*d^2*f^2*g^6/(d*x + c)^2 - 84
*(b*x + a)^3*a*c^5*d^2*f^2*g^6/(d*x + c)^3 + 28*(b*x + a)^4*c^6*d^2*f^2*g^6
/(d*x + c)^4 - 4*a^3*b*c^4*f*g^7 + 12*(b*x + a)*a^2*b*c^5*f*g^7/(d*x + c) -
12*(b*x + a)^2*a*b*c^6*f*g^7/(d*x + c)^2 + 4*(b*x + a)^3*b*c^7*f*g^7/(d*x
+ c)^3 - 4*a^4*c^3*d*f*g^7 + 20*(b*x + a)*a^3*c^4*d*f*g^7/(d*x + c) - 36*(b

*x + a)^2*a^2*c^5*d*f*g^7/(d*x + c)^2 + 28*(b*x + a)^3*a*c^6*d*f*g^7/(d*x +
 c)^3 - 8*(b*x + a)^4*c^7*d*f*g^7/(d*x + c)^4 + a^4*c^4*g^8 - 4*(b*x + a)*a
^3*c^5*g^8/(d*x + c) + 6*(b*x + a)^2*a^2*c^6*g^8/(d*x + c)^2 - 4*(b*x + a)^
3*a*c^7*g^8/(d*x + c)^3 + (b*x + a)^4*c^8*g^8/(d*x + c)^4) - 6*(4*B*b^5*c^2
*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^3*n - 6*B*b^5*c^3*d^
2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d^4*f^2*g*n - 6*B*a^3
*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 4*B*
a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b^5*c^5*g^3*n + B*a*b^4*c
^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a)/(d*x + c)
)/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*
g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3
- 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3
 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 +
16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2
*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2
*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4
*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + (36*B*b^8*c^3*d^2*f^5*g*n - 108
*B*a*b^7*c^2*d^3*f^5*g*n - 108*(b*x + a)*B*b^7*c^3*d^3*f^5*g*n/(d*x + c) +
108*B*a^2*b^6*c*d^4*f^5*g*n + 324*(b*x + a)*B*a*b^6*c^2*d^4*f^5*g*n/(d*x +
c) + 108*(b*x + a)^2*B*b^6*c^3*d^4*f^5*g*n/(d*x + c)^2 - 36*B*a^3*b^5*d^5*f
^5*g*n - 324*(b*x + a)*B*a^2*b^5*c*d^5*f^5*g*n/(d*x + c) - 324*(b*x + a)^2*
B*a*b^5*c^2*d^5*f^5*g*n/(d*x + c)^2 - 36*(b*x + a)^3*B*b^5*c^3*d^5*f^5*g*n/
(d*x + c)^3 + 108*(b*x + a)*B*a^3*b^4*d^6*f^5*g*n/(d*x + c) + 324*(b*x + a)
^2*B*a^2*b^4*c*d^6*f^5*g*n/(d*x + c)^2 + 108*(b*x + a)^3*B*a*b^4*c^2*d^6*f^
5*g*n/(d*x + c)^3 - 108*(b*x + a)^2*B*a^3*b^3*d^7*f^5*g*n/(d*x + c)^2 - 108
*(b*x + a)^3*B*a^2*b^3*c*d^7*f^5*g*n/(d*x + c)^3 + 36*(b*x + a)^3*B*a^3*b^2
*d^8*f^5*g*n/(d*x + c)^3 - 36*B*b^8*c^4*d*f^4*g^2*n - 36*B*a*b^7*c^3*d^2*f^
4*g^2*n + 204*(b*x + a)*B*b^7*c^4*d^2*f^4*g^2*n/(d*x + c) + 324*B*a^2*b^6*c
^2*d^3*f^4*g^2*n - 276*(b*x + a)*B*a*b^6*c^3*d^3*f^4*g^2*n/(d*x + c) - 300*
(b*x + a)^2*B*b^6*c^4*d^3*f^4*g^2*n/(d*x + c)^2 - 396*B*a^3*b^5*c*d^4*f^4*g
^2*n - 396*(b*x + a)*B*a^2*b^5*c^2*d^4*f^4*g^2*n/(d*x + c) + 660*(b*x + a)^
2*B*a*b^5*c^3*d^4*f^4*g^2*n/(d*x + c)^2 + 132*(b*x + a)^3*B*b^5*c^4*d^4*f^4
*g^2*n/(d*x + c)^3 + 144*B*a^4*b^4*d^5*f^4*g^2*n + 804*(b*x + a)*B*a^3*b^4*
c*d^5*f^4*g^2*n/(d*x + c) - 180*(b*x + a)^2*B*a^2*b^4*c^2*d^5*f^4*g^2*n/(d*
x + c)^2 - 348*(b*x + a)^3*B*a*b^4*c^3*d^5*f^4*g^2*n/(d*x + c)^3 - 336*(b*x
 + a)*B*a^4*b^3*d^6*f^4*g^2*n/(d*x + c) - 420*(b*x + a)^2*B*a^3*b^3*c*d^6*f
^4*g^2*n/(d*x + c)^2 + 252*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f^4*g^2*n/(d*x + c
)^3 + 240*(b*x + a)^2*B*a^4*b^2*d^7*f^4*g^2*n/(d*x + c)^2 + 12*(b*x + a)^3*
B*a^3*b^2*c*d^7*f^4*g^2*n/(d*x + c)^3 - 48*(b*x + a)^3*B*a^4*b*d^8*f^4*g^2*
n/(d*x + c)^3 + 11*B*b^8*c^5*f^3*g^3*n + 89*B*a*b^7*c^4*d*f^3*g^3*n - 122*(
b*x + a)*B*b^7*c^5*d*f^3*g^3*n/(d*x + c) - 106*B*a^2*b^6*c^3*d^2*f^3*g^3*n
- 206*(b*x + a)*B*a*b^6*c^4*d^2*f^3*g^3*n/(d*x + c) + 297*(b*x + a)^2*B*b^6
*c^5*d^2*f^3*g^3*n/(d*x + c)^2 - 326*B*a^3*b^5*c^2*d^3*f^3*g^3*n + 964*(b*x
 + a)*B*a^2*b^5*c^3*d^3*f^3*g^3*n/(d*x + c) - 285*(b*x + a)^2*B*a*b^5*c^4*d
^3*f^3*g^3*n/(d*x + c)^2 - 186*(b*x + a)^3*B*b^5*c^5*d^3*f^3*g^3*n/(d*x + c

)^3 + 559*B*a^4*b^4*c*d^4*f^3*g^3*n - 436*(b*x + a)*B*a^3*b^4*c^2*d^4*f^3*g
^3*n/(d*x + c) - 750*(b*x + a)^2*B*a^2*b^4*c^3*d^4*f^3*g^3*n/(d*x + c)^2 +
402*(b*x + a)^3*B*a*b^4*c^4*d^4*f^3*g^3*n/(d*x + c)^3 - 227*B*a^5*b^3*d^5*f
^3*g^3*n - 586*(b*x + a)*B*a^4*b^3*c*d^5*f^3*g^3*n/(d*x + c) + 990*(b*x + a
)^2*B*a^3*b^3*c^2*d^5*f^3*g^3*n/(d*x + c)^2 - 108*(b*x + a)^3*B*a^2*b^3*c^3
*d^5*f^3*g^3*n/(d*x + c)^3 + 386*(b*x + a)*B*a^5*b^2*d^6*f^3*g^3*n/(d*x + c
) - 75*(b*x + a)^2*B*a^4*b^2*c*d^6*f^3*g^3*n/(d*x + c)^2 - 228*(b*x + a)^3*
B*a^3*b^2*c^2*d^6*f^3*g^3*n/(d*x + c)^3 - 177*(b*x + a)^2*B*a^5*b*d^7*f^3*g
^3*n/(d*x + c)^2 + 102*(b*x + a)^3*B*a^4*b*c*d^7*f^3*g^3*n/(d*x + c)^3 + 18
*(b*x + a)^3*B*a^5*d^8*f^3*g^3*n/(d*x + c)^3 - 33*B*a*b^7*c^5*f^2*g^4*n + 2
6*(b*x + a)*B*b^7*c^6*f^2*g^4*n/(d*x + c) - 51*B*a^2*b^6*c^4*d*f^2*g^4*n +
210*(b*x + a)*B*a*b^6*c^5*d*f^2*g^4*n/(d*x + c) - 126*(b*x + a)^2*B*b^6*c^6
*d*f^2*g^4*n/(d*x + c)^2 + 174*B*a^3*b^5*c^3*d^2*f^2*g^4*n - 216*(b*x + a)*
B*a^2*b^5*c^4*d^2*f^2*g^4*n/(d*x + c) - 135*(b*x + a)^2*B*a*b^5*c^5*d^2*f^2
*g^4*n/(d*x + c)^2 + 126*(b*x + a)^3*B*b^5*c^6*d^2*f^2*g^4*n/(d*x + c)^3 +
114*B*a^4*b^4*c^2*d^3*f^2*g^4*n - 676*(b*x + a)*B*a^3*b^4*c^3*d^3*f^2*g^4*n
/(d*x + c) + 765*(b*x + a)^2*B*a^2*b^4*c^4*d^3*f^2*g^4*n/(d*x + c)^2 - 198*
(b*x + a)^3*B*a*b^4*c^5*d^3*f^2*g^4*n/(d*x + c)^3 - 381*B*a^5*b^3*c*d^4*f^2
*g^4*n + 834*(b*x + a)*B*a^4*b^3*c^2*d^4*f^2*g^4*n/(d*x + c) - 270*(b*x + a
)^2*B*a^3*b^3*c^3*d^4*f^2*g^4*n/(d*x + c)^2 - 108*(b*x + a)^3*B*a^2*b^3*c^4
*d^4*f^2*g^4*n/(d*x + c)^3 + 177*B*a^6*b^2*d^5*f^2*g^4*n + 18*(b*x + a)*B*a
^5*b^2*c*d^5*f^2*g^4*n/(d*x + c) - 540*(b*x + a)^2*B*a^4*b^2*c^2*d^5*f^2*g^
4*n/(d*x + c)^2 + 252*(b*x + a)^3*B*a^3*b^2*c^3*d^5*f^2*g^4*n/(d*x + c)^3 -
 196*(b*x + a)*B*a^6*b*d^6*f^2*g^4*n/(d*x + c) + 261*(b*x + a)^2*B*a^5*b*c*
d^6*f^2*g^4*n/(d*x + c)^2 - 18*(b*x + a)^3*B*a^4*b*c^2*d^6*f^2*g^4*n/(d*x +
 c)^3 + 45*(b*x + a)^2*B*a^6*d^7*f^2*g^4*n/(d*x + c)^2 - 54*(b*x + a)^3*B*a
^5*c*d^7*f^2*g^4*n/(d*x + c)^3 + 33*B*a^2*b^6*c^5*f*g^5*n - 52*(b*x + a)*B*
a*b^6*c^6*f*g^5*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^7*f*g^5*n/(d*x + c)^2
 - 21*B*a^3*b^5*c^4*d*f*g^5*n - 54*(b*x + a)*B*a^2*b^5*c^5*d*f*g^5*n/(d*x +
c) + 105*(b*x + a)^2*B*a*b^5*c^6*d*f*g^5*n/(d*x + c)^2 - 42*(b*x + a)^3*B*b
^5*c^7*d*f*g^5*n/(d*x + c)^3 - 66*B*a^4*b^4*c^3*d^2*f*g^5*n + 234*(b*x + a)
*B*a^3*b^4*c^4*d^2*f*g^5*n/(d*x + c) - 180*(b*x + a)^2*B*a^2*b^4*c^5*d^2*f*
g^5*n/(d*x + c)^2 + 42*(b*x + a)^3*B*a*b^4*c^6*d^2*f*g^5*n/(d*x + c)^3 - 6*
B*a^5*b^3*c^2*d^3*f*g^5*n + 104*(b*x + a)*B*a^4*b^3*c^3*d^3*f*g^5*n/(d*x +
c) - 210*(b*x + a)^2*B*a^3*b^3*c^4*d^3*f*g^5*n/(d*x + c)^2 + 72*(b*x + a)^3
*B*a^2*b^3*c^5*d^3*f*g^5*n/(d*x + c)^3 + 129*B*a^6*b^2*c*d^4*f*g^5*n - 396*
(b*x + a)*B*a^5*b^2*c^2*d^4*f*g^5*n/(d*x + c) + 345*(b*x + a)^2*B*a^4*b^2*c
^3*d^4*f*g^5*n/(d*x + c)^2 - 48*(b*x + a)^3*B*a^3*b^2*c^4*d^4*f*g^5*n/(d*x
+ c)^3 - 69*B*a^7*b*d^5*f*g^5*n + 126*(b*x + a)*B*a^6*b*c*d^5*f*g^5*n/(d*x
+ c) + 9*(b*x + a)^2*B*a^5*b*c^2*d^5*f*g^5*n/(d*x + c)^2 - 78*(b*x + a)^3*B
*a^4*b*c^3*d^5*f*g^5*n/(d*x + c)^3 + 38*(b*x + a)*B*a^7*d^6*f*g^5*n/(d*x +
c) - 90*(b*x + a)^2*B*a^6*c*d^6*f*g^5*n/(d*x + c)^2 + 54*(b*x + a)^3*B*a^5*
c^2*d^6*f*g^5*n/(d*x + c)^3 - 11*B*a^3*b^5*c^5*g^6*n + 26*(b*x + a)*B*a^2*b
^5*c^6*g^6*n/(d*x + c) - 21*(b*x + a)^2*B*a*b^5*c^7*g^6*n/(d*x + c)^2 + 6*(
b*x + a)^3*B*b^5*c^8*g^6*n/(d*x + c)^3 + 19*B*a^4*b^4*c^4*d*g^6*n - 34*(b*x

+ a)*B*a^3*b^4*c^5*d*g^6*n/(d*x + c) + 21*(b*x + a)^2*B*a^2*b^4*c^6*d*g^6*
n/(d*x + c)^2 - 6*(b*x + a)^3*B*a*b^4*c^7*d*g^6*n/(d*x + c)^3 - 2*B*a^5*b^3
*c^3*d^2*g^6*n - 16*(b*x + a)*B*a^4*b^3*c^4*d^2*g^6*n/(d*x + c) + 18*(b*x +
a)^2*B*a^3*b^3*c^5*d^2*g^6*n/(d*x + c)^2 + 2*B*a^6*b^2*c^2*d^3*g^6*n - 8*(
b*x + a)*B*a^5*b^2*c^3*d^3*g^6*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b^2*c^4*d
^3*g^6*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a^3*b^2*c^5*d^3*g^6*n/(d*x + c)^3 -
19*B*a^7*b*c*d^4*g^6*n + 70*(b*x + a)*B*a^6*b*c^2*d^4*g^6*n/(d*x + c) - 93
*(b*x + a)^2*B*a^5*b*c^3*d^4*g^6*n/(d*x + c)^2 + 42*(b*x + a)^3*B*a^4*b*c^4
*d^4*g^6*n/(d*x + c)^3 + 11*B*a^8*d^5*g^6*n - 38*(b*x + a)*B*a^7*c*d^5*g^6*
n/(d*x + c) + 45*(b*x + a)^2*B*a^6*c^2*d^5*g^6*n/(d*x + c)^2 - 18*(b*x + a)
^3*B*a^5*c^3*d^5*g^6*n/(d*x + c)^3 + 24*B*b^8*c^2*d^3*f^6*log(e) - 48*B*a*b
^7*c*d^4*f^6*log(e) - 72*(b*x + a)*B*b^7*c^2*d^4*f^6*log(e)/(d*x + c) + 24*
B*a^2*b^6*d^5*f^6*log(e) + 144*(b*x + a)*B*a*b^6*c*d^5*f^6*log(e)/(d*x + c)
+ 72*(b*x + a)^2*B*b^6*c^2*d^5*f^6*log(e)/(d*x + c)^2 - 72*(b*x + a)*B*a^2
*b^5*d^6*f^6*log(e)/(d*x + c) - 144*(b*x + a)^2*B*a*b^5*c*d^6*f^6*log(e)/(d
x + c)^2 - 24(b*x + a)^3*B*b^5*c^2*d^6*f^6*log(e)/(d*x + c)^3 + 72*(b*x +
a)^2*B*a^2*b^4*d^7*f^6*log(e)/(d*x + c)^2 + 48*(b*x + a)^3*B*a*b^4*c*d^7*f
^6*log(e)/(d*x + c)^3 - 24*(b*x + a)^3*B*a^2*b^3*d^8*f^6*log(e)/(d*x + c)^3
- 36*B*b^8*c^3*d^2*f^5*g*log(e) - 36*B*a*b^7*c^2*d^3*f^5*g*log(e) + 144*(b
*x + a)*B*b^7*c^3*d^3*f^5*g*log(e)/(d*x + c) + 180*B*a^2*b^6*c*d^4*f^5*g*lo
g(e) - 180*(b*x + a)^2*B*b^6*c^3*d^4*f^5*g*log(e)/(d*x + c)^2 - 108*B*a^3*b
^5*d^5*f^5*g*log(e) - 432*(b*x + a)*B*a^2*b^5*c*d^5*f^5*g*log(e)/(d*x + c)
+ 108*(b*x + a)^2*B*a*b^5*c^2*d^5*f^5*g*log(e)/(d*x + c)^2 + 72*(b*x + a)^3
*B*b^5*c^3*d^5*f^5*g*log(e)/(d*x + c)^3 + 288*(b*x + a)*B*a^3*b^4*d^6*f^5*g
*log(e)/(d*x + c) + 324*(b*x + a)^2*B*a^2*b^4*c*d^6*f^5*g*log(e)/(d*x + c)^
2 - 72*(b*x + a)^3*B*a*b^4*c^2*d^6*f^5*g*log(e)/(d*x + c)^3 - 252*(b*x + a)
^2*B*a^3*b^3*d^7*f^5*g*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a^2*b^3*c*d^7*
f^5*g*log(e)/(d*x + c)^3 + 72*(b*x + a)^3*B*a^3*b^2*d^8*f^5*g*log(e)/(d*x +
c)^3 + 24*B*b^8*c^4*d*f^4*g^2*log(e) + 84*B*a*b^7*c^3*d^2*f^4*g^2*log(e) -
96*(b*x + a)*B*b^7*c^4*d^2*f^4*g^2*log(e)/(d*x + c) - 36*B*a^2*b^6*c^2*d^3
*f^4*g^2*log(e) - 336*(b*x + a)*B*a*b^6*c^3*d^3*f^4*g^2*log(e)/(d*x + c) +
144*(b*x + a)^2*B*b^6*c^4*d^3*f^4*g^2*log(e)/(d*x + c)^2 - 276*B*a^3*b^5*c*
d^4*f^4*g^2*log(e) + 504*(b*x + a)*B*a^2*b^5*c^2*d^4*f^4*g^2*log(e)/(d*x +
c) + 324*(b*x + a)^2*B*a*b^5*c^3*d^4*f^4*g^2*log(e)/(d*x + c)^2 - 72*(b*x +
a)^3*B*b^5*c^4*d^4*f^4*g^2*log(e)/(d*x + c)^3 + 204*B*a^4*b^4*d^5*f^4*g^2*
log(e) + 384*(b*x + a)*B*a^3*b^4*c*d^5*f^4*g^2*log(e)/(d*x + c) - 756*(b*x
+ a)^2*B*a^2*b^4*c^2*d^5*f^4*g^2*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a*b^
4*c^3*d^5*f^4*g^2*log(e)/(d*x + c)^3 - 456*(b*x + a)*B*a^4*b^3*d^6*f^4*g^2*
log(e)/(d*x + c) - 36*(b*x + a)^2*B*a^3*b^3*c*d^6*f^4*g^2*log(e)/(d*x + c)^
2 + 288*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f^4*g^2*log(e)/(d*x + c)^3 + 324*(b*x
+ a)^2*B*a^4*b^2*d^7*f^4*g^2*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a^3*b^2
*c*d^7*f^4*g^2*log(e)/(d*x + c)^3 - 72*(b*x + a)^3*B*a^4*b*d^8*f^4*g^2*log(
e)/(d*x + c)^3 - 6*B*b^8*c^5*f^3*g^3*log(e) - 66*B*a*b^7*c^4*d*f^3*g^3*log(
e) + 24*(b*x + a)*B*b^7*c^5*d*f^3*g^3*log(e)/(d*x + c) - 36*B*a^2*b^6*c^3*d
^2*f^3*g^3*log(e) + 264*(b*x + a)*B*a*b^6*c^4*d^2*f^3*g^3*log(e)/(d*x + c)

- 36*(b*x + a)^2*B*b^6*c^5*d^2*f^3*g^3*log(e)/(d*x + c)^2 + 84*B*a^3*b^5*c^
2*d^3*f^3*g^3*log(e) + 144*(b*x + a)*B*a^2*b^5*c^3*d^3*f^3*g^3*log(e)/(d*x
+ c) - 396*(b*x + a)^2*B*a*b^5*c^4*d^3*f^3*g^3*log(e)/(d*x + c)^2 + 24*(b*x
 + a)^3*B*b^5*c^5*d^3*f^3*g^3*log(e)/(d*x + c)^3 + 234*B*a^4*b^4*c*d^4*f^3*
g^3*log(e) - 816*(b*x + a)*B*a^3*b^4*c^2*d^4*f^3*g^3*log(e)/(d*x + c) + 144
*(b*x + a)^2*B*a^2*b^4*c^3*d^4*f^3*g^3*log(e)/(d*x + c)^2 + 168*(b*x + a)^3
*B*a*b^4*c^4*d^4*f^3*g^3*log(e)/(d*x + c)^3 - 210*B*a^5*b^3*d^5*f^3*g^3*log
(e) + 24*(b*x + a)*B*a^4*b^3*c*d^5*f^3*g^3*log(e)/(d*x + c) + 864*(b*x + a)
^2*B*a^3*b^3*c^2*d^5*f^3*g^3*log(e)/(d*x + c)^2 - 192*(b*x + a)^3*B*a^2*b^3
*c^3*d^5*f^3*g^3*log(e)/(d*x + c)^3 + 360*(b*x + a)*B*a^5*b^2*d^6*f^3*g^3*l
og(e)/(d*x + c) - 396*(b*x + a)^2*B*a^4*b^2*c*d^6*f^3*g^3*log(e)/(d*x + c)^
2 - 192*(b*x + a)^3*B*a^3*b^2*c^2*d^6*f^3*g^3*log(e)/(d*x + c)^3 - 180*(b*x
 + a)^2*B*a^5*b*d^7*f^3*g^3*log(e)/(d*x + c)^2 + 168*(b*x + a)^3*B*a^4*b*c*
d^7*f^3*g^3*log(e)/(d*x + c)^3 + 24*(b*x + a)^3*B*a^5*d^8*f^3*g^3*log(e)/(d
*x + c)^3 + 18*B*a*b^7*c^5*f^2*g^4*log(e) + 54*B*a^2*b^6*c^4*d*f^2*g^4*log(
e) - 72*(b*x + a)*B*a*b^6*c^5*d*f^2*g^4*log(e)/(d*x + c) - 36*B*a^3*b^5*c^3
*d^2*f^2*g^4*log(e) - 216*(b*x + a)*B*a^2*b^5*c^4*d^2*f^2*g^4*log(e)/(d*x +
 c) + 108*(b*x + a)^2*B*a*b^5*c^5*d^2*f^2*g^4*log(e)/(d*x + c)^2 - 36*B*a^4
*b^4*c^2*d^3*f^2*g^4*log(e) + 144*(b*x + a)*B*a^3*b^4*c^3*d^3*f^2*g^4*log(e
)/(d*x + c) + 324*(b*x + a)^2*B*a^2*b^4*c^4*d^3*f^2*g^4*log(e)/(d*x + c)^2
- 72*(b*x + a)^3*B*a*b^4*c^5*d^3*f^2*g^4*log(e)/(d*x + c)^3 - 126*B*a^5*b^3
*c*d^4*f^2*g^4*log(e) + 504*(b*x + a)*B*a^4*b^3*c^2*d^4*f^2*g^4*log(e)/(d*x
 + c) - 576*(b*x + a)^2*B*a^3*b^3*c^3*d^4*f^2*g^4*log(e)/(d*x + c)^2 - 72*(
b*x + a)^3*B*a^2*b^3*c^4*d^4*f^2*g^4*log(e)/(d*x + c)^3 + 126*B*a^6*b^2*d^5
*f^2*g^4*log(e) - 216*(b*x + a)*B*a^5*b^2*c*d^5*f^2*g^4*log(e)/(d*x + c) -
216*(b*x + a)^2*B*a^4*b^2*c^2*d^5*f^2*g^4*log(e)/(d*x + c)^2 + 288*(b*x + a
)^3*B*a^3*b^2*c^3*d^5*f^2*g^4*log(e)/(d*x + c)^3 - 144*(b*x + a)*B*a^6*b*d^
6*f^2*g^4*log(e)/(d*x + c) + 324*(b*x + a)^2*B*a^5*b*c*d^6*f^2*g^4*log(e)/(
d*x + c)^2 - 72*(b*x + a)^3*B*a^4*b*c^2*d^6*f^2*g^4*log(e)/(d*x + c)^3 + 36
*(b*x + a)^2*B*a^6*d^7*f^2*g^4*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a^5*c*
d^7*f^2*g^4*log(e)/(d*x + c)^3 - 18*B*a^2*b^6*c^5*f*g^5*log(e) - 6*B*a^3*b^
5*c^4*d*f*g^5*log(e) + 72*(b*x + a)*B*a^2*b^5*c^5*d*f*g^5*log(e)/(d*x + c)
+ 24*B*a^4*b^4*c^3*d^2*f*g^5*log(e) + 24*(b*x + a)*B*a^3*b^4*c^4*d^2*f*g^5*
log(e)/(d*x + c) - 108*(b*x + a)^2*B*a^2*b^4*c^5*d^2*f*g^5*log(e)/(d*x + c)
^2 - 96*(b*x + a)*B*a^4*b^3*c^3*d^3*f*g^5*log(e)/(d*x + c) - 36*(b*x + a)^2
*B*a^3*b^3*c^4*d^3*f*g^5*log(e)/(d*x + c)^2 + 72*(b*x + a)^3*B*a^2*b^3*c^5*
d^3*f*g^5*log(e)/(d*x + c)^3 + 42*B*a^6*b^2*c*d^4*f*g^5*log(e) - 144*(b*x +
 a)*B*a^5*b^2*c^2*d^4*f*g^5*log(e)/(d*x + c) + 324*(b*x + a)^2*B*a^4*b^2*c^
3*d^4*f*g^5*log(e)/(d*x + c)^2 - 72*(b*x + a)^3*B*a^3*b^2*c^4*d^4*f*g^5*log
(e)/(d*x + c)^3 - 42*B*a^7*b*d^5*f*g^5*log(e) + 120*(b*x + a)*B*a^6*b*c*d^5
*f*g^5*log(e)/(d*x + c) - 108*(b*x + a)^2*B*a^5*b*c^2*d^5*f*g^5*log(e)/(d*x
 + c)^2 - 72*(b*x + a)^3*B*a^4*b*c^3*d^5*f*g^5*log(e)/(d*x + c)^3 + 24*(b*x
 + a)*B*a^7*d^6*f*g^5*log(e)/(d*x + c) - 72*(b*x + a)^2*B*a^6*c*d^6*f*g^5*l
og(e)/(d*x + c)^2 + 72*(b*x + a)^3*B*a^5*c^2*d^6*f*g^5*log(e)/(d*x + c)^3 +
 6*B*a^3*b^5*c^5*g^6*log(e) - 6*B*a^4*b^4*c^4*d*g^6*log(e) - 24*(b*x + a)*B

*a^3*b^4*c^5*d*g^6*log(e)/(d*x + c) + 24*(b*x + a)*B*a^4*b^3*c^4*d^2*g^6*lo
g(e)/(d*x + c) + 36*(b*x + a)^2*B*a^3*b^3*c^5*d^2*g^6*log(e)/(d*x + c)^2 -
36*(b*x + a)^2*B*a^4*b^2*c^4*d^3*g^6*log(e)/(d*x + c)^2 - 24*(b*x + a)^3*B*
a^3*b^2*c^5*d^3*g^6*log(e)/(d*x + c)^3 - 6*B*a^7*b*c*d^4*g^6*log(e) + 24*(b
*x + a)*B*a^6*b*c^2*d^4*g^6*log(e)/(d*x + c) - 36*(b*x + a)^2*B*a^5*b*c^3*d
^4*g^6*log(e)/(d*x + c)^2 + 48*(b*x + a)^3*B*a^4*b*c^4*d^4*g^6*log(e)/(d*x
+ c)^3 + 6*B*a^8*d^5*g^6*log(e) - 24*(b*x + a)*B*a^7*c*d^5*g^6*log(e)/(d*x
+ c) + 36*(b*x + a)^2*B*a^6*c^2*d^5*g^6*log(e)/(d*x + c)^2 - 24*(b*x + a)^3
*B*a^5*c^3*d^5*g^6*log(e)/(d*x + c)^3 + 24*A*b^8*c^2*d^3*f^6 - 48*A*a*b^7*c
*d^4*f^6 - 72*(b*x + a)*A*b^7*c^2*d^4*f^6/(d*x + c) + 24*A*a^2*b^6*d^5*f^6
+ 144*(b*x + a)*A*a*b^6*c*d^5*f^6/(d*x + c) + 72*(b*x + a)^2*A*b^6*c^2*d^5*
f^6/(d*x + c)^2 - 72*(b*x + a)*A*a^2*b^5*d^6*f^6/(d*x + c) - 144*(b*x + a)^
2*A*a*b^5*c*d^6*f^6/(d*x + c)^2 - 24*(b*x + a)^3*A*b^5*c^2*d^6*f^6/(d*x + c
)^3 + 72*(b*x + a)^2*A*a^2*b^4*d^7*f^6/(d*x + c)^2 + 48*(b*x + a)^3*A*a*b^4
*c*d^7*f^6/(d*x + c)^3 - 24*(b*x + a)^3*A*a^2*b^3*d^8*f^6/(d*x + c)^3 - 36*
A*b^8*c^3*d^2*f^5*g - 36*A*a*b^7*c^2*d^3*f^5*g + 144*(b*x + a)*A*b^7*c^3*d^
3*f^5*g/(d*x + c) + 180*A*a^2*b^6*c*d^4*f^5*g - 180*(b*x + a)^2*A*b^6*c^3*d
^4*f^5*g/(d*x + c)^2 - 108*A*a^3*b^5*d^5*f^5*g - 432*(b*x + a)*A*a^2*b^5*c*
d^5*f^5*g/(d*x + c) + 108*(b*x + a)^2*A*a*b^5*c^2*d^5*f^5*g/(d*x + c)^2 + 7
2*(b*x + a)^3*A*b^5*c^3*d^5*f^5*g/(d*x + c)^3 + 288*(b*x + a)*A*a^3*b^4*d^6
*f^5*g/(d*x + c) + 324*(b*x + a)^2*A*a^2*b^4*c*d^6*f^5*g/(d*x + c)^2 - 72*(
b*x + a)^3*A*a*b^4*c^2*d^6*f^5*g/(d*x + c)^3 - 252*(b*x + a)^2*A*a^3*b^3*d^
7*f^5*g/(d*x + c)^2 - 72*(b*x + a)^3*A*a^2*b^3*c*d^7*f^5*g/(d*x + c)^3 + 72
*(b*x + a)^3*A*a^3*b^2*d^8*f^5*g/(d*x + c)^3 + 24*A*b^8*c^4*d*f^4*g^2 + 84*
A*a*b^7*c^3*d^2*f^4*g^2 - 96*(b*x + a)*A*b^7*c^4*d^2*f^4*g^2/(d*x + c) - 36
*A*a^2*b^6*c^2*d^3*f^4*g^2 - 336*(b*x + a)*A*a*b^6*c^3*d^3*f^4*g^2/(d*x + c
) + 144*(b*x + a)^2*A*b^6*c^4*d^3*f^4*g^2/(d*x + c)^2 - 276*A*a^3*b^5*c*d^4
*f^4*g^2 + 504*(b*x + a)*A*a^2*b^5*c^2*d^4*f^4*g^2/(d*x + c) + 324*(b*x + a
)^2*A*a*b^5*c^3*d^4*f^4*g^2/(d*x + c)^2 - 72*(b*x + a)^3*A*b^5*c^4*d^4*f^4*
g^2/(d*x + c)^3 + 204*A*a^4*b^4*d^5*f^4*g^2 + 384*(b*x + a)*A*a^3*b^4*c*d^5
*f^4*g^2/(d*x + c) - 756*(b*x + a)^2*A*a^2*b^4*c^2*d^5*f^4*g^2/(d*x + c)^2
- 72*(b*x + a)^3*A*a*b^4*c^3*d^5*f^4*g^2/(d*x + c)^3 - 456*(b*x + a)*A*a^4*
b^3*d^6*f^4*g^2/(d*x + c) - 36*(b*x + a)^2*A*a^3*b^3*c*d^6*f^4*g^2/(d*x + c
)^2 + 288*(b*x + a)^3*A*a^2*b^3*c^2*d^6*f^4*g^2/(d*x + c)^3 + 324*(b*x + a)
^2*A*a^4*b^2*d^7*f^4*g^2/(d*x + c)^2 - 72*(b*x + a)^3*A*a^3*b^2*c*d^7*f^4*g
^2/(d*x + c)^3 - 72*(b*x + a)^3*A*a^4*b*d^8*f^4*g^2/(d*x + c)^3 - 6*A*b^8*c
^5*f^3*g^3 - 66*A*a*b^7*c^4*d*f^3*g^3 + 24*(b*x + a)*A*b^7*c^5*d*f^3*g^3/(d
*x + c) - 36*A*a^2*b^6*c^3*d^2*f^3*g^3 + 264*(b*x + a)*A*a*b^6*c^4*d^2*f^3*
g^3/(d*x + c) - 36*(b*x + a)^2*A*b^6*c^5*d^2*f^3*g^3/(d*x + c)^2 + 84*A*a^3
*b^5*c^2*d^3*f^3*g^3 + 144*(b*x + a)*A*a^2*b^5*c^3*d^3*f^3*g^3/(d*x + c) -
396*(b*x + a)^2*A*a*b^5*c^4*d^3*f^3*g^3/(d*x + c)^2 + 24*(b*x + a)^3*A*b^5*
c^5*d^3*f^3*g^3/(d*x + c)^3 + 234*A*a^4*b^4*c*d^4*f^3*g^3 - 816*(b*x + a)*A
*a^3*b^4*c^2*d^4*f^3*g^3/(d*x + c) + 144*(b*x + a)^2*A*a^2*b^4*c^3*d^4*f^3*
g^3/(d*x + c)^2 + 168*(b*x + a)^3*A*a*b^4*c^4*d^4*f^3*g^3/(d*x + c)^3 - 210
*A*a^5*b^3*d^5*f^3*g^3 + 24*(b*x + a)*A*a^4*b^3*c*d^5*f^3*g^3/(d*x + c) + 8

64*(b*x + a)^2*A*a^3*b^3*c^2*d^5*f^3*g^3/(d*x + c)^2 - 192*(b*x + a)^3*A*a^
2*b^3*c^3*d^5*f^3*g^3/(d*x + c)^3 + 360*(b*x + a)*A*a^5*b^2*d^6*f^3*g^3/(d*
x + c) - 396*(b*x + a)^2*A*a^4*b^2*c*d^6*f^3*g^3/(d*x + c)^2 - 192*(b*x + a
)^3*A*a^3*b^2*c^2*d^6*f^3*g^3/(d*x + c)^3 - 180*(b*x + a)^2*A*a^5*b*d^7*f^3
*g^3/(d*x + c)^2 + 168*(b*x + a)^3*A*a^4*b*c*d^7*f^3*g^3/(d*x + c)^3 + 24*(
b*x + a)^3*A*a^5*d^8*f^3*g^3/(d*x + c)^3 + 18*A*a*b^7*c^5*f^2*g^4 + 54*A*a^
2*b^6*c^4*d*f^2*g^4 - 72*(b*x + a)*A*a*b^6*c^5*d*f^2*g^4/(d*x + c) - 36*A*a
^3*b^5*c^3*d^2*f^2*g^4 - 216*(b*x + a)*A*a^2*b^5*c^4*d^2*f^2*g^4/(d*x + c)
+ 108*(b*x + a)^2*A*a*b^5*c^5*d^2*f^2*g^4/(d*x + c)^2 - 36*A*a^4*b^4*c^2*d^
3*f^2*g^4 + 144*(b*x + a)*A*a^3*b^4*c^3*d^3*f^2*g^4/(d*x + c) + 324*(b*x +
a)^2*A*a^2*b^4*c^4*d^3*f^2*g^4/(d*x + c)^2 - 72*(b*x + a)^3*A*a*b^4*c^5*d^3
*f^2*g^4/(d*x + c)^3 - 126*A*a^5*b^3*c*d^4*f^2*g^4 + 504*(b*x + a)*A*a^4*b^
3*c^2*d^4*f^2*g^4/(d*x + c) - 576*(b*x + a)^2*A*a^3*b^3*c^3*d^4*f^2*g^4/(d*
x + c)^2 - 72*(b*x + a)^3*A*a^2*b^3*c^4*d^4*f^2*g^4/(d*x + c)^3 + 126*A*a^6
*b^2*d^5*f^2*g^4 - 216*(b*x + a)*A*a^5*b^2*c*d^5*f^2*g^4/(d*x + c) - 216*(b
*x + a)^2*A*a^4*b^2*c^2*d^5*f^2*g^4/(d*x + c)^2 + 288*(b*x + a)^3*A*a^3*b^2
*c^3*d^5*f^2*g^4/(d*x + c)^3 - 144*(b*x + a)*A*a^6*b*d^6*f^2*g^4/(d*x + c)
+ 324*(b*x + a)^2*A*a^5*b*c*d^6*f^2*g^4/(d*x + c)^2 - 72*(b*x + a)^3*A*a^4*
b*c^2*d^6*f^2*g^4/(d*x + c)^3 + 36*(b*x + a)^2*A*a^6*d^7*f^2*g^4/(d*x + c)^
2 - 72*(b*x + a)^3*A*a^5*c*d^7*f^2*g^4/(d*x + c)^3 - 18*A*a^2*b^6*c^5*f*g^5
 - 6*A*a^3*b^5*c^4*d*f*g^5 + 72*(b*x + a)*A*a^2*b^5*c^5*d*f*g^5/(d*x + c) +
 24*A*a^4*b^4*c^3*d^2*f*g^5 + 24*(b*x + a)*A*a^3*b^4*c^4*d^2*f*g^5/(d*x + c
) - 108*(b*x + a)^2*A*a^2*b^4*c^5*d^2*f*g^5/(d*x + c)^2 - 96*(b*x + a)*A*a^
4*b^3*c^3*d^3*f*g^5/(d*x + c) - 36*(b*x + a)^2*A*a^3*b^3*c^4*d^3*f*g^5/(d*x
 + c)^2 + 72*(b*x + a)^3*A*a^2*b^3*c^5*d^3*f*g^5/(d*x + c)^3 + 42*A*a^6*b^2
*c*d^4*f*g^5 - 144*(b*x + a)*A*a^5*b^2*c^2*d^4*f*g^5/(d*x + c) + 324*(b*x +
 a)^2*A*a^4*b^2*c^3*d^4*f*g^5/(d*x + c)^2 - 72*(b*x + a)^3*A*a^3*b^2*c^4*d^
4*f*g^5/(d*x + c)^3 - 42*A*a^7*b*d^5*f*g^5 + 120*(b*x + a)*A*a^6*b*c*d^5*f*
g^5/(d*x + c) - 108*(b*x + a)^2*A*a^5*b*c^2*d^5*f*g^5/(d*x + c)^2 - 72*(b*x
 + a)^3*A*a^4*b*c^3*d^5*f*g^5/(d*x + c)^3 + 24*(b*x + a)*A*a^7*d^6*f*g^5/(d
x + c) - 72(b*x + a)^2*A*a^6*c*d^6*f*g^5/(d*x + c)^2 + 72*(b*x + a)^3*A*a
^5*c^2*d^6*f*g^5/(d*x + c)^3 + 6*A*a^3*b^5*c^5*g^6 - 6*A*a^4*b^4*c^4*d*g^6
- 24*(b*x + a)*A*a^3*b^4*c^5*d*g^6/(d*x + c) + 24*(b*x + a)*A*a^4*b^3*c^4*d
^2*g^6/(d*x + c) + 36*(b*x + a)^2*A*a^3*b^3*c^5*d^2*g^6/(d*x + c)^2 - 36*(b
*x + a)^2*A*a^4*b^2*c^4*d^3*g^6/(d*x + c)^2 - 24*(b*x + a)^3*A*a^3*b^2*c^5*
d^3*g^6/(d*x + c)^3 - 6*A*a^7*b*c*d^4*g^6 + 24*(b*x + a)*A*a^6*b*c^2*d^4*g^
6/(d*x + c) - 36*(b*x + a)^2*A*a^5*b*c^3*d^4*g^6/(d*x + c)^2 + 48*(b*x + a)
^3*A*a^4*b*c^4*d^4*g^6/(d*x + c)^3 + 6*A*a^8*d^5*g^6 - 24*(b*x + a)*A*a^7*c
*d^5*g^6/(d*x + c) + 36*(b*x + a)^2*A*a^6*c^2*d^5*g^6/(d*x + c)^2 - 24*(b*x
 + a)^3*A*a^5*c^3*d^5*g^6/(d*x + c)^3)/(b^7*d^4*f^11 - 4*(b*x + a)*b^6*d^5*
f^11/(d*x + c) + 6*(b*x + a)^2*b^5*d^6*f^11/(d*x + c)^2 - 4*(b*x + a)^3*b^4
*d^7*f^11/(d*x + c)^3 + (b*x + a)^4*b^3*d^8*f^11/(d*x + c)^4 - 4*b^7*c*d^3*
f^10*g - 7*a*b^6*d^4*f^10*g + 20*(b*x + a)*b^6*c*d^4*f^10*g/(d*x + c) + 24*
(b*x + a)*a*b^5*d^5*f^10*g/(d*x + c) - 36*(b*x + a)^2*b^5*c*d^5*f^10*g/(d*x
 + c)^2 - 30*(b*x + a)^2*a*b^4*d^6*f^10*g/(d*x + c)^2 + 28*(b*x + a)^3*b^4*

c*d^6*f^10*g/(d*x + c)^3 + 16*(b*x + a)^3*a*b^3*d^7*f^10*g/(d*x + c)^3 - 8*
(b*x + a)^4*b^3*c*d^7*f^10*g/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*d^8*f^10*g/(
d*x + c)^4 + 6*b^7*c^2*d^2*f^9*g^2 + 28*a*b^6*c*d^3*f^9*g^2 - 40*(b*x + a)*
b^6*c^2*d^3*f^9*g^2/(d*x + c) + 21*a^2*b^5*d^4*f^9*g^2 - 120*(b*x + a)*a*b^
5*c*d^4*f^9*g^2/(d*x + c) + 90*(b*x + a)^2*b^5*c^2*d^4*f^9*g^2/(d*x + c)^2
- 60*(b*x + a)*a^2*b^4*d^5*f^9*g^2/(d*x + c) + 180*(b*x + a)^2*a*b^4*c*d^5*
f^9*g^2/(d*x + c)^2 - 84*(b*x + a)^3*b^4*c^2*d^5*f^9*g^2/(d*x + c)^3 + 60*(
b*x + a)^2*a^2*b^3*d^6*f^9*g^2/(d*x + c)^2 - 112*(b*x + a)^3*a*b^3*c*d^6*f^
9*g^2/(d*x + c)^3 + 28*(b*x + a)^4*b^3*c^2*d^6*f^9*g^2/(d*x + c)^4 - 24*(b*
x + a)^3*a^2*b^2*d^7*f^9*g^2/(d*x + c)^3 + 24*(b*x + a)^4*a*b^2*c*d^7*f^9*g
^2/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*d^8*f^9*g^2/(d*x + c)^4 - 4*b^7*c^3*d*
f^8*g^3 - 42*a*b^6*c^2*d^2*f^8*g^3 + 40*(b*x + a)*b^6*c^3*d^2*f^8*g^3/(d*x
+ c) - 84*a^2*b^5*c*d^3*f^8*g^3 + 240*(b*x + a)*a*b^5*c^2*d^3*f^8*g^3/(d*x
+ c) - 120*(b*x + a)^2*b^5*c^3*d^3*f^8*g^3/(d*x + c)^2 - 35*a^3*b^4*d^4*f^8
g^3 + 300(b*x + a)*a^2*b^4*c*d^4*f^8*g^3/(d*x + c) - 450*(b*x + a)^2*a*b^
4*c^2*d^4*f^8*g^3/(d*x + c)^2 + 140*(b*x + a)^3*b^4*c^3*d^4*f^8*g^3/(d*x +
c)^3 + 80*(b*x + a)*a^3*b^3*d^5*f^8*g^3/(d*x + c) - 360*(b*x + a)^2*a^2*b^3
*c*d^5*f^8*g^3/(d*x + c)^2 + 336*(b*x + a)^3*a*b^3*c^2*d^5*f^8*g^3/(d*x + c
)^3 - 56*(b*x + a)^4*b^3*c^3*d^5*f^8*g^3/(d*x + c)^4 - 60*(b*x + a)^2*a^3*b
^2*d^6*f^8*g^3/(d*x + c)^2 + 168*(b*x + a)^3*a^2*b^2*c*d^6*f^8*g^3/(d*x + c
)^3 - 84*(b*x + a)^4*a*b^2*c^2*d^6*f^8*g^3/(d*x + c)^4 + 16*(b*x + a)^3*a^3
*b*d^7*f^8*g^3/(d*x + c)^3 - 24*(b*x + a)^4*a^2*b*c*d^7*f^8*g^3/(d*x + c)^4
- (b*x + a)^4*a^3*d^8*f^8*g^3/(d*x + c)^4 + b^7*c^4*f^7*g^4 + 28*a*b^6*c^3
*d*f^7*g^4 - 20*(b*x + a)*b^6*c^4*d*f^7*g^4/(d*x + c) + 126*a^2*b^5*c^2*d^2
*f^7*g^4 - 240*(b*x + a)*a*b^5*c^3*d^2*f^7*g^4/(d*x + c) + 90*(b*x + a)^2*b
^5*c^4*d^2*f^7*g^4/(d*x + c)^2 + 140*a^3*b^4*c*d^3*f^7*g^4 - 600*(b*x + a)*
a^2*b^4*c^2*d^3*f^7*g^4/(d*x + c) + 600*(b*x + a)^2*a*b^4*c^3*d^3*f^7*g^4/(
d*x + c)^2 - 140*(b*x + a)^3*b^4*c^4*d^3*f^7*g^4/(d*x + c)^3 + 35*a^4*b^3*d
^4*f^7*g^4 - 400*(b*x + a)*a^3*b^3*c*d^4*f^7*g^4/(d*x + c) + 900*(b*x + a)^
2*a^2*b^3*c^2*d^4*f^7*g^4/(d*x + c)^2 - 560*(b*x + a)^3*a*b^3*c^3*d^4*f^7*g
^4/(d*x + c)^3 + 70*(b*x + a)^4*b^3*c^4*d^4*f^7*g^4/(d*x + c)^4 - 60*(b*x +
a)*a^4*b^2*d^5*f^7*g^4/(d*x + c) + 360*(b*x + a)^2*a^3*b^2*c*d^5*f^7*g^4/(
d*x + c)^2 - 504*(b*x + a)^3*a^2*b^2*c^2*d^5*f^7*g^4/(d*x + c)^3 + 168*(b*x
+ a)^4*a*b^2*c^3*d^5*f^7*g^4/(d*x + c)^4 + 30*(b*x + a)^2*a^4*b*d^6*f^7*g^
4/(d*x + c)^2 - 112*(b*x + a)^3*a^3*b*c*d^6*f^7*g^4/(d*x + c)^3 + 84*(b*x +
a)^4*a^2*b*c^2*d^6*f^7*g^4/(d*x + c)^4 - 4*(b*x + a)^3*a^4*d^7*f^7*g^4/(d*
x + c)^3 + 8*(b*x + a)^4*a^3*c*d^7*f^7*g^4/(d*x + c)^4 - 7*a*b^6*c^4*f^6*g^
5 + 4*(b*x + a)*b^6*c^5*f^6*g^5/(d*x + c) - 84*a^2*b^5*c^3*d*f^6*g^5 + 120*
(b*x + a)*a*b^5*c^4*d*f^6*g^5/(d*x + c) - 36*(b*x + a)^2*b^5*c^5*d*f^6*g^5/
(d*x + c)^2 - 210*a^3*b^4*c^2*d^2*f^6*g^5 + 600*(b*x + a)*a^2*b^4*c^3*d^2*f
^6*g^5/(d*x + c) - 450*(b*x + a)^2*a*b^4*c^4*d^2*f^6*g^5/(d*x + c)^2 + 84*(
b*x + a)^3*b^4*c^5*d^2*f^6*g^5/(d*x + c)^3 - 140*a^4*b^3*c*d^3*f^6*g^5 + 80
0*(b*x + a)*a^3*b^3*c^2*d^3*f^6*g^5/(d*x + c) - 1200*(b*x + a)^2*a^2*b^3*c^
3*d^3*f^6*g^5/(d*x + c)^2 + 560*(b*x + a)^3*a*b^3*c^4*d^3*f^6*g^5/(d*x + c)
^3 - 56*(b*x + a)^4*b^3*c^5*d^3*f^6*g^5/(d*x + c)^4 - 21*a^5*b^2*d^4*f^6*g^

5 + 300*(b*x + a)*a^4*b^2*c*d^4*f^6*g^5/(d*x + c) - 900*(b*x + a)^2*a^3*b^2
*c^2*d^4*f^6*g^5/(d*x + c)^2 + 840*(b*x + a)^3*a^2*b^2*c^3*d^4*f^6*g^5/(d*x
 + c)^3 - 210*(b*x + a)^4*a*b^2*c^4*d^4*f^6*g^5/(d*x + c)^4 + 24*(b*x + a)*
a^5*b*d^5*f^6*g^5/(d*x + c) - 180*(b*x + a)^2*a^4*b*c*d^5*f^6*g^5/(d*x + c)
^2 + 336*(b*x + a)^3*a^3*b*c^2*d^5*f^6*g^5/(d*x + c)^3 - 168*(b*x + a)^4*a^
2*b*c^3*d^5*f^6*g^5/(d*x + c)^4 - 6*(b*x + a)^2*a^5*d^6*f^6*g^5/(d*x + c)^2
 + 28*(b*x + a)^3*a^4*c*d^6*f^6*g^5/(d*x + c)^3 - 28*(b*x + a)^4*a^3*c^2*d^
6*f^6*g^5/(d*x + c)^4 + 21*a^2*b^5*c^4*f^5*g^6 - 24*(b*x + a)*a*b^5*c^5*f^5
*g^6/(d*x + c) + 6*(b*x + a)^2*b^5*c^6*f^5*g^6/(d*x + c)^2 + 140*a^3*b^4*c^
3*d*f^5*g^6 - 300*(b*x + a)*a^2*b^4*c^4*d*f^5*g^6/(d*x + c) + 180*(b*x + a)
^2*a*b^4*c^5*d*f^5*g^6/(d*x + c)^2 - 28*(b*x + a)^3*b^4*c^6*d*f^5*g^6/(d*x
+ c)^3 + 210*a^4*b^3*c^2*d^2*f^5*g^6 - 800*(b*x + a)*a^3*b^3*c^3*d^2*f^5*g^
6/(d*x + c) + 900*(b*x + a)^2*a^2*b^3*c^4*d^2*f^5*g^6/(d*x + c)^2 - 336*(b*
x + a)^3*a*b^3*c^5*d^2*f^5*g^6/(d*x + c)^3 + 28*(b*x + a)^4*b^3*c^6*d^2*f^5
*g^6/(d*x + c)^4 + 84*a^5*b^2*c*d^3*f^5*g^6 - 600*(b*x + a)*a^4*b^2*c^2*d^3
*f^5*g^6/(d*x + c) + 1200*(b*x + a)^2*a^3*b^2*c^3*d^3*f^5*g^6/(d*x + c)^2 -
 840*(b*x + a)^3*a^2*b^2*c^4*d^3*f^5*g^6/(d*x + c)^3 + 168*(b*x + a)^4*a*b^
2*c^5*d^3*f^5*g^6/(d*x + c)^4 + 7*a^6*b*d^4*f^5*g^6 - 120*(b*x + a)*a^5*b*c
*d^4*f^5*g^6/(d*x + c) + 450*(b*x + a)^2*a^4*b*c^2*d^4*f^5*g^6/(d*x + c)^2
- 560*(b*x + a)^3*a^3*b*c^3*d^4*f^5*g^6/(d*x + c)^3 + 210*(b*x + a)^4*a^2*b
*c^4*d^4*f^5*g^6/(d*x + c)^4 - 4*(b*x + a)*a^6*d^5*f^5*g^6/(d*x + c) + 36*(
b*x + a)^2*a^5*c*d^5*f^5*g^6/(d*x + c)^2 - 84*(b*x + a)^3*a^4*c^2*d^5*f^5*g
^6/(d*x + c)^3 + 56*(b*x + a)^4*a^3*c^3*d^5*f^5*g^6/(d*x + c)^4 - 35*a^3*b^
4*c^4*f^4*g^7 + 60*(b*x + a)*a^2*b^4*c^5*f^4*g^7/(d*x + c) - 30*(b*x + a)^2
*a*b^4*c^6*f^4*g^7/(d*x + c)^2 + 4*(b*x + a)^3*b^4*c^7*f^4*g^7/(d*x + c)^3
- 140*a^4*b^3*c^3*d*f^4*g^7 + 400*(b*x + a)*a^3*b^3*c^4*d*f^4*g^7/(d*x + c)
 - 360*(b*x + a)^2*a^2*b^3*c^5*d*f^4*g^7/(d*x + c)^2 + 112*(b*x + a)^3*a*b^
3*c^6*d*f^4*g^7/(d*x + c)^3 - 8*(b*x + a)^4*b^3*c^7*d*f^4*g^7/(d*x + c)^4 -
 126*a^5*b^2*c^2*d^2*f^4*g^7 + 600*(b*x + a)*a^4*b^2*c^3*d^2*f^4*g^7/(d*x +
 c) - 900*(b*x + a)^2*a^3*b^2*c^4*d^2*f^4*g^7/(d*x + c)^2 + 504*(b*x + a)^3
*a^2*b^2*c^5*d^2*f^4*g^7/(d*x + c)^3 - 84*(b*x + a)^4*a*b^2*c^6*d^2*f^4*g^7
/(d*x + c)^4 - 28*a^6*b*c*d^3*f^4*g^7 + 240*(b*x + a)*a^5*b*c^2*d^3*f^4*g^7
/(d*x + c) - 600*(b*x + a)^2*a^4*b*c^3*d^3*f^4*g^7/(d*x + c)^2 + 560*(b*x +
 a)^3*a^3*b*c^4*d^3*f^4*g^7/(d*x + c)^3 - 168*(b*x + a)^4*a^2*b*c^5*d^3*f^4
*g^7/(d*x + c)^4 - a^7*d^4*f^4*g^7 + 20*(b*x + a)*a^6*c*d^4*f^4*g^7/(d*x +
c) - 90*(b*x + a)^2*a^5*c^2*d^4*f^4*g^7/(d*x + c)^2 + 140*(b*x + a)^3*a^4*c
^3*d^4*f^4*g^7/(d*x + c)^3 - 70*(b*x + a)^4*a^3*c^4*d^4*f^4*g^7/(d*x + c)^4
 + 35*a^4*b^3*c^4*f^3*g^8 - 80*(b*x + a)*a^3*b^3*c^5*f^3*g^8/(d*x + c) + 60
*(b*x + a)^2*a^2*b^3*c^6*f^3*g^8/(d*x + c)^2 - 16*(b*x + a)^3*a*b^3*c^7*f^3
*g^8/(d*x + c)^3 + (b*x + a)^4*b^3*c^8*f^3*g^8/(d*x + c)^4 + 84*a^5*b^2*c^3
*d*f^3*g^8 - 300*(b*x + a)*a^4*b^2*c^4*d*f^3*g^8/(d*x + c) + 360*(b*x + a)^
2*a^3*b^2*c^5*d*f^3*g^8/(d*x + c)^2 - 168*(b*x + a)^3*a^2*b^2*c^6*d*f^3*g^8
/(d*x + c)^3 + 24*(b*x + a)^4*a*b^2*c^7*d*f^3*g^8/(d*x + c)^4 + 42*a^6*b*c^
2*d^2*f^3*g^8 - 240*(b*x + a)*a^5*b*c^3*d^2*f^3*g^8/(d*x + c) + 450*(b*x +
a)^2*a^4*b*c^4*d^2*f^3*g^8/(d*x + c)^2 - 336*(b*x + a)^3*a^3*b*c^5*d^2*f^3*

```
g^8/(d*x + c)^3 + 84*(b*x + a)^4*a^2*b*c^6*d^2*f^3*g^8/(d*x + c)^4 + 4*a^7*
c*d^3*f^3*g^8 - 40*(b*x + a)*a^6*c^2*d^3*f^3*g^8/(d*x + c) + 120*(b*x + a)^
2*a^5*c^3*d^3*f^3*g^8/(d*x + c)^2 - 140*(b*x + a)^3*a^4*c^4*d^3*f^3*g^8/(d*
x + c)^3 + 56*(b*x + a)^4*a^3*c^5*d^3*f^3*g^8/(d*x + c)^4 - 21*a^5*b^2*c^4*
f^2*g^9 + 60*(b*x + a)*a^4*b^2*c^5*f^2*g^9/(d*x + c) - 60*(b*x + a)^2*a^3*b
^2*c^6*f^2*g^9/(d*x + c)^2 + 24*(b*x + a)^3*a^2*b^2*c^7*f^2*g^9/(d*x + c)^3
 - 3*(b*x + a)^4*a*b^2*c^8*f^2*g^9/(d*x + c)^4 - 28*a^6*b*c^3*d*f^2*g^9 + 1
20*(b*x + a)*a^5*b*c^4*d*f^2*g^9/(d*x + c) - 180*(b*x + a)^2*a^4*b*c^5*d*f^
2*g^9/(d*x + c)^2 + 112*(b*x + a)^3*a^3*b*c^6*d*f^2*g^9/(d*x + c)^3 - 24*(b
*x + a)^4*a^2*b*c^7*d*f^2*g^9/(d*x + c)^4 - 6*a^7*c^2*d^2*f^2*g^9 + 40*(b*x
 + a)*a^6*c^3*d^2*f^2*g^9/(d*x + c) - 90*(b*x + a)^2*a^5*c^4*d^2*f^2*g^9/(d
*x + c)^2 + 84*(b*x + a)^3*a^4*c^5*d^2*f^2*g^9/(d*x + c)^3 - 28*(b*x + a)^4
*a^3*c^6*d^2*f^2*g^9/(d*x + c)^4 + 7*a^6*b*c^4*f*g^10 - 24*(b*x + a)*a^5*b*
c^5*f*g^10/(d*x + c) + 30*(b*x + a)^2*a^4*b*c^6*f*g^10/(d*x + c)^2 - 16*(b*
x + a)^3*a^3*b*c^7*f*g^10/(d*x + c)^3 + 3*(b*x + a)^4*a^2*b*c^8*f*g^10/(d*x
 + c)^4 + 4*a^7*c^3*d*f*g^10 - 20*(b*x + a)*a^6*c^4*d*f*g^10/(d*x + c) + 36
*(b*x + a)^2*a^5*c^5*d*f*g^10/(d*x + c)^2 - 28*(b*x + a)^3*a^4*c^6*d*f*g^10
/(d*x + c)^3 + 8*(b*x + a)^4*a^3*c^7*d*f*g^10/(d*x + c)^4 - a^7*c^4*g^11 +
4*(b*x + a)*a^6*c^5*g^11/(d*x + c) - 6*(b*x + a)^2*a^5*c^6*g^11/(d*x + c)^2
 + 4*(b*x + a)^3*a^4*c^7*g^11/(d*x + c)^3 - (b*x + a)^4*a^3*c^8*g^11/(d*x +
 c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 2569, normalized size of antiderivative = 6.62

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^5,x)
```

```
[Out] ((x^3*(B*a^3*d^3*g^6*n - B*b^3*c^3*g^6*n - 3*B*a^2*b*d^3*f*g^5*n + 3*B*b^3*
c^2*d*f*g^5*n + 3*B*a*b^2*d^3*f^2*g^4*n - 3*B*b^3*c*d^2*f^2*g^4*n))/(a^3*c^
3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*
f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 +
 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*
a^2*b*c^2*d*f^2*g^4) - (6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3
*g^3 - 6*A*b^3*c^3*f^3*g^3 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^
2 + 18*A*a^3*c*d^2*f^2*g^4 + 18*A*b^3*c^2*d*f^4*g^2 - 11*B*a^3*d^3*f^3*g^3*
n + 11*B*b^3*c^3*f^3*g^3*n - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^3*f^5*g -
18*A*a^3*c^2*d*f*g^5 - 18*A*b^3*c*d^2*f^5*g + 2*B*a^2*b*c^3*f*g^5*n - 26*B*
a*b^2*d^3*f^5*g*n - 2*B*a^3*c^2*d*f*g^5*n + 26*B*b^3*c*d^2*f^5*g*n + 54*A*a
*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3 +
54*A*a^2*b*c^2*d*f^2*g^4 - 7*B*a*b^2*c^3*f^2*g^4*n + 31*B*a^2*b*d^3*f^4*g^2
```

*n + 7*B*a^3*c*d^2*f^2*g^4*n - 31*B*b^3*c^2*d*f^4*g^2*n + 15*B*a*b^2*c^2*d*
f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3
*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*
a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4
*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 -
9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (
x^2*(B*a*b^2*c^3*g^6*n - B*a^3*c*d^2*g^6*n + 7*B*a^3*d^3*f*g^5*n - 7*B*b^3*
c^3*f*g^5*n + 20*B*a*b^2*d^3*f^3*g^3*n - 21*B*a^2*b*d^3*f^2*g^4*n - 20*B*b^
3*c*d^2*f^3*g^3*n + 21*B*b^3*c^2*d*f^2*g^4*n - 3*B*a*b^2*c^2*d*f*g^5*n + 3*
B*a^2*b*c*d^2*f*g^5*n))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b
^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5
- 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d
^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^
3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(13*B*a^3*d^3*
f^2*g^4*n - 13*B*b^3*c^3*f^2*g^4*n - B*a^2*b*c^3*g^6*n + B*a^3*c^2*d*g^6*n
+ 5*B*a*b^2*c^3*f*g^5*n - 5*B*a^3*c*d^2*f*g^5*n + 34*B*a*b^2*d^3*f^4*g^2*n
- 38*B*a^2*b*d^3*f^3*g^3*n - 34*B*b^3*c*d^2*f^4*g^2*n + 38*B*b^3*c^2*d*f^3*
g^3*n - 12*B*a*b^2*c^2*d*f^2*g^4*n + 12*B*a^2*b*c*d^2*f^2*g^4*n))/(3*(a^3*c
^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^
5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3
*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2
+ 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9
*a^2*b*c^2*d*f^2*g^4)))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f*g^4*x^3
+ 24*f^2*g^3*x^2) + (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2*n - 6*B*b^4*c^2*d
^2*f^2*n) - g^2*(4*B*a^3*b*d^4*f*n - 4*B*b^4*c^3*d*f*n) + g^3*(B*a^4*d^4*n
- B*b^4*c^4*n) - 4*B*a*b^3*d^4*f^3*n + 4*B*b^4*c*d^3*f^3*n))/(4*a^4*c^4*g^8
 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f
^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f
^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*
b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^
3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 64*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*
f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 64*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^
2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*
c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^2*f^4*g^4) - (B*log(e*((a + b*x)/(c + d
*x))^n))/(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) +
(B*b^4*n*log(a + b*x))/(4*a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2
*b^2*f^2*g^3 - 16*a^3*b*f*g^4) - (B*d^4*n*log(c + d*x))/(4*c^4*g^5 + 4*d^4*
f^4*g - 16*c*d^3*f^3*g^2 + 24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)

3.67 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 923

$$\begin{aligned}
\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= \frac{B^2(bc-ad)^3g^3n^2x}{6b^3d^3} \\
&+ \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)n^2x}{4b^3d^3} + \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d^4} \\
&- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^3} \\
&- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4} \\
&- \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4} \\
&- \frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} \\
&- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^4} \\
&+ \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4} \\
&+ \frac{B^2(bc-ad)^4g^3n^2\log(c+dx)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log(c+dx)}{4b^4d^4} \\
&+ \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2\log(c+dx)}{2b^4d^4} \\
&- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}
\end{aligned}$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b
*c*g+4*b*d*f)*n^2*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/b^2/d^4
```

```
-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*
d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/4*B*(
-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/b/d^4-1/4*(-a*g+b*f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+
2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+1/6*B^2*(-a*
d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/6*B^2*(-a*d+b*c)^4*g^
3*n^2*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n
^2*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*
d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*n^2*ln(d*x+c)/b^4/d^4-1/2*B^2*(-a
*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*
d*f*g+2*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 923, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules

used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{B^2g^3n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{6b^4d^4} + \frac{B^2g^3n^2\log(c+dx)(bc-ad)^4}{6b^4d^4} + \frac{B^2g^3n^2x(bc-ad)^3}{6b^3d^3} + \frac{B^2g^2(4bdf-3bcg-adg)n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^3}{4b^4d^4} + \frac{B^2g^2(4bdf-3bcg-adg)n^2\log(c+dx)(bc-ad)^3}{4b^4d^4} + \frac{B^2g^3n^2(c+dx)^2(bc-ad)^2}{12b^2d^4} + \frac{B^2g^2(4bdf-3bcg-adg)n^2x(bc-ad)^2}{4b^3d^3} + \frac{B^2g\left((6d^2f^2-8cdgf+3c^2g^2)b^2-2adg(2df-cg)b+a^2d^2g^2\right)n^2\log(c+dx)(bc-ad)^2}{2b^4d^4} - \frac{Bg^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{6bd^4} - \frac{Bg^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{4b^2d^4} - \frac{Bg\left((6d^2f^2-8cdgf+3c^2g^2)b^2-2adg(2df-cg)b+a^2d^2g^2\right)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{2b^4d^3} - \frac{B(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\right.}{2b^4d^4} - \frac{B^2(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-}{2b^4d^4} - \frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^3*g^3*n^2*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f
 - 3*b*c*g - a*d*g)*n^2*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c + d*
x)^2)/(12*b^2*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g
) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*n*(a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g -
 a*d*g)*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d^4) -
 (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
6*b*d^4) - ((b*f - a*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b^4*
g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g) - (B*(b*c
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2
*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(
b*c - a*d)/(b*(c + d*x))])/(2*b^4*d^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[(a
+ b*x)/(c + d*x)])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g
- a*d*g)*n^2*Log[(a + b*x)/(c + d*x)])/(4*b^4*d^4) + (B^2*(b*c - a*d)^4*g^3
```

```
*n^2*Log[c + d*x])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g
- a*d*g)*n^2*Log[c + d*x])/(4*b^4*d^4) + (B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2
- 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*n^2*Lo
g[c + d*x])/(2*b^4*d^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b
*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b^4*d^4)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

<u>Rule 2338</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

<u>Rule 2351</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

<u>Rule 2356</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^3\left(A+B\log\left(ex^n\right)\right)^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(Bn)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^4(A+B\log(ex^n))}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(Bn)\text{Subst}\left(\int \left(\frac{(bf-ag)^4(A+B\log(ex^n))}{b^4x} + \frac{(bc-ad)^4g^4(A+B\log(ex^n))}{bd^3(b-dx)^4} + \frac{(bc-ad)^3g^3(4bdf-3bcg-adg)(A+B\log(ex^n))}{b^2d^3(b-dx)^3}\right.\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)^4g^3n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^3} - \frac{(B(bf-ag)^4n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^4g} - \frac{(B(bc-ad)^3g^2(4bdf-3bcg-adg)n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b^2d^3} + \frac{(B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n)\,\mathrm{Subst}\left(\int\frac{A+}{}\right.}{2b^4d^3} - \frac{(B(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\right.}{2b^3d^3}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\right.\right.\right.}{2b^4d^3} - \frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4} - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4} - \frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\right.\right.\right.}{2b^4d^4} + \frac{(B^2(bc-ad)^4g^3n^2)\,\mathrm{Subst}\left(\int\frac{1}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{6bd^4} + \frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2)\,\mathrm{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4b^2d^4} + \frac{(B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2)\,\mathrm{Subst}\left(\int\frac{\mathrm{lo}}{}\right.}{2b^4d^4} + \frac{(B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2)\,\mathrm{Subst}\left(\int\frac{1}{b-dx}\,dx,x\right.}{2b^4d^3}$$

$$
\begin{aligned}
= & -\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^3} \\
& -\frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4} \\
& -\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4} \\
& -\frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} \\
& -\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^4} \\
& +\frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2\log(c+dx)}{2b^4d^4} \\
& -\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4} \\
& +\frac{(B^2(bc-ad)^4g^3n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{6bd^4} \\
& +\frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{4b^2d^4}
\end{aligned}
$$

$$= \frac{B^2(bc-ad)^3g^3n^2x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)n^2x}{4b^3d^3}$$
$$+ \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d^4}$$
$$- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e(\right.\right.}{2b^4d^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4}$$
$$- \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4}$$
$$- \frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e(\right.\right.}{2b^4d^4}$$
$$+ \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$
$$+ \frac{B^2(bc-ad)^4g^3n^2\log(c+dx)}{6b^4d^4}$$
$$+ \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log(c+dx)}{4b^4d^4}$$
$$+ \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2\log(c+dx)}{2b^4d^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 757, normalized size of antiderivative = 0.82

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{Bn\left(6Abd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+6Bd(bc-ad)g^2(\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(6*A*b*d*(b*c
- a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f
*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c
*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log
```

```
[e*((a + b*x)/(c + d*x))^n]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*
(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*Log[c + d*x] - 6*
b^4*(d*f - c*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(
b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log
[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g
 + a*d*g)*n*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c
+ d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b
^4*B*(d*f - c*g)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g
)
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x + c
))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(e
*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2651 vs. 2(892) = 1784.

Time = 0.71 (sec) , antiderivative size = 2651, normalized size of antiderivative = 2.87

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*A*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*g^3*x^4 +
2*A*B*f*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g^2*x^3 + 3*
A*B*f^2*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*f^2*g*x^2 -
1/12*A*B*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A
*B*f^2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*
d)) + 2*A*B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^3*x*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f^3*x - 1/12*(6*a^3*c*d^3*g^3*n^2 -
 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2*g*n^2 - 6*
c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 + (24*c*d^3*f^3*n*log(e) - (11*g^3
*n^2 + 6*g^3*n*log(e))*c^4 + 12*(3*f*g^2*n^2 + 2*f*g^2*n*log(e))*c^3*d - 36
*(f^2*g*n^2 + f^2*g*n*log(e))*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/
2*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 -
a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^
2 - c^4*g^3*n^2)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^
4*log(e)^2 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 -
 c^4*g^3*n^2)*B^2*b^4*log(b*x + a)*log(d*x + c) - 3*(4*c*d^3*f^3*n^2 - 6*c^
2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*log(d*x + c)^2 +
 2*(a*b^3*d^4*g^3*n*log(e) - (c*d^3*g^3*n*log(e) - 6*d^4*f*g^2*log(e)^2)*b^
4)*B^2*x^3 + ((g^3*n^2 - 3*g^3*n*log(e))*a^2*b^2*d^4 - 2*(c*d^3*g^3*n^2 - 6
```

```
*d^4*f*g^2*n*log(e))*a*b^3 - (12*c*d^3*f*g^2*n*log(e) - 18*d^4*f^2*g*log(e)
^2 - (g^3*n^2 + 3*g^3*n*log(e))*c^2*d^2)*b^4)*B^2*x^2 - 3*(4*a*b^3*d^4*f^3*
n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 - a^4*d^4*g^3*n^2)*B^
2*log(b*x + a)^2 - ((5*g^3*n^2 - 6*g^3*n*log(e))*a^3*b*d^4 - (5*c*d^3*g^3*n
^2 + 12*(f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^2*b^2 + (24*c*d^3*f*g^2*n^2 -
 5*c^2*d^2*g^3*n^2 - 36*d^4*f^2*g*n*log(e))*a*b^3 + (36*c*d^3*f^2*g*n*log(e
) - 12*d^4*f^3*log(e)^2 + (5*g^3*n^2 + 6*g^3*n*log(e))*c^3*d - 12*(f*g^2*n^
2 + 2*f*g^2*n*log(e))*c^2*d^2)*b^4)*B^2*x + ((11*g^3*n^2 - 6*g^3*n*log(e))*
a^4*d^4 - 2*(c*d^3*g^3*n^2 + 6*(3*f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^3*b
+ 3*(4*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2 + 12*(f^2*g*n^2 - f^2*g*n*log(e))*
d^4)*a^2*b^2 - 6*(6*c*d^3*f^2*g*n^2 - 4*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2 -
 4*d^4*f^3*n*log(e))*a*b^3)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B
^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((
b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^
4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*
g^3*x^4*log(e) - 6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c
^4*g^3*n)*B^2*b^4*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4
*f*g^2*log(e))*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n -
(4*c*d^3*f*g^2*n - c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6
*a*b^3*d^4*f^2*g*n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2
*g*n - 4*c^2*d^2*f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(
4*a*b^3*d^4*f^3*n - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g
^3*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) -
6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c^4*g^3*n)*B^2*b^4
*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4*f*g^2*log(e))*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n - (4*c*d^3*f*g^2*n
- c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g*
n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2*g*n - 4*c^2*d^2*
f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(4*a*b^3*d^4*f^3*n
 - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g^3*n)*B^2*log(b*x
 + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^
2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d^4
)
```

Giac [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (f+g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.68 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 565

$$\begin{aligned}
&\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} - \frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2} \\
&\quad - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3} \\
&\quad - \frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} \\
&\quad + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^3d^3} \\
&\quad + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{3b^3d^3} \\
&\quad + \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2\log(c+dx)}{3b^3d^3} \\
&\quad + \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2/d^2-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3
*b*d*f)*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/3*B*(-a*d+b*c)*
g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/3*(-a*g+b*f)^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/g+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-
3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*
x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/3*
B^2*(-a*d+b*c)^3*g^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2
```

```
*b*c*g+3*b*d*f)*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d
*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/
b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\begin{aligned}\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx\\ =\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d^3}\\ +\frac{2B^2n^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}\\ -\frac{2Bgn(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d^2}\\ -\frac{(bf-ag)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b^3g}-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd^3}\\ +\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g}\\ +\frac{2B^2gn^2(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}\\ +\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2g^2n^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2g^2n^2x(bc-ad)^2}{3b^2d^2}\end{aligned}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b^2*d^2) - (2*B*(b*c - a*d)*g*(3*b*d*f - 2
*b*c*g - a*d*g)*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3*
d^2) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*b*d^3) - ((b*f - a*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*g) + (
2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c
*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)
/(b*(c + d*x))])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[(a + b*x)/(c
+ d*x)])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[c + d*x])/(3*b^3*d^3)
 + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - 2*b*c*g - a*d*g)*n^2*Log[c + d*x])/(3*
b^3*d^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3
*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
```

```
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^2\left(A+B\log\left(ex^n\right)\right)^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2Bn)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^3(A+B\log(ex^n))}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2Bn)\text{Subst}\left(\int \left(\frac{(bf-ag)^3(A+B\log(ex^n))}{b^3x} + \frac{(bc-ad)^3g^3(A+B\log(ex^n))}{bd^2(b-dx)^3} + \frac{(bc-ad)^2g^2(3bdf-2bcg-adg)(A+B\log(ex^n\right.}{b^2d^2(b-dx)^2}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g}$$

$$-\frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^2}$$

$$-\frac{(2B(bf-ag)^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3g}$$

$$-\frac{(2B(bc-ad)^2g(3bdf-2bcg-adg)n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2d^2}$$

$$-\frac{(2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x\right.}{3b^3d^2}$$

$$= -\frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2}$$

$$-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3}$$

$$-\frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g}$$

$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log}{3b^3d^3}$$

$$+\frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$+\frac{(2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3d^2}$$

$$-\frac{(2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,\right.}{3b^3d^3}$$

$$= -\frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2}$$
$$-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3}$$
$$-\frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{lo}}{3b^3d^3}$$
$$+\frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2\log(c+dx)}{3b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$
$$+\frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$=\frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2}$$
$$-\frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2}$$
$$-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3}$$
$$-\frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{lo}}{3b^3d^3}$$
$$+\frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{3b^3d^3}$$
$$+\frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2\log(c+dx)}{3b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.90

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b
*c*g - a*d*g)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*d^2*(b*c - a*d
)*g^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*g^2*(-3
*b*d*f + b*c*g + a*d*g)*n*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*(b*c - a*d)*g^3*n*(a^2*d^2*Log[a
+ b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3
*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2*Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Maple [F]

$$\int (gx+f)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
 A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. 2(544) = 1088.

Time = 0.69 (sec) , antiderivative size = 1659, normalized size of antiderivative = 2.94

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
[Out] 2/3*A*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*g^2*x^3 +
2*A*B*f*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g*x^2 + 1/3*A*
B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a*log(
b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 - c^2*d*
g^2*n^2)*a*b - (6*c*d^2*f^2*n*log(e) + (3*g^2*n^2 + 2*g^2*n*log(e))*c^3 - 6
*(f*g*n^2 + f*g*n*log(e))*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*(3*a
*b^2*d^3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d^2*f^2*n^2
 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*
b^3*d^3*g^2*x^3*log(e)^2 + 2*(3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n
^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2
+ c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (c*d^2*g^
2*n*log(e) - 3*d^3*f*g*log(e)^2)*b^3)*B^2*x^2 - (3*a*b^2*d^3*f^2*n^2 - 3*a^
2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2 - 2*g^2*n
*log(e))*a^2*b*d^3 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n*log(e))*a*b^2 - (6*c*d^
2*f*g*n*log(e) - 3*d^3*f^2*log(e)^2 - (g^2*n^2 + 2*g^2*n*log(e))*c^2*d)*b^3
)*B^2*x - ((3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3 - (c*d^2*g^2*n^2 + 6*(f*g*n
^2 - f*g*n*log(e))*d^3)*a^2*b + 2*(3*c*d^2*f*g*n^2 - c^2*d*g^2*n^2 - 3*d^3*
f^2*n*log(e))*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^
3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^2*x^3
```

```
+ 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((d*x + c)^n)^2 + (2*B^2*
b^3*d^3*g^2*x^3*log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*
b^3*log(d*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*log(e))*b^3)
*B^2*x^2 + 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*
g^2*n - 3*d^3*f^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f
*g*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (2*B^2*b^3*d^3*g
^2*x^3*log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*b^3*log(d
*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*log(e))*b^3)*B^2*x^2
+ 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*g^2*n - 3
*d^3*f^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f*g*n + a^
3*d^3*g^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*
x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3)
```

Giac [F]

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (f+g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.69 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 290

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d} - \frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} + \frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/g+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+B^2*(-a*d+b*c)^2*g*n^2*ln(
d*x+c)/b^2/d^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*polylog(2,d*(b*x+a
)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{Bn(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d^2}$$
$$-\frac{(bf-ag)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g}$$
$$-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g}$$
$$+\frac{B^2n^2(bc-ad)(-adg-bcg+2bdf)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$
$$+\frac{B^2gn^2(bc-ad)^2\log(c+dx)}{b^2d^2}$$

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2
*d)) - ((b*f - a*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*g) +
 ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*g) + (B*(b*c - a
*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[
(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*
x])/(b^2*d^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)\left(A+B\log\left(ex^n\right)\right)^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(Bn)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^2(A+B\log(ex^n))}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(Bn)\text{Subst}\left(\int \left(\frac{(bf-ag)^2(A+B\log(ex^n))}{b^2x} + \frac{(bc-ad)^2g^2(A+B\log(ex^n))}{bd(b-dx)^2} + \frac{(bc-ad)g(2bdf-bcg-adg)(A+B\log(ex^n))}{b^2d(b-dx)}\right) d\right.}{g}$$

$$= \frac{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)^2gn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd} - \frac{(B(bf-ag)^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(B(bc-ad)(2bdf-bcg-adg)n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d}$$

$$= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d} - \frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} + \frac{(B^2(bc-ad)^2gn^2)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d} - \frac{(B^2(bc-ad)(2bdf-bcg-adg)n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^2}$$

$$= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d} - \frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} + \frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+\right.\right.}{}}{}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n] + 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 2*B*(b*c - a*d)^2*g^2*n*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*n*(Log[a + b*
x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*Log[(d*(a + b*x))/(-(b*
c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])))/(b^2*d^2))/(2*g)
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n)
^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(285) = 570$.

Time = 0.68 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.10

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=ABgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}A^2gx^2-ABgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+2ABfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+2ABfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+A^2fx-\frac{(acdgn^2+(2cdfn\log(e)-(gn^2+gn\log(e))c^2)b)B^2\log(dx+c)}{bd^2}+\frac{(2abd^2fn^2-a^2d^2gn^2-(2cdfn^2-c^2gn^2)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}+\frac{B^2b^2d^2gx^2\log(e)^2+2(2cdfn^2-c^2gn^2)B^2b^2\log(bx+a)\log(dx+c)-(2cdfn^2-c^2gn^2)B^2b^2\log(dx+$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
[Out] A*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*g*x^2 - A*B*g*n*
(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B
*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*f*x - (a*c*d*g*n^2 + (2*c*d*f*n*log(e) - (g*n^2 +
g*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*d^2*g
*n^2 - (2*c*d*f*n^2 - c^2*g*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b
^2*d^2*g*x^2*log(e)^2 + 2*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(b*x + a)*lo
g(d*x + c) - (2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(d*x + c)^2 - (2*a*b*d^2*
f*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*log(e) - (c*d*g*
n*log(e) - d^2*f*log(e)^2)*b^2)*B^2*x + 2*((g*n^2 - g*n*log(e))*a^2*d^2 - (
c*d*g*n^2 - 2*d^2*f*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 +
2*B^2*b^2*d^2*f*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*
f*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*
```

```
n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^
2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*
(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*log(d*x + c) + (a
*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d
^2*g*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x
 + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (f+g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.70 $\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx = \frac{(a+bx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log \left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{2B^2(bc-ad)n^2 \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+2*B*(-a*d+b*c)*n*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d+2*B^2*(-a*d+b*c)*n^2*polylog
(2,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2535, 2543, 2458, 2378, 2370, 2352}

$$\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx = \frac{2Bn(bc-ad) \log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd} + \frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b} + \frac{2B^2n^2(bc-ad) \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/b + (2*B*(b*c - a*d)*n
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(b*
d) + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2543

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d)
/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c*
g, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} - \frac{(2B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{b}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} - \frac{(2B^2(bc-ad)^2n^2)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{bd}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} - \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx, x, c+dx\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{2B^2(bc-ad)n^2\,\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.67

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\Big(2ad\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2bc\Big(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\Big)\log(c+dx)-aBdn\Big(\log(a-}{}$$

[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]

[Out] x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)

Maple [F]

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

Fricas [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2, x)

Sympy [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**2, x)
```

Maxima [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
[Out] 2*A*B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*x*log(e*((b*x + a)/(d
*x + c))^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c) - b*c*n^2*l
og(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)^n)^2 + 2*(a*
d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log(e))*log((b*x + a)^n) - 2*
(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b*d*x*l
og(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2*d*x^2*log(e)^2 + a*b*c*log
(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*log(e) + log(e)^2)*a*b*d)*x -
 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b*x + a))/(b^2*d*x^2 + a*b*c
 + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx=\int \left(A+B\,\ln \left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 \,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.71 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 297

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
g+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*n^2*polylog(3,(-c*g+d*f)
*(b*x+a)/(-a*g+b*f)/(d*x+c))/g
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2553, 2404, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\frac{2Bn\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g}-\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g}-\frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}+\frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]

[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])
/g) + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/g - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (2*B*n*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/g + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (2*B^2*
n^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= (bc-ad)\text{Subst}\left(\int \left(\frac{d(A+B\log(ex^n))^2}{(bc-ad)g(b-dx)} + \frac{(-df+cg)\left(A+B\log(ex^n)\right)^2}{(bc-ad)g(bf-ag-(df-cg)x)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{g} + \frac{((-bc+ad)(df-cg))\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g}
\end{aligned}
$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}$$
$$+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$
$$+\frac{(2Bn)\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$
$$-\frac{(2Bn)\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$
$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}$$
$$+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$
$$-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g}$$
$$+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$
$$+\frac{(2B^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$
$$-\frac{(2B^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$
$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}$$
$$+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$
$$-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g}$$
$$+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$
$$+\frac{2B^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g}-\frac{2B^2n^2\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1441 vs. 2(297) = 594.

Time = 0.30 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.85

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$$
$$= \frac{-B^2n^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)+A^2\log(f+gx)-2ABn\log\left(\frac{a}{b}+x\right)\log(f+gx)+B^2n^2\log^2\left(\frac{a}{b}\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

```
[Out] (-(B^2*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*n*Log[a/b + x]*Log[f +
 g*x] + B^2*n^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*n*Log[c/d + x]*Log[f +
g*x] - 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*n^2*Log[c/d +
 x]^2*Log[f + g*x] + 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*
B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + 2*B^2*n*Lo
g[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + B^2*Log[e*((a + b*
x)/(c + d*x))^n]^2*Log[f + g*x] + 2*A*B*n*Log[a/b + x]*Log[(b*(f + g*x))/(b
*f - a*g)] - B^2*n^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*
n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)
] + 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*
x))/(b*f - a*g)] - B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f +
g*x))/(b*f - a*g)] + 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f
- a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] -
 B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f +
 g*x))/(b*f - a*g)] - 2*A*B*n*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] +
 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] - B^2*n
^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n*Log[c/d + x]*Log
[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n^2*Log[
a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] +
 B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + g*x))/(d*f - c*g)]
 - 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/
((d*f - c*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*f - c*g)] + B^2*n^2*Log[((b*f
 - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(f + g*x)
)/((d*f - c*g)*(a + b*x))] + 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] +
B*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a
+ b*x))/(-(b*f) + a*g)] - 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n
*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(c + d
*x))/(-(d*f) + c*g)] - 2*B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(
a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*Log[((b*f -
a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))] + 2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
```

```
)] - 2*B^2*n^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])
/g
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{gx+f}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{f+gx}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n)
- 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(g*x + f),
x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{f+g\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x), x)
```

3.72 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*g+d*f)*(b*x+a)
/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{2553, 2355, 2354, 2438\}$

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(bf-ag)(df-cg)} + \frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)(bf-ag)} + \frac{2B^2n^2(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*f - a*g)*(f + g*x)
) + (2*B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f
 - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) + (2
*B^2*(b*c - a*d)*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d
*x))])/((b*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)} - \frac{(2B(bc-ad)n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bf-ag}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} - \frac{(2B^2(bc-ad)n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

$$= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)n^2\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(206) = 412.

Time = 0.28 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx} + \frac{Bn\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(bf-ag)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2(bc-ad)\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)) + (B*n*(2*b*(d*f - c
*g)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(b*f - a*g)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - b*B*(d*f - c*g)*n*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
```

```
 + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*n*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*n*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c
 + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*
g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{2}}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{2}}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{2}}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="maxima
")
```

```
[Out] 2*A*B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) +
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B^2*(log(
(d*x + c)^n)^2/(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 +
(d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b*x +
 a)^n) + 2*(d*f*n + (g*n - g*log(e))*d*x - c*g*log(e) - (d*g*x + c*g)*log((
b*x + a)^n))*log((d*x + c)^n))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x
^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - 2*A*B*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n)/(g^2*x + f*g) - A^2/(g^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(f+g\,x)^2}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2, x)
```

3.73 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 389

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx \\
&= \frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2} \\
&\quad - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{B^2(bc-ad)^2gn^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad + \frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}
\end{aligned}
$$

```
[Out] B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*pol
ylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$$
$$=\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}$$
$$+\frac{Bn(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(bf-ag)^2(df-cg)^2}$$
$$-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(f+gx)^2}$$
$$+\frac{B^2n^2(bc-ad)(-adg-bcg+2bdf)\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$
$$+\frac{B^2gn^2(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f
 - a*g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2)/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*g*(
f + g*x)^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[(f + g*x)/(c + d*x)])/((b*f - a*
g)^2*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c
*g - a*d*g)*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]
)/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)\left(A+B\log\left(ex^n\right)\right)^2}{(bf-ag-(df-cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(Bn)\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(Bn)\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))}{(bf-ag)^2x} + \frac{(bc-ad)^2g^2(A+B\log(ex^n))}{(bf-ag)(df-cg)(bf-ag-(df-cg)x)^2} + \frac{(bc-ad)g(-2bdf+bcg+adg)(A+B\log(ex^n)}{(bf-ag)^2(df-cg)(bf-ag-(df-cg)x)}\right.\right.}{g}$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(b^2Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{g(bf-ag)^2} + \frac{(B(bc-ad)^2gn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)} - \frac{(B(bc-ad)(2bdf-bcg-adg)n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)}$$

$$= \frac{B(bc-ad)gn(a+bx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} - \frac{(B^2(bc-ad)^2gn^2)\,\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)} - \frac{(B^2(bc-ad)(2bdf-bcg-adg)n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

$$= \frac{B(bc-ad)gn(a+bx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{B^2(bc-ad)^2gn^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) + 2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 2*B*(b*c - a*d)*g*n*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*n*(f + g*x)*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) - B*d^2*(b*f - a*g)^2*n*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*n*(f + g*x)*((Log[(g*(
a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x]
 + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f -
c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{\left(gx+f\right)^3}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x))*A*B*n - 1/2*B^2*(log((d*x + c)^n)^2/(g^3*x^
2 + 2*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g
*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^
n) + (d*f*n + (g*n - 2*g*log(e))*d*x - 2*c*g*log(e) - 2*(d*g*x + c*g)*log((
b*x + a)^n))*log((d*x + c)^n))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x
^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - A*B*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A^
2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(f+g\,x)^3}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3, x)
```

3.74 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 747

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx \\
&= \frac{B^2(bc-ad)^2g^2n^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2} \\
&\quad + \frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad - \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\right.}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}
$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-1/3*
B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(-c
*g+d*f)^3/(g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*n*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))^2/g/(g*x+f)^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/(-a*
g+b*f)^3/(-c*g+d*f)^3-1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((g*x+f)/(d*x+c))/(-a*
```

```
g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*n^2*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2
-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$$
$$=\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2n^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bf-ag)^3}$$
$$-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(f+gx)^3}$$
$$+\frac{2Bgn(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}$$
$$+\frac{B^2g^2n^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2gn^2(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g
*x)) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3
*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2/(3*g*(f + g*x)^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[(a + b*x)/(c + d*x)])/
(3*(b*f - a*g)^3*(d*f - c*g)^3) - (B^2*(b*c - a*d)^3*g^2*n^2*Log[(f + g*x)/
```

```
(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d
*f - b*c*g - 2*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f -
 c*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d
^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*
f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2
*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^2 (A+B\log(ex^n))^2}{(bf-ag-(df-cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2Bn)\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))}{x(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2Bn)\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))}{(bf-ag)^3x} + \frac{(-bc+ad)^3g^3(A+B\log(ex^n))}{(bf-ag)(df-cg)^2(bf-ag-(df-cg)x)^3} + \frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(A+B\log(ex}{(bf-ag)^2(df-cg)^2(bf-ag-(df-cg)x)^2}\right.\right.}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2b^3Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g(bf-ag)^3}$$
$$- \frac{(2B(bc-ad)^3g^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^2}$$
$$+ \frac{(2B(bc-ad)^2g(3bdf-bcg-2adg)n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)^2(df-cg)^2}$$
$$- \frac{(2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{bf-ag+(-df+cg)x\right.}{3(bf-ag)^3(df-cg)^2}$$

$$= -\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+ \frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+ \frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{lo}}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int \frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^3}$$
$$- \frac{(2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2)\,\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^2}$$
$$- \frac{(2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)}{bf-ag}\right.}{x}\right.}{3(bf-ag)^3(df-cg)^3}$$

$$= -\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{(B^2(bc-ad)^3g^2n^2)\,\text{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)}\right)dx,x,}{3(bf-ag)(df-cg)^3}$$

$$=\frac{B^2(bc-ad)^2g^2n^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx =$$
$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bd}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*((b*c - a*d
)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*
(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*
(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*(b*c -
a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x]
- 2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
) + B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c
+ d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) +
 b^3*B*(d*f - c*g)^3*n*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d
^3*(b*f - a*g)^3*n*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*
c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2))*n*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(
c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^
3))/(g*(f + g*x)^3)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(gx+f)^4}\,dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
```

```
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*A*B*n - 1/3*B^2*(log((d*x + c)^n
)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*(3*d*g
*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log((b*x + a)^n)^2 + 6*(d*g*
x*log(e) + c*g*log(e))*log((b*x + a)^n) + 2*(d*f*n + (g*n - 3*g*log(e))*d*x
 - 3*c*g*log(e) - 3*(d*g*x + c*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^
5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3
 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 2/
3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2
*g^2*x + f^3*g) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^4}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4, x)
```

3.75 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1208

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = -\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$-\frac{B^2(bc-ad)^3g^3n^2(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)n^2(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}$$
$$-\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n^2\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a}{(bf-ag)(c}\right.}{2(bf-ag)^4(df-cg)^4}$$

```
[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^
2-1/6*B^2*(-a*d+b*c)^3*g^3*n^2*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/
4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*(d*x+c)/(-a*g+b*f)^3/(-
c*g+d*f)^4/(g*x+f)+1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b
*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*
g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)
+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^4-1/6*B^
2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/4*B^
2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b
*f)^4/(-c*g+d*f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b
```

```
*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln
((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d
^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g
+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f
*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*
(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}$$
$$+\frac{B(bc-ad)g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4} - \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^2g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)n^2\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)n^2\,\text{PolyLog}\left(2,\frac{(df-c}{(bf-a}\right.}{2(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)n^2(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$-\frac{B^2(bc-ad)^3g^3n^2(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)} - \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*
(f + g*x)^2) - (B^2*(b*c - a*d)^3*g^3*n^2*(c + d*x))/(6*(b*f - a*g)^3*(d*f
- c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^
2*(c + d*x))/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3
*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*(b*f - a*g)*(d*f
- c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*f - a*g)^2*(d*f - c
*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c
*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*(a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2/(4*g*(f + g*x)^4) - (B^2*(b*c - a*d)^4*g^3*n^2*Lo
g[(a + b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^
3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^2*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a
*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[(f + g*x)/(c + d*x)])
/(6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g
- 3*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*f - c*g)^4) +
(B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*
f^2 - 4*c*d*f*g + c^2*g^2))*n^2*Log[(f + g*x)/(c + d*x)])/(2*(b*f - a*g)^4*
(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g -
a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]
)/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d
*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n
^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*f - a
*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
```

```
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^3 (A+B\log(ex^n))^2}{(bf-ag-(df-cg)x)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(Bn)\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))}{x(bf-ag+(-df+cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(Bn)\text{Subst}\left(\int \left(\frac{b^4(A+B\log(ex^n))}{(bf-ag)^4x} + \frac{(bc-ad)^4g^4(A+B\log(ex^n))}{(bf-ag)(df-cg)^3(bf-ag-(df-cg)x)^4} + \frac{(bc-ad)^3g^3(-4bdf+bcg+3adg)(A+B\log(}{(bf-ag)^2(df-cg)^3(bf-ag-(df-cg)x)}\right.\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(b^4Bn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2g(bf-ag)^4}$$

$$+ \frac{(B(bc-ad)^4g^3n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bf-ag)(df-cg)^3}$$

$$- \frac{(B(bc-ad)^3g^2(4bdf-bcg-3adg)n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bf-ag)^2(df-cg)^3}$$

$$+ \frac{(B(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))n)\text{Subst}\left(\int \frac{A+B\log(e}{(bf-ag+(-df+}\right.}{2(bf-ag)^3(df-cg)^3}$$

$$+ \frac{(B(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))n)\text{Subst}\left(\int \frac{A}{bf-}\right.}{2(bf-ag)^4(df-cg)^3}$$

$$= \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$

$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a-}{c+}\right.\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$

$$+\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{2(bf-ag)^4(df-cg)^4}$$

$$-\frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{6(bf-ag)(df-cg)^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2)\,\text{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4(bf-ag)^2(df-cg)^4}$$

$$-\frac{(B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n^2)\,\text{Subst}\left(\int\frac{1}{bf-ag+(-df+\right.}{2(bf-ag)^4(df-cg)^3}$$

$$+\frac{(B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2)\,\text{Subst}\left(\int\frac{\log}{}\right.}{2(bf-ag)^4(df-cg)^4}$$

$$= \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$

$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\right.\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$

$$+\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}$$

$$+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n^2\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\right.\right.\right.}{2(bf-ag)^4(df-cg)^4}$$

$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\text{Li}_2\left(\frac{(df-cg)(a\cdots}{(bf-ag)(c\cdots}\right.}{2(bf-ag)^4(df-cg)^4}$$

$$-\frac{(B^2(bc-ad)^4g^3n^2)\,\text{Subst}\left(\int\left(\frac{1}{(bf-ag)^3x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^3}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)^2}+\frac{\cdots}{(bf\cdots}\right.\right.}{6(bf-ag)(df-cg)^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2)\,\text{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{\cdots}{(bf-ag)^2(\cdots}\right.\right.}{4(bf-ag)^2(df-cg)^4}$$

$$= -\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2} - \frac{B^2(bc-ad)^3g^3n^2(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+ \frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)n^2(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+ \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+ \frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a\cdots}{c\cdots}\right.\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+ \frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}$$
$$- \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} + \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} - \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n^2\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a\cdots}{c\cdots}\right.\right.\right.}{2(bf-ag)^4(df-cg)^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\operatorname{Li}_2\left(\frac{(df-cg)(a+b\cdots}{(bf-ag)(c+d\cdots}\right.}{2(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 1329, normalized size of antiderivative = 1.10

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx =$$
$$-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2b\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c
- a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(
f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c - a*d)*g*(b*f - a*
g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c
```

```
*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b
^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f
*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2
*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(f
 + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (
b*c - a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*
(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f +
 g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)
*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*n*(
f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f
- a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c
*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d*
x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*Log[f + g*x]) + 3*b^4*B*(d*f - c*g)^4*
n*(f + g*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)
]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*B*d^4*(b*f - a*g)^4*n*
(f + g*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 6*B*(b*c - a*d)*g*(-2*b*d*
f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f
*g + c^2*g^2))*n*(f + g*x)^3*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(
c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f - c*g)^
4))/(g*(f + g*x)^4)
```

Maple [F]

$$\int \frac{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{5}}dx$$

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)

[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2
+ 5*f^4*g*x + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
```

```
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*A*B*n - 1/4*B^2*(log((d*x +
 c)^n)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) + 4*
integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2*(d*g*x + c*g)*log((b*
x + a)^n)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^n) + (d*f*n + (g*
n - 4*g*log(e))*d*x - 4*c*g*log(e) - 4*(d*g*x + c*g)*log((b*x + a)^n))*log(
(d*x + c)^n))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g
^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3
*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/2*A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^
4*g) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g
)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(f+g\,x\right)^5}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5, x)
```

3.76 $\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log(e*((b*x + a)/(d*x + c))^n) + A),
x)
```

Sympy [N/A]

Not integrable

Time = 28.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 26.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+g\,x)^2}{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.77 $\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\operatorname{Int}\left(\frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 16.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 16.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.78 $\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral(1/(A + B*log(e*((a + b*x)/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.79 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 17.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.80 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{\left(gx+f\right)^{2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 26.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{{(f+g\,x)}^2\,\left(A+B\,\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.81 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^{3}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 35.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.82 $\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 +
2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 51.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x
)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 10.69

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
 + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*
b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*
c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c*n - a*d*n
)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*
d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.83 $\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
[Out] integral((g*x + f)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*
x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 60.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c
*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (
b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.84 $\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
[Out] integral(1/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(
d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 8.12

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b
*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) -
a*d*n*log(e))*B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^2*l
og((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*
B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.85 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))
^n)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [N/A]

Not integrable

Time = 146.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)),
x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 500, normalized size of antiderivative = 15.62

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n*log(e)
 - a*d*f*n*log(e))*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2)*x + ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*l
og((b*x + a)^n) - ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log
((d*x + c)^n)) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/
((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*log(e) - a*d*f^2*n*log(e))*B^2 +
((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*log(e) - a*d*g^2*n*log(e))*B^2)*x
^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n*log(e) - a*d*f*g*n*log(e))
*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*
x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n
)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*
log((d*x + c)^n)), x)
```

Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.86 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.75

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 752, normalized size of antiderivative = 23.50

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*
log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*
log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c
*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2
 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x
+ a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*
x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n)) - integrate(-(b*c*f + (d
*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B
+ (b*c*g^3*n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*
A*B + (b*c*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g
^2*n)*A*B + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^
2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x
 + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2
 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((
b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2
*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)
*B^2)*log((d*x + c)^n)), x)
```

Giac [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.87 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.03

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo
g(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 1001, normalized size of antiderivative = 31.28

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*n
*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c*f
^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B +
(b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^2*g*n - a*d*
f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x + ((b*c*g^3
*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^
2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((b*x + a)^n)
 - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2
 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((d
*x + c)^n)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g - (
d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^4*n*log(e) - a*d*g^
4*n*log(e))*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*c*f*g^3*n*lo
g(e) - a*d*f*g^3*n*log(e))*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*A*B + (b*c*f^
4*n*log(e) - a*d*f^4*n*log(e))*B^2 + 6*((b*c*f^2*g^2*n - a*d*f^2*g^2*n)*A*B
 + (b*c*f^2*g^2*n*log(e) - a*d*f^2*g^2*n*log(e))*B^2)*x^2 + 4*((b*c*f^3*g*n
 - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n*log(e) - a*d*f^3*g*n*log(e))*B^2)*x + ((
b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*
(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2
*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((b*x + a)^n) - ((b*c*g^4*n - a*d*g^4*
n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d
*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*
d*f^4*n)*B^2)*log((d*x + c)^n)), x)
```

Giac [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.88 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+1/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g
^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/5*B*(-a*d+b*c)^5*g^4*ln(d*x+c)
/b/d^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

$= \{2548, 21, 45\}$

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{g^4(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b} - \frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5} + \frac{Bg^4x(bc-ad)^4}{5d^4} - \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bg^4(a+bx)^4(bc-ad)}{20bd}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*d
^3) + (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a
+ b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(5*b) - (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{(B(bc-ad))\int \frac{(ag+bgx)^5}{(a+bx)(c+dx)}\,dx}{5bg}$$
$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{(B(bc-ad)g^4)\int \frac{(a+bx)^4}{c+dx}\,dx}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{(B(bc-ad)g^4)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}$$

$$= \frac{B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - \frac{B(bc-ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12}{12d^5}\right.}{5b}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(-1
2*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c) + a
*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(12*d
^5)))/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(168) = 336.

Time = 1.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.47

method	result
risch	$\frac{g^4b^3Bac^2x^2}{2d^2} - \frac{2g^4bBa^3cx}{d} + \frac{2g^4b^2Ba^2c^2x}{d^2} - \frac{g^4b^3Bac^3x}{d^3} + \frac{g^4B\ln(dx+c)a^5}{5b} + \frac{g^4b^4Ax^5}{5} + g^4b^3Aax^4 +$
parallelrisch	$\frac{16Bx^3a^2b^3d^5g^4+4Bx^3b^5c^2d^3g^4+120Ax^2a^3b^2d^5g^4+36Bx^2a^3b^2d^5g^4-6Bx^2b^5c^3d^2g^4+60Axa^4bd^5g^4+48Bxa^4bd^5g^4}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g^4/d^2*b^3*B*a*c^2*x^2-2*g^4/d*b*B*a^3*c*x+2*g^4/d^2*b^2*B*a^2*c^2*x-g
^4/d^3*b^3*B*a*c^3*x+1/5*g^4/b*B*ln(d*x+c)*a^5+1/5*g^4*b^4*A*x^5+g^4*b^3*A*
a*x^4+1/20*g^4*b^3*B*a*x^4-1/20*g^4/d*b^4*B*c*x^4+2*g^4*b^2*A*a^2*x^3+4/15*
g^4*b^2*B*a^2*x^3+1/15*g^4/d^2*b^4*B*c^2*x^3+2*g^4*b*A*a^3*x^2+3/5*g^4*b*B*
a^3*x^2-1/10*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+4/5*g^4*B*a^4*x+1/5*g^4/d^4*
b^4*B*c^4*x-g^4/d*B*ln(d*x+c)*a^4*c-1/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+2*g^4/d
^2*b*B*ln(d*x+c)*a^3*c^2-2*g^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+g^4/d^4*b^3*B*ln
(d*x+c)*a*c^4-1/3*g^4/d*b^3*B*a*c*x^3-g^4/d*b^2*B*a^2*c*x^2+1/5*(b*x+a)^5*g
^4*B/b*ln(e*(b*x+a)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(168) = 336$.

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{12\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4\log(bx+a)-3\,(Bb^5cd^4-(20\,A+B)ab^4d^5)g^4x^4+4\,(Bb^5c^2d^3-5\,Bab^4cd^4+\cdots}{\cdots}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2*(
15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A + 4*B
)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.52 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$=\frac{Ab^4g^4x^5}{5}+\frac{Ba^5g^4\log\left(x+\frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b}$$

$$-\frac{Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{6Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba}{}\right.}{5d^5}$$

$$+x^4\left(Aab^3g^4+\frac{Bab^3g^4}{20}-\frac{Bb^4cg^4}{20d}\right)+x^3\cdot\left(2Aa^2b^2g^4+\frac{4Ba^2b^2g^4}{15}-\frac{Bab^3cg^4}{3d}+\frac{Bb^4c^2g^4}{15d^2}\right)$$

$$+x^2\cdot\left(2Aa^3bg^4+\frac{3Ba^3bg^4}{5}-\frac{Ba^2b^2cg^4}{d}+\frac{Bab^3c^2g^4}{2d^2}-\frac{Bb^4c^3g^4}{10d^3}\right)$$

$$+x\left(Aa^4g^4+\frac{4Ba^4g^4}{5}-\frac{2Ba^3bcg^4}{d}+\frac{2Ba^2b^2c^2g^4}{d^2}-\frac{Bab^3c^3g^4}{d^3}+\frac{Bb^4c^4g^4}{5d^4}\right)$$

$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
b**3*g**4 + B*a*b**3*g**4/20 - B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
g**4 + 4*B*a**2*b**2*g**4/15 - B*a*b**3*c*g**4/(3*d) + B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 + 3*B*a**3*b*g**4/5 - B*a**2*b**2*c*g**4/d
+ B*a*b**3*c**2*g**4/(2*d**2) - B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 + 4*B*a**4*g**4/5 - 2*B*a**3*b*c*g**4/d + 2*B*a**2*b**2*c**2*g**4/d**2 -
B*a*b**3*c**3*g**4/d**3 + B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(168) = 336$.

Time = 0.21 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.46

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2\,Aa^2b^2g^4x^3$$
$$+ 2\,Aa^3bg^4x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^4g^4$$
$$+2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^3bg^4$$
$$+\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6\,x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log(bx+a)}{b^4}+\frac{6\,c^4\log(dx+c)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2c}{b^3}\right.$$
$$+\frac{1}{60}\left(12\,x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(}{}\right.$$
$$+ Aa^4g^4x$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(
b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
*a*b^3*g^4 + 1/60*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log
(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4036 vs. $2(168) = 336$.

Time = 0.53 (sec) , antiderivative size = 4036, normalized size of antiderivative = 22.42

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/60*(12*(B*b^10*c^6*e^6*g^4 - 6*B*a*b^9*c^5*d*e^6*g^4 + 15*B*a^2*b^8*c^4*d
^2*e^6*g^4 - 20*B*a^3*b^7*c^3*d^3*e^6*g^4 + 15*B*a^4*b^6*c^2*d^4*e^6*g^4 -
6*B*a^5*b^5*c*d^5*e^6*g^4 + B*a^6*b^4*d^6*e^6*g^4 - 5*(b*e*x + a*e)*B*b^9*c
^6*d*e^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^4/(d*x + c)
 - 75*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^4/(d*x + c) + 100*(b*e*x + a*e)
*B*a^3*b^6*c^3*d^4*e^5*g^4/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e
^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^4/(d*x + c) - 5*(
b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^4/(d*x + c) + 10*(b*e*x + a*e)^2*B*b^8*c^6
*d^2*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(d*x
+ c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(d*x + c)^2 - 200*(b
*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B
*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e
^4*g^4/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^4/(d*x + c)^2 -
 10*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 + 60*(b*e*x + a*e)^3*
B*a*b^6*c^5*d^4*e^3*g^4/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^2*b^5*c^4*d^5
*e^3*g^4/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g^4/(d*x +
 c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g^4/(d*x + c)^3 + 60*(b*e
*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g^4/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a^6*
b*d^9*e^3*g^4/(d*x + c)^3 + 5*(b*e*x + a*e)^4*B*b^6*c^6*d^4*e^2*g^4/(d*x +
c)^4 - 30*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g^4/(d*x + c)^4 + 75*(b*e*x +
 a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g^4/(d*x + c)^4 - 100*(b*e*x + a*e)^4*B*a^3*b
^3*c^3*d^7*e^2*g^4/(d*x + c)^4 + 75*(b*e*x + a*e)^4*B*a^4*b^2*c^2*d^8*e^2*g
^4/(d*x + c)^4 - 30*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g^4/(d*x + c)^4 + 5*(
b*e*x + a*e)^4*B*a^6*d^10*e^2*g^4/(d*x + c)^4)*log((b*e*x + a*e)/(d*x + c))
/(b^5*d^5*e^5 - 5*(b*e*x + a*e)*b^4*d^6*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*
b^3*d^7*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2*d^8*e^2/(d*x + c)^3 + 5*(b
*e*x + a*e)^4*b*d^9*e/(d*x + c)^4 - (b*e*x + a*e)^5*d^10/(d*x + c)^5) + (12
*A*b^10*c^6*e^6*g^4 + 25*B*b^10*c^6*e^6*g^4 - 72*A*a*b^9*c^5*d*e^6*g^4 - 15
0*B*a*b^9*c^5*d*e^6*g^4 + 180*A*a^2*b^8*c^4*d^2*e^6*g^4 + 375*B*a^2*b^8*c^4
*d^2*e^6*g^4 - 240*A*a^3*b^7*c^3*d^3*e^6*g^4 - 500*B*a^3*b^7*c^3*d^3*e^6*g^
4 + 180*A*a^4*b^6*c^2*d^4*e^6*g^4 + 375*B*a^4*b^6*c^2*d^4*e^6*g^4 - 72*A*a^
5*b^5*c*d^5*e^6*g^4 - 150*B*a^5*b^5*c*d^5*e^6*g^4 + 12*A*a^6*b^4*d^6*e^6*g^
4 + 25*B*a^6*b^4*d^6*e^6*g^4 - 60*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g^4/(d*x +
c) - 113*(b*e*x + a*e)*B*b^9*c^6*d*e^5*g^4/(d*x + c) + 360*(b*e*x + a*e)*A*
a*b^8*c^5*d^2*e^5*g^4/(d*x + c) + 678*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^4
```

/(d*x + c) - 900*(b*e*x + a*e)*A*a^2*b^7*c^4*d^3*e^5*g^4/(d*x + c) - 1695*(
b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^4/(d*x + c) + 1200*(b*e*x + a*e)*A*a^3
*b^6*c^3*d^4*e^5*g^4/(d*x + c) + 2260*(b*e*x + a*e)*B*a^3*b^6*c^3*d^4*e^5*g
^4/(d*x + c) - 900*(b*e*x + a*e)*A*a^4*b^5*c^2*d^5*e^5*g^4/(d*x + c) - 1695
*(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e^5*g^4/(d*x + c) + 360*(b*e*x + a*e)*A*a^
5*b^4*c*d^6*e^5*g^4/(d*x + c) + 678*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^4/(
d*x + c) - 60*(b*e*x + a*e)*A*a^6*b^3*d^7*e^5*g^4/(d*x + c) - 113*(b*e*x +
a*e)*B*a^6*b^3*d^7*e^5*g^4/(d*x + c) + 120*(b*e*x + a*e)^2*A*b^8*c^6*d^2*e^
4*g^4/(d*x + c)^2 + 196*(b*e*x + a*e)^2*B*b^8*c^6*d^2*e^4*g^4/(d*x + c)^2 -
 720*(b*e*x + a*e)^2*A*a*b^7*c^5*d^3*e^4*g^4/(d*x + c)^2 - 1176*(b*e*x + a*
e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(d*x + c)^2 + 1800*(b*e*x + a*e)^2*A*a^2*b^6*c
^4*d^4*e^4*g^4/(d*x + c)^2 + 2940*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4
/(d*x + c)^2 - 2400*(b*e*x + a*e)^2*A*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 -
 3920*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 1800*(b*e*x +
 a*e)^2*A*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 + 2940*(b*e*x + a*e)^2*B*a^4*
b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 - 720*(b*e*x + a*e)^2*A*a^5*b^3*c*d^7*e^4*g
^4/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e^4*g^4/(d*x + c)^2 +
 120*(b*e*x + a*e)^2*A*a^6*b^2*d^8*e^4*g^4/(d*x + c)^2 + 196*(b*e*x + a*e)^
2*B*a^6*b^2*d^8*e^4*g^4/(d*x + c)^2 - 120*(b*e*x + a*e)^3*A*b^7*c^6*d^3*e^3
*g^4/(d*x + c)^3 - 156*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 +
720*(b*e*x + a*e)^3*A*a*b^6*c^5*d^4*e^3*g^4/(d*x + c)^3 + 936*(b*e*x + a*e)
^3*B*a*b^6*c^5*d^4*e^3*g^4/(d*x + c)^3 - 1800*(b*e*x + a*e)^3*A*a^2*b^5*c^4
*d^5*e^3*g^4/(d*x + c)^3 - 2340*(b*e*x + a*e)^3*B*a^2*b^5*c^4*d^5*e^3*g^4/(
d*x + c)^3 + 2400*(b*e*x + a*e)^3*A*a^3*b^4*c^3*d^6*e^3*g^4/(d*x + c)^3 + 3
120*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g^4/(d*x + c)^3 - 1800*(b*e*x + a
*e)^3*A*a^4*b^3*c^2*d^7*e^3*g^4/(d*x + c)^3 - 2340*(b*e*x + a*e)^3*B*a^4*b^
3*c^2*d^7*e^3*g^4/(d*x + c)^3 + 720*(b*e*x + a*e)^3*A*a^5*b^2*c*d^8*e^3*g^4
/(d*x + c)^3 + 936*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g^4/(d*x + c)^3 - 12
0*(b*e*x + a*e)^3*A*a^6*b*d^9*e^3*g^4/(d*x + c)^3 - 156*(b*e*x + a*e)^3*B*a
^6*b*d^9*e^3*g^4/(d*x + c)^3 + 60*(b*e*x + a*e)^4*A*b^6*c^6*d^4*e^2*g^4/(d*
x + c)^4 + 48*(b*e*x + a*e)^4*B*b^6*c^6*d^4*e^2*g^4/(d*x + c)^4 - 360*(b*e*
x + a*e)^4*A*a*b^5*c^5*d^5*e^2*g^4/(d*x + c)^4 - 288*(b*e*x + a*e)^4*B*a*b^
5*c^5*d^5*e^2*g^4/(d*x + c)^4 + 900*(b*e*x + a*e)^4*A*a^2*b^4*c^4*d^6*e^2*g
^4/(d*x + c)^4 + 720*(b*e*x + a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g^4/(d*x + c)^4
 - 1200*(b*e*x + a*e)^4*A*a^3*b^3*c^3*d^7*e^2*g^4/(d*x + c)^4 - 960*(b*e*x +
 a*e)^4*B*a^3*b^3*c^3*d^7*e^2*g^4/(d*x + c)^4 + 900*(b*e*x + a*e)^4*A*a^4*b
^2*c^2*d^8*e^2*g^4/(d*x + c)^4 + 720*(b*e*x + a*e)^4*B*a^4*b^2*c^2*d^8*e^2*
g^4/(d*x + c)^4 - 360*(b*e*x + a*e)^4*A*a^5*b*c*d^9*e^2*g^4/(d*x + c)^4 - 2
88*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g^4/(d*x + c)^4 + 60*(b*e*x + a*e)^4*A
*a^6*d^10*e^2*g^4/(d*x + c)^4 + 48*(b*e*x + a*e)^4*B*a^6*d^10*e^2*g^4/(d*x
+ c)^4)/(b^5*d^5*e^5 - 5*(b*e*x + a*e)*b^4*d^6*e^4/(d*x + c) + 10*(b*e*x +
a*e)^2*b^3*d^7*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2*d^8*e^2/(d*x + c)^3
 + 5*(b*e*x + a*e)^4*b*d^9*e/(d*x + c)^4 - (b*e*x + a*e)^5*d^10/(d*x + c)^5
) + 12*(B*b^6*c^6*e*g^4 - 6*B*a*b^5*c^5*d*e*g^4 + 15*B*a^2*b^4*c^4*d^2*e*g^
4 - 20*B*a^3*b^3*c^3*d^3*e*g^4 + 15*B*a^4*b^2*c^2*d^4*e*g^4 - 6*B*a^5*b*c*d

```
^5*e*g^4 + B*a^6*d^6*e*g^4)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^5) -
 12*(B*b^6*c^6*e*g^4 - 6*B*a*b^5*c^5*d*e*g^4 + 15*B*a^2*b^4*c^4*d^2*e*g^4 -
 20*B*a^3*b^3*c^3*d^3*e*g^4 + 15*B*a^4*b^2*c^2*d^4*e*g^4 - 6*B*a^5*b*c*d^5*
e*g^4 + B*a^6*d^6*e*g^4)*log((b*e*x + a*e)/(d*x + c))/(b*d^5))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 1009, normalized size of antiderivative = 5.61

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}\right.}{10\,b\,d}\right.$$

$$+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$+\frac{a\,c\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d - B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
+ 2*B*a*d - 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*
a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c +
 B*a*d - B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*
x)*(B*b^4*c^5*g^4 + 5*B*a^4*c*d^4*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 10*B*a^2*b
^2*c^3*d^2*g^4 - 5*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5
*g^4*log(a + b*x))/(5*b)
```

3.89 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} -\frac{B(bc-ad)g^3(a+bx)^3}{12bd} +\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} +\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/12*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))
/b+1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3x(bc-ad)^3}{4d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)}{12bd}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*x)/d^3 + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/(4*
b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}-\frac{(B(bc-ad))\int\frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{4bg}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}-\frac{(B(bc-ad)g^3)\int\frac{(a+bx)^3}{c+dx}\,dx}{4b}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}\\
&\quad-\frac{(B(bc-ad)g^3)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{4b}\\
&= -\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{12bd}\\
&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$
\int(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx
$$
$$
=\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}
$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(6*
b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3
- 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(139) = 278.

Time = 0.88 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b}+\frac{g^3b^3A\,x^4}{4}+g^3b^2Aa\,x^3+\frac{g^3b^2Ba\,x^3}{12}-\frac{g^3b^3Bc\,x^3}{12d}+\frac{3g^3bA\,a^2x^2}{2}+\frac{3g^3bB\,a^2x^2}{8}$
parallelrisch	$\frac{24Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bd^4g^3+24B\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^3c^3dg^3-24B\ln(bx+a)a^3bcd^3g^3+9Ba^3bcd^3g^3+24Ba^2b^2c^2d^2g^3-21Bc}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)/(d*x+c))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*x
^3+1/12*g^3*b^2*B*a*x^3-1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/8*g^3*
b*B*a^2*x^2-1/2*g^3*b^2/d*B*a*c*x^2+1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*x+1
/4*g^3/b*B*ln(d*x+c)*a^4-g^3/d*B*ln(d*x+c)*a^3*c+3/2*g^3*b/d^2*B*ln(d*x+c)*
a^2*c^2-g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4+3/4*g
^3*B*a^3*x-3/2*g^3*b/d*B*a^2*c*x+g^3*b^2/d^2*B*a*c^2*x-1/4*g^3*b^3/d^3*B*c^
3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(139) = 278$.

Time = 0.28 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.13

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3\log(bx+a)-2\,(Bb^4cd^3-(12\,A+B)ab^3d^4)g^3x^3+3\,(Bb^4c^2d^2-4\,Bab^3cd^3+3\,(}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(4*
A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.04 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$
$$-\frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g}\right.}{4d^4}$$
$$+x^3\left(Aab^2g^3+\frac{Bab^2g^3}{12}-\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{8}-\frac{Bab^2cg^3}{2d}+\frac{Bb^3c^2g^3}{8d^2}\right)$$
$$+x\left(Aa^3g^3+\frac{3Ba^3g^3}{4}-\frac{3Ba^2bcg^3}{2d}+\frac{Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{4d^3}\right)$$
$$+\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/12 - B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*
b*g**3/8 - B*a*b**2*c*g**3/(2*d) + B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 + 3*B*a**3*g**3/4 - 3*B*a**2*b*c*g**3/(2*d) + B*a*b**2*c**2*g**3/d**2 -
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(139) = 278.

Time = 0.20 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.95

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^3g^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2bg^3$$
$$+\frac{1}{2}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2c$$
$$+\frac{1}{24}\left(6\,x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log(bx+a)}{b^4}+\frac{6\,c^4\log(dx+c)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d}{b^3}\right.$$
$$+Aa^3g^3x$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2
*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d
^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. 2(139) = 278.

Time = 0.45 (sec) , antiderivative size = 2776, normalized size of antiderivative = 18.63

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d^
2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*a^
5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(b*e
```

*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^5*c^
3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(d*x +
 c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*
B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^3*g^3/(
d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2 + 60*(b
*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*
a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^
3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 4*(
b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x + a*e)^3*B*a*b
^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g
^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 -
 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B
*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4*e^4 - 4
*(b*e*x + a*e)*b^3*d^5*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^2/(d*x +
 c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^8/(d*x +
c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*b^8*c^5*e^5*g^3 - 30*A*a*b^7*c^4*d*e^5*
g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 + 60*A*a^2*b^6*c^3*d^2*e^5*g^3 + 110*B*a^2*b
^6*c^3*d^2*e^5*g^3 - 60*A*a^3*b^5*c^2*d^3*e^5*g^3 - 110*B*a^3*b^5*c^2*d^3*e
^5*g^3 + 30*A*a^4*b^4*c*d^4*e^5*g^3 + 55*B*a^4*b^4*c*d^4*e^5*g^3 - 6*A*a^5*
b^3*d^5*e^5*g^3 - 11*B*a^5*b^3*d^5*e^5*g^3 - 24*(b*e*x + a*e)*A*b^7*c^5*d*e
^4*g^3/(d*x + c) - 38*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 120*(b*
e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) + 190*(b*e*x + a*e)*B*a*b^6*c^
4*d^2*e^4*g^3/(d*x + c) - 240*(b*e*x + a*e)*A*a^2*b^5*c^3*d^3*e^4*g^3/(d*x
+ c) - 380*(b*e*x + a*e)*B*a^2*b^5*c^3*d^3*e^4*g^3/(d*x + c) + 240*(b*e*x +
 a*e)*A*a^3*b^4*c^2*d^4*e^4*g^3/(d*x + c) + 380*(b*e*x + a*e)*B*a^3*b^4*c^2
*d^4*e^4*g^3/(d*x + c) - 120*(b*e*x + a*e)*A*a^4*b^3*c*d^5*e^4*g^3/(d*x + c
) - 190*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 24*(b*e*x + a*e)*
A*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 38*(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*g^3/(d
x + c) + 36(b*e*x + a*e)^2*A*b^6*c^5*d^2*e^3*g^3/(d*x + c)^2 + 45*(b*e*x
+ a*e)^2*B*b^6*c^5*d^2*e^3*g^3/(d*x + c)^2 - 180*(b*e*x + a*e)^2*A*a*b^5*c^
4*d^3*e^3*g^3/(d*x + c)^2 - 225*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*
x + c)^2 + 360*(b*e*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 + 450*
(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 360*(b*e*x + a*e)^2
*A*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 - 450*(b*e*x + a*e)^2*B*a^3*b^3*c^2*
d^5*e^3*g^3/(d*x + c)^2 + 180*(b*e*x + a*e)^2*A*a^4*b^2*c*d^6*e^3*g^3/(d*x
+ c)^2 + 225*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*g^3/(d*x + c)^2 - 36*(b*e*
x + a*e)^2*A*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 45*(b*e*x + a*e)^2*B*a^5*b*d^7
*e^3*g^3/(d*x + c)^2 - 24*(b*e*x + a*e)^3*A*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3
 - 18*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 120*(b*e*x + a*e)
^3*A*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 + 90*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4
*e^2*g^3/(d*x + c)^3 - 240*(b*e*x + a*e)^3*A*a^2*b^3*c^3*d^5*e^2*g^3/(d*x +
 c)^3 - 180*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g^3/(d*x + c)^3 + 240*(b*
e*x + a*e)^3*A*a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 + 180*(b*e*x + a*e)^3*B*
a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 - 120*(b*e*x + a*e)^3*A*a^4*b*c*d^7*e^2
*g^3/(d*x + c)^3 - 90*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 2

```
4*(b*e*x + a*e)^3*A*a^5*d^8*e^2*g^3/(d*x + c)^3 + 18*(b*e*x + a*e)^3*B*a^5*
d^8*e^2*g^3/(d*x + c)^3)/(b^4*d^4*e^4 - 4*(b*e*x + a*e)*b^3*d^5*e^3/(d*x +
c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e/
(d*x + c)^3 + (b*e*x + a*e)^4*d^8/(d*x + c)^4) + 6*(B*b^5*c^5*e*g^3 - 5*B*a
*b^4*c^4*d*e*g^3 + 10*B*a^2*b^3*c^3*d^2*e*g^3 - 10*B*a^3*b^2*c^2*d^3*e*g^3
+ 5*B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log(-b*e + (b*e*x + a*e)*d/(d*x
+ c))/(b*d^4) - 6*(B*b^5*c^5*e*g^3 - 5*B*a*b^4*c^4*d*e*g^3 + 10*B*a^2*b^3*c
^3*d^2*e*g^3 - 10*B*a^3*b^2*c^2*d^3*e*g^3 + 5*B*a^4*b*c*d^4*e*g^3 - B*a^5*d
^5*e*g^3)*log((b*e*x + a*e)/(d*x + c))/(b*d^4))*(b*c/((b*c*e - a*d*e)*(b*c
- a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^2 \cdots}{d \cdots}\right.}{4\,b\,d}\right.$$

$$+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{2\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)\,(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$+\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{4\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c + 3*B*a*d - 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c + B*a*d - B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(a +
b*x))/(c + d*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.90 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b/d^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b} - \frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3} + \frac{Bg^2x(bc-ad)^2}{3d^2} - \frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) +
 (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a
*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}-\frac{(B(bc-ad))\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{3bg}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}-\frac{(B(bc-ad)g^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}-\frac{(B(bc-ad)g^2)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= \frac{B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{6bd}\\
&\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}-\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*
(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x])
)/(2*d^3)))/(3*b)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b}+\frac{g^2 b^2 A x^3}{3}+g^2 b A a x^2+\frac{g^2 b B a x^2}{6}-\frac{g^2 b^2 B c x^2}{6d}+g^2 A a^2 x+\frac{g^2 B \ln(dx+c) a^3}{3b} \cdots$
parallelrisch	$\frac{2B x^3 \ln\left(\frac{e(bx+a)}{dx+c}\right) a b^3 c d^3 g^2+2A x^3 a b^3 c d^3 g^2+6B x^2 \ln\left(\frac{e(bx+a)}{dx+c}\right) a^2 b^2 c d^3 g^2+6A x^2 a^2 b^2 c d^3 g^2+B x^2 a^2 b^2 c d^3 g^2-B x^2 c \cdots}{\cdots}$
parts	$\frac{A g^2 (bx+a)^3}{3b}-\frac{B g^2 (ad-cb) e \left(2be\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}+\cdots\right.\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(A d^2 g^2\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{b^2e^2}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{1}{d^3\left(be-\left(\frac{be}{d}+\cdots\right.\right.}\cdots\right.\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(A d^2 g^2\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{b^2e^2}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{1}{d^3\left(be-\left(\frac{be}{d}+\cdots\right.\right.}\cdots\right.\right.}{\cdots}$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^2
+1/6*g^2*b*B*a*x^2-1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+1/3*g^2/b*B*ln(d*x+c)*
```

```
a^3-g^2/d*B*ln(d*x+c)*a^2*c+g^2*b/d^2*B*ln(d*x+c)*a*c^2-1/3*g^2*b^2/d^3*B*l
n(d*x+c)*c^3+2/3*g^2*B*a^2*x-g^2*b/d*B*a*c*x+1/3*g^2*b^2/d^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 + 2\,Ba^3d^3g^2\log(bx+a) - (Bb^3cd^2 - (6\,A+B)ab^2d^3)g^2x^2 + 2\,(Bb^3c^2d - 3\,Bab^2cd^2 + (3\,A\cdots}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (6
*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*B)
*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.31 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$
$$- \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot(3a^2d^2-3\cdots}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2+\frac{Babg^2}{6}-\frac{Bb^2cg^2}{6d}\right) + x\left(Aa^2g^2+\frac{2Ba^2g^2}{3}-\frac{Babcg^2}{d}+\frac{Bb^2c^2g^2}{3d^2}\right)$$
$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*g**2*x**3/3 + B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
```

```
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 + B*a*b*g**2/6 - B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 + 2*B*a**2*g**2/3 - B*a*b*c*g**2/d + B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2c}{b^2d^2} + Aa^2g^2x$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(110) = 220$.

Time = 0.42 (sec) , antiderivative size = 1742, normalized size of antiderivative = 14.76

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*e
^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a*e
)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g^2/
(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(b*e*
x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d^5*e^
3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2 - 12*
(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*
a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*
g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)*log((b*e
*x + a*e)/(d*x + c))/(b^3*d^3*e^3 - 3*(b*e*x + a*e)*b^2*d^4*e^2/(d*x + c) +
 3*(b*e*x + a*e)^2*b*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^6/(d*x + c)^3) +
 (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8*A*a*b^5*c^3*d*e^4*g^2 - 12*
B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e^4*g^2 + 18*B*a^2*b^4*c^2*d^2
*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a^3*b^3*c*d^3*e^4*g^2 + 2*A*a^4
*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 - 6*(b*e*x + a*e)*A*b^5*c^4*d*e^
3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 24*(b*e*x
 + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e)*B*a*b^4*c^3*d^
2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c)
- 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 24*(b*e*x + a*e)*A
*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2
/(d*x + c) - 6*(b*e*x + a*e)*A*a^4*b*d^5*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e
)*B*a^4*b*d^5*e^3*g^2/(d*x + c) + 6*(b*e*x + a*e)^2*A*b^4*c^4*d^2*e^2*g^2/(
d*x + c)^2 + 4*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2 - 24*(b*e*
x + a*e)^2*A*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 - 16*(b*e*x + a*e)^2*B*a*b^3
*c^3*d^3*e^2*g^2/(d*x + c)^2 + 36*(b*e*x + a*e)^2*A*a^2*b^2*c^2*d^4*e^2*g^2
/(d*x + c)^2 + 24*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 2
4*(b*e*x + a*e)^2*A*a^3*b*c*d^5*e^2*g^2/(d*x + c)^2 - 16*(b*e*x + a*e)^2*B*
a^3*b*c*d^5*e^2*g^2/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^4*d^6*e^2*g^2/(d*x
+ c)^2 + 4*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)/(b^3*d^3*e^3 - 3*
(b*e*x + a*e)*b^2*d^4*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e/(d*x + c)^2
 - (b*e*x + a*e)^3*d^6/(d*x + c)^3) + 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*
e*g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2
)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^3) - 2*(B*b^4*c^4*e*g^2 - 4*B*
a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a
^4*d^4*e*g^2)*log((b*e*x + a*e)/(d*x + c))/(b*d^3))*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right) - \frac{\ln(c+d\,x)\,(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c + B*a*d - B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a + b*x))/(c + d
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 + (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.91 $\int (ag+bgx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx = -\frac{B(bc-ad)gx}{2d} + \frac{g(a+bx)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} + \frac{B(bc-ad)^2 g \log(c+dx)}{2bd^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/2*B
*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2548, 21, 45}

$$\int (ag+bgx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx = \frac{g(a+bx)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b} + \frac{Bg(bc-ad)^2 \log(c+dx)}{2bd^2} - \frac{Bgx(bc-ad)}{2d}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*x)/d + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}-\frac{(B(bc-ad))\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{2bg}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}\,dx}{2b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx}{2b}\\
&= -\frac{B(bc-ad)gx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{gbAx^2}{2}+gAax+\frac{Ba^2g\ln(-bx-a)}{2b}-\frac{gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{2d^2}+\frac{gB}{2}$
parallelrisch	$\frac{Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2g+Ax^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)abd^2g+2Axabd^2g+B\ln(bx+a)a^2d^2g-2B\ln(bx+a)abcdg+B\ln}{2}$
parts	$Ag\left(\frac{1}{2}bx^2+ax\right)-\frac{Bg(ad-cb)e\left(bde(ad-cb)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}+\frac{\ln\left(\frac{be}{d}\right.}{}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-Ad^2g(ad-cb)\left(\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-Bd^2g(ad-cb)\left(-\frac{be\left(-\frac{\ln(b}{}\right.}{}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(-Ad^2g(ad-cb)\left(\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-Bd^2g(ad-cb)\left(-\frac{be\left(-\frac{\ln(b}{}\right.}{}\right.\right.}{}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)/(d*x+c))+1/2*g*b*A*x^2+g*A*a*x+1/2*B*a^2*g
/b*ln(-b*x-a)-g/d*B*ln(d*x+c)*a*c+1/2*g*b/d^2*B*ln(d*x+c)*c^2+1/2*g*B*a*x-1
/2*g*b/d*B*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+Ba^2d^2g\log(bx+a)-(Bb^2cd-(2A+B)abd^2)gx+(Bb^2c^2-2Babcd)g\log(dx+c)+(Bb^2d}{2bd^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a*
b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 +
2*B*a*b*d^2*g*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.87 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}$$
$$-\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}$$
$$+x\left(Aag+\frac{Bag}{2}-\frac{Bbcg}{2d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/(2*d**2) + x*(A*a*g + B*a*g/2 - B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}\,Abgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bag$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbg$$
$$+Aagx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
- c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(75) = 150.

Time = 0.37 (sec) , antiderivative size = 869, normalized size of antiderivative = 10.73

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=$$
$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3e^3g-3\,Bab^3c^2de^3g+3\,Ba^2b^2cd^2e^3g-Ba^3bd^3e^3g-\frac{2\,(bex+ae)Bb^3c^3de^2g}{dx+c}+\frac{6\,(bex+ae)Bab^2c^2d^2e^2g}{dx+c}\right.}{b^2d^2e^2-\frac{2\,(bex+ae)bd^3e}{dx+c}+\frac{(bex+ae)^2d^4}{(dx+c)^2}}\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*x
+ a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*
g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a*e)/
(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x + a*e)
^2*d^4/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g - 3*A*a*b^3*c^2*d*
e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g + 3*B*a^2*b^2*c*d^2
*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*b^3*c^3*
d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*x +
a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2*
```

```
g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) - 3*(b*e*x + a*
e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*A*a^3*d^4*e^2*g/(d*x + c
) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)
*b*d^3*e/(d*x + c) + (b*e*x + a*e)^2*d^4/(d*x + c)^2) + (B*b^3*c^3*e*g - 3*
B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-b*e + (b*e*x
+ a*e)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^
2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((b*e*x + a*e)/(d*x + c))/(b*d^2))*(b*c/(
(b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{\ln\left(c+d\,x\right)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,\ln\left(a+b\,x\right)}{2\,b}$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2
*d)) + log((e*(a + b*x))/(c + d*x))*((B*b*g*x^2)/2 + B*a*g*x) + (log(c + d*
x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*log(a + b*
x))/(2*b)
```

3.92 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} + \frac{B\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g+B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = \frac{B\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(b*g)) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg}+\frac{(B(bc-ad))\int\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{bg}$$

$$=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg}+\frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx,x,a+bx\right)}{b^2g}$$

$$=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg}-\frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(B(bc-ad))\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}\,dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} + \frac{B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$$
$$= \frac{\log(g(a+bx))\left(-B\log(g(a+bx))+2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2B\,\text{PolyLog}\left(2,\frac{d(a+b}{-bc+c}\right.}{2bg}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(-(B*Log[g*(a + b*x)]) + 2*(A + B*Log[(e*(a + b*x))/(c +
d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(2*b*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(79) = 158.

Time = 1.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.86

method	result
parts	$\frac{A\ln(bx+a)}{gb}-\frac{B(ad-cb)e\left(-\frac{d^{2}\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2(ad-cb)be}+\frac{d^{3}\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\cdots\right.}{\cdots}\right)}{d}\cdots\right)}{(ad-cb)be}\cdots\right)}{g\,d^{2}}\cdots$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^{2}A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)}-\frac{d^{2}B\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{d^{2}}\cdots$
default	$-\frac{e(ad-cb)\left(-\frac{d^{2}A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)}-\frac{d^{2}B\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{d^{2}}\cdots$
risch	$\frac{A\ln(bx+a)}{gb}+\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}a}{2g(ad-cb)b}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}c}{2g(ad-cb)}-\frac{Bd\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)a}{g(ad-cb)b}+\frac{B\operatorname{dilog}\cdots}{\cdots}$

[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)

[Out] A/g*ln(b*x+a)/b-B/g/d^2*(a*d-b*c)*e*(-1/2/(a*d-b*c)*d^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^3/b/e*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="fricas")

[Out] integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(b*g*x + a*g), x)

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)

[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="maxima")

[Out] -B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e) + (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x), x)
```

3.93 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx=-\frac{B}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2550, 2341}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx=-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) - ((c + d*x)*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\ &= -\frac{B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = \frac{-Abc-bBc+aAd+aBd-Bd(a+bx)\log(a+bx)+(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right)+aBd\log(c+dx)+}{b(bc-ad)g^2(a+bx)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] (-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*Log[a + b*x] + (-(b*B*c)
+ a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + a*B*d*Log[c + d*x] + b*B*d*x*Log[c
+ d*x])/(b*(b*c - a*d)*g^2*(a + b*x))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result	s
norman	$\frac{\frac{(A+B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}}{g(bx+a)}$	8
parallelrisch	$-\frac{-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2-B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd+Aa\,b^2d^2-A\,b^3cd+Ba\,b^2d^2-B\,b^3cd}{g^2(bx+a)b^3d(ad-cb)}$	
parts	$-\frac{A}{g^2(bx+a)b}-\frac{Be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{g^2(ad-cb)}$	
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{b\,g^2(bx+a)}-\frac{-B\ln(-bx-a)bdx+B\ln(dx+c)bdx-B\ln(-bx-a)ad+B\ln(dx+c)ad+Aad-Abc+Bad-Bbc}{(bx+a)g^2b(ad-cb)}$	
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}\right)}{d^2}$	
default	$-\frac{e(ad-cb)\left(-\frac{d^2A}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}\right)}{d^2}$	

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A+B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)/(d*x+c))+B*d/(a*d-b*c)/g*x*ln(e*(
b*x+a)/(d*x+c)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx=-\frac{(A+B)bc-(A+B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A + B)*b*c - (A + B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(d*x + c)
))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.

Time = 0.63 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.70

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x} - \frac{Bd\log\left(x + \frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{Bd\log\left(x + \frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A-B}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) - B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A - B)/(a
*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2} + \frac{1}{b^2g^2x+abg^2} + \frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2} - \frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2
*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2}+\frac{(Ae^2+Be^2)(dx+c)}{(bex+ae)g^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2) + (A*e^2
 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d))
 - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x))/(c + d*x)))/(b^2*g^2
*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a
*d - b*c))
```

3.94 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3} -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}$$

```
[Out] -1/4*B/b/g^3/(b*x+a)^2+1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*ln(b*x+a)
/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-1/2*B*
d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2} +\frac{Bd}{2bg^3(a+bx)(bc-ad)}-\frac{B}{4bg^3(a+bx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*B/(b*g^3*(a + b*x)^2) + (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2
*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)
])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^2}\,dx}{2bg}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}\\
&\quad+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{2bg^3}\\
&= -\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}\\
&\quad-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]

[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(-3*a*d + b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

method	result
norman	$\frac{\frac{Ba\,d^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{2Aabd-2A\,b^2c+3Babd-B\,b^2c}{4g\,b^2(ad-cb)}-\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{(bx+a)^2g^2}$
parallelrisch	$-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^4d^3-4B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^4c\,d^2-2B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^3+2Bxa\,b^4d^3-2Bx\,b^5c\,d^2+2B\ln\left(\frac{e(bx+a)}{dx+c}\right.}{4g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)ab\,d^2x-4B\ln(-bx-a)ab\,d^2x+2B\ln(dx+}{4(a^2d^2-2ab}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B(ad-cb)e\left(\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3}-\frac{d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}\right.}{(ad-cb)^3}\right.}{g^3d^2}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right.}{(ad-cb)^3g^3}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right.}{(ad-cb)^3g^3}\right.}{d^2}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (B*a*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*x*ln(e*(b*x+a)/(d*x+c))-1/4*(2*A*a*b
*d-2*A*b^2*c+3*B*a*b*d-B*b^2*c)/g/b^2/(a*d-b*c)-1/2/g*B*d/(a*d-b*c)*x+1/2*B
*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))+1/2*B*d^
2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*b*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/g^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.51

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{(2\,A+B)b^2c^2 - 4\,(A+B)abcd + (2\,A+3\,B)a^2d^2 - 2\,(Bb^2cd - Babd^2)x - 2\,(Bb^2d^2x^2 + 2\,Babd^2x -}{4\,((b^5c^2 - 2\,ab^4cd + a^2b^3d^2)g^3x^2 + 2\,(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2\,a^3b^2c}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + (2*A + 3*B)*a^2*d^2 - 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^
2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(124) = 248.

Time = 1.06 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx &= -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\ &\quad - \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} \\ &\quad + \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} \\ &\quad + \frac{-2Aad+2Abc-3Bad+Bbc-2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot\left(4ab^3dg^3-4b^4cg^3\right)+x\left(8a^2b^2dg^3-8ab^3cg^3\right)}\end{aligned}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2\,d^2}{(b^3c^2-2\right.$$
$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx=$$
$$-\frac{1}{4}\left(\frac{2\left(Bbe^3-\frac{2\,(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\,Abe^3+Bbe^3-\frac{4\,(bex+ae)Ade^2}{dx+c}-\frac{4\,(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right)\left(\frac{b}{(bce-ade\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*
x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d*x
 + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x + c) - 4*(b*
e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x
 + a*e)^2*a*d*g^3/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((
b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d -
 b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log((e*(a + b*x))
/(c + d*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B*d^2*atanh((2*b^3*c^2*
g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b
*g^3*(a*d - b*c)^2)
```

3.95 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2} -\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

```
[Out] -1/9*B/b/g^4/(b*x+a)^3+1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3+1/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3} +\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} +\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}-\frac{B}{9bg^4(a+bx)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*B/(b*g^4*(a + b*x)^3) + (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) - (B*d
^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*Log[a + b*x])/(3*b*(b*c - a*
d)^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*b*g^4*(a + b*x)^3) + (B
*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^3}\,dx}{3bg} \\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4} \\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3} \\
&\quad +\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)}{3bg^4}
\end{aligned}$$

$$= -\frac{B}{9bg^4(a+bx)^3} + \frac{Bd}{6b(bc-ad)g^4(a+bx)^2} - \frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)} - \frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3} + \frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + \frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
[Out] -1/18*(6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(11*a^2*d^2
 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(a +
b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^
4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(166) = 332.

Time = 1.20 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.06

method	result
parts	$-\frac{A}{3g^4(bx+a)^3b}-\frac{B(ad-cb)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^4}-\frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+c}\right.}\right.}{(ad-cb)^4}\right.}{g^4d^2}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln(dx+c)a\,b^2d^3x^2-}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-18B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^5d^4-18Bxa\,b^6c\,d^3+15Bx\,a^2b^5d^4+}{}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{6A\,a^2b\,d^2-12Aa\,b^2cd+6A\,b^3c^2+9B\,a^2b\,d^2-7Ba\,b^2cd+}{18g\,b^2\left(a^2d^2-2abcd+b^2c^2\right)}}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A\,b^2e^2}{3(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A\,b^2e^2}{3(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/g^4/(b*x+a)^3/b-B/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4*(-1/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d
/(d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^2/(a*d-b
*c)^4*e^2*b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{2\,(3\,A+B)b^3c^3-9\,(2\,A+B)ab^2c^2d+18\,(A+B)a^2bcd^2-(6\,A+11\,B)a^3d^3+6\,(Bb^3cd^2-Bab^2d^3)x^2}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d
^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^
5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b
*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.

Time = 1.62 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} - \frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2-11Ba^2d^2+7Babcd-2Bb^2c^2-6B}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-1}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 -
11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6*B*b**2*d**2*x**2 + x*(-15*
B*a*b*d**2 + 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 +
18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4
```

```
+ 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**
4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g*
*4 + 54*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2\right. - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] -1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(163) = 326$.

Time = 0.46 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.38

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\left(\frac{6\left(Bb^2e^4 - \frac{3\,(bex+ae)Bbde^3}{dx+c} + \frac{3\,(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bex+ae)^3abcdg^4}{(dx+c)^3} + \frac{(bex+ae)^3a^2d^2g^4}{(dx+c)^3}} + \frac{6\,Ab^2e^4 + 2\,Bb^2e^4 - \frac{18\,(bex+ae)Abde^3}{dx+c} - }{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3} - }\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e)
^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b^2
*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*x +
 a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e^4 - 18*(b*e*x +
 a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 18*(b*e*x
 + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^2*e^2/(d*x + c)^2)
/((b*e*x + a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(
d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e
)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{5\,B\,a\,d^2\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,b\,g^4\,(a+b\,x)^3} + \frac{7\,B\,a\,c\,d}{18\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,c\,d\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (
A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(18*b*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
 - (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x))/(
c + d*x)))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b
*x)^3) + (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.96 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3} -\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} +\frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5}$$

```
[Out] -1/16*B/b/g^5/(b*x+a)^4+1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2/b/(-a
*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*d^4*ln(b
*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^5/(b*x+a)^4-1
/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4} +\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} +\frac{Bd}{12bg^5(a+bx)^3(bc-ad)}-\frac{B}{16bg^5(a+bx)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/16*B/(b*g^5*(a + b*x)^4) + (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B
*d^2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(4*b*(b*c - a*d)^3*g^5*
(a + b*x)) + (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(
a + b*x))/(c + d*x)])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(4*b*(b*
c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^4}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}$$
$$+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\right.}{4bg^5}$$

$$= -\frac{B}{16bg^5(a+bx)^4} + \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} + \frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4} - \frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4} + \frac{B\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4) + (B*((-3*(b*c - a*d)^
4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a
 + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^4*L
og[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(195) = 390$.

Time = 2.16 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{A}{4g^5(bx+a)^4 b} - \frac{B(ad-cb)e\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^5} - \frac{3d^4be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}\right.}{(ad-cb)^5}\right.}{}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b\,g^5(bx+a)^4} - \frac{-48Ba\,b^3c\,d^3x^2-72B\,a^2b^2c\,d^3x+24Ba\,b^3c^2d^2x-48A\,a^3bc\,d^3+72A\,a^2b^2c^2d^2-48Aa\,b^3c^3d-48B\ln}{}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A\,b^3e^3}{4(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{d^3A\,b^2e^2}{(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^5A}{(ad-cb)^5g^5\left(\frac{be}{d}\right.}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A\,b^3e^3}{4(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{d^3A\,b^2e^2}{(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^5A}{(ad-cb)^5g^5\left(\frac{be}{d}\right.}\right.}{}$
parallelrisch	$\frac{12B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^3c\,d^4+48B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7b^2c\,d^4+72B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8bc\,d^4+12A\,x^4a^2b^7c^5+3B\,x^4a^2b^7c^5+4}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,b^2d^4B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(4A\,a^3d^3-12A\,a^2bc\,d^2+12Aa\,b^2\right.}{4ga\left(\right.}$

[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)

[Out] -1/4*A/g^5/(b*x+a)^4/b-B/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/(a*d-b*c)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(192) = 384$.

Time = 0.26 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.05

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(4\,A+B)b^4c^4-16\,(3\,A+B)ab^3c^3d+36\,(2\,A+B)a^2b^2c^2d^2-48\,(A+B)a^3bcd^3+(12\,A+25\,B)a^4d}{48\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7c^3}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas")
[Out] -1/48*(3*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^
2*c^2*d^2 - 48*(A + B)*a^3*b*c*d^3 + (12*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*
b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*
(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*
b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4
*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.

Time = 2.41 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}$$
$$-\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5\left(ad-bc\right)^4}$$
$$+\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5\left(ad-bc\right)^4}$$
$$+\frac{-12Aa^3d^3+36Aa^2bcd^2-36Aab^2c^2}{48a^7bd^3g^5-144a^6b^2cd^2g^5+144a^5b^3c^2dg^5-48a^4b^4c^3g^5+x^4\cdot(48a^3b^5d^3g^5-144a^2b^6cd^2g^5+144ab^7c^2a}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*log(x +
 (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + B*d**4*log(x
 + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 - 25*B*a**
3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B*b**
3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**2*b*
d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a*
*6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 +
x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*
d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5
*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(28
8*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g*
*5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*
c*d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(192) = 384.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = \frac{1}{48}B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2\cdots}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6\,(a^2b\cdots}\right. - \frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] 1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
```

```
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(192) = 384$.

Time = 0.49 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}\left(\frac{12\left(Bb^3e^5 - \frac{4\,(bex+ae)Bb^2de^4}{dx+c} + \frac{6\,(bex+ae)^2Bbd^2e^3}{(dx+c)^2} - \frac{4\,(bex+ae)^3Bd^3e^2}{(dx+c)^3}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^3c^3g^5}{(dx+c)^4} - \frac{3\,(bex+ae)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3\,(bex+ae)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bex+ae)^4a^3d^3g^5}{(dx+c)^4}} + \frac{12\,Ab^3e^5 + 3\,Bb^3e^5}{}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/48*(12*(B*b^3*e^5 - 4*(b*e*x + a*e)*B*b^2*d*e^4/(d*x + c) + 6*(b*e*x + a
*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)*lo
g((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*
e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^2*g^
5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (12*A*b^3*e^5 +
3*B*b^3*e^5 - 48*(b*e*x + a*e)*A*b^2*d*e^4/(d*x + c) - 16*(b*e*x + a*e)*B*b
^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*b*d^2*e^3/(d*x + c)^2 + 36*(b*e*x
 + a*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 48*(b*e*x + a*e)^3*A*d^3*e^2/(d*x + c)^
3 - 48*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)/((b*e*x + a*e)^4*b^3*c^3*g^5/
(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*
e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4)
)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}-\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 36*A*a*b^2*c
^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^
2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3
*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*b^2*g^5*x + 16*a*b^4
*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/
(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4)
```

3.97 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 365

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad+\frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} \\
&\quad-\frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} \\
&\quad+\frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} \\
&\quad+\frac{B(bc-ad)^5g^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12A+25B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^5} \\
&\quad+\frac{2B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/5*g^4*(b
*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/30*B*(-a*d+b*c)^2*g^4*(b*x+a)^3*(
4*A+B+4*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/60*B*(-a*d+b*c)^3*g^4*(b*x+a)^2*(1
2*A+7*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+1/30*B*(-a*d+b*c)^4*g^4*(b*x+a)*(
12*A+13*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*ln((-a*
d+b*c)/b/(d*x+c))*(12*A+25*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^5+2/5*B^2*(-a*
d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+25B\right)}{30bd^5}$$
$$+\frac{Bg^4(a+bx)(bc-ad)^4\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+13B\right)}{30bd^4}$$
$$-\frac{Bg^4(a+bx)^2(bc-ad)^3\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+7B\right)}{60bd^3}$$
$$+\frac{Bg^4(a+bx)^3(bc-ad)^2\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{30bd^2}$$
$$-\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10bd}$$
$$+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b}+\frac{2B^2g^4(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/10*(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b) + (B
*(b*c - a*d)^2*g^4*(a + b*x)^3*(4*A + B + 4*B*Log[(e*(a + b*x))/(c + d*x)])
)/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(12*A + 7*B + 12*B*Log[(e*(
a + b*x))/(c + d*x)]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*(a + b*x)*(12*A +
13*B + 12*B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b*d^4) + (B*(b*c - a*d)^5*g^
4*Log[(b*c - a*d)/(b*(c + d*x))]*(12*A + 25*B + 12*B*Log[(e*(a + b*x))/(c +
 d*x)]))/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{x^4(A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{x^4(A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{\left(B(bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{x^3(4A+B+4B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10bd}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{(B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{x^2(4B+3(4A+B)+12B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30bd^2}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} + \frac{(B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{x(12B+2(4B+3(4A+B))+24B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{60bd^3}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} - \frac{(B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{36B+2(4B+3(4A+B))+24B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{60bd^4}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} + \frac{B(bc-ad)^5g^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12A+25B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^5} - \frac{(2B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{5bd^5}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} + \frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} + \frac{B(bc-ad)^5g^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12A+25B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^5} + \frac{2B^2(bc-ad)^5g^4\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.40

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-12d^2(bc-ad)^2(a+bx)^2\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a
*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d
*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x
^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2389 vs. $2(350) = 700$.

Time = 0.32 (sec) , antiderivative size = 2389, normalized size of antiderivative = 6.55

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x - 1/30*((12*g^4*log(e) + 25*g^4)*b^4*c^5 - (60*g^4*log(e) + 113*g^4)*a*b
^3*c^4*d + 4*(30*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(10*g^4*log(e) +
 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) + 4*g^4)*a^4*c*d^4)*B^2*log(d*x +
 c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 1
0*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 6*(b^5*c*d^4*g^4*log(e) - (1
0*g^4*log(e)^2 + g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 2*((4*g^4*log(e) + g^4)*b
^5*c^2*d^3 - 2*(10*g^4*log(e) + g^4)*a*b^4*c*d^4 + (60*g^4*log(e)^2 + 16*g^
4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 - ((12*g^4*log(e) + 7*g^4)*b^5*c^3*d^2
 - 3*(20*g^4*log(e) + 9*g^4)*a*b^4*c^2*d^3 + 3*(40*g^4*log(e) + 11*g^4)*a^2
```

```
*b^3*c*d^4 - (120*g^4*log(e)^2 + 72*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^2*x
^2 + 2*((12*g^4*log(e) + 13*g^4)*b^5*c^4*d - (60*g^4*log(e) + 59*g^4)*a*b^4
*c^3*d^2 + 6*(20*g^4*log(e) + 17*g^4)*a^2*b^3*c^2*d^3 - (120*g^4*log(e) + 7
9*g^4)*a^3*b^2*c*d^4 + (30*g^4*log(e)^2 + 48*g^4*log(e) + 23*g^4)*a^4*b*d^5
)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^
3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^
5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*
a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(12*B^2*b^
5*d^5*g^4*x^5*log(e) - 3*(b^5*c*d^4*g^4 - (20*g^4*log(e) + g^4)*a*b^4*d^5)*
B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*log(e) + 2*g^4
)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*
b^3*c*d^4*g^4 - 2*(10*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^
4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g
^4 + (5*g^4*log(e) + 4*g^4)*a^4*b*d^5)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54*a^2
*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + (12*g^4*lo
g(e) + 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) - 2*(12*B^2*b^5*d^5*g^4*x^5*log(e
) - 3*(b^5*c*d^4*g^4 - (20*g^4*log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^
2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*log(e) + 2*g^4)*a^2*b^3*d^5)*B^2*
x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(
10*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4 - 5*a*b^4*c
^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + (5*g^4*log(e)
+ 4*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^4\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.98 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 309

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$+ \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2}$$
$$- \frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3}$$
$$- \frac{B(bc-ad)^4g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^4}$$
$$- \frac{B^2(bc-ad)^4g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/4*g^3*(b*
x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(3
*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/12*B*(-a*d+b*c)^3*g^3*(b*x+a)*(6*A+
5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3-1/12*B*(-a*d+b*c)^4*g^3*ln((-a*d+b*c)/
b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B^2*(-a*d+b*c)^4*
g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{12bd^4} \\
&\quad -\frac{Bg^3(a+bx)(bc-ad)^3\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{12bd^3} \\
&\quad +\frac{Bg^3(a+bx)^2(bc-ad)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{12bd^2} \\
&\quad -\frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6bd} \\
&\quad +\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b}-\frac{B^2g^3(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b) + (B*
(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))
/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b
*x))/(c + d*x)]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*Log[(b*c - a*d)/(b*(c +
 d*x))]*(6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(12*b*d^4) - (B^2*
(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{\left(B(bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd}$$

$$=-\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2}-\frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{x(3B+2(3A+B)+6B\log(ex))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{12bd^2}$$

$$=-\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2}-\frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3}+\frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{9B+2(3A+B)+6B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{12bd^3}$$

$$=-\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2}-\frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3}-\frac{B(bc-ad)^4g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^4}+\frac{(B^2(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^4}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}$$
$$+ \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$+ \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2}$$
$$- \frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3}$$
$$- \frac{B(bc-ad)^4g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^4}$$
$$- \frac{B^2(bc-ad)^4g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.27

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c
- a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(
b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

Maple [F]

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^
3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. $2(296) = 592$.

Time $= 0.32$ (sec) , antiderivative size $= 1732$, normalized size of antiderivative $= 5.61$

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*g^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/12*((6*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(12*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) + 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 2*(b^4*c*d^3*g^3*log(e) - (6*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*x^3 + ((3*g^3*log(e) + g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) + g^3)*a*b^3*c*d^3 + (18*g^3*log(e)^2 + 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - ((6*g^3*log(e) + 5*g^3)*b^4*c^3*d - (24*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) + 19*g^3)*a^2*b^2*c*d^3 - (12*g^3*log(e)^2 + 18*g^3*log(e) + 7*g^3)*a^3*b*d^4)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (4*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 - (6*g^3*log(e) + 11*g^3)*a^4*d^4)*B^2)*log(b*x + a) - (6*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (4*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log \left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln \left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.99 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 253

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b}+\frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2}+\frac{B(bc-ad)^3g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3}+\frac{2B^2(bc-ad)^3g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/3*B*(-a*d+b*c)^2*g^2*(b*x+a)*(2*A+
B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2*ln((-a*d+b*c)/b/(
d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$
$$=\frac{Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{3bd^3}$$
$$+\frac{Bg^2(a+bx)(bc-ad)^2\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{3bd^2}$$
$$-\frac{Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd}$$
$$+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b}+\frac{2B^2g^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) + (B*
(b*c - a*d)^2*g^2*(a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(
3*b*d^2) + (B*(b*c - a*d)^3*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B +
 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{\left(B(bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{x(2A+B+2B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}$$

$$= -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2} - \frac{\left(B(bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{2A+3B+2B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2}$$

$$= -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2} + \frac{B(bc-ad)^3g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} - \frac{(2B^2(bc-ad)^3g^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$= -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2} + \frac{B(bc-ad)^3g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} + \frac{2B^2(bc-ad)^3g^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(a}{c}\right.\right.\right.}{}}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a
*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*(b*c
```

```
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(242) = 484.

Time = 0.29 (sec) , antiderivative size = 1165, normalized size of antiderivative = 4.60

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x - 1/3*((2*g^2*log(e) + 3*g^2)*b^2*c^3 - (6*g^2*log(e) + 7*g^2)*a*b*c^2*d
 + 2*(3*g^2*log(e) + 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (b^3*c*d^2*g^2*log(e) - (3*g^2
*log(e)^2 + g^2*log(e))*a*b^2*d^3)*B^2*x^2 + ((2*g^2*log(e) + g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) + g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^
3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x +
 (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (2*g^2*log(e) + 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*
log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (
3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.100 $\int (ag+bgx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \, dx = -\frac{B(bc-ad)g(a+bx) \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{g(a+bx)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B(bc-ad)^2 g \log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd^2} - \frac{B^2(bc-ad)^2 g \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/2*g*(b*x+a)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/b-B*(-a*d+b*c)^2*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B
+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d
*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{2550, 2381, 2384, 2354, 2438\}$

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{bd^2} - \frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd} + \frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b} - \frac{B^2g(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*d)) +
 (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b) - (B*(b*c - a
*d)^2*g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x))/(c + d*
x)]))/(b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^2g\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{\left(B(bc-ad)^2g\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}$$

$$= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} + \frac{\left(B(bc-ad)^2g\right)\text{Subst}\left(\int \frac{A+B+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}$$

$$= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B(bc-ad)^2g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd^2} + \frac{\left(B^2(bc-ad)^2g\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}$$

$$= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B(bc-ad)^2g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd^2} - \frac{B^2(bc-ad)^2g\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B(bc-ad)\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\left(\right.\right.\right.}{}\right.}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Lo
g[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.39

$$\begin{aligned}
&\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx\\
&=\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABag\\
&\quad+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABbg\\
&\quad+A^2agx+\frac{((g\log(e)+g)bc^2-(2g\log(e)+g)acd)B^2\log(dx+c)}{d^2}\\
&\quad+\frac{(b^2c^2g-2abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}\\
&\quad+\frac{B^2b^2d^2gx^2\log(e)^2-2\left(b^2cdg\log(e)-\left(g\log(e)^2+g\log(e)\right)abd^2\right)B^2x+\left(B^2b^2d^2gx^2+2B^2abd^2gx+\ldots\right.}{}
\end{aligned}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a
)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b
*g + A^2*a*g*x + ((g*log(e) + g)*b*c^2 - (2*g*log(e) + g)*a*c*d)*B^2*log(d*
x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x
 + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) +
 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(b^2*c*d*g*log(e) - (g*log(e)^2 + g*lo
g(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2
*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g -
2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(
e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) + g)*a^2*d^2 - a*b*c*d*g)*B
^2)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) + g)*a*b*d^2
- b^2*c*d*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g
)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.101 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x
+c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2550, 2379, 2421, 6724}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{2B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{bg} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x
))])/(b*g)) + (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))])/(b*g) + (2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))
])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{(2B)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{(2B^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(128) = 256$.

Time = 0.94 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.58

$$\int\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{3A^2\log(a+bx)+3AB\left(\log^2\left(\frac{a}{b}+x\right)-2\log(a+bx)\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2\left(\mathrm{lo}\right.\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (3*A^2*Log[a + b*x] + 3*A*B*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x]
- Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*(Lo
g[a/b + x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a
 + b*x]*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3
*Log[a/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/
b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a +
b*x))/(c + d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*
c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a +
 b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(128) = 256$.

Time = 1.18 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.62

method	result
parts	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2 e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3be} + \frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right) + 2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d(\cdots)}{\cdots}\right)\cdots}{be}\cdots\right)}{g}\cdots$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)} - \frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)}{be}\right)\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)} - \frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)}{be}\right)\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
risch	$\frac{A^2\ln(bx+a)}{gb}+\frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3gb}-\frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{gb}-\frac{2B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)\cdots}{d(dx+c\cdots}\right)\cdots}{\cdots}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] A^2/g*ln(b*x+a)/b-B^2/g*e*(-1/3/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+1/b/e
*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+
c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))))-2*A*B/g/d
^2*(a*d-b*c)*e*(-1/2/(a*d-b*c)*d^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/
(a*d-b*c)*d^3/b/e*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/
d))
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c +
 d*x))**2/(a + b*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x +
 a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x + (2*B^2*b*d*x + (b*c + a*d)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x), x)
```

3.102 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 126

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=-\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/
g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2550, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=-\frac{2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B*(c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= -\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(2B)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= -\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)} \\
&\quad - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+B\log\left(\frac{e(a}{c+}\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40

method	result
norman	$\frac{\frac{\left(A^2+2BA+2B^2\right)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{2cB(A+B)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{2Bd(A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{-2AB\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd-2AB\,b^3cd+A^2a\,b^2d^2-A^2b^3cd+2B^2a\,b^2d^2-2B^2b^3cd+2ABa\,b^2d^2-2ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2}{g^2(bx+a)b^3d(ad-cb}$
parts	$-\frac{A^2}{g^2(bx+a)b}-\frac{B^2e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{g^2(ad-cb)}-\frac{2BAe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\right.}{g^2(ad-cb)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right.}{}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right.}{}\right.}{d^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{}{d(}\right.}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A^2+2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2+B^2*d/g/
(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c))^2+2*c*B*(A+B)/g/(a*d-b*c)*ln(e*(b*x+a)/(d
*x+c))+2*B*d*(A+B)/g/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{(A^2+2\,AB+2\,B^2)bc-(A^2+2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\left((AB+B^2)bdx+\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 + 2*A*B + 2*B^2)*b*c - (A^2 + 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b*e*x + a*e)/(d*x + c))^2 + 2*((A*B + B^2)*b*d*x + (A*B + B^2)*b*
c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*
b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(105) = 210$.

Time = 1.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.44

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd+\frac{2Ba^2d^3(A+B)}{ad-bc}-\frac{4Babcd^2(A+B)}{ad-bc}+\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{(-2AB-2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2-2AB-2B^2}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d + 2*B**2*a*d**2 + 2*B**2
*b*c*d - 2*B*a**2*d**3*(A + B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A + B)/(a*d -
b*c) - 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b*d**2 + 4*B**2*b*d**2))
/(b*g**2*(a*d - b*c)) + 2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d +
 2*B**2*a*d**2 + 2*B**2*b*c*d + 2*B*a**2*d**3*(A + B)/(a*d - b*c) - 4*B*a*b
*c*d**2*(A + B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b
*d**2 + 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B - 2*B**2)*log(e*(a +
 b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(a +
b*x)/(c + d*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x
) + (-A**2 - 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(126) = 252$.

Time = 0.21 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.30

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+ad)\log(bx+c\ldots}{\ldots}\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] -(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((b
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1
/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x +
 c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.52

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2}+\frac{2\,(ABe^2+B^2e^2)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2}+\frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(dx\ldots}{(bex+ae)g^2}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2) + 2*
(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g
^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2 + 2*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*(a + b*x))/(c + d*x)
)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d - b*c))) - (log((e*(a +
b*x))/(c + d*x))*((2*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d + a/(b*d
)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c)
)*(A + B)*4i)/(b*g^2*(a*d - b*c))
```

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 268

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx &= \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} \\ &\quad+\frac{2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3(a+bx)} \\ &\quad-\frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} \\ &\quad+\frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} \\ &\quad-\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\end{aligned}$$

```
[Out] 2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g
^3/(b*x+a)^2+2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*
x+a)-1/2*b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2} + \frac{2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{bB^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2} + \frac{2B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (2*B^2*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/(4*
(b*c - a*d)^2*g^3*(a + b*x)^2) + (2*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*(c + d*x)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^2*g^3*(a
+ b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
```

```
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{x^3} - \frac{d(A+B\log(ex))^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2} \\
&\quad + \frac{(bB)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{(2Bd)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} \\
&= \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} \\
&\quad + \frac{2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} \\
&\quad + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-4d^2(a+bx)^2\log(a+bx)}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(
a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) -
2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*
(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.81

method	result
norman	$\frac{\frac{Bd(2Aad+2Bad+Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc+4ABad-2ABbc+4B^2ad-B^2bc\right)x}{2ag(ad-cb)}+\frac{Bc(4Aad-}{}}{}$
parallelrisch	$-\frac{-4A^2a\,b^4c\,d^2+6AB\,a^2b^3d^3+2AB\,b^5c^2d-8B^2a\,b^4c\,d^2-8ABa\,b^4c\,d^2-4B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c\,d^2-4B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a}{}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2(ad-cb)e\left(\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3}-\frac{d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-}{d(d}\right.}{2\left(\frac{be}{d}+\frac{(ad-}{d(d}\right.}\right.}{}\right.}{g^3d^2}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A^2be}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}\right.}{(ad-cb)^3g^3}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A^2be}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}\right.}{(ad-cb)^3g^3}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (B/g*d*(2*A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(
d*x+c))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2+1
/2*(2*A^2*a*d-2*A^2*b*c+4*A*B*a*d-2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-b*c
)*x+1/2*B*c*(4*A*a*d-2*A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*l
n(e*(b*x+a)/(d*x+c))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln
(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c+6*A*B*a*d-2*A*B*b*c+7*B^2*a*
d-B^2*b*c)/g/a^2*b/(a*d-b*c)*x^2+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2
)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*B*b/g*d^2*(2*A+3*B)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/g^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.37

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(2\,A^2 + 2\,AB + B^2)b^2c^2 - 4\,(A^2 + 2\,AB + 2\,B^2)abcd + (2\,A^2 + 6\,AB + 7\,B^2)a^2d^2 - 2\,(B^2b^2d^2x^2 + 2\,B^2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B + 3*B^2)
*b^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2 - (2
*A*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B + B^2
)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*
b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. 2(241) = 482.

Time = 2.11 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.34

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{Bd^2\cdot(2A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2-\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}+\frac{Bb^3c^3}{(}}{4ABbd^3+6B^2bd^3}\right.}{2bg^3\,(ad-bc)^2} + \frac{Bd^2\cdot(2A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2+\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}-\frac{Bb^3c^3}{(}}{4ABbd^3+6B^2bd^3}\right.}{2bg^3\,(ad-bc)^2} + \frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2} + \frac{(-2ABad+2ABbc-3B^2ad+B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2} + \frac{-2A^2ad+2A^2bc-6ABad+2ABbc-7B^2ad+B^2bc+x(-4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3
+ 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d
**4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*
c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2
*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d
**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A +
 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a
*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A + 3*B)/(
a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) +
(2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a
+ b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*
g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*
g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**
3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*l
og(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b*
*2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2
) + (-2*A**2*a*d + 2*A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 7*B**2*a*d + B**2*b
*c + x*(-4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x
**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*
g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(262) = 524$.

Time = 0.24 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{1}{4}\left(2\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2}{(b^3c^2-}\right.\right.$$
$$+\frac{1}{2}AB\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{}{(b^3c}\right.$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2
```

```
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a
*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2
 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*
b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*
d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)
*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d
^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c
^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*((2*b*d*x -
 b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
(a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^
3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*g^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(B^2be^3-\frac{2\,(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\left(2\,ABbe^3+B^2be^3-\frac{4\,(bex+ae)ABde^2}{dx+c}-\frac{4\,(bex+ae)B^2de^2}{dx+c}\right)\log}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^
3/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*B*d*e^2/(d*
x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/
((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d*x + c)^2)
 + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A^2*d*e^2/(d*x
+ c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*B^2*d*e^2/(d*x
 + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d*x
+ c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a
*d)))
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{\frac{2A^2ad-2A^2bc+7B^2ad-B^2bc+6ABad-2ABbc}{2(ad-bc)}+\frac{x(3bdB^2+2AbdB)}{ad-bc}}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}$$
$$-\ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2b^2g^3\left(2ax+bx^2+\frac{a^2}{b}\right)}-\frac{B^2d^2}{2bg^3\left(a^2d^2-2abcd+b^2c^2\right)}\right)$$
$$-\frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{b^2dg^3}+\frac{B^2x(ad-bc)}{bg^3(a^2d^2-2abcd+b^2c^2)}+\frac{B^2d^2\left(\frac{2a^2d^2-3abcd+b^2c^2}{2bd^3}+\frac{a(ad-bc)}{2bd^2}\right)}{bg^3(a^2d^2-2abcd+b^2c^2)}\right)}{\frac{bx^2}{d}+\frac{a^2}{bd}+\frac{2ax}{d}}$$
$$-\frac{Bd^2\,\mathrm{atan}\left(\frac{Bd^2\left(2bdx-\frac{b^3c^2g^3-a^2bd^2g^3}{bg^3(ad-bc)}\right)(2A+3B)\,1\mathrm{i}}{(ad-bc)(3B^2d^2+2ABd^2)}\right)(2A+3B)\,1\mathrm{i}}{bg^3(ad-bc)^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2
*(a*d - b*c)) + (x*(3*B^2*b*d + 2*A*B*b*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2*b
^3*g^3*x^2 + 4*a*b^2*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*b^2*g^
3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b^2*d*g^3) + (B^2*x*(a*d - b
*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((B
*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A + 3
*B)*1i)/((a*d - b*c)*(3*B^2*d^2 + 2*A*B*d^2)))*(2*A + 3*B)*1i)/(b*g^3*(a*d
- b*c)^2)
```

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 418

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =& -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}\end{aligned}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-2*B*d^2
*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)+b*B*d*(d*x+c)
```

```
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^2-2/9*b^2*B*(d*x+c)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}dx = -\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3} - \frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3} - \frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4(a+bx)(bc-ad)^3} + \frac{bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3} + \frac{bBd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3} - \frac{2b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3} - \frac{2B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3} + \frac{bB^2d(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*(c + d*x)^2
)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) - (2*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B*d*(c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B*(c + d*x)^3*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3) - (
d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g^4*(a
 + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c -
```

```
 a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex))^2}{x^4}-\frac{2bd(A+B\log(ex))^2}{x^3}+\frac{d^2(A+B\log(ex))^2}{x^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}-\frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}\\
&\quad+\frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}
\end{aligned}$$

$$= -\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3} + \frac{(2b^2B)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^4}dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^4} - \frac{(2bBd)\text{Subst}\left(\int\frac{A+B\log(ex)}{x^3}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4} + \frac{(2Bd^2)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^2}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)} + \frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.39

$$\int\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(12A(bc-ad)^3+4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+bx\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(12*A*(b*c - a*d)^3 +
 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*
(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*
x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x
] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*
x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)
] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^
3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x
)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3
*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[
```

```
(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2
 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*
d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*
x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x
)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(410) = 820$.

Time = 1.34 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3 b} - \frac{B^2(ad-cb)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}} - \frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}} - \frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^4} - \frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right.}{2\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right.}\right.}{\cdots}\right.}{\cdots}$
norman	$\frac{\frac{B^2a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g} + \frac{B^2ab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)} - \frac{18A^2a^2b^2d^2-36A^2a\,b^3cd+18A^2b^4c^2+66AB\,a^2b^2d^2-42\cdots}{54g(a\cdots}}{\cdots}$
parallelrisch	$-\frac{66AB\,a^3b^4d^4-12AB\,b^7c^3d-108B^2a^2b^5c\,d^3+27B^2a\,b^6c^2d^2-108AB\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^6d^4-108ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^5\cdots}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A^2/g^4/(b*x+a)^3/b-B^2/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4*(-1/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x
+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^2/(a*d-b*c)^4*e^2*
b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4
*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d
+(a*d-b*c)*e/d/(d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2)+d^2/(a*d-b*c)^4*e^2*b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2\,(9\,A^2+6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2+2\,AB+B^2)ab^2c^2d+54\,(A^2+2\,AB+2\,B^2)a^2bcd^2-(18\,A^2+}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a^
3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
- 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 3
*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*B + 49
*B^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^2)*b^3*
c^3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*(2*B^2*b
^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^
2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
- 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3
 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 -
3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. 2(384) = 768.

Time = 11.68 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d**
4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3*b
*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*B)/
(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b**4*
```

```
c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4))/(9
*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B
*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 11*B)/(a
*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**
2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A + 11*B)
/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d
**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*
B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**
3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6*d**3*g**4 -
9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*
a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**
4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**
3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2
- 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*
g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6
*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d - 15*B**2*a*b*d**2*x -
2*B**2*b**2*c**2 + 3*B**2*b**2*c*d*x - 6*B**2*b**2*d**2*x**2)*log(e*(a + b*
x)/(c + d*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d*
*2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*
d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*
a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x*
*3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**
2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a*b*c*d + 12*A*B*b**2*c**2 + 85*B**
2*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(90*A*B*a*b*d**2 - 18*A*B*b**2*c*d + 147*B**2*a*b*d
**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 54
*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4 +
 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g**
4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d*
g**4 + 162*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(410) = 820$.

Time = 0.29 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.39

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] -1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
```

```
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 - 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*
a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/(
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log
(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.73

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{54}\left(\frac{18\left(B^2b^2e^4-\frac{3\,(bex+ae)B^2bde^3}{dx+c}+\frac{3\,(bex+ae)^2B^2d^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(6\,ABb^2e^4+2\,B^2b^2e^4-\frac{18\,(bex+\cdots}{dx\cdots}\right.}{(be\cdots}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x +
 a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a
```

```
*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 +
 (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + 6*(6*A*B*b^2*e^4 + 2*B^2*b^2*e^
4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*b*d*e^3/(d
*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B
^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b^2*c
^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*x + a
*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (18*A^2*b^2*e^4 + 12*A*B*b^2*e^4 + 4*B^2*b
^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*x + c) - 54*(b*e*x + a*e)*A*B*b*d*
e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 54*(b*e*x + a*e)^2
*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 10
8*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4/(d*
x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*
d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*
d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.55

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2-42\,A\,B\,a\,b\,c\,d+12\,A\,B\,b^2\,c^2+85\,B^2\,a^2\,d^2-23\,B^2\,a\,b\,c\,d+4\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x\,(-5\,c\,B^2\,b^2\,d+49\,a\,B^{2}}{}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\,d\right.}$$

$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{3\,b^2\,g^4\,\left(3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2\right)}-\frac{B^2\,d^3}{3\,b\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,A\,B}{3\,b^2\,d\,g^4}+\frac{2\,B^2\,d^3\,\left(a\,\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{6\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{3\,a^3\,d^3-6\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d-b^3\,c^3}{3\,b\,d^4}\right)}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{2\,B^2\,d^3\,x^2\,\left(\frac{b^2\,c-a}{3\,d^2}\right.}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d}\right.}{\frac{3\,a^2\,x}{d}+\frac{a^3}{b\,d}+\frac{b^2\,x^3}{d}+\frac{3\,a\,b}{d}}$$

$$-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\,\left(\frac{a^3\,b\,d^3\,g^4-a^2\,b^2\,c\,d^2\,g^4-a\,b^3\,c^2\,d\,g^4+b^4\,c^3\,g^4}{a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4}+2\,b\,d\,x\right)\,(6\,A+11\,B)\,\left(a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4\right)\,1\mathrm{i}}{b\,g^4\,(a\,d-b\,c)^3\,(11\,B^2\,d^3+6\,A\,B\,d^3)}\right)\,(6\,A+11\,B}{9\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 + 66*A*B
*a^2*d^2 + 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d - 42*A*B*a*b*c*
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 30*A*B*a*b*d^2 -
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d + 6*A*B*b^2*d))/(a*d
```

```
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*((2*A*B)/(3*b^2
*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) - (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A + 11*B)*(b^3*c^2*g^4 +
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 + 6*A
*B*d^3)))*(6*A + 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 575

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} + \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4} + \frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{d^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4} + \frac{2Bd^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^5(a+bx)(bc-ad)^4} - \frac{3bd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4} - \frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4} - \frac{b^3B^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4} + \frac{2b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4} + \frac{2B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4} - \frac{3bB^2d^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32*(b*c - a*d)^4*g^5*(a
+ b*x)^4) + (2*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c
- a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B*d*(c + d*x)^3*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*
B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)^4*g^5*(a
 + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c -
```

```
 a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*(c +
d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)^4*g^5*(a + b*
x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex))^2}{x^5} - \frac{3b^2d(A+B\log(ex))^2}{x^4} + \frac{3bd^2(A+B\log(ex))^2}{x^3} - \frac{d^3(A+B\log(ex))^2}{x^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4} + \frac{(b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^5} - \frac{(2b^2Bd)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} + \frac{(3bBd^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{(2Bd^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} + \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{72\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(36A(bc-ad)^4+9B(bc-ad)^4+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2(a+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*A*(b*c - a*d)^4
+ 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) +
a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d
)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c)
+ a*d)*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b*
x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a*d)^
4*Log[(e*(a + b*x))/(c + d*x)] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(a + b*x))
/(c + d*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a + b*x))/(c + d
*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + 14
4*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*Log[c + d*x] - 144
*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d
^4*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] + 72*B*d^4*(a + b*
x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/
(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a
*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. 2(559) = 1118.

Time = 2.32 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.05

method	result	size
parts	Expression too large to display	1179
derivativedivides	Expression too large to display	1393
default	Expression too large to display	1393
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	3080

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x
+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/(a*d-b*c)^5*b^
2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e^3*(-1/4/(b*e/d+(a*d
```

```
-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^4))-2*B*A/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c
)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/(a*d-b*c)^5
*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e^3*(-1/4/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{9\,(8\,A^2+4\,AB+B^2)b^4c^4-32\,(9\,A^2+6\,AB+2\,B^2)ab^3c^3d+216\,(2\,A^2+2\,AB+B^2)a^2b^2c^2d^2-288\,(A}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 + 300*A*B + 415*B^2)*a^4*d^4 - 12*((12*A*B + 25*B^
2)*b^4*c*d^3 - (12*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((12*A*B + 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B + 11*B^2)*a*b^3*c*d^3 + (84*A*B + 163*B^2)*a^2*b^2*d^4)
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((12*A*B + 7*B^2)*
b^4*c^3*d - 12*(6*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B + 3*B^2)*a^2*b^2*
c*d^3 - (156*A*B + 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B + 25*B^2)*b^4*d^4*x^
4 - 3*(4*A*B + B^2)*b^4*c^4 + 16*(3*A*B + B^2)*a*b^3*c^3*d - 36*(2*A*B + B^
2)*a^2*b^2*c^2*d^2 + 48*(A*B + B^2)*a^3*b*c*d^3 + 4*(3*B^2*b^4*c*d^3 + 2*(6
*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*
(2*A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 +
 18*B^2*a^2*b^2*c*d^3 + 12*(A*B + B^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x
 + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a
^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^
```

```
4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^
5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3
+ a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(559) = 1118$.

Time = 0.36 (sec) , antiderivative size = 2123, normalized size of antiderivative = 3.69

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
[Out] 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^
3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d
^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*
d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*
g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
```

```
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/24*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c
) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/288*(72*(B^2*b^3*e^5 - 4*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 6*(b*e*
x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*x +
 c)^3)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c
)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2
*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(1
2*A*B*b^3*e^5 + 3*B^2*b^3*e^5 - 48*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x + c) -
16*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*b*d^2*e^3
/(d*x + c)^2 + 36*(b*e*x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 48*(b*e*x + a
```

```
*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 48*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*x + c)^3)
*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*
(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^2
*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (72*A^2*b^3*e
^5 + 36*A*B*b^3*e^5 + 9*B^2*b^3*e^5 - 288*(b*e*x + a*e)*A^2*b^2*d*e^4/(d*x
+ c) - 192*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x + c) - 64*(b*e*x + a*e)*B^2*b^2
*d*e^4/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*b*d^2*e^3/(d*x + c)^2 + 432*(b*e
*x + a*e)^2*A*B*b*d^2*e^3/(d*x + c)^2 + 216*(b*e*x + a*e)^2*B^2*b*d^2*e^3/(
d*x + c)^2 - 288*(b*e*x + a*e)^3*A^2*d^3*e^2/(d*x + c)^3 - 576*(b*e*x + a*e
)^3*A*B*d^3*e^2/(d*x + c)^3 - 576*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*x + c)^3)/
((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^
5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*
e)^4*a^3*d^3*g^5/(d*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b
*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.27

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atan((B*d^4*(12*A + 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*a*
b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2*d
^4 + 12*A*B*d^4)) + (B*d^5*x*(12*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3
*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4
+ 12*A*B*d^4)))*(12*A + 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4) - log((e*(a + b*
x))/(c + d*x))^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 +
 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2
- 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(
2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a
^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b
^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*
d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2
*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c
- a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*
b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(
a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(
```

```
a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)
) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*
(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2
)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) +
 (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/
((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d)
 - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 30
0*A*B*a^3*d^3 - 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2
+ 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 + 156*A*B*a*b^2*c^2*d - 276*A*B*
a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2
+ 84*A*B*a*b^2*d^3 - 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b
*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 + 156*A*B*a^2*b*d^3 + 12*A*B*b^
3*c^2*d - 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 1
2*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 -
192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2
*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5
) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) +
 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
```

3.106 $\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\text{PolyLog}\left(2,\frac{bc-ad}{b(c+dx)}\right)}{df}$$

```
[Out] polylog(2,(-a*d+b*c)/b/(d*x+c))/d/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2497}

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

```
[In] Int[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

```
[Out] PolyLog[2, 1 - (d*(a + b*x))/(b*(c + d*x))]/(d*f)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-2\log\left(\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\right)-2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2df}$$

```
[In] Integrate[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

```
[Out] (Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 2*Lo
g[(d*(a + b*x))/(b*(c + d*x))] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])/(2*d*f)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
default	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
risch	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
parts	$\frac{\ln\left(\frac{d(bx+a)}{b(dx+c)}\right)\ln(dx+c)}{df}-\frac{b\left(-\frac{d^2\ln(dx+c)^2}{2b}+d^2\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}\right)\right)}{d^3f}$	132

```
[In] int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/f*dilog(1+(a*d-b*c)/b/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\mathrm{Li}_2\left(-\frac{bdx+ad}{bdx+bc}+1\right)}{df}$$

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")
[Out] dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)
```

Sympy [F]

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\int \frac{\log\left(\frac{ad}{bc+bdx}+\frac{bdx}{bc+bdx}\right)}{c+dx}\,dx}{f}$$

```
[In] integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
[Out] Integral(log(a*d/(b*c + b*d*x) + b*d*x/(b*c + b*d*x))/(c + d*x), x)/f
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.64

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = -\frac{b\left(\frac{\log(dx+c)^2}{bf}-\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bf}\right)}{2\,d} - \frac{b\left(\frac{d\log(bx+a)}{b}-\frac{d\log(dx+c)}{b}\right)\log(dfx+cf)}{d^2f} + \frac{\log(dfx+cf)\log\left(\frac{(bx+a)d}{(dx+c)b}\right)}{df}$$

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")
[Out] -1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/b
- d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x +
a)*d/((d*x + c)*b))/(d*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(27) = 54$.

Time = 34.96 (sec) , antiderivative size = 1203, normalized size of antiderivative = 42.96

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \text{Too large to display}$$

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*d/(b*c - a*d)^2 - a*b*d^2/(b*c - a*d)^2)^2*((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(log(abs(b*d*x + a*d)/abs(b*d*x + b*c))/(b
^3*d^4*f) - log(abs((b*d*x + a*d)/(b*d*x + b*c) - 1))/(b^3*d^4*f) - 1/(b^3*
d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1))) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log((a + b*((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)
*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c -
 a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b
*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x +
b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))/(b*d/(b*c - a*d)
- (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d)
)*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))*b*d/
((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/
(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c -
a*d))))))*d/(b*(c + ((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)
) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d
/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*
(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c -
a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))*d/(b*d/(b*c - a*d) - (a*d - b
*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*
x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))*b*d/((b*c - a*
d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d
))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d))))))
)/(b^3*d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\mathrm{Li}_2\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right)}{d\,f}$$

```
[In] int(log((d*(a + b*x))/(b*(c + d*x)))/(c*f + d*f*x),x)
```

```
[Out] dilog((d*(a + b*x))/(b*(c + d*x)))/(d*f)
```

3.107 $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx} \, dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,-1/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2497}

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx} \, dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

```
[In] Int[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/b
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

```
[In] Integrate[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
[Out] PolyLog[2, -(a + b*x)^(-1)]/b
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\text{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
default	$\frac{\text{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
risch	$\frac{\text{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
parts	$\frac{\ln\left(1+\frac{1}{bx+a}\right)\ln(bx+a)}{b}+\frac{\frac{\ln(bx+a)^2}{2}-\text{dilog}(bx+a+1)-\ln(bx+a)\ln(bx+a+1)}{b}$	61

```
[In] int(ln(1+1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*dilog(1+1/(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{Li}_2\left(-\frac{bx+a+1}{bx+a}+1\right)}{b}$$

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="fricas")
[Out] dilog(-(b*x + a + 1)/(b*x + a) + 1)/b
```

Sympy [F]

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

```
[In] integrate(ln(1+1/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(1 + 1/(a + b*x))/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.07

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{2\,\log\left(bx+a+1\right)\log\left(bx+a\right)-\log\left(bx+a\right)^{2}}{2\,b}-\frac{\log\left(bx+a+1\right)\log\left(bx+a\right)+\mathrm{Li}_{2}(-bx-a)}{b}$$

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*log(b*x + a + 1)*log(b*x + a) - log(b*x + a)^2)/b - (log(b*x + a + 1
)*log(b*x + a) + dilog(-b*x - a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(14) = 28$.

Time = 3.75 (sec) , antiderivative size = 320, normalized size of antiderivative = 21.33

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

$$= \frac{1}{2}\left((a+1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a|}\right)}{b^4} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a}-1\right|\right)}{b^4} - \frac{1}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)} - \frac{\log\left(\frac{1}{a-\frac{\left(\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}+\cdots\right)\right.}{\cdots}}\right.}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)\cdots}\right.$$

[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")

[Out] 1/2*((a + 1)*b - a*b)^2*(log(abs(b*x + a + 1)/abs(b*x + a))/b^4 - log(abs((b*x + a + 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a + 1)/(b*x + a) - 1)) -

```
log(1/(a - ((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1)*b/(b*
x + a) - b) + 1)*a/(a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1
)*b/(b*x + a) - b)) - a - 1)*b/((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/
((b*x + a + 1)*b/(b*x + a) - b) + 1)*b/(a - ((b*x + a + 1)*a/(b*x + a) - a
- 1)*b/((b*x + a + 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a + 1)/(b*x
 + a) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\mathrm{polylog}\left(2,-\frac{1}{a+b\,x}\right)}{b}$$

```
[In] int(log(1/(a + b*x) + 1)/(a + b*x),x)
[Out] polylog(2, -1/(a + b*x))/b
```

3.108 $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,1/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2497}

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

```
[In] Int[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
[Out] PolyLog[2, (a + b*x)^(-1)]/b
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral}=\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

```
[In] Integrate[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
[Out] PolyLog[2, (a + b*x)^(-1)]/b
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
default	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
risch	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
parts	$\frac{\ln\left(1-\frac{1}{bx+a}\right)\ln(bx+a)}{b}-\frac{-\frac{\ln(bx+a)^{2}}{2}-\operatorname{dilog}(bx+a)}{b}$	48

```
[In] int(ln(1-1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*dilog(1-1/(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\operatorname{Li}_2\left(-\frac{bx+a-1}{bx+a}+1\right)}{b}$$

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")
[Out] dilog(-(b*x + a - 1)/(b*x + a) + 1)/b
```

Sympy [F]

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

```
[In] integrate(ln(1-1/(b*x+a))/(b*x+a),x)
[Out] Integral(log(1 - 1/(a + b*x))/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.54

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = -\frac{\log(bx+a)^2 - 2\log(bx+a)\log(bx+a-1)}{2b} - \frac{\log(bx+a)\log(-bx-a+1) + \mathrm{Li}_2(bx+a)}{b}$$

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")
[Out] -1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*l
og(-b*x - a + 1) + dilog(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(12) = 24$.

Time = 3.57 (sec) , antiderivative size = 322, normalized size of antiderivative = 24.77

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = -\frac{1}{2}\left((a-1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a-1|}{|bx+a|}\right)}{b^4} - \frac{\log\left(\left|\frac{bx+a-1}{bx+a}-1\right|\right)}{b^4} - \frac{1}{b^4\left(\frac{bx+a-1}{bx+a}-1\right)} - \frac{\log\left(-\frac{1}{\cdots}\right)}{b^4\left(\frac{bx+a-1}{bx+a}\cdots\right.}\right.$$

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((a - 1)*b - a*b)^2*(log(abs(b*x + a - 1)/abs(b*x + a))/b^4 - log(abs(
(b*x + a - 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a - 1)/(b*x + a) - 1)) -
```

```
 log(-1/(a - ((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a - 1)*b/(
b*x + a) - b) - 1)*a/(a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a -
 1)*b/(b*x + a) - b)) - a + 1)*b/((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*
b/((b*x + a - 1)*b/(b*x + a) - b) - 1)*b/(a - ((b*x + a - 1)*a/(b*x + a) -
a + 1)*b/((b*x + a - 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a - 1)/(b
*x + a) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\mathrm{polylog}\left(2,\frac{1}{a+b\,x}\right)}{b}$$

```
[In] int(log(1 - 1/(a + b*x))/(a + b*x),x)
```

```
[Out] polylog(2, 1/(a + b*x))/b
```

3.109 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \operatorname{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*e*x + a*e)/(d*x +
c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(
2*a*b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 13.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.110 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\text{integral} = \int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] g*(Integral(a/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(b
*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.111 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c)
)), x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A*a + A*b*x + B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*x*
log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 9.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.112 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 50

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)g^2}$$

```
[Out] e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)/g^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2550, 2346, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B*(b*
c - a*d)*g^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2} \\
&= \frac{e\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2} \\
&= \frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)g^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2-aBdg^2}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(b*B*c
*g^2 - a*B*d*g^2)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{e\,e^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61
default	$\frac{e\,e^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61
risch	$\frac{e\,e^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
[Out] e/(a*d-b*c)/g^2/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{\frac{A}{B}}\,\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bbc-Bad)g^2}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")
[Out] e*e^(A/B)*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e))/((B*b*c - B*a*d)*g
^2)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b
*e*x/(c + d*x)) + 2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x
**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.113 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{be^2e^{\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B(bc-ad)^2g^3} - \frac{dee^{A/B}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)^2g^3}$$

```
[Out] b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3-d*
e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2550, 2395, 2346, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{be^2e^{\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{Bg^3(bc-ad)^2} - \frac{dee^{A/B}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^3(bc-ad)^2}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
B])/(B*(b*c - a*d)^2*g^3) - (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a +
 b*x))/(c + d*x)])/B)])/(B*(b*c - a*d)^2*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex))}-\frac{d}{x^2(A+B\log(ex))}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{(de)\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3} + \frac{(be^2)\,\text{Subst}\left(\int \frac{e^{-2x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3}$$

$$= \frac{be^2e^{\frac{2A}{B}}\mathrm{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B(bc-ad)^2g^3} - \frac{dee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)^2g^3}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$
$$= \frac{ee^{A/B}\left(bee^{A/B}\,\mathrm{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right) - d\,\mathrm{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)\right)}{B(bc-ad)^2g^3}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (e*E^(A/B)*(b*e*E^(A/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/B] - d*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)]))/(B*(
b*c - a*d)^2*g^3)
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{Ei}_1\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(ad-cb)^2g^3}$	117
default	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{Ei}_1\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(ad-cb)^2g^3}$	117
risch	$-\frac{e^2b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{Ei}_1\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{(ad-cb)^2g^3B}+\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)^2g^3B}$	131

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] e/(a*d-b*c)^2/g^3*(d/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)-b
*e/B*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{be^2e^{\left(\frac{2A}{B}\right)}\operatorname{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2A}{B}\right)}}{b^2e^2x^2+2\,abe^2x+a^2e^2}\right)-dee^{\frac{A}{B}}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3}$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")
```

```
[Out] (b*e^2*e^(2*A/B)*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2
*x^2 + 2*a*b*e^2*x + a^2*e^2)) - d*e*e^(A/B)*log_integral((d*x + c)*e^(-A/B
)/(b*e*x + a*e)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}}{g^3}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b*e
*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*
*3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.114 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \operatorname{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}, x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b*e*x + a*e)/(d*x
+ c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 9.53

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) -
 a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2
+ 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*
c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B
+ (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.115 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e
*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.89 (sec) , antiderivative size = 274, normalized size of antiderivative = 9.13

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$

$$-\frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2b^2cx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{3b^2dx^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,d\right.}{B\,(ad-bc)}$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c +
 d*x))) - g*(Integral(a**2*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(2*a*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + In
tegral(2*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integr
al(3*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integra
l(4*a*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c
))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.57

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b
*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) -
 (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))
*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.116 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.81

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{c+dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{d\int \frac{1}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg\,(ad-bc)}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] (c + d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(a + b*x
)/(c + d*x))) - d*Integral(1/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x)/(B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x) - (d
*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c
) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.117 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)g^2} - \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

```
[Out] -e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)/g^2+(-d*x-c)/
B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2550, 2343, 2346, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^2(bc-ad)} - \frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] -((e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B^2
*(b*c - a*d)*g^2)) - (c + d*x)/(B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}$$

$$= -\frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)g^2}$$

$$= -\frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{e\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)g^2}$$

$$= -\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)g^2} - \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)+\frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}}{B^2(-bc+ad)g^2}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)] + (B*(c
 + d*x))/((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(B^2*(-(b*c) + a
*d)*g^2)
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}-\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^2B^2(ad-cb)}$	113
derivativedivides	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132
default	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)/B/(b*x+a)*(d*x+c)/g^2/(A+B*ln(e*(b*x+a)/(d*x+c)))-1/g^2/B^2*e/(
a*d-b*c)*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{Bdx+Bc+\left((Bbex+Bae)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Abex+Aae)e^{\frac{A}{B}}\right)\log\_integral\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(AB^2b^2c-AB^2abd)g^2x+(AB^2abc-AB^2a^2d)g^2+((B^3b^2c-B^3abd)g^2x+(B^3abc-B^3a^2d)g^2)\log\left(\right.}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*d*x + B*c + ((B*b*e*x + B*a*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (
A*b*e*x + A*a*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e)))/(
(A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((B^3
*b^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*log((b*e*x + a*e)/
(d*x + c)))
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

$$= \frac{c+dx}{ABa^2dg^2-ABabcg^2+ABabdg^2x-ABb^2cg^2x+(B^2a^2dg^2-B^2abcg^2+B^2abdg^2x-B^2b^2cg^2x)\log\left(\frac{e(a}{c}\right.} - \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (c + d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*c
*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b*
*2*c*g**2*x)*log(e*(a + b*x)/(c + d*x))) - Integral(1/(A*a**2 + 2*A*a*b*x +
 A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 2*B*a*b*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c
 + d*x))), x)/(B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log
(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*log
(e))*B^2)*x + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)
*log(b*x + a) - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^
2)*log(d*x + c)) + integrate(-1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2*b^
2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*x +
(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) - (B^2*b^2*g
^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.118 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 212

$$\begin{aligned}\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx &= -\frac{2be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2(bc-ad)^2g^3}\\
&\quad+\frac{dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)^2g^3}\\
&\quad+\frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\\
&\quad-\frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\end{aligned}$$

```
[Out] -2*b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g
^3+d*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g^3+d*(
d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))-b*(d*x+c)^2/B
/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2550, 2395, 2343, 2346, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$=-\frac{2be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2g^3(bc-ad)^2}+\frac{dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}+\frac{d(c+dx)}{Bg^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/B])/(B^2*(b*c - a*d)^2*g^3) + (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e
*(a + b*x))/(c + d*x)])/B)])/(B^2*(b*c - a*d)^2*g^3) + (d*(c + d*x))/(B*(b*
c - a*d)^2*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])) - (b*(c + d*
x)^2)/(B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex))^2} - \frac{d}{x^2(A+B\log(ex))^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{(2b)\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3} + \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3}$$

$$= \frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} + \frac{(de)\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^2g^3} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{e^{-2x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^2g^3}$$

$$= -\frac{2be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{-2be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right) + dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right) - \frac{B(bc-\ldots}{(a+bx)^2\left(A+\ldots\right.}}{B^2(bc-ad)^2g^3}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/B] + d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)
] - (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/(B^2*(b*c - a*d)^2*g^3)
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

method	result
risch	$\frac{dx+c}{(ad-cb)B(bx+a)^2g^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}+\frac{2e^2b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{Ei}_1\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{g^3B^2(ad-cb)^2}-\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\right.\right.}{g^3B^2(ad-cb)^2}$
derivativedivides	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2B\left(A\right.}\right.\right.}{(ad-cb)^2g^3}$
default	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2B\left(A\right.}\right.\right.}{(ad-cb)^2g^3}$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)/B/(b*x+a)^2*(d*x+c)/g^3/(A+B*ln(e*(b*x+a)/(d*x+c)))+2*e^2/g^3/B
^2/(a*d-b*c)^2*b*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B)-e
/g^3/B^2/(a*d-b*c)^2*d*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(210) = 420.

Time = 0.27 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.69

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx =$$

$$-\frac{Bbc^2-Bacd+(Bbcd-Bad^2)x-\left((Bb^2dex^2+2\,Babdex+Ba^2de)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Ab^2dex^2+2\,A\right.}{(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2)g^3x^2+2\,(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2)g^3x+}$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x - ((B*b^2*d*e*x^2 + 2*B*a*b*d*e
*x + B*a^2*d*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (A*b^2*d*e*x^2 + 2*A
*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a
*e)) + 2*((B*b^3*e^2*x^2 + 2*B*a*b^2*e^2*x + B*a^2*b*e^2)*e^(2*A/B)*log((b*
e*x + a*e)/(d*x + c)) + (A*b^3*e^2*x^2 + 2*A*a*b^2*e^2*x + A*a^2*b*e^2)*e^(
2*A/B))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2*x^2 + 2*
a*b*e^2*x + a^2*e^2)))/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*
d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*
g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^3 + ((B^3
```

```
*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*b^3*c^2 -
2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c
*d + B^3*a^4*d^2)*g^3)*log((b*e*x + a*e)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3
*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*d
*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3*l
og(e) - a^2*b*d*g^3*log(e))*B^2)*x + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2
*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x
 + a) - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*
B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate((b*d*x + 2
*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a*b^3*d*g
^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3*log(e) -
 a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3*c*g^
3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3
)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + ((b^4*c*g^3 -
a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2
*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) -
 ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x
^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)
*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.119 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 182

$$\begin{aligned}\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx &= \frac{2B(bc-ad)^4g^4x}{5d^4}-\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3}\\ &\quad+\frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}-\frac{B(bc-ad)g^4(a+bx)^4}{10bd}\\ &\quad+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b}-\frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}\end{aligned}$$

```
[Out] 2/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+2/15*B*
(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g^
4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/5*B*(-a*d+b*c)^5*g^4*ln(d*x
+c)/b/d^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\begin{aligned}\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx &= \frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{2Bg^4x(bc-ad)^4}{5d^4}\\ &\quad-\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}+\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)}{10bd}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*
d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*
(a + b*x)^4)/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]))/(5*b) - (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{(2B(bc-ad))\int\frac{(ag+bgx)^5}{(a+bx)(c+dx)}\,dx}{5bg}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{(2B(bc-ad)g^4)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b}\\
&\quad - \frac{(2B(bc-ad)g^4)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2B(bc-ad)^4 g^4 x}{5d^4} - \frac{B(bc-ad)^3 g^4 (a+bx)^2}{5bd^3} \\
&\quad + \frac{2B(bc-ad)^2 g^4 (a+bx)^3}{15bd^2} - \frac{B(bc-ad) g^4 (a+bx)^4}{10bd} \\
&\quad + \frac{g^4 (a+bx)^5 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{2B(bc-ad)^5 g^4 \log(c+dx)}{5bd^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$
\int (ag+bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx
= \frac{g^4 \left((a+bx)^5 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) + \frac{B(bc-ad)\left(12bd(bc-ad)^3 x - 6d^2(bc-ad)^2(a+bx)^2 + 4d^3(bc-ad)(a+bx)^3 - 3d^4(a+bx)^4 - 12(b\right.}{6d^5}}{5b}
$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(b*c - a*d)
*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a
*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(6*d^
5)))/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(170) = 340$.

Time = 1.20 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.45

method	result
risch	$-\frac{4g^4bBa^3cx}{d} + \frac{4g^4b^2Ba^2c^2x}{d^2} - \frac{2g^4b^3Bac^3x}{d^3} - \frac{4g^4b^2B\ln(dx+c)a^2c^3}{d^3} + \frac{2g^4b^3B\ln(dx+c)ac^4}{d^4} + \frac{4g^4bB\ln(dx+}{d^2}$
parallelrisch	$\frac{6Bx^5\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^5cd^5g^4 + 6B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^5c^6g^4 + 12B\ln(bx+a)a^6cd^5g^4 - 12B\ln(bx+a)ab^5c^6g^4 + 4Bx^3ab^5c^3}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*g^4/d*b*B*a^3*c*x+4*g^4/d^2*b^2*B*a^2*c^2*x-2*g^4/d^3*b^3*B*a*c^3*x-4*g^
4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+2*g^4/d^4*b^3*B*ln(d*x+c)*a*c^4+4*g^4/d^2*b*B
*ln(d*x+c)*a^3*c^2+1/5*(b*x+a)^5*g^4*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)-2/3*g^4/
d*b^3*B*a*c*x^3-2*g^4/d*b^2*B*a^2*c*x^2+g^4/d^2*b^3*B*a*c^2*x^2+2/15*g^4/d^
```

```
2*b^4*B*c^2*x^3+8/15*g^4*b^2*B*a^2*x^3+2*g^4*b*A*a^3*x^2+6/5*g^4*b*B*a^3*x^
2-1/5*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+8/5*g^4*B*a^4*x+2/5*g^4/d^4*b^4*B*c
^4*x-2*g^4/d*B*ln(d*x+c)*a^4*c-2/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+1/5*g^4*b^4*
A*x^5+g^4*b^3*A*a*x^4+1/10*g^4*b^3*B*a*x^4-1/10*g^4/d*b^4*B*c*x^4+2*g^4*b^2
*A*a^2*x^3+2/5*g^4/b*B*ln(d*x+c)*a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(170) = 340$.

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{6\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4\log(bx+a) - 3\,(Bb^5cd^4 - (10\,A+B)ab^4d^5)g^4x^4 + 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 + }{}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
- (10*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + (15*
A + 4*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 - 2*(5*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A + 8*B)*
a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^
2 + 2*c*d*x + c^2)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 3.61 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{Ab^4g^4x^5}{5}$$
$$+\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}$$
$$-\frac{2Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{12Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-1\cdots}{\cdots}\right.}{5d\cdots}$$
$$+x^4\left(Aab^3g^4+\frac{Bab^3g^4}{10}-\frac{Bb^4cg^4}{10d}\right)+x^3\cdot\left(2Aa^2b^2g^4+\frac{8Ba^2b^2g^4}{15}-\frac{2Bab^3cg^4}{3d}+\frac{2Bb^4c^2g^4}{15d^2}\right)$$
$$+x^2\cdot\left(2Aa^3bg^4+\frac{6Ba^3bg^4}{5}-\frac{2Ba^2b^2cg^4}{d}+\frac{Bab^3c^2g^4}{d^2}-\frac{Bb^4c^3g^4}{5d^3}\right)$$
$$+x\left(Aa^4g^4+\frac{8Ba^4g^4}{5}-\frac{4Ba^3bcg^4}{d}+\frac{4Ba^2b^2c^2g^4}{d^2}-\frac{2Bab^3c^3g^4}{d^3}+\frac{2Bb^4c^4g^4}{5d^4}\right)$$
$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) - 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/10 - B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 + 8*B*a**2*b**2*g**4/15 - 2*B*a*b**3*c*g
**4/(3*d) + 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 + 6*B*a**
3*b*g**4/5 - 2*B*a**2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/d**2 - B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 + 8*B*a**4*g**4/5 - 4*B*a**3*b*c*g**4/d +
4*B*a**2*b**2*c**2*g**4/d**2 - 2*B*a*b**3*c**3*g**4/d**3 + 2*B*b**4*c**4*g*
```

```
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(a + b*x)**2/(c + d*x)
**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(170) = 340$.

Time = 0.22 (sec) , antiderivative size = 885, normalized size of antiderivative = 4.86

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$$
$$= \frac{1}{5} Ab^4g^4x^5 + Aab^3g^4x^4 + 2\,Aa^2b^2g^4x^3 + 2\,Aa^3bg^4x^2$$
$$+ \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx -}{d}\right.$$
$$+2\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log(}{d}\right.$$
$$+2\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(}{d}\right.$$
$$+\frac{1}{3}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log(bx+a)}{b^4} + \frac{6\,c^4\log}{}\right.$$
$$+\frac{1}{30}\left(6\,x^5\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{12\,a^5\log(bx+a)}{b^5} - \frac{12\,c}{}\right.$$
$$+ Aa^4g^4x$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*
d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*
a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^
3*b*g^4 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*
x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
```

```
) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(170) = 340$.

Time = 59.29 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.69

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{1}{5}Ab^4g^4x^5 + \frac{2\,Ba^5g^4\log(bx+a)}{5\,b} - \frac{(Bb^4cg^4 - 10\,Aab^3dg^4 - Bab^3dg^4)x^4}{10\,d}$$
$$+ \frac{2\,(Bb^4c^2g^4 - 5\,Bab^3cdg^4 + 15\,Aa^2b^2d^2g^4 + 4\,Ba^2b^2d^2g^4)x^3}{15\,d^2}$$
$$+ \frac{1}{5}\left(Bb^4g^4x^5 + 5\,Bab^3g^4x^4 + 10\,Ba^2b^2g^4x^3 + 10\,Ba^3bg^4x^2 + 5\,Ba^4g^4x\right)\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right)$$
$$- \frac{(Bb^4c^3g^4 - 5\,Bab^3c^2dg^4 + 10\,Ba^2b^2cd^2g^4 - 10\,Aa^3bd^3g^4 - 6\,Ba^3bd^3g^4)x^2}{5\,d^3}$$
$$+ \frac{(2\,Bb^4c^4g^4 - 10\,Bab^3c^3dg^4 + 20\,Ba^2b^2c^2d^2g^4 - 20\,Ba^3bcd^3g^4 + 5\,Aa^4d^4g^4 + 8\,Ba^4d^4g^4)x}{5\,d^4}$$
$$- \frac{2\,(Bb^4c^5g^4 - 5\,Bab^3c^4dg^4 + 10\,Ba^2b^2c^3d^2g^4 - 10\,Ba^3bc^2d^3g^4 + 5\,Ba^4cd^4g^4)\log(-dx-c)}{5\,d^5}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + 2/5*B*a^5*g^4*log(b*x + a)/b - 1/10*(B*b^4*c*g^4 - 10*A
*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d*g
^4 + 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^4*x
^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*B*a^
4*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) - 1
/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 - 10*A*a^3
*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 + 1/5*(2*B*b^4*c^4*g^4 - 10*B*a*b^3
*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 + 5*A*a^4*d^4*
g^4 + 8*B*a^4*d^4*g^4)*x/d^4 - 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*d*g^4 + 1
0*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*g^4)*log(-
d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 1025, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= x^2\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c}{d}\right.}{10\,b\,d}\right.$$

$$\left.+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}\right.$$

$$\left.-\frac{a\,c\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+4\,B\,a\,d-4\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$+\frac{a\,c\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d -
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 4*B*a*d - 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*
d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(a + b*x)^2)/(c + d*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(2*B*b^4*c^5*g^4
 + 10*B*a^4*c*d^4*g^4 - 20*B*a^3*b*c^2*d^3*g^4 + 20*B*a^2*b^2*c^3*d^2*g^4 -
 10*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (2*B*a^5*g^4*log(a +
b*x))/(5*b)
```

3.120 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 151

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = -\frac{B(bc-ad)^3g^3x}{2d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}$$

```
[Out] -1/2*B*(-a*d+b*c)^3*g^3*x/d^3+1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/6*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^
2))/b+1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4} - \frac{Bg^3x(bc-ad)^3}{2d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)}{6bd}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)^3*g^3*x)/d^3 + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) + (g^3*(a + b*x)^4*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/
(2*b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}-\frac{(B(bc-ad))\int\frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{2bg}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}-\frac{(B(bc-ad)g^3)\int\frac{(a+bx)^3}{c+dx}\,dx}{2b}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}\\
&\quad -\frac{(B(bc-ad)g^3)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{2b}\\
&= -\frac{B(bc-ad)^3g^3x}{2d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{6bd}\\
&\quad +\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}\right)}{4b}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(b*c - a*d)
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4)))/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(141) = 282.

Time = 0.88 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b}+\frac{g^3b^3A\,x^4}{4}+g^3b^2Aa\,x^3+\frac{g^3b^2Ba\,x^3}{6}-\frac{g^3b^3Bc\,x^3}{6d}+\frac{3g^3bA\,a^2x^2}{2}+\frac{3g^3bB\,a^2}{4}$
parallelrisch	$\frac{-24B\ln(bx+a)a^3bc\,d^3g^3+12B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^3bc\,d^3g^3-18B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2b^2c^2d^2g^3+12B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a\,b^3c^3d\,g^3}{}$
parts	$\frac{A\,g^3(bx+a)^4}{4b}-\frac{B\,g^3\left(\left(\left(-\frac{(dx+c)^4\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^4}-\frac{(-ad+cb)(dx}{2b^2}\right.\right.\right.\right.}{}$
derivativedivides	$-\frac{\frac{A\,g^3\left(-\frac{b^3(dx+c)^4}{4}-\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(dx+c)-\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)^2}{2}-b^2(ad-cb)(dx+c)^3\right)}{d^3}+\frac{B\,g^3\left(\right.}{}}{}$
default	$-\frac{\frac{A\,g^3\left(-\frac{b^3(dx+c)^4}{4}-\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(dx+c)-\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)^2}{2}-b^2(ad-cb)(dx+c)^3\right)}{d^3}+\frac{B\,g^3\left(\right.}{}}{}$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/4*g^3*b^3*A*x^4+g^3*b^2*A
*a*x^3+1/6*g^3*b^2*B*a*x^3-1/6*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/4*g^
```

```
3*b*B*a^2*x^2-g^3*b^2/d*B*a*c*x^2+1/4*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*x+1/2
*g^3/b*B*ln(d*x+c)*a^4-2*g^3/d*B*ln(d*x+c)*a^3*c+3*g^3*b/d^2*B*ln(d*x+c)*a^
2*c^2-2*g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/2*g^3*b^3/d^4*B*ln(d*x+c)*c^4+3/2*g
^3*B*a^3*x-3*g^3*b/d*B*a^2*c*x+2*g^3*b^2/d^2*B*a*c^2*x-1/2*g^3*b^3/d^3*B*c^
3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(141) = 282.

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.26

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{3\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3\log\left(bx+a\right)-2\left(Bb^4cd^3-(6\,A+B)ab^3d^4\right)g^3x^3+3\left(Bb^4c^2d^2-4\,Bab^3cd^3+3\,(2$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (6*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(2*A
 + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 - (2*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.04 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}$$
$$- \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)\ldots}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2\ldots}\right.}{2d^4}$$
$$+ x^3\left(Aab^2g^3+\frac{Bab^2g^3}{6}-\frac{Bb^3cg^3}{6d}\right) + x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{4}-\frac{Bab^2cg^3}{d}+\frac{Bb^3c^2g^3}{4d^2}\right)$$
$$+ x\left(Aa^3g^3+\frac{3Ba^3g^3}{2}-\frac{3Ba^2bcg^3}{d}+\frac{2Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{2d^3}\right)$$
$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/6 - B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*b*
g**3/4 - B*a*b**2*c*g**3/d + B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 +
3*B*a**3*g**3/2 - 3*B*a**2*b*c*g**3/d + 2*B*a*b**2*c**2*g**3/d**2 - B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(141) = 282.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.28

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx+\right.}{d}\right.$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log\left(\right.}{d^2}\right.$$
$$+\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx\right.}{d^3}\right.$$
$$+\frac{1}{12}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{6\,a^4\log\left(bx+a\right)}{b^4}+\frac{6\,c^4\log}{}\right.$$
$$+\,Aa^3g^3x$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b
^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e
*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(141) = 282$.

Time = 10.23 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.35

$$\begin{aligned}
&\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx \\
&= \frac{1}{4}\,Ab^3g^3x^4 + \frac{Ba^4g^3\log\left(bx+a\right)}{2\,b} - \frac{(Bb^3cg^3 - 6\,Aab^2dg^3 - Bab^2dg^3)x^3}{6\,d} \\
&\quad + \frac{1}{4}\left(Bb^3g^3x^4 + 4\,Bab^2g^3x^3 + 6\,Ba^2bg^3x^2 + 4\,Ba^3g^3x\right)\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right) \\
&\quad + \frac{(Bb^3c^2g^3 - 4\,Bab^2cdg^3 + 6\,Aa^2bd^2g^3 + 3\,Ba^2bd^2g^3)x^2}{4\,d^2} \\
&\quad - \frac{(Bb^3c^3g^3 - 4\,Bab^2c^2dg^3 + 6\,Ba^2bcd^2g^3 - 2\,Aa^3d^3g^3 - 3\,Ba^3d^3g^3)x}{2\,d^3} \\
&\quad + \frac{(Bb^3c^4g^3 - 4\,Bab^2c^3dg^3 + 6\,Ba^2bc^2d^2g^3 - 4\,Ba^3cd^3g^3)\log\left(dx+c\right)}{2\,d^4}
\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + 1/2*B*a^4*g^3*log(b*x + a)/b - 1/6*(B*b^3*c*g^3 - 6*A*a
*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^3
+ 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d
^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 + 6*A*a^2
*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*B*a*b^2*c^
2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/d^3 +
1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a^3*
c*d^3*g^3)*log(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}\right.$$
$$\left.-\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+x\left(\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+A}{2\,b\,d}\right.$$
$$+\frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{d}$$
$$\left.-\frac{a\,c\,\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right)$$
$$+\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}$$
$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*
a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
```

```
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c + 3*B*a*d - 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.121 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{2B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b} - \frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

```
[Out] 2/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b
/d^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b} - \frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3} + \frac{2Bg^2x(bc-ad)^2}{3d^2} - \frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d)
 + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*
(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{(2B(bc-ad))\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{3bg}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{(2B(bc-ad)g^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{(2B(bc-ad)g^2)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= \frac{2B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{3bd}\\
&\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}\right)}{3b}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) + a
*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/d^3))/(3*b)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b}+\frac{g^2b^2A\,x^3}{3}+g^2bAa\,x^2+\frac{g^2bBa\,x^2}{3}-\frac{g^2b^2Bc\,x^2}{3d}+g^2A\,a^2x+\frac{2g^2B\ln(dx+c)}{3b}$
parts	$\frac{A\,g^2(bx+a)^3}{3b}-\frac{B\,g^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right)\right.}{}$
parallelrisch	$\frac{-12B\ln(bx+a)a^2bc\,d^2g^2+12B\ln(bx+a)a\,b^2c^2d\,g^2+6B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a\,b^2d^3g^2+6Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2b\,d^3g^2+10Ba\,c}{}$
derivativedivides	$-\frac{\frac{A\,g^2\left(-\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)-\frac{b^2(dx+c)^3}{3}-b(ad-cb)(dx+c)^2\right)}{d^2}+\frac{B\,g^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb\right.\right.}{}}{}$
default	$-\frac{\frac{A\,g^2\left(-\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)-\frac{b^2(dx+c)^3}{3}-b(ad-cb)(dx+c)^2\right)}{d^2}+\frac{B\,g^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb\right.\right.}{}}{}$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*b^2*A*x^3+g^2*b*A*a
*x^2+1/3*g^2*b*B*a*x^2-1/3*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+2/3*g^2/b*B*ln(d*x
+c)*a^3-2*g^2/d*B*ln(d*x+c)*a^2*c+2*g^2*b/d^2*B*ln(d*x+c)*a*c^2-2/3*g^2*b^2
```

```
/d^3*B*ln(d*x+c)*c^3+4/3*g^2*B*a^2*x-2*g^2*b/d*B*a*c*x+2/3*g^2*b^2/d^2*B*c^
2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{Ab^3d^3g^2x^3 + 2\,Ba^3d^3g^2\log(bx+a) - (Bb^3cd^2 - (3\,A+B)ab^2d^3)g^2x^2 + (2\,Bb^3c^2d - 6\,Bab^2cd^2 + (3\,A+}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (3*A
 + B)*a*b^2*d^3)*g^2*x^2 + (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + (3*A + 4*B)*a
^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.39 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx &= \frac{Ab^2g^2x^3}{3} + \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b} \\ &- \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3a^2}{}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3} \\ &+ x^2\left(Aabg^2+\frac{Babg^2}{3}-\frac{Bb^2cg^2}{3d}\right) + x\left(Aa^2g^2+\frac{4Ba^2g^2}{3}-\frac{2Babcg^2}{d}+\frac{2Bb^2c^2g^2}{3d^2}\right) \\ &+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\end{aligned}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 + 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
*2))/(3*b) - 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 +
B*a*b*g**2/3 - B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 + 4*B*a**2*g**2/3 - 2*
B*a*b*c*g**2/d + 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(112) = 224$.

Time = 0.36 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.64

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{1}{3} Ab^2g^2x^3 + Aabg^2x^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx+\cdots}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log(dx\cdots}{d^2}\right.$$
$$+\frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(\cdots}{d^3}\right.$$
$$+ Aa^2g^2x$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b^2*e*x^2
/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d
^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2
- 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(112) = 224$.

Time = 1.81 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.05

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$= \frac{1}{3}Ab^2g^2x^3 + \frac{2Ba^3g^2\log(bx+a)}{3b} - \frac{(Bb^2cg^2-3Aabdg^2-Babdg^2)x^2}{3d} + \frac{1}{3}\left(Bb^2g^2x^3+3Babg^2x^2+3Ba^2g^2x\right)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{(2Bb^2c^2g^2-6Babcdg^2+3Aa^2d^2g^2+4Ba^2d^2g^2)x}{3d^2} - \frac{2(Bb^2c^3g^2-3Babc^2dg^2+3Ba^2cd^2g^2)\log(-dx-c)}{3d^3}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + 2/3*B*a^3*g^2*log(b*x + a)/b - 1/3*(B*b^2*c*g^2 - 3*A*a
*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*B*
a^2*g^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) +
 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2*g^2
)*x/d^2 - 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*log(-
d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$-\frac{\ln(c+d\,x)\,\left(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a
*d - 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a +
b*x)^2)/(c + d*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 + (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.122 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=-\frac{B(bc-ad)gx}{d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}$$

```
[Out] -B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b+B*(-a
*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}-\frac{Bgx(bc-ad)}{d}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}-\frac{(B(bc-ad))\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{bg}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}\,dx}{b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{b}\\
&= -\frac{B(bc-ad)gx}{d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(-(b*c) + a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{2gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{d^2}+\frac{B\,a^2g\ln(-bx-a)}{b}+$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab\,d^2g+2Axab\,d^2g+2B\ln(bx+a)a^2d^2g-4B\ln(bx+a)abcdg+}{}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{Bg\left(\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-\frac{dx+c}{b}+\right.\right.\right.}{}$
derivativedivides	$-\frac{Ag\left(-\frac{b(dx+c)^2}{2}-(ad-cb)(dx+c)\right)}{d}+\frac{Bg\left(\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-\right.\right.\right.}{}$
default	$-\frac{Ag\left(-\frac{b(dx+c)^2}{2}-(ad-cb)(dx+c)\right)}{d}+\frac{Bg\left(\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-\right.\right.\right.}{}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*g*b*A*x^2+g*A*a*x-2*g/d*B
*ln(d*x+c)*a*c+g*b/d^2*B*ln(d*x+c)*c^2+B*a^2*g/b*ln(-b*x-a)+g*B*a*x-g*b/d*B
*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+2\,Ba^2d^2g\log\left(bx+a\right)-2\left(Bb^2cd-(A+B)abd^2\right)gx+2\left(Bb^2c^2-2\,Babcd\right)g\log\left(dx+c\right)+(B}{2\,bd^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*log(b*x + a) - 2*(B*b^2*c*d - (A + B)*
a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
2 + 2*B*a*b*d^2*g*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.84 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}$$
$$-\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}$$
$$+x\left(Aag+Bag-\frac{Bbcg}{d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g + B*a*g - B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a + b*x
)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Abgx^2 + \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx\ \cdots}{d} \cdots + \frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log \cdots}{\cdots} \cdots + Aagx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)
/b - 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.65

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Abgx^2+\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{(Bbcg-Aadg-Badg)x}{d}+\frac{(Bbc^2g-2\,Bacdg)\log(dx+c)}{d^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/2*A*b*g*x^2 + B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A*a*d
*g - B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c+B\,a\,d-B\,b\,c)}{d} - \frac{A\,g\,(a\,d+b\,c)}{d}\right) + \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) + \frac{A\,b\,g\,x^2}{2} + \frac{B\,a^2\,g\,\ln(a+b\,x)}{b} - \frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c + B*a*d - B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(a + b*x)^2)/(c + d*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 + (B*a^
2*g*log(a + b*x))/b - (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.123 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg} + \frac{2B\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g+2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = \frac{2B\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]))/(b*g)) + (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg}+\frac{(2B(bc-ad))\int\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{bg}$$

$$=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg}+\frac{(2B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx,x,a+bx\right)}{b^2g}$$

$$=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg}-\frac{(2B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg} - \frac{(2B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg} + \frac{2B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx$$
$$= \frac{\log(a+bx)\left(A-B\log(a+bx)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A - B*Log[a + b*x] + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
B*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(82) = 164.

Time = 0.70 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right)}{b}-\frac{\left(\ln\right.}{}\right.}{g}$
derivativedivides	$-\frac{dA\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+\frac{dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right.}{b}\right.}{d}$
default	$-\frac{dA\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+\frac{dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right.}{b}\right.}{d}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{gb}+\frac{2B\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)ad}{gb(ad-cb)}-\frac{2B\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)c}{g(ad-cb)}+\frac{2B\ln}{}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] A/g*ln(b*x+a)/b+B/g*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2
)-(2*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*ln(
((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/b-(ln((a*d-b*c)/(d*x+c)+b)/(a*d-b*c)*l
n(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/(a*d-b*c)*ln((a*d-b*c)/(d*x+c)+b)^
2)*(-a*d+b*c)/b)
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="fricas
")
[Out] integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g),x)
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x*
*2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
+ d**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="maxima
")
[Out] -B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e
) + 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{bgx + ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{a\,g + b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x), x)
```

3.124 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 65

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{2B}{bg^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)/g^2/(
b*x+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2550, 2341}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}$$

$$= -\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)} + \frac{2B(bc-ad)\left(-\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x))) + (2*B*(b*c -
 a*d)*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

method	result
norman	$\frac{\frac{(A+2B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parts	$-\frac{A}{g^2(bx+a)b}+\frac{\frac{2Bx}{ag}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{-2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2-2B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3cd+2Aab^2d^2-2Ab^3cd+4Bab^2d^2-4Bb^3cd}{2g^2(bx+a)b^3d(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{bg^2(bx+a)}-\frac{-2B\ln(-bx-a)bdx+2B\ln(dx+c)bdx-2B\ln(-bx-a)ad+2B\ln(dx+c)ad+Aad-Abc+2Bad-}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{-\frac{d^2A}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
default	$-\frac{-\frac{d^2A}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A+2*B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/g*B*d/(a*d-b*c)*
x*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(A+2\,B)bc-(A+2\,B)ad+(Bbdx+Bbc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A + 2*B)*b*c - (A + 2*B)*a*d + (B*b*d*x + B*b*c)*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*
c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x} - \frac{2Bd\log\left(x + \frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{2Bd\log\left(x + \frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A-2B}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) - 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
+ 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A - 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2} + \frac{2}{b^2g^2x+abg^2} + \frac{2\,d\log(bx+a)}{(b^2c-abd)g^2} - \frac{2\,d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
+ c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*g^
2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log(d*x + c)/
((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(65) = 130.

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$$
$$= \left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{(b^2cg^2-abdg^2)(bgx+ag)bg}\right)-\frac{\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)}{(bgx+ag)bg}\right)$$
$$-\frac{A}{(bgx+ag)bg}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] (2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g^
2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) - log((b*x + a)^2*e/(d*x + c)^2)/((b*g*x
 + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A+2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}$$
$$-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + 2*B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(b
^2*g^2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*
g^2*(a*d - b*c))
```

3.125 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}$$

```
[Out] -1/2*B/b/g^3/(b*x+a)^2+B*d/b/(-a*d+b*c)/g^3/(b*x+a)+B*d^2*ln(b*x+a)/b/(-a*d
+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^3/(b*x+a)^2-B*d^2*ln(d
*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2} - \frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}+\frac{Bd}{bg^3(a+bx)(bc-ad)}-\frac{B}{2bg^3(a+bx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*B/(b*g^3*(a + b*x)^2) + (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*L
og[a + b*x])/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^2}\,dx}{bg}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}\\
&\quad+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{bg^3}\\
&= -\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}\\
&\quad-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\frac{B((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx))}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
[Out] -1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + (B*((b*c - a*d)*(-3*a*d + b*
(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d
*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

method	result
parallelrisch	$-\frac{2Bxab^4d^3-2Bxb^5cd^2-Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5d^3+Aa^2b^3d^3+Ab^5c^2d+3Ba^2b^3d^3+Bb^5c^2d-2Aab^4cd^2-4Bab^4cd^2+}{2g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2bg^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)abd^2x-4B\ln(-bx-a)abd^2x+2B\ln(dx}{2(a^2d^2-2ab}$
parts	$-\frac{A}{2g^3(bx+a)^2b}+\frac{\frac{(2Bad-Bbc)x}{ag(ad-cb)}+\frac{Bad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(a^2d^2-2abcd+b^2c^2)g}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{(3Bad-Bbc)bx^2}{2ga^2(ad-cb)}+\frac{Bd^2bx^2\ln\left(\frac{e(bx+}{(dx+}\right.}{2(a^2d^2-2abcd+b^2}}{g^2(bx+a)^2}$
norman	$\frac{\frac{(Aad-Abc+2Bad-Bbc)x}{ag(ad-cb)}+\frac{Bad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(a^2d^2-2abcd+b^2c^2)g}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{(Aad-Abc+3Bad-Bbc)bx^2}{2a^2g(ad-cb)}+\frac{Bd^2bx^2\ln\left(\frac{e(bx+}{(dx+}\right.}{2(a^2d^2-2abcd+b^2}}{g^2(bx+a)^2}$
derivativedivides	$-\frac{\frac{d^3A\left(\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{Bd^3}{g(ad-cb)(dx+c)}+\frac{3Bd^3}{2bg(dx+c)^2}-\frac{bBd^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+}\right.}{d^2}\right.}{2g(a^2d^2-2abcd+b^2}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
default	$-\frac{\frac{d^3A\left(\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{Bd^3}{g(ad-cb)(dx+c)}+\frac{3Bd^3}{2bg(dx+c)^2}-\frac{bBd^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+}\right.}{d^2}\right.}{2g(a^2d^2-2abcd+b^2}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*B*x*a*b^4*d^3-2*B*x*b^5*c*d^2-B*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)*b^5*d
^3+A*a^2*b^3*d^3+A*b^5*c^2*d+3*B*a^2*b^3*d^3+B*b^5*c^2*d-2*A*a*b^4*c*d^2-4*
B*a*b^4*c*d^2+B*ln(e*(b*x+a)^2/(d*x+c)^2)*b^5*c^2*d-2*B*x*ln(e*(b*x+a)^2/(d
*x+c)^2)*a*b^4*d^3-2*B*ln(e*(b*x+a)^2/(d*x+c)^2)*a*b^4*c*d^2)/g^3/(b*x+a)^2
/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{(A+B)b^2c^2-2\,(A+2\,B)abcd+(A+3\,B)a^2d^2-2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-Bb^2}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A + B)*b^2*c^2 - 2*(A + 2*B)*a*b*c*d + (A + 3*B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c
^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(122) = 244.

Time = 1.03 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.03

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}$$
$$-\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2}$$
$$+\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2}$$
$$+\frac{-Aad+Abc-3Bad+Bbc-2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\,(4a^2b^2dg^3-4ab^3cg^3)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(134) = 268.

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.22

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{}{d^2x^2}\right.}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}\right.$$
$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b
^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^
3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} + \frac{2\,Bbdx-Abc-Bbc+Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2*
log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*a
*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - B*b*c + A*a*d + 3*B*a*d)
/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a
^2*b^2*c*g^3 - a^3*b*d*g^3)
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{A\,a\,d-A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((A*a*d - A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c
))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*log((e*(a + b*x)^2)/(c +
d*x)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (2*B*d^2*atanh((b^3*c^2*g^3
- a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*g^3*(a*
d - b*c)^2)
```

3.126 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

```
[Out] -2/9*B/b/g^4/(b*x+a)^3+1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-2/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)^2/(d*x+c)^2))/b/g^4/(b*x+a)^3+2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3} + \frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} + \frac{Bd}{3bg^4(a+bx)^2(bc-ad)}-\frac{2B}{9bg^4(a+bx)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B)/(9*b*g^4*(a + b*x)^3) + (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) - (2
*B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (2*B*d^3*Log[a + b*x])/(3*b*(b*
c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*b*g^4*(a + b*
x)^3) + (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^3}\,dx}{3bg} \\
&= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4} \\
&= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3} \\
&\quad+\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)\,d}{3bg^4}
\end{aligned}$$

$$= -\frac{2B}{9bg^4(a+bx)^3} + \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} - \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3} + \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) + \frac{B\left(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+d\right.}{(bc-ad)^3}}{9bg^4(a+bx)^3}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(2*(b*c - a*d)^3 - 3*
d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)
^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^4*(a
 + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(168) = 336.

Time = 1.57 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln(dx+c)a\,b^2d^3x^2-}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-36Bxa\,b^6c\,d^3-18B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2b^5d^4-18B\ln\left(\frac{e(bx-}{(dx+}\right.}{}$
derivativedivides	$-\frac{\frac{d^4A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\,d^4\ln}{3g\left(a^3d^3-\right.}}{}}{}$
default	$-\frac{\frac{d^4A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\,d^4\ln}{3g\left(a^3d^3-\right.}}{}}{}$
parts	$-\frac{A}{3g^4(bx+a)^3b}+\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(6B\,a^2d^2-6Babcd+2B\,b^2c^2\right)x}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}+}{}$
norman	$\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(3A\,a^2d^2-6Aabcd+3A\,b^2c^2+6B\,a^2d^2-6Babcd+2B\,b^2c^2\right.}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*B/b/g^4/(b*x+a)^3*ln(e*(b*x+a)^2/(d*x+c)^2)-1/9*(-6*B*ln(-b*x-a)*b^3*d
^3*x^3+6*B*ln(d*x+c)*b^3*d^3*x^3-18*B*ln(-b*x-a)*a*b^2*d^3*x^2+18*B*ln(d*x+
c)*a*b^2*d^3*x^2-18*B*ln(-b*x-a)*a^2*b*d^3*x+18*B*ln(d*x+c)*a^2*b*d^3*x+6*B
*a*b^2*d^3*x^2-6*B*b^3*c*d^2*x^2-6*B*ln(-b*x-a)*a^3*d^3+6*B*ln(d*x+c)*a^3*d
^3+15*B*a^2*b*d^3*x-18*B*a*b^2*c*d^2*x+3*B*b^3*c^2*d*x+3*A*a^3*d^3-9*A*a^2*
b*c*d^2+9*A*a*b^2*c^2*d-3*A*b^3*c^3+11*B*a^3*d^3-18*B*a^2*b*c*d^2+9*B*a*b^2
*c^2*d-2*B*c^3*b^3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/g^4/(b*x+
a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(165) = 330$.

Time = 0.27 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.43

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{(3\,A+2\,B)b^3c^3-9\,(A+B)ab^2c^2d+9\,(A+2\,B)a^2bcd^2-(3\,A+11\,B)a^3d^3+6\,(Bb^3cd^2-Bab^2d^3)x^2-}{9\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^2
 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*
b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3
*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.

Time = 1.66 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} - \frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\left(ad-bc\right)^3} + \frac{2Bd^3\log\left(x+\frac{\frac{2Ba^4d^7}{(ad-bc)^3}-\frac{8Ba^3bcd^6}{(ad-bc)^3}+\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4+\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\left(ad-bc\right)^3} + \frac{-3Aa^2d^2+6Aabcd-3Ab^2c^2-11Ba^2d^2+7Babcd-2Bb^2c^2-6Bb^2}{9a^5bd^2g^4-18a^4b^2cdg^4+9a^3b^3c^2g^4+x^3\cdot\left(9a^2b^4d^2g^4-18ab^5cdg^4+9b^6c^2g^4\right)+x^2\cdot\left(27a^3b^3d^2g^4-54a^2\right.}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6
*B*b**2*d**2*x**2 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4
```

```
- 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g*
*4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4
- 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4
- 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(165) = 330$.

Time = 0.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{1}{9}\,B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)\right. - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(165) = 330$.

Time = 0.34 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,Bd^3\log(bx+a)}{3\,(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4)} + \frac{2\,Bd^3\log(dx+c)}{3\,(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4)} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)} - \frac{6\,Bb^2d^2x^2-3\,Bb^2cdx+15\,Babd^2x+3\,Ab^2c^2+2\,Bb^2c^2-6\,Aabcd-7\,\ldots}{9\,(b^6c^2g^4x^3-2\,ab^5cdg^4x^3+a^2b^4d^2g^4x^3+3\,ab^5c^2g^4x^2-6\,a^2b^4cdg^4x^2+3\,a^3b^3d^2g^4x^2+3\,a^2b^4c^2g^4x-6\,\ldots}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*
g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*
g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*e*x^2 + 2*a*b*e
*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b
*d^2*x + 3*A*b^2*c^2 + 2*B*b^2*c^2 - 6*A*a*b*c*d - 7*B*a*b*c*d + 3*A*a^2*d^
2 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*
x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 +
 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*
c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3} + \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) -
 (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(9*b*g^4*
(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3
) - (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x)
^2)/(c + d*x)^2))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*
(a + b*x)^3) + (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.127 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx =& -\frac{B}{8bg^5(a+bx)^4}+\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}\\&-\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)}\\&+\frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5}\end{aligned}$$

```
[Out] -1/8*B/b/g^5/(b*x+a)^4+1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/4*B*d^2/b/(-a*d
+b*c)^2/g^5/(b*x+a)^2+1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/2*B*d^4*ln(b*x
+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^5/(b*x+a)^4
-1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx =& -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}\\&+\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}\\&+\frac{Bd}{6bg^5(a+bx)^3(bc-ad)}-\frac{B}{8bg^5(a+bx)^4}\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/8*B/(b*g^5*(a + b*x)^4) + (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d
^2)/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a
 + b*x)) + (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(2*b*(
b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^4}\,dx}{2bg}$$

$$=-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$=-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}$$

$$+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\right.}{2bg^5}$$

$$= -\frac{B}{8bg^5(a+bx)^4} + \frac{Bd}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)} + \frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4} - \frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{6\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) + \frac{B\left(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(\right.}{(bc-ad)^4}}{24bg^5(a+bx)^4}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/24*(6*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(3*(b*c - a*d)^4 + 4
*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-
(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*
x)^4*Log[c + d*x]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(197) = 394.

Time = 2.71 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.53

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{-48Ba\,b^3c\,d^3x^2-72B\,a^2b^2c\,d^3x+24Ba\,b^3c^2d^2x-24A\,a^3bc\,d^3+36A\,a^2b^2c^2d^2-24Aa\,b^3c^3d-48B}{}$
derivativedivides	$-\frac{d^5A\left(-\frac{1}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}-\frac{b^2}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{3b}{2(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\right.}}{g^5}$
default	$-\frac{d^5A\left(-\frac{1}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}-\frac{b^2}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{3b}{2(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\right.}}{g^5}$
parts	$-\frac{A}{4g^5(bx+a)^4b}+\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{(4B\,a^3d^3}{2ga(a}$
parallelrisch	$\frac{6B\,x^4\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^6b^3c\,d^4+24B\,x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^7b^2c\,d^4+36B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^8bc\,d^4+24Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^9c\,d^4}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{(2A\,a^3d^3-6A\,a^2bc\,d^2+6Aa\,b^2c^2}{2ga(a}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
[Out] -1/4*B/b/g^5/(b*x+a)^4*ln(e*(b*x+a)^2/(d*x+c)^2)-1/24*(-48*B*a*b^3*c*d^3*x^
2-72*B*a^2*b^2*c*d^3*x+24*B*a*b^3*c^2*d^2*x-24*A*a^3*b*c*d^3+36*A*a^2*b^2*c
^2*d^2-24*A*a*b^3*c^3*d-48*B*ln(-b*x-a)*a*b^3*d^4*x^3+48*B*ln(d*x+c)*a*b^3*
d^4*x^3-72*B*ln(-b*x-a)*a^2*b^2*d^4*x^2+72*B*ln(d*x+c)*a^2*b^2*d^4*x^2-48*B
*ln(-b*x-a)*a^3*b*d^4*x+48*B*ln(d*x+c)*a^3*b*d^4*x+12*B*ln(d*x+c)*a^4*d^4+1
2*B*a*b^3*d^4*x^3-12*B*b^4*c*d^3*x^3+42*B*a^2*b^2*d^4*x^2+6*B*b^4*c^2*d^2*x
^2+52*B*a^3*b*d^4*x-4*B*b^4*c^3*d*x-12*B*ln(-b*x-a)*a^4*d^4+3*B*b^4*c^4+36*
B*a^2*b^2*c^2*d^2+6*A*a^4*d^4+25*B*a^4*d^4-16*B*a*b^3*c^3*d+6*A*b^4*c^4-48*
B*a^3*b*c*d^3-12*B*ln(-b*x-a)*b^4*d^4*x^4+12*B*ln(d*x+c)*b^4*d^4*x^4)/(a^4*
d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/g^5/(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(194) = 388$.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{3\,(2\,A+B)b^4c^4-8\,(3\,A+2\,B)ab^3c^3d+36\,(A+B)a^2b^2c^2d^2-24\,(A+2\,B)a^3bcd^3+(6\,A+25\,B)a^4d^4-}{24\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A + B)*b^4*c^4 - 8*(3*A + 2*B)*a*b^3*c^3*d + 36*(A + B)*a^2*b^2
*c^2*d^2 - 24*(A + 2*B)*a^3*b*c*d^3 + (6*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*
c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5
*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*
b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4
*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^
4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.

Time = 2.42 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}$$
$$-\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5\left(ad-bc\right)^4}$$
$$+\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5\left(ad-bc\right)^4}$$
$$+\frac{-6Aa^3d^3+18Aa^2bcd^2-18Aab^2c^2d+}{24a^7bd^3g^5-72a^6b^2cd^2g^5+72a^5b^3c^2dg^5-24a^4b^4c^3g^5+x^4\cdot(24a^3b^5d^3g^5-72a^2b^6cd^2g^5+72ab^7c^2dg^5-}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*l
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + B*d**4
```

```
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 - 25*B
*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B
*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**
2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72
*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5
+ x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*
d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*
c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*
a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5
 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d
**2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.22 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{24}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 23\,a}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(a^2b}\right.$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 6*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*
x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
```

```
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(194) = 388.

Time = 0.66 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.01

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)} + \frac{Bd^3}{2\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg} - \frac{Bd^2}{4\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2} - \frac{B\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{4\,(bgx+ag)^4bg} + \frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2} - \frac{2\,Ab^3g^3+Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*
d^4*g^5) + 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3
- a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a
^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log(b^2*e/(b^2*c^2*g^2/(b*g*x + a*
g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*
d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)^4*b*g) +
 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B*b^3*g^3
)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}$$

$$-\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((6*A*a^3*d^3 - 6*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 18*A*a*b^2*c^2
*d - 18*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^2*
d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c
^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5
*x^3 + 12*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b^2*c*d^3*g^5)/
(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4)
```

3.128 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 377

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} \\ &+ \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} \\ &- \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} \\ &+ \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} \\ &+ \frac{2B(bc-ad)^5g^4\left(6A+25B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{15bd^5} \\ &+ \frac{8B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}\end{aligned}$$

```
[Out] -1/5*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/5*g^4
*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+2/15*B*(-a*d+b*c)^2*g^4*(b*x
+a)^3*(2*A+B+2*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/15*B*(-a*d+b*c)^3*g^4*(
b*x+a)^2*(6*A+7*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3+2/15*B*(-a*d+b*c)^4*
g^4*(b*x+a)*(6*A+13*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4+2/15*B*(-a*d+b*c
)^5*g^4*(6*A+25*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b
/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{2Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+25B\right)}{15bd^5}+\frac{2Bg^4(a+bx)(bc-ad)^4\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+13B\right)}{15bd^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+7B\right)}{15bd^3}+\frac{2Bg^4(a+bx)^3(bc-ad)^2\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{15bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5bd}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b}+\frac{8B^2g^4(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] -1/5*(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(5*
b) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(2*A + B + 2*B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(15*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(6*A + 7*B + 6
*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^3) + (2*B*(b*c - a*d)^4*g^4*(
a + b*x)*(6*A + 13*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^4) +
(2*B*(b*c - a*d)^5*g^4*(6*A + 25*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*
Log[(b*c - a*d)/(b*(c + d*x))])/(15*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 g^4\right) \text{Subst}\left(\int \frac{x^4 (A+B\log(ex^2))^2}{(b-dx)^6}\, dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^4(a+bx)^5 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)^5 g^4)\,\text{Subst}\left(\int \frac{x^4(A+B\log(ex^2))}{(b-dx)^5}\, dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} + \frac{(B(bc-ad)^5 g^4)\,\text{Subst}\left(\int \frac{x^3(4A+2B+4B\log(ex^2))}{(b-dx)^4}\, dx, x, \frac{a+bx}{c+dx}\right)}{5bd}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} + \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} - \frac{\left(B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{x^2(8B+3(4A+2B)+12B\log(ex^2))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{15bd^2}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} + \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} + \frac{\left(B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{x(24B+2(8B+3(4A+2B))+24B\log(ex^2))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{30bd^3}$$

$$= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} + \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} + \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} - \frac{\left(B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{72B+2(8B+3(4A+2B))+24B\log(ex^2)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{30bd^4}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} \\
&+ \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} \\
&- \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} \\
&+ \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} \\
&+ \frac{2B(bc-ad)^5g^4\left(6A+25B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{15bd^5} \\
&- \frac{(8B^2(bc-ad)^5g^4)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{5bd^5} \\
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} \\
&+ \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} \\
&- \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} \\
&+ \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} \\
&+ \frac{2B(bc-ad)^5g^4\left(6A+25B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{15bd^5} \\
&+ \frac{8B^2(bc-ad)^5g^4\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.39

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B(bc-ad)\left(12Abd(bc-ad)^3x+12Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-6d^2(bc-ad)^2(a+bx}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*
x)^2)/(c + d*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*(b*c - a*d)^4*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*
(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*
(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Lo
g[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

Maple [F]

$$\int \left(bgx+ag\right)^{4}\left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2}dx$$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^4\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*e*x^2 + 2*a
*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(362) = 724$.

Time = 0.35 (sec) , antiderivative size = 2650, normalized size of antiderivative = 7.03

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*
x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/
b - 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x
/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
```

c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(b^2*e*x^2/(d^2
*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^
2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (
3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 +
A^2*a^4*g^4*x - 2/15*((6*g^4*log(e) + 25*g^4)*b^4*c^5 - (30*g^4*log(e) + 1
13*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(5*g^
4*log(e) + 13*g^4)*a^3*b*c^2*d^3 + 6*(5*g^4*log(e) + 8*g^4)*a^4*c*d^4)*B^2*
log(d*x + c)/d^5 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^
2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^5) + 1/15*(3*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 3*(b^5*c*d^4*g^4*log
(e) - (5*g^4*log(e)^2 + g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 2*((2*g^4*log(e) +
g^4)*b^5*c^2*d^3 - 2*(5*g^4*log(e) + g^4)*a*b^4*c*d^4 + (15*g^4*log(e)^2 +
8*g^4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 - ((6*g^4*log(e) + 7*g^4)*b^5*c^3
d^2 - 3(10*g^4*log(e) + 9*g^4)*a*b^4*c^2*d^3 + 3*(20*g^4*log(e) + 11*g^4)
*a^2*b^3*c*d^4 - (30*g^4*log(e)^2 + 36*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^
2*x^2 + (2*(6*g^4*log(e) + 13*g^4)*b^5*c^4*d - 2*(30*g^4*log(e) + 59*g^4)*a
*b^4*c^3*d^2 + 12*(10*g^4*log(e) + 17*g^4)*a^2*b^3*c^2*d^3 - 2*(60*g^4*log(
e) + 79*g^4)*a^3*b^2*c*d^4 + (15*g^4*log(e)^2 + 48*g^4*log(e) + 46*g^4)*a^4
*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*
a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x +
B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5
*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*
a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4
- 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(6*B
^2*b^5*d^5*g^4*x^5*log(e) - 3*(b^5*c*d^4*g^4 - (10*g^4*log(e) + g^4)*a*b^4*
d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + (15*g^4*log(e) + 4*
g^4)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a
^2*b^3*c*d^4*g^4 - 2*(5*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 6*(2*b^5
*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d
^4*g^4 + (5*g^4*log(e) + 8*g^4)*a^4*b*d^5)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54
*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + (6*g^4
*log(e) + 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) - 2*(6*B^2*b^5*d^5*g^4*x^5*log
(e) - 3*(b^5*c*d^4*g^4 - (10*g^4*log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*
c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + (15*g^4*log(e) + 4*g^4)*a^2*b^3*d^5)*B^2*
x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(
5*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*
c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 + (5*g^4*log(e)
+ 8*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*
x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b
*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)

Giac [F]

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (bgx + ag)^4 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (a\,g + b\,g\,x)^4 \left(A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.129 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 319

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}$$
$$+\frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2}$$
$$-\frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3}$$
$$-\frac{B(bc-ad)^4g^3\left(3A+11B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^4}$$
$$-\frac{2B^2(bc-ad)^4g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+1/6*B*(-a*d+b*c)^2*g^3*(b*x+
a)^2*(3*A+2*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/3*B*(-a*d+b*c)^3*g^3*(
b*x+a)*(3*A+5*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*B*(-a*d+b*c)^4*g^3
*(3*A+11*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^4-2*
B^2*(-a*d+b*c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= -\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+11B\right)}{3bd^4} - \frac{Bg^3(a+bx)(bc-ad)^3\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+5B\right)}{3bd^3} + \frac{Bg^3(a+bx)^2(bc-ad)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{6bd^2} - \frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd} + \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b} - \frac{2B^2g^3(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*
b) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + 2*B + 3*B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(6*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(3*A + 5*B + 3*B*
Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^3) - (B*(b*c - a*d)^4*g^3*(3*A +
11*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))]
)/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/(b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^2))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^2))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} + \frac{\left(B(bc-ad)^4g^3\right)\text{Subst}\left(\int \frac{x^2(3A+2B+3B\log(ex^2))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2} - \frac{(B(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int \frac{x(6B+2(3A+2B)+6B\log(ex^2))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd^2}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2} - \frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} + \frac{(B(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int \frac{18B+2(3A+2B)+6B\log(ex^2)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd^3}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2} - \frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} - \frac{B(bc-ad)^4g^3\left(3A+11B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^4} + \frac{(2B^2(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^4}$$

$$= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2} - \frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} - \frac{B(bc-ad)^4g^3\left(3A+11B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^4} - \frac{2B^2(bc-ad)^4g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.26

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3d^2(-bc+ad)(a+bx)\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d)^2*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)]))))/(3*d^4)))/(4*b)
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(308) = 616$.

Time = 0.33 (sec) , antiderivative size = 1948, normalized size of antiderivative = 6.11

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) +
 a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*
x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c
*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2
*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*((3*g^3*log(e) + 11*g^3)*b^3*c^4 - 2
*(6*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) + 5*g^3)*a^2*b*c^2*d
^2 - 6*(2*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*
g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d
^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 4*(
b^4*c*d^3*g^3*log(e) - (3*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*x^3 + 2
*((3*g^3*log(e) + 2*g^3)*b^4*c^2*d^2 - 4*(3*g^3*log(e) + g^3)*a*b^3*c*d^3 +
 (9*g^3*log(e)^2 + 9*g^3*log(e) + 2*g^3)*a^2*b^2*d^4)*B^2*x^2 - 4*((3*g^3*l
og(e) + 5*g^3)*b^4*c^3*d - (12*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (18*g^3
*log(e) + 19*g^3)*a^2*b^2*c*d^3 - (3*g^3*log(e)^2 + 9*g^3*log(e) + 7*g^3)*a
^3*b*d^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2
*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a
)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4
*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2
*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 + 4*(3*B^2*b^4*d^
4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (6*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x
^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(2*g^3*log(e) + g^3)*a^2*b^
2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g
^3 - (2*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*
b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 - (3*g^3*log(e) + 11*g^3)*a^4*d^4)*B^2
```

```
)*log(b*x + a) - 4*(3*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (6*g^
3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
 + 3*(2*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3
*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (2*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*
x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c)
)/(b*d^4)
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.130 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 255

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} + \frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2} + \frac{4B(bc-ad)^3g^2\left(A+3B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} + \frac{8B^2(bc-ad)^3g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[Out] -2/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4/3*B*(-a*d+b*c)^2*g^2*(b*x+
a)*(A+B+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2+4/3*B*(-a*d+b*c)^3*g^2*(A+3*B+B*
ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c
)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$$
$$=\frac{4Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+3B\right)}{3bd^3}$$
$$+\frac{4Bg^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{3bd^2}$$
$$-\frac{2Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd}$$
$$+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b}+\frac{8B^2g^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*
b) + (4*B*(b*c - a*d)^2*g^2*(a + b*x)*(A + B + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(3*b*d^2) + (4*B*(b*c - a*d)^3*g^2*(A + 3*B + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^3) + (8*B^2*(b*c - a
*d)^3*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^2))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^2))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}\\
&= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}\\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b}\\
&\quad + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{x(2A+2B+2B\log(ex^2))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}\\
&= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}\\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b}\\
&\quad + \frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2}\\
&\quad - \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{2A+6B+2B\log(ex^2)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2}
\end{aligned}$$

$$= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} + \frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2} + \frac{4B(bc-ad)^3g^2\left(A+3B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} - \frac{(8B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} + \frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2} + \frac{4B(bc-ad)^3g^2\left(A+3B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} + \frac{8B^2(bc-ad)^3g^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.17

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{2B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-d^2(a+bx)^2\left(A+B\log\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x)^
2)/(c + d*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
- 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x)^
```

```
2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

Maple [F]

$$\int (bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*
x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^
2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. 2(244) = 488.

Time = 0.32 (sec) , antiderivative size = 1326, normalized size of antiderivative = 5.20

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x
 + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(b^2*e*x^2/(d^
2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x
^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x - 4/3*((g^2*log(e) + 3*g^2)*b^2*c^3 - (3*g^2*log(e) + 7*g^2)*
a*b*c^2*d + (3*g^2*log(e) + 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 8/3*(b
^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (2*b^3*c*d^2*g^2*log(e)
 - (3*g^2*log(e)^2 + 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + (4*(g^2*log(e) + g^
2)*b^3*c^2*d - 4*(3*g^2*log(e) + 2*g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 8*g
^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*
d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(
B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^
3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + 4*
(B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (3*g^2*log(e) + g^2)*a*b^2*d
^3)*B^2*x^2 + (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 4*g^2)
*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (g^2*log(e) +
3*g^2)*a^3*d^3)*B^2)*log(b*x + a) - 4*(B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*
d^2*g^2 - (3*g^2*log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + (2*b^3*c^2*d*g^2 - 6*a*
b^2*c*d^2*g^2 + (3*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 2*(B^2*b^3*d^3*g^
2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*
log(b*x + a))*log(d*x + c))/(b*d^3)
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.131 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 188

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}\\ &\quad-\frac{2B(bc-ad)^2g\left(A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2}\\ &\quad-\frac{4B^2(bc-ad)^2g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b-2*B*(-a*d+b*c)^2*g*(A+2*B+B*ln(e*(b*
x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used

$= \{2550, 2381, 2384, 2354, 2438\}$

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= -\frac{2Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+2B\right)}{bd^2} - \frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd} + \frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b} - \frac{4B^2g(bc-ad)^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*
d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b) - (2*
B*(b*c - a*d)^2*g*(A + 2*B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c -
 a*d)/(b*(c + d*x))])/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^2g\right)\text{Subst}\left(\int\frac{x\left(A+B\log\left(ex^2\right)\right)^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)$$

$$=\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}-\frac{\left(2B(bc-ad)^2g\right)\text{Subst}\left(\int\frac{x\left(A+B\log\left(ex^2\right)\right)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b}$$

$$=-\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}+\frac{\left(2B(bc-ad)^2g\right)\text{Subst}\left(\int\frac{A+2B+B\log\left(ex^2\right)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd}$$

$$=-\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}-\frac{2B(bc-ad)^2g\left(A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2}+\frac{\left(4B^2(bc-ad)^2g\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^2}$$

$$=-\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}-\frac{2B(bc-ad)^2g\left(A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2}-\frac{4B^2(bc-ad)^2g\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{4B(bc-ad)\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-2B(bc-ad)\log(c+dx)-(bc-ad)\left(A+B\log\right.\right.}{}\right.}{2b}$$

[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]

[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(b*c - a*d)*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] - 2*B*(b*c - a*d)*Log[c + d*x] - (b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(185) = 370.

Time = 0.31 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.87

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\frac{1}{2}A^2bgx^2$$
$$+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx\right.}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log\left(d\right.}{d^2}\right.$$
$$+A^2agx+\frac{2\left((g\log\left(e\right)+2\,g)bc^2-2\,(g\log\left(e\right)+g)acd\right)B^2\log\left(dx+c\right)}{d^2}$$
$$+\frac{4\left(b^2c^2g-2\,abcdg+a^2d^2g\right)\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+\frac{B^2b^2d^2gx^2\log\left(e\right)^2-2\left(2\,b^2cdg\log\left(e\right)-\left(g\log\left(e\right)^2+2\,g\log\left(e\right)\right)abd^2\right)B^2x+4\left(B^2b^2d^2gx^2+2\,B^2abd^2g\right.}{}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x
+ a)/b - 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x + 2*((g*log(e) + 2*g)*b*c^2 - 2*(g*log(e) + g)*a
*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(2*b^2*c*d*g*log(e)
- (g*log(e)^2 + 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 4*(B^2*b^2*d^2
*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) +
```

```
 2*g)*a^2*d^2 - 2*a*b*c*d*g)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e
) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2
*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.132 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+4*B*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(
3,b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used

$= \{2550, 2379, 2421, 6724\}$

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{4B\,\mathrm{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{bg} + \frac{8B^2\,\mathrm{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/(b*g)) + (4*B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/(b*g) + (8*B^2*PolyLog[3, (b*(c + d*x))/(d*(a
 + b*x))])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{(4B)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^2))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{(8B^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{-2AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(a+bx)}{(c+dx)}\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
[Out] (-2*A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] - 2*A*B*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2] - B^2*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2]^2 - 4*A*B*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*A*B*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*B^2*Log[(e*(a + b*x)^2)/(c + d*x)^2]
*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*x))/(
d*(a + b*x))])/(b*g)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{bgx+ag}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}}{g}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a**2*e/(c**2 + 2*c*d*x + d
**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*
c*d*x + d**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log(b
*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + 2*(2*B^2*b*d*x + (b*c + a*d)*B^2)*l
og(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x), x)
```

3.133 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{4B(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-4*B*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+
c)^2))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a
*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2550, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{4B(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)} - \frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-8*B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (4*B*(c + d*x)*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= -\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(4B)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= -\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{4B(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}\\
&\quad - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left((bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(a+bx)\left(A+B\log\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*((b*c - a*d)*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2]) + d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2]) - d*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{\frac{(A^2+4BA+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{2cB(A+2B)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{2Bd(A+2B)x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2A^2ab^2d^2-2A^2b^3cd-2B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2b^3d^2-8B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2+16B^2ab^2d^2-16B^2b^3cd-4ABx\ln\left(\frac{e(bx}{(dx}\right.}{2g^2(bx+a)b^3}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}+\frac{4I}{}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}+\frac{4I}{}$
derivativedivides	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{\frac{4d^2AB}{bg(dx+c)}-\frac{2c}{}}{}}{d}$
default	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{\frac{4d^2AB}{bg(dx+c)}-\frac{2c}{}}{}}{d}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] ((A^2+4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)^2/(d*x+c)^2)^2+B^2*
d/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^2)^2+2*c*B*(A+2*B)/g/(a*d-b*c)*ln(e*
(b*x+a)^2/(d*x+c)^2)+2*B*d*(A+2*B)/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^2))
/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.54

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{(A^2+4\,AB+8\,B^2)bc-(A^2+4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\left((AB+2\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 + 4*A*B + 8*B^2)*b*c - (A^2 + 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(
(A*B + 2*B^2)*b*d*x + (A*B + 2*B^2)*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d
)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(112) = 224$.

Time = 1.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.49

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd^2(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd+\frac{4Ba^2d^3(A+2B)}{ad-bc}-\frac{8Babcd^2(A+2B)}{ad-bc}+\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{(-2AB-4B^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2-4AB-8B^2}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B*
*2*b*c*d - 4*B*a**2*d**3*(A + 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A + 2*B)/(
a*d - b*c) - 4*B*b**2*c**2*d*(A + 2*B)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B**2
*b*d**2))/(b*g**2*(a*d - b*c)) + 4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*
A*B*b*c*d + 8*B**2*a*d**2 + 8*B**2*b*c*d + 4*B*a**2*d**3*(A + 2*B)/(a*d - b
*c) - 8*B*a*b*c*d**2*(A + 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A + 2*B)/(a*d
 - b*c))/(8*A*B*b*d**2 + 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B -
4*B**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
 + B**2*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2
+ a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 - 4*A*B - 8*B**2)/(a*b*g**2 + b**2
*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(130) = 260$.

Time = 0.23 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.42

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -4\left(\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)\right.\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2}+\frac{2}{b^2g^2x+abg^2}+\frac{2\,d\log(bx+a)}{(b^2c-abd)g^2}-\frac{2\,d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{B^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - ((
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
- 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a
))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))
*B^2 - 2*A*B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
+ 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) +
 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log
(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
)^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(130) = 260$.

Time = 0.72 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2 + \frac{4\left(ABd+2\,B^2d\right)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2} - \frac{2\left(AB+2\,B^2\right)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{(bgx+ag)bg} - \frac{A^2+4\,AB+8\,B^2}{(bgx+ag)bg}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log(b^2*e/(b^2*c
^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x
 + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2 + 4
*(A*B*d + 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(b^2*
c*g^2 - a*b*d*g^2) - 2*(A*B + 2*B^2)*log(b^2*e/(b^2*c^2*g^2/(b*g*x + a*g)^2
 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/
(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)*b*g) - (A^2
+ 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{A^2+4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{4\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+2\,B)\,8\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*
g^2*(a*d - b*c))) - (A^2 + 8*B^2 + 4*A*B)/(b^2*g^2*x + a*b*g^2) - (log((e*(
a + b*x)^2)/(c + d*x)^2)*((4*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d
+ a/(b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*
d - b*c))*(A + 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 272

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2} + \frac{4Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

```
[Out] 8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(
b*x+a)^2+4*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*
x+a)-b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2
*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2}+\frac{4Bd(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2(c+dx)^2}{g^3(a+bx)^2(bc-ad)^2}+\frac{8B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (8*B^2*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/((b
*c - a*d)^2*g^3*(a + b*x)^2) + (4*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*
x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)^2*g^3*(a + b*x)
) - (b*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*
d)^2*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
```

```
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)\left(A+B\log\left(ex^2\right)\right)^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b\left(A+B\log\left(ex^2\right)\right)^2}{x^3}-\frac{d\left(A+B\log\left(ex^2\right)\right)^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{b\text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}-\frac{d\text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)}-\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\\
&\quad+\frac{(2bB)\text{Subst}\left(\int \frac{A+B\log\left(ex^2\right)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}-\frac{(4Bd)\text{Subst}\left(\int \frac{A+B\log\left(ex^2\right)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}\\
&= \frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}\\
&\quad+\frac{4Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)}-\frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2}\\
&\quad+\frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)}-\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B\left((bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-2d^2(a+bx)^2\log(}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*((b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x
)^2)/(c + d*x)^2])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*
Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) +
 a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
+ d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.80

method	result
norman	$\frac{\frac{\left(A^2ad-A^2bc+4ABad-2ABbc+8B^2ad-2B^2bc\right)x}{ag(ad-cb)}+\frac{Bc(2Aad-Abc+4Bad-Bbc)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bB\,d^2(\cdots}{g\cdots}}{\cdots}$
parallelrisch	$-\frac{-2AB\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5d^3-2B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2a\,b^4d^3-8B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a\,b^4d^3-4B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5c\,d^2-2\cdots}{\cdots}$
derivativedivides	$-\frac{\frac{d^3A^2\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3b\,B^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\cdots}{g(a\cdots}}{\cdots}}{\cdots}$
default	$-\frac{\frac{d^3A^2\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3b\,B^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\cdots}{g(a\cdots}}{\cdots}}{\cdots}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{(4ad-cb)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)^2}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(2a\cdots}{2g(\cdots}}{g^2(\cdots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{(4ad-cb)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)^2}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(2a\cdots}{2g(\cdots}}{g^2(\cdots}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((A^2*a*d-A^2*b*c+4*A*B*a*d-2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x+
B*c*(2*A*a*d-A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a
)^2/(d*x+c)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)^2/(d*
x+c)^2)^2+b*B/g*d^2*(A+3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)^2/
(d*x+c)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)
^2/(d*x+c)^2)^2+1/2*(A^2*a*d-A^2*b*c+6*A*B*a*d-2*A*B*b*c+14*B^2*a*d-2*B^2*b
*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b*c*d+b^2
*c^2)*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)^2)/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(A^2+2\,AB+2\,B^2)b^2c^2-2\,(A^2+4\,AB+8\,B^2)abcd+(A^2+6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2a}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 + 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 + 4*A*B + 8*B^2)*a*b*c*d + (A^
2 + 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((A*B
+ 3*B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d + 2*(B
^2*b^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*
b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(252) = 504$.

Time = 2.08 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.23

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2Bd^2(A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2-\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A}{(ad-bc)}}{4ABbd^3+12B^2bd^3}\right.}{bg^3(ad-bc)^2} + \frac{2Bd^2(A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2+\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2(A}{(ad-bc)}}{4ABbd^3+12B^2bd^3}\right.}{bg^3(ad-bc)^2} + \frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2} + \frac{(-ABad+ABbc-3B^2ad+B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a^3bdg^3-a^2b^2cg^3+2a^2b^2dg^3x-2ab^3cg^3x+ab^3dg^3x^2-b^4cg^3x^2} + \frac{-A^2ad+A^2bc-6ABad+2ABbc-14B^2ad+2B^2bc+x(-4ABbd-12B^2bd)}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\,(4a^2b^2dg^3-4ab^3cg^3)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3
+ 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d
**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**
2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*
d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 +
 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A + 3*B)
/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c
**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c
)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*
c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a + b*x)**2/
(c + d*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3
*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3
*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x*
*2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*log(e*(a
+ b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g
**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2
*a*d + A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(-4*
A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b
**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(270) = 540.

Time = 0.26 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \left(\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log{(bx+a)}}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\,\mathrm{l}}{(b^3c^2-2\,c}\right.\right.$$

$$+AB\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\right.}{b^3g^3x^2+2\,ab^2g^3x+a^2bg}\right.$$

$$-\frac{B^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] (((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*g^3))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^
2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a*b*c*
d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2
)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x
^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(
b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^
3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^
3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*x - b*c + 3*
a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*
c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*
a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)
) - 1/2*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^3*g^3*x^2 + 2*a*b^2*
g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.85

$$\begin{aligned}&\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx\\&=-\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,b\,d\,B^2+A\,b\,d\,B)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}\\&\quad-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\\&\quad-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{A\,B}{b^2\,d\,g^3}+\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}\\&\quad-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(A+3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2+2\,A\,B\,d^2)}\right)\,(A+3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}\end{aligned}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^3,x)
[Out] - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2*
(a*d - b*c)) + (2*x*(3*B^2*b*d + A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^
3*x^2 + 2*a*b^2*g^3*x) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(2*b^2*g^3
*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(b^2*d*g^3) + (2*B^2*x*(a*
d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 +
b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((
B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A + 3*
B)*2i)/((a*d - b*c)*(6*B^2*d^2 + 2*A*B*d^2)))*(A + 3*B)*4i)/(b*g^3*(a*d - b
*c)^2)
```

3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 429

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =& -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}\end{aligned}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-4*B*d^2*(
```

```
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B*d*(d*
x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2-4/9*b^2*B
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(
d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}dx = -\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3} - \frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3} - \frac{4Bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3} + \frac{bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3} + \frac{2bBd(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3} - \frac{8b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3} - \frac{8B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3} + \frac{2bB^2d(c+dx)^2}{g^4(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B^2*d*(c + d*x)
^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (8*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) - (4*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B*d*(c + d*x)^2*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (4*b^2*B*(c
 + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(9*(b*c - a*d)^3*g^4*(a
 + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b
```

```
*c - a*d)^3*g^4*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c
 + d*x)^2])^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2\left(A+B\log\left(ex^2\right)\right)^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2\left(A+B\log\left(ex^2\right)\right)^2}{x^4} - \frac{2bd\left(A+B\log\left(ex^2\right)\right)^2}{x^3} + \frac{d^2\left(A+B\log\left(ex^2\right)\right)^2}{x^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4} - \frac{(2bd)\text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4} + \frac{d^2\text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= -\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3} + \frac{(4b^2B)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x^4}dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^4} - \frac{(4bBd)\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x^3}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4} + \frac{(4Bd^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x^2}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^4}$$

$$= -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} + \frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} - \frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.39

$$\int\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}dx =$$

$$-\frac{9\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{2B\left(6A(bc-ad)^3+4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx)^2+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/27*(9*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*A*(b*c - a*d)
^3 + 4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)
^2*(a + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a +
 b*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a +
b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 6*B*(b*c - a*d)^3*Log[(e*(a +
b*x)^2)/(c + d*x)^2] - 9*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c
 + d*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)
^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x)^2)/(c + d*x)^2] -
18*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*
B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 18*B*d^3
```

```
*(a + b*x)^3*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[c + d*x] - 18*B*d^3*(a +
b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))
/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*
d)^3)/(b*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(423) = 846.

Time = 2.11 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.38

method	result	size
derivativedivides	Expression too large to display	1019
default	Expression too large to display	1019
parallelrisch	Expression too large to display	1025
norman	Expression too large to display	1054
risch	Expression too large to display	1309
parts	Expression too large to display	1309

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(d^4/g^4*A^2*(-1/3*b^2/(a*d-b*c)^3/(a*d/(d*x+c)-b*c/(d*x+c)+b)^3+b/(a*
d-b*c)^3/(a*d/(d*x+c)-b*c/(d*x+c)+b)^2-1/(a*d-b*c)^3/(a*d/(d*x+c)-b*c/(d*x+
c)+b))+(170/27*B^2/b*d^4/g/(d*x+c)^3-22/9*b^2*B^2*d^4/g/(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+44/9*B^2
*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+98/9*B^2*d^4/g/(a*d-b*c)/(d*x+
c)^2-4*B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2
)-1/3*B^2*b^2*d^4/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d
/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a*d/(d
*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-6*B^2*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*
x+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-B^2*b*d^4/g/(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(d*x+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2)/(a*d/(d*x+c)-b
*c/(d*x+c)+b)^3/g^3+(22/9*A*B/b*d^4/g/(d*x+c)^3-2/3*b^2*A*B*d^4/g/(a^3*d^3-
3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2
)+4/3*A*B*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+10/3*A*B*d^4/g/(a*d-b
*c)/(d*x+c)^2-2*A*B*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)
+b)^2/d^2)-2*A*B*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln(e*(a*d/(d*x
+c)-b*c/(d*x+c)+b)^2/d^2))/(a*d/(d*x+c)-b*c/(d*x+c)+b)^3/g^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{(9\,A^2+12\,AB+8\,B^2)b^3c^3-27\,(A^2+2\,AB+2\,B^2)ab^2c^2d+27\,(A^2+4\,AB+8\,B^2)a^2bcd^2-(9\,A^2+66}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a^3
*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*B + 3*
B^2)*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B^2)*b^3
*d^3*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*(A*B + 2
*B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3
*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*x)*log((b^
2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^
6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5
*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^
4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c
^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. 2(406) = 812.

Time = 12.15 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a
*d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16*B
*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A
```

```
+ 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3
- 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*
b*d**4))/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*
a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a**4*d**
7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)
**3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c*
*3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d
- b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3
*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**
3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)**2/(c + d*x)
**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9
*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4
*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c
*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27
*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*
g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a*
*2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2*a**2*d**2 + 14*B**2*a*
b*c*d - 30*B**2*a*b*d**2*x - 4*B**2*b**2*c**2 + 6*B**2*b**2*c*d*x - 12*B**2
*b**2*d**2*x**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(9*a**5*b*d**2*g**4 - 18*
a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*
a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4
*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c*
*2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a
**2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a
*b*c*d + 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b
**2*c**2 + x**2*(36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(90*A*B*a*b*d**
2 - 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d**
2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b**
4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3*
d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**2
*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(423) = 846$.

Time = 0.31 (sec) , antiderivative size = 1575, normalized size of antiderivative = 3.67

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
```

```
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27*a*b^2*c
^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 1
8*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2
- 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a
*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 +
33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d
^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*
g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*
g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a
*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)
*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5
*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 1
1*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^
2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b
^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d
+ a^5*b*d^2)*g^4) + 3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(
d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c
^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)
/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")

[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2+66ABa^2d^2-42ABabcd+12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)} + \frac{2x(-5cB^2b^2d+49a}{}}{x\,(27a^2b^3cg^4-27a^3b^2dg^4)-x^2\,(27a^2b^3dg^4-27ab^4cg^4)+x^3\,(9b^5cg^4-9ab^4}$$
$$-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{3\,b^2\,g^4\,\left(3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2\right)} - \frac{B^2\,d^3}{3\,b\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{2\,A\,B}{3\,b^2\,d\,g^4}+\frac{2\,B^2\,d^3\left(a\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{3\,b\,d^3}+\frac{2\,a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{2\left(3\,a^3\,d^3-6\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,b\,d^4}\right)}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{2\,B^2\,d^3\,x^2\,(}{3\,b\,g^4\,(a^3\,d^3}\right.}{\frac{3\,a^2\,x}{d}+\frac{a^3}{b\,d}+\frac{b^2\,x^3}{d}}$$
$$-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\left(\frac{a^3\,b\,d^3\,g^4-a^2\,b^2\,c\,d^2\,g^4-a\,b^3\,c^2\,d\,g^4+b^4\,c^3\,g^4}{a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4}+2\,b\,d\,x\right)(3\,A+11\,B)\,\left(a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4\right)\,4\mathrm{i}}{b\,g^4\,(a\,d-b\,c)^3\,\left(44\,B^2\,d^3+12\,A\,B\,d^3\right)}\right)\,(3\,A+11\,}{9\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 66*A*B*
a^2*d^2 + 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d - 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 15*A*B*a*b*d^2 -
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d + 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((2*A*B)
/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
```

```
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
+ 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 + 12*A*B*d^3)))*(3*A + 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 587

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} + \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} + \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+4*B*d^3*(d*x+c)*(A+B*ln(e*(b*x+a)
^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^2+4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*B*(d*x+c)^4*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+
B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2395, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4} + \frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{d^3(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4} + \frac{4Bd^3(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^5(a+bx)(bc-ad)^4} - \frac{3bd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4} - \frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^5(a+bx)^2(bc-ad)^4} - \frac{b^3B^2(c+dx)^4}{8g^5(a+bx)^4(bc-ad)^4} + \frac{8b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4} + \frac{8B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4} - \frac{3bB^2d^2(c+dx)^2}{g^5(a+bx)^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (8*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/((b*c - a*d)^4*g^5*(a + b*x)^2) + (8*b^2*B^2*d*(c + d*x)^3)/(9*(b*c -
a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(b*c - a*d)^4*g^5*(a + b
*x)^4) + (4*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c
 - a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]))/((b*c - a*d)^4*g^5*(a + b*x)^2) + (4*b^2*B*d*(c + d*x)^3*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3)
- (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*(b*c - a*
d)^4*g^5*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
```

```
^2])^2)/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (b^2*d*(c
 + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)^4*g^5*(a
 + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(
4*(b*c - a*d)^4*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3\left(A+B\log\left(ex^2\right)\right)^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^3\left(A+B\log\left(ex^2\right)\right)^2}{x^5} - \frac{3b^2d\left(A+B\log\left(ex^2\right)\right)^2}{x^4} + \frac{3bd^2\left(A+B\log\left(ex^2\right)\right)^2}{x^3} - \frac{d^3\left(A+B\log\left(ex^2\right)\right)^2}{x^2}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4} + \frac{(b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{(4b^2Bd)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} + \frac{(6bBd^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5} - \frac{(4Bd^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^5}$$

$$= \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} + \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} + \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B\left(18A(bc-ad)^4+9B(bc-ad)^4+24Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+36Ad^2(bc-ad)^2(a\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/72*(18*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*A*(b*c - a*d)
^4 + 9*B*(b*c - a*d)^4 + 24*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c)
 + a*d)^3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c -
a*d)^2*(a + b*x)^2 + 72*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c
) + a*d)*(a + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b
*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 18*B*(b*c - a*d)
^4*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 24*B*d*(-(b*c) + a*d)^3*(a + b*x)*Log
[(e*(a + b*x)^2)/(c + d*x)^2] + 36*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(
a + b*x)^2)/(c + d*x)^2] + 72*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a +
b*x)^2)/(c + d*x)^2] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x)^2
)/(c + d*x)^2] + 72*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*
Log[c + d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] + 72*B*d^4*(a + b*x)^4*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[c + d*
x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. 2(575) = 1150.

Time = 3.50 (sec) , antiderivative size = 1486, normalized size of antiderivative = 2.53

method	result	size
derivativedivides	Expression too large to display	1486
default	Expression too large to display	1486
norman	Expression too large to display	1816
parallelrisch	Expression too large to display	2110
risch	Expression too large to display	2235
parts	Expression too large to display	2235

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(d^5/g^5*A^2*(-1/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)+1/4*b^3/(a*d-
b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)^4-b^2/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+
c)+b)^3+3/2*b/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)^2)+(415/72*B^2/b*d^5/
g/(d*x+c)^4-25/12*b^3*B^2*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*
a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+25/6*B^2*b^2*d
^5/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+163/12*B^2*b*d^5
/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2+271/18*B^2*d^5/g/(a*d-b*c)/(d*x+c)
^3-4*B^2*d^5/g/(a*d-b*c)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-
1/4*B^2*b^3*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^
4*c^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^5/g/(a*d-b*c)/(d*x+c
)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-9*B^2*d^5*b/g/(a^2*d^2-2*a*b*
c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-22/3*B^2*d^5
*b^2/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a*d/(d*x
+c)-b*c/(d*x+c)+b)^2/d^2)-3/2*B^2*b*d^5/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+
c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*b^2*d^5/g/(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/
d^2)^2)/(a*d/(d*x+c)-b*c/(d*x+c)+b)^4/g^4+(A*B*b^2*d^5/g/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+25/12*A*B/b*d^5/g/(d*x+c)^4-1/2*b^3*A*B
*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e
*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+7/2*A*B*b*d^5/g/(a^2*d^2-2*a*b*c*d+b^2*
c^2)/(d*x+c)^2+13/3*A*B*d^5/g/(a*d-b*c)/(d*x+c)^3-2*A*B*d^5/g/(a*d-b*c)/(d*
x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-3*A*B*d^5*b/g/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-2*A*B*d^5*b
^2/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a*d/(d*x+c
)-b*c/(d*x+c)+b)^2/d^2))/(a*d/(d*x+c)-b*c/(d*x+c)+b)^4/g^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{9\,(2\,A^2+2\,AB+B^2)b^4c^4-8\,(9\,A^2+12\,AB+8\,B^2)ab^3c^3d+108\,(A^2+2\,AB+2\,B^2)a^2b^2c^2d^2-72\,(A^2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^2)*
b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c^2*d
^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)*x^2
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2))^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c^2*d^2 +
108*(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x - 6*((6*A
*B + 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B + 2*B^2)*a*b^
3*c^3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3*b*c*d^3 + 4
*(3*B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2
- 8*B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d -
 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2)*a^3*b*d^4)*x)
*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^9*c^4
- 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^
4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2
 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*
d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4
 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(575) = 1150$.

Time = 0.37 (sec) , antiderivative size = 2279, normalized size of antiderivative = 3.88

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
[Out] 1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c
^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b^2*e*x^2
/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d
^2*x^2 + 2*c*d*x + c^2)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^
2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13
*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4
*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^
2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x +
 a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*
c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d
^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a
*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4
*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*l
og(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b
```

```
^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b
^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^
5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4
*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*
c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*
x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*
a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/12*A*B*((12*b^3*d^3*x^3 -
3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7
*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3
 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 -
 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
- 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3
*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 6*log(b^2*e*x^2/(d^2*x
^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2
+ 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^
3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)
/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4)*g^5)) - 1/4*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$=\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcd}{(bgx+a}}\right.$$

$$+\frac{1}{12}\left(\frac{12\,B^2d^3}{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}-\frac{6\,B^2d^2}{(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2}\right.$$

$$-\frac{(6\,ABd^4+25\,B^2d^4)\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{6\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}$$

$$+\frac{6\,ABd^3+25\,B^2d^3}{6\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}$$

$$-\frac{6\,ABbd^2+13\,B^2bd^2}{12\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2b^2g^2}$$

$$+\frac{6\,ABb^2dg+7\,B^2b^2dg}{18\,(bgx+ag)^3(bc-ad)b^3g^3}-\frac{2\,A^2b^3g^3+2\,ABb^3g^3+B^2b^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log(b^2*e/(b
^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b
*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2
 + 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
 a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^
2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^
2) - 3*(2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log(b^2*e/(
b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(
b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))
- 1/6*(6*A*B*d^4 + 25*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*
g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^
2*c*d^3*g^5 + a^4*b*d^4*g^5) + 1/6*(6*A*B*d^3 + 25*B^2*d^3)/((b^3*c^3*g^3 -
 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) -
1/12*(6*A*B*b*d^2 + 13*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b
*g*x + a*g)^2*b^2*g^2) + 1/18*(6*A*B*b^2*d*g + 7*B^2*b^2*d*g)/((b*g*x + a*g
)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 2*A*B*b^3*g^3 + B^2*b^3*g^3
)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 1883, normalized size of antiderivative = 3.21

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^5,x)
[Out] (B*d^4*atan((B*d^4*(6*A + 25*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4
*c^3*d*g^5 + 12*a^3*b^2*c*d^3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 +
 6*A*B*d^4)) + (B*d^5*x*(6*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3
*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 + 6*A*
B*d^4)))*(6*A + 25*B)*1i)/(3*b*g^5*(a*d - b*c)^4) - log((e*(a + b*x)^2)/(c
+ d*x)^2)^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*
b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a
*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(2
*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*
c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)
/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*
d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/
d^2) + (b^3*c - a*b^2*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 +
b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 -
b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 +
 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d
) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - ((18*A^2*a^3*d^3 - 1
8*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 150*A*B*a^3*d^3 - 18*A*B*
b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 16
1*B^2*a^2*b*c*d^2 + 78*A*B*a*b^2*c^2*d - 138*A*B*a^2*b*c*d^2)/(12*(a*d - b*
c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 + 42*A*B*a*b^2*d^3 - 6*A*B
*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53
*B^2*a*b^2*c*d^2 + 78*A*B*a^2*b*d^3 + 6*A*B*b^3*c^2*d - 30*A*B*a*b^2*c*d^2)
)/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 6*A*B*b^3*d^2))/(a*d - b*c))/(
x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*b^3*c*d*g^5) + x^3*(24*
a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*g^5) + x^4*(6*b^7*c^2*g
```

```
^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36*a^2*b^5*c^2*g^5 + 36*a
^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g^5 + 6*a^4*b^3*c^2*g^5
- 12*a^5*b^2*c*d*g^5)
```

3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}, x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*
e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)
)), x) + Integral(b**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2))), x))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
[Out] integral((b*g*x + a*g)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*
c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] g*(Integral(a/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)
 + Integral(b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/
(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bbx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
*2)) + B*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*
c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)}$$

```
[Out] 1/2*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x
+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2550, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] (E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi[-1/2
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(b*c - a*d)*g^2*(a + b*x
))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}\\
&= \frac{\left(\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bc-ad)g^2(a+bx)}\\
&= \frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)\cdots}}{g^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
*2))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 152

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = -\frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)} + \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B(bc-ad)^2g^3}$$

```
[Out] 1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3-
1/2*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b
*x+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)^2/g^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

= {2550, 2395, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2Bg^3(bc-ad)^2}$$
$$-\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] (b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B)])/
(2*B*(b*c - a*d)^2*g^3) - (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*
(c + d*x)*ExpIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(
2*B*(b*c - a*d)^2*g^3*(a + b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
```

```
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex^2))} - \frac{d}{x^2(A+B\log(ex^2))}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{(be)\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bc-ad)^2g^3} - \frac{\left(d\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bc-ad)^2g^3(a+bx)}$$

$$= \frac{bee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B(bc-ad)^2g^3} - \frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

$$= \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Ba^2bx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\cdots}}{g\cdots}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*
x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*c*d*x +
 d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 308, normalized size of antiderivative = 9.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*
log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2
+ a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*
g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
[Out] integral((b*g*x + a*g)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*
c*d*x + c^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.19 (sec) , antiderivative size = 558, normalized size of antiderivative = 17.44

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right.}{}$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a + b
*x)**2/(c + d*x)**2)) - g*(Integral(a**2*d/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(a**2*e/(c**2 +
2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2
/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*a*b*d
*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d
*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d
 - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 7.19

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*
d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*d*g
*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*
log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(
e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*
a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log
((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{c+dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{d\int\frac{1}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e*
(a + b*x)**2/(c + d*x)**2)) - d*Integral(1/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 4.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x
) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B
^2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 7.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.145 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 150

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = -\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)} - \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

```
[Out] 1/2*(-d*x-c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))-1/4*e
xp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x+a)^2
/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used

$= \{2550, 2343, 2347, 2209\}$

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = -\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)} - \frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] -1/4*(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi
[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(B^2*(b*c - a*d)*g^2*(a
+ b*x)) - (c + d*x)/(2*B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}$$

$$= -\frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{2B(bc-ad)g^2}$$

$$= -\frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{\left(\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4B(bc-ad)g^2(a+bx)}$$

$$= -\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)} - \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d
^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$=\frac{c+dx}{2ABa^2dg^2-2ABabcg^2+2ABabdg^2x-2ABb^2cg^2x+(2B^2a^2dg^2-2B^2abcg^2+2B^2abdg^2x-2B^2b^2cg^2x)}$$
$$-\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x}\right.}}{2Bg^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A*
B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g*
*2*x - 2*B**2*b**2*c*g**2*x)*log(e*(a + b*x)**2/(c + d*x)**2)) - Integral(1
/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*c*d*x + d*
*2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c
*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c*
*2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/
(2*B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2
*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2
*log(e))*B^2)*x + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2
)*B^2)*log(b*x + a) - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d
*g^2)*B^2)*log(d*x + c)) + integrate(-1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2
 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b
*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a)
- 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 266

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)} - \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2g^3} + \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

```
[Out] -1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/g
^3+1/2*d*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
-1/2*b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))+1/4*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e
*(b*x+a)^2/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)^2/g^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2550, 2395, 2343, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}dx$$
$$=\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2}-\frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{2Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}+\frac{d(c+dx)}{2Bg^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] -1/2*(b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/
B)])/(B^2*(b*c - a*d)^2*g^3) + (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x
)^2]*(c + d*x)*ExpIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/
B])/(4*B^2*(b*c - a*d)^2*g^3*(a + b*x)) + (d*(c + d*x))/(2*B*(b*c - a*d)^2*
g^3*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])) - (b*(c + d*x)^2)/(
2*B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b-dx}{x^3(A+B\log(ex^2))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3(A+B\log(ex^2))^2} - \frac{d}{x^2(A+B\log(ex^2))^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^2))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b\text{Subst}\left(\int \frac{1}{x^3(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)^2g^3} + \frac{d\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^2))}\,dx, x, \frac{a+bx}{c+dx}\right)}{2B(bc-ad)^2g^3}$$

$$= \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{(be)\text{Subst}\left(\int \frac{e^{-x}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2B(bc-ad)^2g^3} + \frac{\left(d\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4B(bc-ad)^2g^3(a+bx)}$$

$$= -\frac{bee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2g^3} + \frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)} + \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2
*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^
3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d
*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b
^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g
^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x
^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*l
og(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b
*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(1/
2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e)
 - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*
```

```
g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B +
 (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + 2*
((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^
2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*
b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.147 $\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 171

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{B(bc-ad)^4nx}{5d^4}-\frac{B(bc-ad)^3n(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2n(a+bx)^3}{15bd^2}-\frac{B(bc-ad)n(a+bx)^4}{20bd}-\frac{B(bc-ad)^5n\log(c+dx)}{5bd^5}+\frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b}$$

```
[Out] 1/5*B*(-a*d+b*c)^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*n*(b*x+a)^2/b/d^3+1/15*B*(-a
*d+b*c)^2*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*n*(b*x+a)^4/b/d-1/5*B*(-a*d+b
*c)^5*n*ln(d*x+c)/b/d^5+1/5*(b*x+a)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 45}

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{(a+bx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bnx(bc-ad)^4}{5d^4}-\frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bn(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bn(a+bx)^4(bc-ad)}{20bd}$$

```
[In] Int[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*n*(a + b*x)^2)/(10*b*d^3)
+ (B*(b*c - a*d)^2*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*n*(a + b*x)^4
)/(20*b*d) - (B*(b*c - a*d)^5*n*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*(A +
 B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(5*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b} - \frac{(B(bc-ad)n)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}$$

$$= \frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b} - \frac{(B(bc-ad)n)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}$$

$$= \frac{B(bc-ad)^4nx}{5d^4} - \frac{B(bc-ad)^3n(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2n(a+bx)^3}{15bd^2} - \frac{B(bc-ad)n(a+bx)^4}{20bd} - \frac{B(bc-ad)^5n\log(c+dx)}{5bd^5} + \frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 364 vs. $2(171) = 342$.

Time = 0.54 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.13

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \frac{bdx(12a^4d^4(5A+4Bn)+12a^3bd^3(-10Bcn+10Adx+3Bdnx)+4a^2b^2d^2(30Ad^2x^2+Bn(30c^2-15cdx+$$

```
[In] Integrate[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(12*a^4*d^4*(5*A + 4*B*n) + 12*a^3*b*d^3*(-10*B*c*n + 10*A*d*x + 3*B
*d*n*x) + 4*a^2*b^2*d^2*(30*A*d^2*x^2 + B*n*(30*c^2 - 15*c*d*x + 4*d^2*x^2)
) + b^4*(12*A*d^4*x^4 + B*c*n*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3
)) + a*b^3*d*(60*A*d^3*x^3 + B*n*(-60*c^3 + 30*c^2*d*x - 20*c*d^2*x^2 + 3*d
^3*x^3))) - 48*a^5*B*d^5*n*Log[a + b*x] - 12*B*(b^5*c^5 - 5*a*b^4*c^4*d + 1
0*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 5*a^5*d^5)*n*Log[c
 + d*x] + 12*B*d^5*(5*a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5
*a*b^4*x^4 + b^5*x^5)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b*d^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(159) = 318$.

Time = 134.47 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.87

method	result
parallelrisch	$\frac{120Ax^2a^4b^2cd^5n+48Bxa^5bcd^5n^2-120Bxa^4b^2c^2d^4n^2+120Bxa^3b^3c^3d^3n^2-60Bxa^2b^4c^4d^2n^2+12Bxab^5c^5dn^2+60Axa^5bcd}{}$
risch	Expression too large to display

```
[In] int((b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(120*A*x^2*a^4*b^2*c*d^5*n+48*B*x*a^5*b*c*d^5*n^2-120*B*x*a^4*b^2*c^2*
d^4*n^2+120*B*x*a^3*b^3*c^3*d^3*n^2-60*B*x*a^2*b^4*c^4*d^2*n^2+12*B*x*a*b^5
*c^5*d*n^2+60*A*x*a^5*b*c*d^5*n+60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b*c^2*
d^4*n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^2*c^3*d^3*n+120*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^3*b^3*c^4*d^2*n-60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^4
*c^5*d*n-60*B*ln(b*x+a)*a^5*b*c^2*d^4*n^2+120*B*ln(b*x+a)*a^4*b^2*c^3*d^3*n
^2-120*B*ln(b*x+a)*a^3*b^3*c^4*d^2*n^2+60*B*ln(b*x+a)*a^2*b^4*c^5*d*n^2+12*
B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^6*n+12*B*ln(b*x+a)*a^6*c*d^5*n^2-12*B
*ln(b*x+a)*a*b^5*c^6*n^2+12*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*n
+12*A*x^5*a*b^5*c*d^5*n+3*B*x^4*a^2*b^4*c*d^5*n^2-3*B*x^4*a*b^5*c^2*d^4*n^2
+60*A*x^4*a^2*b^4*c*d^5*n+16*B*x^3*a^3*b^3*c*d^5*n^2-20*B*x^3*a^2*b^4*c^2*d
^4*n^2+4*B*x^3*a*b^5*c^3*d^3*n^2+120*A*x^3*a^3*b^3*c*d^5*n+36*B*x^2*a^4*b^2
```

```
*c*d^5*n^2-60*B*x^2*a^3*b^3*c^2*d^4*n^2+30*B*x^2*a^2*b^4*c^3*d^3*n^2-6*B*x^
2*a*b^5*c^4*d^2*n^2+60*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^4*c*d^5*n+12
0*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^3*c*d^5*n+120*B*x^2*ln(e*(b*x+a)^
n/((d*x+c)^n))*a^4*b^2*c*d^5*n+60*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b*c*d
^5*n)/b/a/c/d^5/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(159) = 318$.

Time = 0.26 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.29

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{12\,Ab^5d^5x^5+3\,(20\,Aab^4d^5-(Bb^5cd^4-Bab^4d^5)n)x^4+4\,(30\,Aa^2b^3d^5+(Bb^5c^2d^3-5\,Bab^4cd^4+4\,Ba^2b^3d\cdots}{\cdots}$$

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*A*b^5*d^5*x^5 + 3*(20*A*a*b^4*d^5 - (B*b^5*c*d^4 - B*a*b^4*d^5)*n)
*x^4 + 4*(30*A*a^2*b^3*d^5 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 4*B*a^2*b^3
*d^5)*n)*x^3 + 6*(20*A*a^3*b^2*d^5 - (B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*n)*x^2 + 12*(5*A*a^4*b*d^5 + (B*b^5*c^
4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a
^4*b*d^5)*n)*x + 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d
^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + B*a^5*d^5*n)*log(b*
x + a) - 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d^5*n*x^3
 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + (B*b^5*c^5 - 5*B*a*b^4*c^4*
d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*n)*log(d
*x + c) + 12*(B*b^5*d^5*x^5 + 5*B*a*b^4*d^5*x^4 + 10*B*a^2*b^3*d^5*x^3 + 10
*B*a^3*b^2*d^5*x^2 + 5*B*a^4*b*d^5*x)*log(e))/(b*d^5)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.92

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{1}{5}Bb^4x^5\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{5}Ab^4x^5 + Bab^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^3x^4 + 2Ba^2b^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2Aa^2b^2x^3 + 2Ba^3bx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2Aa^3bx^2 + Ba^4x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^4x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^4}{e} - \frac{2\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Ba^3b}{e} + \frac{\left(\frac{2a^3en\log(bx+a)}{b^3} - \frac{2c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Ba^2b^2}{e} - \frac{\left(\frac{6a^4en\log(bx+a)}{b^4} - \frac{6c^4en\log(dx+c)}{d^4} + \frac{2(b^3cd^2en-ab^2d^3en)x^3-3(b^3c^2den-a^2bd^3en)x^2+6(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bab^3}{6e} + \frac{\left(\frac{12a^5en\log(bx+a)}{b^5} - \frac{12c^5en\log(dx+c)}{d^5} - \frac{3(b^4cd^3en-ab^3d^4en)x^4-4(b^4c^2d^2en-a^2b^2d^4en)x^3+6(b^4c^3den-a^3bd^4en)x^2-12(b^4c^4en-a^4\ldots}{b^4d^4}\ldots}{60e}$$

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*b^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*b^4*x^5 + B*a*b^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^3*x^4 + 2*B*a^2*b^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*a^2*b^2*x^3 + 2*B*a^3*b*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*a^3*b*x^2 + B*a^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a^4*x
 + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^4/e - 2*(a^2*e*n*log(b
*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*a^3
*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d
*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*a^2
*b^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*
(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2
 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*a*b^3/e + 1/60*(12*a^5*e*n
*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3*e*n - a*b^3
*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*(b^4*c^3*d*e*
n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b^4*d^4))*B*b^4
/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(159) = 318$.

Time = 7.18 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.96

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{Ba^5n\log(bx+a)}{5\,b} + \frac{1}{5}\left(Bb^4\log(e)+Ab^4\right)x^5 - \frac{(Bb^4cn - Bab^3dn - 20\,Bab^3d\log(e) - 20\,Aab^3d)x^4}{20\,d} + \frac{(Bb^4c^2n - 5\,Bab^3cdn + 4\,Ba^2b^2d^2n + 30\,Ba^2b^2d^2\log(e) + 30\,Aa^2b^2d^2)x^3}{15\,d^2} + \frac{1}{5}\left(Bb^4nx^5 + 5\,Bab^3nx^4 + 10\,Ba^2b^2nx^3 + 10\,Ba^3bnx^2 + 5\,Ba^4nx\right)\log(bx+a) - \frac{1}{5}\left(Bb^4nx^5 + 5\,Bab^3nx^4 + 10\,Ba^2b^2nx^3 + 10\,Ba^3bnx^2 + 5\,Ba^4nx\right)\log(dx+c) - \frac{(Bb^4c^3n - 5\,Bab^3c^2dn + 10\,Ba^2b^2cd^2n - 6\,Ba^3bd^3n - 20\,Ba^3bd^3\log(e) - 20\,Aa^3bd^3)x^2}{10\,d^3} + \frac{(Bb^4c^4n - 5\,Bab^3c^3dn + 10\,Ba^2b^2c^2d^2n - 10\,Ba^3bcd^3n + 4\,Ba^4d^4n + 5\,Ba^4d^4\log(e) + 5\,Aa^4d^4)x}{5\,d^4} - \frac{(Bb^4c^5n - 5\,Bab^3c^4dn + 10\,Ba^2b^2c^3d^2n - 10\,Ba^3bc^2d^3n + 5\,Ba^4cd^4n)\log(-dx-c)}{5\,d^5}$$

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/5*B*a^5*n*log(b*x + a)/b + 1/5*(B*b^4*log(e) + A*b^4)*x^5 - 1/20*(B*b^4*c
*n - B*a*b^3*d*n - 20*B*a*b^3*d*log(e) - 20*A*a*b^3*d)*x^4/d + 1/15*(B*b^4*
c^2*n - 5*B*a*b^3*c*d*n + 4*B*a^2*b^2*d^2*n + 30*B*a^2*b^2*d^2*log(e) + 30*
A*a^2*b^2*d^2)*x^3/d^2 + 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*b^2*
n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(b*x + a) - 1/5*(B*b^4*n*x^5 + 5
*B*a*b^3*n*x^4 + 10*B*a^2*b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(d
*x + c) - 1/10*(B*b^4*c^3*n - 5*B*a*b^3*c^2*d*n + 10*B*a^2*b^2*c*d^2*n - 6*
B*a^3*b*d^3*n - 20*B*a^3*b*d^3*log(e) - 20*A*a^3*b*d^3)*x^2/d^3 + 1/5*(B*b^
4*c^4*n - 5*B*a*b^3*c^3*d*n + 10*B*a^2*b^2*c^2*d^2*n - 10*B*a^3*b*c*d^3*n +
 4*B*a^4*d^4*n + 5*B*a^4*d^4*log(e) + 5*A*a^4*d^4)*x/d^4 - 1/5*(B*b^4*c^5*n
 - 5*B*a*b^3*c^4*d*n + 10*B*a^2*b^2*c^3*d^2*n - 10*B*a^3*b*c^2*d^3*n + 5*B*
a^4*c*d^4*n)*log(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 936, normalized size of antiderivative = 5.47

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= x^4 \left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{20\,d} - \frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{20\,d}\right)$$

$$- x^3 \left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d} - \frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{15\,b\,d} - \frac{a\,b^2\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d} + \frac{A\,a\,b^3\,c}{3\,d}\right)$$

$$+ \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^4\,x + 2\,B\,a^3\,b\,x^2 + 2\,B\,a^2\,b^2\,x^3 + B\,a\,b^3\,x^4 + \frac{B\,b^4\,x^5}{5}\right)$$

$$+ x \left(\frac{a^3\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{d} - \frac{(5\,a\,d+5\,b\,c)\,\left(\frac{2\,a^2\,b\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d} + \frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d} - \frac{A\,b^3\,(5\,a\,d+5\,b\,c}{5\,d}\right.}{5\,b\,d}\right.}{5\,b\,d}\right.}{5\,b\,d} + \frac{a\,c\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d} - \frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{5\,b\,d} - \frac{a\,b^2\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d} + \frac{A\,a\,b^3\,c}{d}\right)}{b\,d}\right)$$

$$+ x^2 \left(\frac{a^2\,b\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d} - \frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d} - \frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{5\,b\,d} - \frac{a\,b^2\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d} + \frac{A\,a\,b^3}{\ldots}\right.}{\ldots}\right.$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^4,x)
```

```
[Out] x^4*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(20*d) - (A*b^3*(5*a*d
+ 5*b*c))/(20*d)) - x^3*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(15*b*d) - (a*b^2*(1
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3*c)/(3*d)) + log((e
*(a + b*x)^n)/(c + d*x)^n)*((B*b^4*x^5)/5 + B*a^4*x + 2*B*a^3*b*x^2 + B*a*b
^3*x^4 + 2*B*a^2*b^2*x^3) + x*((a^3*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b
*c*n))/d - ((5*a*d + 5*b*c)*((2*a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*
n))/d + ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*
d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(1
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(5*b*d) - (a*c*
((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b
*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d)
- (a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(b*d
)) + x^2*((a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + ((5*a*d + 5*b
*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(10*A*a*d + 5*A*b*c + B
*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(10*b*d) - (a*c*((b^3*(25*A*a*d + 5*
A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)
) + (A*b^4*x^5)/5 - (log(c + d*x)*(B*b^4*c^5*n + 5*B*a^4*c*d^4*n + 10*B*a^2
*b^2*c^3*d^2*n - 5*B*a*b^3*c^4*d*n - 10*B*a^3*b*c^2*d^3*n))/(5*d^5) + (B*a^
5*n*log(a + b*x))/(5*b)
```

3.148 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 142

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = -\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)n(a+bx)^3}{12bd} + \frac{B(bc-ad)^4n\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*n*(b*x+a)^2/b/d^2-1/12*B*(-a
*d+b*c)*n*(b*x+a)^3/b/d+1/4*B*(-a*d+b*c)^4*n*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 45}

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \frac{(a+bx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}+\frac{Bn(bc-ad)^4\log(c+dx)}{4bd^4} - \frac{Bnx(bc-ad)^3}{4d^3}+\frac{Bn(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bn(a+bx)^3(bc-ad)}{12bd}$$

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*n*x)/d^3 + (B*(b*c - a*d)^2*n*(a + b*x)^2)/(8*b*d^2)
- (B*(b*c - a*d)*n*(a + b*x)^3)/(12*b*d) + (B*(b*c - a*d)^4*n*Log[c + d*x])
/(4*b*d^4) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b} - \frac{(B(bc-ad)n)\int \frac{(a+bx)^3}{c+dx}\,dx}{4b} \\
&= \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b} \\
&\quad - \frac{(B(bc-ad)n)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{4b} \\
&= -\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)n(a+bx)^3}{12bd} \\
&\quad + \frac{B(bc-ad)^4n\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.92

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{bdx(6a^3d^3(4A+3Bn)+9a^2bd^2(-4Bcn+4Adx+Bdnx)+b^3(6Ad^3x^3+Bcn(-6c^2+3cdx-2d^2x^2))+}{}$$

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(6*a^3*d^3*(4*A + 3*B*n) + 9*a^2*b*d^2*(-4*B*c*n + 4*A*d*x + B*d*n*x
) + b^3*(6*A*d^3*x^3 + B*c*n*(-6*c^2 + 3*c*d*x - 2*d^2*x^2)) + 2*a*b^2*d*(1
2*A*d^2*x^2 + B*n*(12*c^2 - 6*c*d*x + d^2*x^2))) - 18*a^4*B*d^4*n*Log[a + b
*x] + 6*B*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 4*
a^4*d^4)*n*Log[c + d*x] + 6*B*d^4*(4*a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*
b^3*x^3 + b^4*x^4)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(132) = 264.

Time = 52.80 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.68

method	result
parallelrisch	$\frac{-6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{4}c^{4}n+3B\,x^{2}b^{4}c^{2}d^{2}n^{2}+18Bx\,a^{3}b\,d^{4}n^{2}-6Bx\,b^{4}c^{3}d\,n^{2}+24Ax\,a^{3}b\,d^{4}n+36A\,x^{2}a^{2}b^{2}d^{4}n+9B\,a^{3}bc}{}$
risch	Expression too large to display

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*n+3*B*x^2*b^4*c^2*d^2*n^2
+18*B*x*a^3*b*d^4*n^2-6*B*x*b^4*c^3*d*n^2+24*A*x*a^3*b*d^4*n+36*A*x^2*a^2*b
^2*d^4*n+9*B*a^3*b*c*d^3*n^2+24*B*a^2*b^2*c^2*d^2*n^2-21*B*a*b^3*c^3*d*n^2-
60*A*a^3*b*c*d^3*n+2*B*x^3*a*b^3*d^4*n^2-2*B*x^3*b^4*c*d^3*n^2+24*A*x^3*a*b
^3*d^4*n+9*B*x^2*a^2*b^2*d^4*n^2-18*B*a^4*d^4*n^2+6*B*b^4*c^4*n^2-24*A*a^4*
d^4*n+24*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*n+36*B*x^2*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^2*b^2*d^4*n-12*B*x^2*a*b^3*c*d^3*n^2+24*B*x*ln(e*(b*x+a)
^n/((d*x+c)^n))*a^3*b*d^4*n-36*B*x*a^2*b^2*c*d^3*n^2+24*B*x*a*b^3*c^2*d^2*n
^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*c*d^3*n-36*B*ln(e*(b*x+a)^n/((d*x
+c)^n))*a^2*b^2*c^2*d^2*n+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*c^3*d*n-24
*B*ln(b*x+a)*a^3*b*c*d^3*n^2+36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*n^2-24*B*ln(b*x
+a)*a*b^3*c^3*d*n^2+6*A*x^4*b^4*d^4*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c
^4*n+6*B*ln(b*x+a)*a^4*d^4*n^2+6*B*ln(b*x+a)*b^4*c^4*n^2)/d^4/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(132) = 264.

Time = 0.28 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.94

$$\int (a+bx)^{3}\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)\,dx = \frac{6\,Ab^{4}d^{4}x^{4}+2\,(12\,Aab^{3}d^{4}-(Bb^{4}cd^{3}-Bab^{3}d^{4})n)x^{3}+3\,(12\,Aa^{2}b^{2}d^{4}+(Bb^{4}c^{2}d^{2}-4\,Bab^{3}cd^{3}+3\,Ba^{2}b^{2}d^{4})}{}$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*A*b^4*d^4*x^4 + 2*(12*A*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*b^3*d^4)*n)*
x^3 + 3*(12*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*
d^4)*n)*x^2 + 6*(4*A*a^3*b*d^4 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2
*b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3
 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^4*d^4*n)*log(b*x + a) -
6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*
```

```
b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*
c*d^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2
*d^4*x^2 + 4*B*a^3*b*d^4*x)*log(e))/(b*d^4)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(132) = 264$.

Time = 0.21 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.29

$$\begin{aligned}
&\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx \\
&= \frac{1}{4}Bb^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4}Ab^3x^4 + Bab^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^2x^3 \\
&+ \frac{3}{2}Ba^2bx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2}Aa^2bx^2 + Ba^3x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^3x \\
&+ \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^3}{e} - \frac{3\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Ba^2b}{2e} \\
&+ \frac{\left(\frac{2a^3en\log(bx+a)}{b^3} - \frac{2c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bab^2}{2e} \\
&- \frac{\left(\frac{6a^4en\log(bx+a)}{b^4} - \frac{6c^4en\log(dx+c)}{d^4} + \frac{2(b^3cd^2en-ab^2d^3en)x^3-3(b^3c^2den-a^2bd^3en)x^2+6(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bb^3}{24e}
\end{aligned}$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
[Out] 1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + B*a^3*x*log((b*x + a)^n*e/(d*x + c)^
n) + A*a^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^3/e - 3/2*
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*a^2*b/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x +
c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x
```

```
)/(b^2*d^2))*B*a*b^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d
*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a
^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*b^3/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(132) = 264$.

Time = 2.07 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.56

$$
\begin{aligned}
&\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx \\
&= \frac{Ba^4n\log(bx+a)}{4\,b} + \frac{1}{4}\left(Bb^3\log(e)+Ab^3\right)x^4 \\
&\quad - \frac{(Bb^3cn - Bab^2dn - 12\,Bab^2d\log(e) - 12\,Aab^2d)x^3}{12\,d} \\
&\quad + \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(bx+a) \\
&\quad - \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(dx+c) \\
&\quad + \frac{(Bb^3c^2n - 4\,Bab^2cdn + 3\,Ba^2bd^2n + 12\,Ba^2bd^2\log(e) + 12\,Aa^2bd^2)x^2}{8\,d^2} \\
&\quad - \frac{(Bb^3c^3n - 4\,Bab^2c^2dn + 6\,Ba^2bcd^2n - 3\,Ba^3d^3n - 4\,Ba^3d^3\log(e) - 4\,Aa^3d^3)x}{4\,d^3} \\
&\quad + \frac{(Bb^3c^4n - 4\,Bab^2c^3dn + 6\,Ba^2bc^2d^2n - 4\,Ba^3cd^3n)\log(dx+c)}{4\,d^4}
\end{aligned}
$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/4*B*a^4*n*log(b*x + a)/b + 1/4*(B*b^3*log(e) + A*b^3)*x^4 - 1/12*(B*b^3*c
*n - B*a*b^2*d*n - 12*B*a*b^2*d*log(e) - 12*A*a*b^2*d)*x^3/d + 1/4*(B*b^3*n
*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1/4*
(B*b^3*n*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x + c
) + 1/8*(B*b^3*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*B*a^2*b*d^2*l
og(e) + 12*A*a^2*b*d^2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*n + 6*
B*a^2*b*c*d^2*n - 3*B*a^3*d^3*n - 4*B*a^3*d^3*log(e) - 4*A*a^3*d^3)*x/d^3 +
 1/4*(B*b^3*c^4*n - 4*B*a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a^3*c*d^3
*n)*log(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.66

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= x^3\left(\frac{b^2\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,d}-\frac{A\,b^2\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^3\,x+\frac{3\,B\,a^2\,b\,x^2}{2}+B\,a\,b^2\,x^3+\frac{B\,b^3\,x^4}{4}\right)$$

$$+x\left(\frac{a^2\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{b^2\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{4\,b\,d}-\frac{a\,b\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b^2}{d}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^2\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{b^2\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{8\,b\,d}\right.$$

$$\left.-\frac{a\,b\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}+\frac{A\,a\,b^2\,c}{2\,d}\right)$$

$$+\frac{A\,b^3\,x^4}{4}+\frac{\ln(c+d\,x)\,\left(-4\,B\,n\,a^3\,c\,d^3+6\,B\,n\,a^2\,b\,c^2\,d^2-4\,B\,n\,a\,b^2\,c^3\,d+B\,n\,b^3\,c^4\right)}{4\,d^4}$$

$$+\frac{B\,a^4\,n\,\ln(a+b\,x)}{4\,b}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^3,x)
```

```
[Out] x^3*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*(4*a*d
+ 4*b*c))/(12*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^3*x^4)/4 + B*a^3
*x + (3*B*a^2*b*x^2)/2 + B*a*b^2*x^3) + x*((a^2*(8*A*a*d + 12*A*b*c + 3*B*a
*d*n - 3*B*b*c*n))/(2*d) + ((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((b^2*(16*A*a
*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/
(4*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*c)/d))
```

```
/(4*b*d) - (a*c*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*
b^2*(4*a*d + 4*b*c))/(4*d)))/(b*d)) - x^2*(((4*a*d + 4*b*c)*((b^2*(16*A*a*d
 + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/(8
*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c)/(
2*d)) + (A*b^3*x^4)/4 + (log(c + d*x)*(B*b^3*c^4*n - 4*B*a^3*c*d^3*n - 4*B*
a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n))/(4*d^4) + (B*a^4*n*log(a + b*x))/(4*b
)
```

3.149 $\int (a+bx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}-\frac{B(bc-ad)^3n\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*n*x/d^2-1/6*B*(-a*d+b*c)*n*(b*x+a)^2/b/d-1/3*B*(-a*d+b*c
)^3*n*ln(d*x+c)/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 45}

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bnx(bc-ad)^2}{3d^2}-\frac{Bn(a+bx)^2(bc-ad)}{6bd}$$

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*n*x)/(3*d^2) - (B*(b*c - a*d)*n*(a + b*x)^2)/(6*b*d) - (B*
(b*c - a*d)^3*n*Log[c + d*x])/(3*b*d^3) + ((a + b*x)^3*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n]))/(3*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}-\frac{(B(bc-ad)n)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}\\
&\quad-\frac{(B(bc-ad)n)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= \frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}-\frac{B(bc-ad)^3n\log(c+dx)}{3bd^3}\\
&\quad+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.72

$$
\int(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx
$$
$$
=\frac{bdx(2a^2d^2(3A+2Bn)+abd(-6Bcn+6Adx+Bdnx)+b^2(2Ad^2x^2+Bcn(2c-dx)))-4a^3Bd^3n\log(a+
$$

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(2*a^2*d^2*(3*A + 2*B*n) + a*b*d*(-6*B*c*n + 6*A*d*x + B*d*n*x) + b^
2*(2*A*d^2*x^2 + B*c*n*(2*c - d*x))) - 4*a^3*B*d^3*n*Log[a + b*x] - 2*B*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - 3*a^3*d^3)*n*Log[c + d*x] + 2*B*d^3
*(3*a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/(6*b*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(105) = 210$.

Time = 18.61 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.09

method	result
parallelrisch	$\frac{6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a^{2}bc\,d^{2}n-6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a\,b^{2}c^{2}dn-6B\ln(bx+a)a^{2}bc\,d^{2}n^{2}+6B\ln(bx+a)a\,b^{2}c^{2}d\,n^{2}+}{}$
risch	Expression too large to display

[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)

[Out] 1/6*(6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c*d^2*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*c^2*d*n-6*B*ln(b*x+a)*a^2*b*c*d^2*n^2+6*B*ln(b*x+a)*a*b^2*c^2*d*n^2+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*d^3*n+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*d^3*n-6*B*x*a*b^2*c*d^2*n^2+B*a^2*b*c*d^2*n^2+5*B*a*b^2*c^2*d*n^2-12*A*a^2*b*c*d^2*n+6*A*x^2*a*b^2*d^3*n+4*B*x*a^2*b*d^3*n^2+2*B*x*b^3*c^2*d*n^2+6*A*x*a^2*b*d^3*n+2*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*n+B*x^2*a*b^2*d^3*n^2-B*x^2*b^3*c*d^2*n^2+2*B*ln(b*x+a)*a^3*d^3*n^2-2*B*ln(b*x+a)*b^3*c^3*n^2+2*A*x^3*b^3*d^3*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^3*n-4*B*a^3*d^3*n^2-2*B*b^3*c^3*n^2-6*A*a^3*d^3*n)/b/d^3/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{2\,Ab^3d^3x^3+(6\,Aab^2d^3-(Bb^3cd^2-Bab^2d^3)n)x^2+2\,(3\,Aa^2bd^3+(Bb^3c^2d-3\,Bab^2cd^2+2\,Ba^2bd^3)n)x+}{}$$

[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")

[Out] 1/6*(2*A*b^3*d^3*x^3 + (6*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 + 2*(3*A*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*n)*x + 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^3*n)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x)*log(e))/(b*d^3)

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.60

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx \\
&= \frac{1}{3} \, B b^2 x^3 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{3} \, A b^2 x^3 + B a b x^2 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A a b x^2 \\
&\quad + B a^2 x \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A a^2 x + \frac{\left(\frac{a e n \log(bx+a)}{b} - \frac{c e n \log(dx+c)}{d}\right) B a^2}{e} \\
&\quad - \frac{\left(\frac{a^2 e n \log(bx+a)}{b^2} - \frac{c^2 e n \log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right) B a b}{e} \\
&\quad + \frac{\left(\frac{2\,a^3 e n \log(bx+a)}{b^3} - \frac{2\,c^3 e n \log(dx+c)}{d^3} - \frac{(b^2 c d e n - a b d^2 e n)x^2 - 2\,(b^2 c^2 e n - a^2 d^2 e n)x}{b^2 d^2}\right) B b^2}{6\,e}
\end{aligned}$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + B*a^2*x*log((b*x + a)^n*e/(d*x +
 c)^n) + A*a^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^2/e -
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*a*b/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)
/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/
(b^2*d^2))*B*b^2/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(105) = 210$.

Time = 0.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.13

$$\begin{aligned}\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= \frac{Ba^3n\log(bx+a)}{3\,b}+\frac{1}{3}\,(Bb^2\log(e)+Ab^2)x^3\\&\quad-\frac{(Bb^2cn-Babdn-6\,Babd\log(e)-6\,Aabd)x^2}{6\,d}\\&\quad+\frac{1}{3}\left(Bb^2nx^3+3\,Babnx^2+3\,Ba^2nx\right)\log(bx+a)\\&\quad-\frac{1}{3}\left(Bb^2nx^3+3\,Babnx^2+3\,Ba^2nx\right)\log(dx+c)\\&\quad+\frac{(Bb^2c^2n-3\,Babcdn+2\,Ba^2d^2n+3\,Ba^2d^2\log(e)+3\,Aa^2d^2)x}{3\,d^2}\\&\quad-\frac{(Bb^2c^3n-3\,Babc^2dn+3\,Ba^2cd^2n)\log(-dx-c)}{3\,d^3}\end{aligned}$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/3*B*a^3*n*log(b*x + a)/b + 1/3*(B*b^2*log(e) + A*b^2)*x^3 - 1/6*(B*b^2*c*
n - B*a*b*d*n - 6*B*a*b*d*log(e) - 6*A*a*b*d)*x^2/d + 1/3*(B*b^2*n*x^3 + 3*
B*a*b*n*x^2 + 3*B*a^2*n*x)*log(b*x + a) - 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x^2
+ 3*B*a^2*n*x)*log(d*x + c) + 1/3*(B*b^2*c^2*n - 3*B*a*b*c*d*n + 2*B*a^2*d^
2*n + 3*B*a^2*d^2*log(e) + 3*A*a^2*d^2)*x/d^2 - 1/3*(B*b^2*c^3*n - 3*B*a*b*
c^2*d*n + 3*B*a^2*c*d^2*n)*log(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.32

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^2\,x+B\,a\,b\,x^2+\frac{B\,b^2\,x^3}{3}\right)$$
$$+x^2\,\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{3\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c}{d}\right)+\frac{A\,b^2\,x^3}{3}$$
$$-\frac{\ln(c+d\,x)\,\left(3\,B\,n\,a^2\,c\,d^2-3\,B\,n\,a\,b\,c^2\,d+B\,n\,b^2\,c^3\right)}{3\,d^3}+\frac{B\,a^3\,n\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^2,x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^2*x^3)/3 + B*a^2*x + B*a*b*x^2) + x^
2*((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*(3*a*d + 3*b*c)
)/(6*d)) - x*((((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(3*d) - (A*b*(3
*a*d + 3*b*c))/(3*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*(3*A*a*d + 3*A*b*c + B*
a*d*n - B*b*c*n))/d + (A*a*b*c)/d) + (A*b^2*x^3)/3 - (log(c + d*x)*(B*b^2*c
^3*n + 3*B*a^2*c*d^2*n - 3*B*a*b*c^2*d*n))/(3*d^3) + (B*a^3*n*log(a + b*x))
/(3*b)
```

3.150 $\int (a+bx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 84

$$\int (a+bx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = -\frac{B(bc-ad)nx}{2d}+\frac{B(bc-ad)^2n\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b}$$

```
[Out] -1/2*B*(-a*d+b*c)*n*x/d+1/2*B*(-a*d+b*c)^2*n*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 45}

$$\int (a+bx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}+\frac{Bn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bnx(bc-ad)}{2d}$$

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n*x)/d + (B*(b*c - a*d)^2*n*Log[c + d*x])/(2*b*d^2) + (
(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b} - \frac{(B(bc-ad)n)\int \frac{a+bx}{c+dx}\,dx}{2b} \\
&= \frac{(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b} - \frac{(B(bc-ad)n)\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{2b} \\
&= -\frac{B(bc-ad)nx}{2d} + \frac{B(bc-ad)^2 n\log(c+dx)}{2bd^2} + \frac{(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$
\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx
= \frac{-a^2Bd^2n\log(a+bx)+B(b^2c^2-2abcd+2a^2d^2)\,n\log(c+dx)+d(bx(2aAd-bBcn+aBdn+Abdx)+B}{2bd^2}
$$

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (-(a^2*B*d^2*n*Log[a + b*x]) + B*(b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*n*Log[c
+ d*x] + d*(b*x*(2*a*A*d - b*B*c*n + a*B*d*n + A*b*d*x) + B*d*(2*a^2 + 2*a*
b*x + b^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(78) = 156.

Time = 5.40 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

method	result
parallelrisch	$\frac{B x^2 \ln\left(e(bx+a)^n (dx+c)^{-n}\right) b^2 d^2 n + A x^2 b^2 d^2 n + B \ln(bx+a) a^2 d^2 n^2 - 2B \ln(bx+a) abcd n^2 + B \ln(bx+a) b^2 c^2 n^2 + 2Bx \ln\left(e(bx\right.}{}$
risch	Expression too large to display

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*n+A*x^2*b^2*d^2*n+B*ln(b*x+a
)*a^2*d^2*n^2-2*B*ln(b*x+a)*a*b*c*d*n^2+B*ln(b*x+a)*b^2*c^2*n^2+2*B*x*ln(e*
(b*x+a)^n/((d*x+c)^n))*a*b*d^2*n+B*x*a*b*d^2*n^2-B*x*b^2*c*d*n^2+2*A*x*a*b*
d^2*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*n-B*ln(e*(b*x+a)^n/((d*x+c)^n
))*b^2*c^2*n-B*a^2*d^2*n^2+B*b^2*c^2*n^2-2*A*a^2*d^2*n-3*A*a*b*c*d*n)/b/d^2
/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.94

$$\int (a+bx) \left(A + B\log\left(e(a+bx)^n (c+dx)^{-n}\right)\right) dx$$
$$= \frac{Ab^2d^2x^2 + (2\,Aabd^2 - (Bb^2cd - Babd^2)n)x + (Bb^2d^2nx^2 + 2\,Babd^2nx + Ba^2d^2n)\log(bx+a) - (Bb^2d^2n}{2\,bd^2}$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*x^2 + (2*A*a*b*d^2 - (B*b^2*c*d - B*a*b*d^2)*n)*x + (B*b^2*d
^2*n*x^2 + 2*B*a*b*d^2*n*x + B*a^2*d^2*n)*log(b*x + a) - (B*b^2*d^2*n*x^2 +
 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + (B*b^2*d^2*x
^2 + 2*B*a*b*d^2*x)*log(e))/(b*d^2)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{1}{2}Bbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}Abx^2+Bax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aax + \frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Ba}{e} - \frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bb}{2\,e}$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
[Out] 1/2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*b*x^2 + B*a*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A*a*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d
)*B*a/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e
*n - a*d*e*n)*x/(b*d))*B*b/e
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{Ba^2n\log(bx+a)}{2\,b}+\frac{1}{2}\left(Bb\log(e)+Ab\right)x^2 + \frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(bx+a)-\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(dx+c) - \frac{(Bbcn-Badn-2\,Bad\log(e)-2\,Aad)x}{2\,d}+\frac{(Bbc^2n-2\,Bacdn)\log(dx+c)}{2\,d^2}$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*B*a^2*n*log(b*x + a)/b + 1/2*(B*b*log(e) + A*b)*x^2 + 1/2*(B*b*n*x^2 +
2*B*a*n*x)*log(b*x + a) - 1/2*(B*b*n*x^2 + 2*B*a*n*x)*log(d*x + c) - 1/2*(B
*b*c*n - B*a*d*n - 2*B*a*d*log(e) - 2*A*a*d)*x/d + 1/2*(B*b*c^2*n - 2*B*a*c
*d*n)*log(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.51

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,b\,x^2}{2}+B\,a\,x\right) + x\left(\frac{4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{2\,d}-\frac{A\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,n-2\,B\,a\,c\,d\,n\right)}{2\,d^2}+\frac{A\,b\,x^2}{2}+\frac{B\,a^2\,n\,\ln(a+b\,x)}{2\,b}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*a*x + (B*b*x^2)/2) + x*((4*A*a*d + 2*A*
b*c + B*a*d*n - B*b*c*n)/(2*d) - (A*(2*a*d + 2*b*c))/(2*d)) + (log(c + d*x)
*(B*b*c^2*n - 2*B*a*c*d*n))/(2*d^2) + (A*b*x^2)/2 + (B*a^2*n*log(a + b*x))/
(2*b)
```

3.151 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b}+\frac{Bn\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b+B*n*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b} \\ &\quad + \frac{(B(bc-ad)n)\int\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{b} \\ &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b} \\ &\quad + \frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx,x,a+bx\right)}{b^2}\end{aligned}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b} - \frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}dx, x, \frac{1}{a+bx}\right)}{b^2}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b} - \frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}dx, x, \frac{1}{a+bx}\right)}{b^2}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b} + \frac{Bn\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}dx = \frac{-Bn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2A\log(a+bx)-2B\log\left(\frac{-bc+ad}{d(a+bx)}\right)\left(n\log\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)+}{2b}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
[Out] (-(B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + 2*A*Log[a + b*x] - 2*B*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*(n*Log[(b*(c + d*x))/(b*c - a*d)] + Log[(e*(a +
 b*x)^n)/(c + d*x)^n]) + 2*B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(2
*b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.59 (sec) , antiderivative size = 523, normalized size of antiderivative = 6.62

method	result
risch	$-\frac{B\ln(bx+a)\ln((dx+c)^n)}{b}+\frac{Bn\,\text{dilog}\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{b}+\frac{Bn\ln(bx+a)\ln\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{b}+\frac{iB\pi\ \text{csgn}(ie)\,\text{csgn}\left(ie(dx+c)^{-}\right.}{2b}$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b*ln(b*x+a)*ln((d*x+c)^n)+1/b*B*n*dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))
+1/b*B*n*ln(b*x+a)*ln((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))+1/2*I/b*B*Pi*csgn(I*
e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I*(b*x+a)^
n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*ln(b*x+a)+A*ln(b*x+a)/b+1/b*
B*ln(e)*ln(b*x+a)+1/2/b*B/n*ln((b*x+a)^n)^2-1/2*I/b*B*Pi*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^3*ln(b*x+a)-1/2*I/b*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3*ln(b
*x+a)-1/2*I/b*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)*ln(b*x+a)-1/2*I/b*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))*ln(b*x+a)
```

Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="fricas")
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{A+B\log\left(e\left(a+bx\right)^n\left(c+dx\right)^{-n}\right)}{a+bx}\,dx$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a),x)
[Out] Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))/(a + b*x), x)
```

Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="maxima")
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + inte
grate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x^2
 + a*b*c + (b^2*c + a*b*d)*x), x)) + A*log(b*x + a)/b
```

Giac [F]

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{a + bx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx + a}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")

[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{a + bx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{a + b\,x}\,dx$$

[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x),x)

[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x), x)

3.152 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 97

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=-\frac{Bn}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}$$

```
[Out] -B*n/b/(b*x+a)-B*d*n*ln(b*x+a)/b/(-a*d+b*c)+B*d*n*ln(d*x+c)/b/(-a*d+b*c)+(-
A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 46}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{Bn}{b(a+bx)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] -((B*n)/(b*(a + b*x))) - (B*d*n*Log[a + b*x])/(b*(b*c - a*d)) + (B*d*n*Log[
c + d*x])/(b*(b*c - a*d)) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*(a
+ b*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{b}\\
&= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx}{b}\\
&= -\frac{Bn}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$$
$$=\frac{-Bdn(a+bx)\log(a+bx)+Bdn(a+bx)\log(c+dx)-(bc-ad)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.}{b(bc-ad)(a+bx)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] (-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a*
d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*x)
)
```

Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

method	result
parallelrisch	$-\frac{-Bx\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}d^{2}n-B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}cdn+Ba\,b^{2}d^{2}n^{2}-B\,b^{3}cd\,n^{2}+Aa\,b^{2}d^{2}n-A\,b^{3}cdn}{(bx+a)b^{3}dn(ad-cb)}$
risch	$\frac{B\ln((dx+c)^{n})}{b(bx+a)}-\frac{2Abc-2Badn+2Bbcn-2Aad-2Bad\ln((bx+a)^{n})-2B\ln(e)ad-2B\ln(dx+c)adn+2B\ln(-bx-a)adn+iB\pi}{}$

[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -(-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n+B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n)/(b*x+a)/b^3/d/n/(a*d-b*c)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbdnx+Bbcn)\log(bx+a)-(Bbdnx+Bbcn)\log(dx+c)+(Bbc-}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="fricas")

[Out] -(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log(b*x + a) - (B*b*d*n*x + B*b*c*n)*log(d*x + c) + (B*b*c - B*a*d)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \text{Timed out}$$

[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)B}{e} - \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{b^2x+ab} - \frac{A}{b^2x+ab}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] -(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) +
 e*n/(b^2*x + a*b))*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) -
A/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Bdn\log(bx+a)}{b^2c-abd}+\frac{Bdn\log(dx+c)}{b^2c-abd} - \frac{Bn\log(bx+a)}{b^2x+ab}+\frac{Bn\log(dx+c)}{b^2x+ab} - \frac{Bn+B\log(e)+A}{b^2x+ab}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -B*d*n*log(b*x + a)/(b^2*c - a*b*d) + B*d*n*log(d*x + c)/(b^2*c - a*b*d) -
B*n*log(b*x + a)/(b^2*x + a*b) + B*n*log(d*x + c)/(b^2*x + a*b) - (B*n + B*
log(e) + A)/(b^2*x + a*b)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=-\frac{A+B\,n}{x\,b^2+a\,b}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{b\,\left(a+b\,x\right)}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^2,x)
```

```
[Out] - (A + B*n)/(a*b + b^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(b*(a + b*
x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c))
```

3.153 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}$$

```
[Out] -1/4*B*n/b/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/(b*x+a)+1/2*B*d^2*n*ln(b*x+a)/b
/(-a*d+b*c)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2+1/2*(-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/b/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 46}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}+\frac{Bdn}{2b(a+bx)(bc-ad)}-\frac{Bn}{4b(a+bx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -1/4*(B*n)/(b*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*(a + b*x)) + (B*d^2*n
*Log[a + b*x])/(2*b*(b*c - a*d)^2) - (B*d^2*n*Log[c + d*x])/(2*b*(b*c - a*d
)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(a + b*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2b}\\
&= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{2b}\\
&= -\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}\\
&\quad-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$
\int\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2A}{(a+bx)^2}+Bn\left(\frac{1+\frac{2d(a+bx)}{-bc+ad}}{(a+bx)^2}-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}}{4b}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -1/4*((2*A)/(a + b*x)^2 + B*n*((1 + (2*d*(a + b*x))/(-(b*c) + a*d))/(a + b*
x)^2 - (2*d^2*Log[a + b*x]])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x]])/(b*c - a*d
)^2) + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]])/(a + b*x)^2)/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(130) = 260.

Time = 19.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

method	result
parallelrisch	$-\frac{-4Bac\,d^2n\,b^4+2B\ln(dx+c)x^2b^5d^3n-2B\ln(bx+a)a^2b^3d^3n+2B\ln(dx+c)a^2b^3d^3n+2Bxa\,b^4d^3n-2Bx\,b^5c\,d^2n-4B\ln\left(e(bx+\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*B*a*c*d^2*n*b^4+2*B*ln(d*x+c)*x^2*b^5*d^3*n-2*B*ln(b*x+a)*a^2*b^3*
d^3*n+2*B*ln(d*x+c)*a^2*b^3*d^3*n+2*B*x*a*b^4*d^3*n-2*B*x*b^5*c*d^2*n-4*B*l
n(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+2*A*a^2*b^3*d^3+2*A*b^5*c^2*d-4*A*a*
b^4*c*d^2-2*B*ln(b*x+a)*x^2*b^5*d^3*n+3*B*a^2*b^3*d^3*n+2*B*ln(e*(b*x+a)^n/
((d*x+c)^n))*a^2*b^3*d^3+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d+B*b^5*c^
2*n*d-4*B*ln(b*x+a)*x*a*b^4*d^3*n+4*B*ln(d*x+c)*x*a*b^4*d^3*n)/(a^2*d^2-2*a
*b*c*d+b^2*c^2)/(b*x+a)^2/b^4/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(127) = 254.

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.16

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx+(Bb^2c^2-4\,Babcd+3\,Ba^2d^2)n-2\,(Bb^2d^2nx^2+}{4\,(a^2b^3c^2-2\,a^3b^2cd+}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2*d^2*n*x^2 + 2*B*a
*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(b*x + a) + 2*(B*b^2*d^2*n*x^2
 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + 2*(B*b^2*c
^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*
d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*
c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.68

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2} - \frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2} + \frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)B}{4\,e} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{A}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="maxima
")
[Out] 1/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n
+ 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*b^2*
x + a^2*b) - 1/2*A/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.77

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \frac{Bd^2n\log(bx+a)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)} - \frac{Bd^2n\log(dx+c)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)} - \frac{Bn\log(bx+a)}{2\,(b^3x^2+2\,ab^2x+a^2b)} + \frac{Bn\log(dx+c)}{2\,(b^3x^2+2\,ab^2x+a^2b)} + \frac{2\,Bbdnx - Bbcn + 3\,Badn - 2\,Bbc\log(e) + 2\,Bad\log(e) - 2\,Abc + 2\,Aad}{4\,(b^4cx^2 - ab^3dx^2 + 2\,ab^3cx - 2\,a^2b^2dx + a^2b^2c - a^3bd)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*d^2*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*d^2*n*
log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*n*log(b*x + a)/(b^
3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*B*n*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a^
2*b) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*B*b*c*log(e) + 2*B*a*d*lo
g(e) - 2*A*b*c + 2*A*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a^2*b^
2*d*x + a^2*b^2*c - a^3*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.40

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2} - \frac{B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (B*log((e*(a + b*x)^n)/(c
+ d*x)^n))/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) - (B*d^2*n*atanh((2*b^3*c^2 - 2*
a^2*b*d^2)/(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

3.154 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 166

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = & -\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2} \\ & -\frac{Bd^2n}{3b(bc-ad)^2(a+bx)} \\ & -\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3} \\ & -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}\end{aligned}$$

```
[Out] -1/9*B*n/b/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/(b*x+a)^2-1/3*B*d^2*n/b/(-a*d+b
*c)^2/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3+1/3*B*d^3*n*ln(d*x+c)/b/
(-a*d+b*c)^3+1/3*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 46}

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = & -\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3} \\ & -\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3} \\ & -\frac{Bd^2n}{3b(a+bx)(bc-ad)^2} \\ & +\frac{Bdn}{6b(a+bx)^2(bc-ad)}-\frac{Bn}{9b(a+bx)^3}\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -1/9*(B*n)/(b*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*(a + b*x)^2) - (B*d^2
*n)/(3*b*(b*c - a*d)^2*(a + b*x)) - (B*d^3*n*Log[a + b*x])/(3*b*(b*c - a*d)
^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3) - (A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/(3*b*(a + b*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3b}\\
&= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)}{3b}\\
&= -\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2(a+bx)}\\
&\quad-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= -\frac{\frac{6A}{(a+bx)^3}+Bn\left(\frac{2+\frac{3d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}}{(a+bx)^3}+\frac{6d^3\log(a+bx)}{(bc-ad)^3}-\frac{6d^3\log(c+dx)}{(bc-ad)^3}\right)+\frac{6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}}{18b}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -1/18*((6*A)/(a + b*x)^3 + B*n*((2 + (3*d*(a + b*x))/(-(b*c) + a*d) + (6*d^
2*(a + b*x)^2)/(b*c - a*d)^2)/(a + b*x)^3 + (6*d^3*Log[a + b*x])/(b*c - a*d
)^3 - (6*d^3*Log[c + d*x])/(b*c - a*d)^3) + (6*B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(a + b*x)^3)/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(157) = 314.

Time = 54.43 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.04

method	result
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-6B\ln(bx+a)x^3b^7d^4n+6B\,x^2a\,b^6d^4n-6B\,x^2b^7c\,d^3n+15Bx\,a^2b^5d^4n+3Bx\,b^7c^2d^2n+6B\ln(d}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(-18*A*a^2*b^5*c*d^3+18*A*a*b^6*c^2*d^2+6*B*ln(e*(b*x+a)^n/((d*x+c)^n
))*a^3*b^4*d^4-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d-6*B*ln(b*x+a)*x^3*
b^7*d^4*n+6*B*x^2*a*b^6*d^4*n-6*B*x^2*b^7*c*d^3*n+15*B*x*a^2*b^5*d^4*n+3*B*
x*b^7*c^2*d^2*n-18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3+18*B*ln(e*(b
*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2+6*B*ln(d*x+c)*x^3*b^7*d^4*n-6*B*ln(b*x+a
)*a^3*b^4*d^4*n+6*B*ln(d*x+c)*a^3*b^4*d^4*n-18*B*x*a*b^6*c*d^3*n-18*B*ln(b*
x+a)*x^2*a*b^6*d^4*n+18*B*ln(d*x+c)*x^2*a*b^6*d^4*n-18*B*ln(b*x+a)*x*a^2*b^
5*d^4*n+18*B*ln(d*x+c)*x*a^2*b^5*d^4*n-18*B*a^2*b^5*c*d^3*n+9*B*a*b^6*c^2*d
^2*n+6*A*a^3*b^4*d^4-6*A*b^7*c^3*d+11*B*a^3*b^4*d^4*n-2*B*b^7*c^3*d*n)/(a^3
*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^3/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(154) = 308.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.25

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(Bb^3c^2d - 6\,Bab^2cd^2 + 5\,B}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*
a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a
^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x +
(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(b*x + a) - 6*(B*b^3*
d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^
2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d
 + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3
*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^
3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(154) = 308$.

Time = 0.20 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.41

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{\left(\frac{6\,d^3en\log(bx+a)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{6\,d^3en\log(dx+c)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} + \frac{6\,b^2d^2enx^2+2\,b^2c^2en-7\,abcden+11\,a^2\cdots}{a^3b^3c^2-2\,a^4b^2cd+a^5bd^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^3+3\,(ab^5c^2\cdots}\right.}{18\,e} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{A}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*
a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c
*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 -
 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x^
2 + 3*a^2*b^2*x + a^3*b) - 1/3*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3
*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(154) = 308$.

Time = 0.29 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.73

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{Bd^3n\log(bx+a)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)} + \frac{Bd^3n\log(dx+c)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)} - \frac{Bn\log(bx+a)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} + \frac{Bn\log(dx+c)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{6\,Bb^2d^2nx^2-3\,Bb^2cdnx+15\,Babd^2nx+2\,Bb^2c^2n-7\,Babcdn+11\,Ba^2d^2n+6\,Bb^2c^2\log(e)-12\,Ba\cdots}{18\,(b^6c^2x^3-2\,ab^5cdx^3+a^2b^4d^2x^3+3\,ab^5c^2x^2-6\,a^2b^4cdx^2+3\,a^3b^3d^2x^2+3\,a^2b^4c^2x-6\,a\cdots}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
```

```
2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x
+ a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*
b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*B*b^2
*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*B*b^2*c^2*log(e) - 12*B*a*b*c*d
*log(e) + 6*B*a^2*d^2*log(e) + 6*A*b^2*c^2 - 12*A*a*b*c*d + 6*A*a^2*d^2)/(b
^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^
4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b
^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{3\,b\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*(a*d - b*c)^2*(a +
 b*x)^3) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(3*b*(a + b*x)^3) - (A*a^2*
d^2)/(3*b*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2*n)/(9*(a*d - b*c)^2*(a + b*
x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*(a
*d - b*c)^3) - (5*B*a*d^2*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3) - (B*b*d^2*n*x
^2)/(3*(a*d - b*c)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*(a*d - b*c)^2*(a + b*
x)^3) - (11*B*a^2*d^2*n)/(18*b*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(
6*(a*d - b*c)^2*(a + b*x)^3)
```

3.155 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5} dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5} dx = -\frac{Bn}{16b(a+bx)^4}+\frac{Bdn}{12b(bc-ad)(a+bx)^3} -\frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2}+\frac{Bd^3n}{4b(bc-ad)^3(a+bx)} +\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4} -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}$$

```
[Out] -1/16*B*n/b/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/(b*x+a)^3-1/8*B*d^2*n/b/(-a*d
+b*c)^2/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/(b*x+a)+1/4*B*d^4*n*ln(b*x+a)/
b/(-a*d+b*c)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4+1/4*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/b/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{2548, 46\}$

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4} + \frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4} - \frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4} + \frac{Bd^3n}{4b(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8b(a+bx)^2(bc-ad)^2} + \frac{Bdn}{12b(a+bx)^3(bc-ad)} - \frac{Bn}{16b(a+bx)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

```
[Out] -1/16*(B*n)/(b*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)*(a + b*x)^3) - (B*d
^2*n)/(8*b*(b*c - a*d)^2*(a + b*x)^2) + (B*d^3*n)/(4*b*(b*c - a*d)^3*(a + b
*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4) - (B*d^4*n*Log[c + d*x])/
(4*b*(b*c - a*d)^4) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*(a + b*
x)^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule 2548

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{4b}$$

$$= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2} + \frac{bd^4}{(bc-ad)^5(a+bx)} - \right.}{4b}$$

$$= -\frac{Bn}{16b(a+bx)^4} + \frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2} + \frac{Bd^3n}{4b(bc-ad)^3(a+bx)} + \frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4} - \frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{\frac{12A}{(a+bx)^4} + Bn\left(\frac{3+\frac{4d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}-\frac{12d^3(a+bx)^3}{(bc-ad)^3}}{(a+bx)^4} - \frac{12d^4\log(a+bx)}{(bc-ad)^4} + \frac{12d^4\log(c+dx)}{(bc-ad)^4}\right) + \frac{12B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}}{48b}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]

[Out] -1/48*((12*A)/(a + b*x)^4 + B*n*((3 + (4*d*(a + b*x))/(-(b*c) + a*d) + (6*d
^2*(a + b*x)^2)/(b*c - a*d)^2 - (12*d^3*(a + b*x)^3)/(b*c - a*d)^3)/(a + b*
x)^4 - (12*d^4*Log[a + b*x])/(b*c - a*d)^4 + (12*d^4*Log[c + d*x])/(b*c - a
*d)^4) + (12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4)/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. 2(184) = 368.

Time = 135.60 (sec) , antiderivative size = 2309, normalized size of antiderivative = 11.84

method	result	size
parallelrisch	Expression too large to display	2309
risch	Expression too large to display	2583

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] 1/48*(48*B*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*n-288*B*ln(b*x+a)*x^3*a^5*b^4*c^3*
d^2*n+192*B*ln(b*x+a)*x^3*a^4*b^5*c^4*d*n-192*B*ln(d*x+c)*x^3*a^6*b^3*c^2*d
^3*n+288*B*ln(d*x+c)*x^3*a^5*b^4*c^3*d^2*n-192*B*ln(d*x+c)*x^3*a^4*b^5*c^4*
d*n+288*B*ln(b*x+a)*x^2*a^7*b^2*c^2*d^3*n-432*B*ln(b*x+a)*x^2*a^6*b^3*c^3*d
^2*n+288*B*ln(b*x+a)*x^2*a^5*b^4*c^4*d*n-288*B*ln(d*x+c)*x^2*a^7*b^2*c^2*d^
3*n+432*B*ln(d*x+c)*x^2*a^6*b^3*c^3*d^2*n-288*B*ln(d*x+c)*x^2*a^5*b^4*c^4*d
*n+192*B*ln(b*x+a)*x*a^8*b*c^2*d^3*n-288*B*ln(b*x+a)*x*a^7*b^2*c^3*d^2*n+19
2*B*ln(b*x+a)*x*a^6*b^3*c^4*d*n-192*B*ln(d*x+c)*x*a^8*b*c^2*d^3*n+288*B*ln(
```

d*x+c)*x*a^7*b^2*c^3*d^2*n-192*B*ln(d*x+c)*x*a^6*b^3*c^4*d*n+12*A*x^4*a^2*b
^7*c^5+48*A*x^3*a^3*b^6*c^5+72*A*x^2*a^4*b^5*c^5+48*A*x*a^9*c*d^4+48*A*x*a^
5*b^4*c^5+48*B*ln(b*x+a)*a^9*c^2*d^3*n-12*B*ln(b*x+a)*a^6*b^3*c^5*n-48*B*ln
(d*x+c)*a^9*c^2*d^3*n+12*B*ln(d*x+c)*a^6*b^3*c^5*n+12*B*x^4*ln(e*(b*x+a)^n/
((d*x+c)^n))*a^2*b^7*c^5+3*B*x^4*a^2*b^7*c^5*n+12*B*x^4*ln(e*(b*x+a)^n/((d*
x+c)^n))*a^6*b^3*c*d^4-48*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^2*d^3
+72*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^3*d^2-48*B*x^4*ln(e*(b*x+a)
^n/((d*x+c)^n))*a^3*b^6*c^4*d+25*B*x^4*a^6*b^3*c*d^4*n-48*B*x^4*a^5*b^4*c^2
*d^3*n+36*B*x^4*a^4*b^5*c^3*d^2*n-16*B*x^4*a^3*b^6*c^4*d*n-192*B*x*ln(e*(b*
x+a)^n/((d*x+c)^n))*a^8*b*c^2*d^3+288*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b
^2*c^3*d^2-192*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^4*d-120*B*x*a^8*b*
c^2*d^3*n+120*B*x*a^7*b^2*c^3*d^2*n-60*B*x*a^6*b^3*c^4*d*n-48*A*x^4*a^5*b^4
*c^2*d^3+72*A*x^4*a^4*b^5*c^3*d^2-48*A*x^4*a^3*b^6*c^4*d+12*A*x^4*a^6*b^3*c
*d^4+72*A*x^2*a^8*b*c*d^4-288*A*x^2*a^7*b^2*c^2*d^3+432*A*x^2*a^6*b^3*c^3*d
^2-288*A*x^2*a^5*b^4*c^4*d-192*A*x*a^8*b*c^2*d^3+288*A*x*a^7*b^2*c^3*d^2-19
2*A*x*a^6*b^3*c^4*d-192*A*x^3*a^6*b^3*c^2*d^3+288*A*x^3*a^5*b^4*c^3*d^2-192
*A*x^3*a^4*b^5*c^4*d+72*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^8*b*c*d^4-288*B
*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c^2*d^3+432*B*x^2*ln(e*(b*x+a)^n/(
(d*x+c)^n))*a^6*b^3*c^3*d^2-288*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c
^4*d+108*B*x^2*a^8*b*c*d^4*n-240*B*x^2*a^7*b^2*c^2*d^3*n+210*B*x^2*a^6*b^3*
c^3*d^2*n-96*B*x^2*a^5*b^4*c^4*d*n-12*B*ln(b*x+a)*x^4*a^2*b^7*c^5*n+12*B*ln
(d*x+c)*x^4*a^2*b^7*c^5*n-48*B*ln(b*x+a)*x^3*a^3*b^6*c^5*n+48*B*ln(d*x+c)*x
^3*a^3*b^6*c^5*n+48*A*x^3*a^7*b^2*c*d^4+48*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n)
)*a^7*b^2*c*d^4-192*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3+288*B
*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^3*d^2-192*B*x^3*ln(e*(b*x+a)^n/(
(d*x+c)^n))*a^4*b^5*c^4*d+88*B*x^3*a^7*b^2*c*d^4*n-180*B*x^3*a^6*b^3*c^2*d^
3*n+144*B*x^3*a^5*b^4*c^3*d^2*n-64*B*x^3*a^4*b^5*c^4*d*n-72*B*ln(b*x+a)*x^2
*a^4*b^5*c^5*n+72*B*ln(d*x+c)*x^2*a^4*b^5*c^5*n-48*B*ln(b*x+a)*x*a^5*b^4*c^
5*n+48*B*ln(d*x+c)*x*a^5*b^4*c^5*n-72*B*ln(b*x+a)*a^8*b*c^3*d^2*n+48*B*ln(b
*x+a)*a^7*b^2*c^4*d*n+72*B*ln(d*x+c)*a^8*b*c^3*d^2*n-48*B*ln(d*x+c)*a^7*b^2
*c^4*d*n+48*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^5+12*B*x^3*a^3*b^6*
c^5*n+72*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^5+18*B*x^2*a^4*b^5*c^5
*n+48*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^9*c*d^4+48*B*x*ln(e*(b*x+a)^n/((d*x
+c)^n))*a^5*b^4*c^5+48*B*x*a^9*c*d^4*n+12*B*x*a^5*b^4*c^5*n-72*B*ln(b*x+a)*
x^4*a^4*b^5*c^3*d^2*n+48*B*ln(b*x+a)*x^4*a^3*b^6*c^4*d*n-48*B*ln(d*x+c)*x^4
*a^5*b^4*c^2*d^3*n+72*B*ln(d*x+c)*x^4*a^4*b^5*c^3*d^2*n-48*B*ln(d*x+c)*x^4*
a^3*b^6*c^4*d*n+192*B*ln(b*x+a)*x^3*a^6*b^3*c^2*d^3*n)/(a^4*d^4-4*a^3*b*c*d
^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^4/a^6/c

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(181) = 362$.

Time = 0.29 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.21

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 + 6\,(Bb^4c}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*
d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3
*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a
*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n - 12
*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b
*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c
*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2
*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2
*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c) + 12*(B*b^4*c^4 - 4*B*a*b^3
*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a^4*d^4)*log(e))/(a^4*b^
5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 +
 (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d
^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*
d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d
^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d
+ 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(181) = 362.

Time = 0.21 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.17

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{\left(\frac{12\,d^4en\log(bx+a)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} - \frac{12\,d^4en\log(dx+c)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} + \frac{12}{a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3+(b^8c}\right.}{}$$
$$- \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}$$
$$- \frac{A}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] 1/48*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n
*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3
*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2
*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^
2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2
 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*B/
e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3
*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2
+ 4*a^3*b^2*x + a^4*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(181) = 362$.

Time = 0.31 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.68

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{Bd^4n\log(bx+a)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)} - \frac{Bd^4n\log(dx+c)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)} - \frac{Bn\log(bx+a)}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)} + \frac{Bn\log(dx+c)}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)} + \frac{12\,Bb^3d^3nx^3-6\,Bb^3cd^2nx^2+42\,Bab^2d^3nx^2+4\,Bb^3c^2dnx-20\,Bab^2cd^2nx+52\,Ba^2bd^3nx-3\,Bb^3c^3n}{48\left(b^8c^3x^4-3\,ab^7c^2dx^4+3\,a^2b^6cd^2x^4-a^3b^5d^3x^4+4\,ab^7c^3x^3-12\,a^2b^6c^2dx^3+12\,a^3b^5cd^2x^3\right.}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a
^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3
*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x + a
)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B*n*l
og(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) +
 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^2 + 4*
B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^3*c^3*n
 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*B*b^3*c^3*
log(e) + 36*B*a*b^2*c^2*d*log(e) - 36*B*a^2*b*c*d^2*log(e) + 12*B*a^3*d^3*l
og(e) - 12*A*b^3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 + 12*A*a^3*d^3)/
(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 +
4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d
^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 -
6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2
*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^
7*b*d^3)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,a\,b\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b\right.}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{4\,b\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)}$$

$$-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,{\left(a\,d-b\,c\right)}^4}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x +
16*a*b^4*x^3 + 24*a^2*b^3*x^2) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(4*b*
(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B*d^4*n*atanh
((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 - 8*a*b^4*c^3*d)/(4*b*(a*d - b*
c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.156 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

Optimal result

Integrand size = 33, antiderivative size = 322

$$\begin{aligned}
&\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)n(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} \\
&\quad+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^2n(a+bx)^2\left(3A+Bn+3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2} \\
&\quad-\frac{B(bc-ad)^3n(a+bx)\left(6A+5Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^3} \\
&\quad-\frac{B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^4} \\
&\quad-\frac{B^2(bc-ad)^4n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

```
[Out] -1/6*B*(-a*d+b*c)*n*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/4*(b*
x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/12*B*(-a*d+b*c)^2*n*(b*x+a)^
2*(3*A+B*n+3*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-1/12*B*(-a*d+b*c)^3*n*(b*
x+a)*(6*A+5*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3-1/12*B*(-a*d+b*c)^4*
n*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d
^4-1/2*B^2*(-a*d+b*c)^4*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2573, 2549, 2381, 2384, 2354, 2438}

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= -\frac{Bn(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+6A+11Bn\right)}{12bd^4} - \frac{Bn(a+bx)(bc-ad)^3\left(6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+6A+5Bn\right)}{12bd^3} + \frac{Bn(a+bx)^2(bc-ad)^2\left(3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3A+Bn\right)}{12bd^2} - \frac{Bn(a+bx)^3(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{6bd} + \frac{(a+bx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4b} - \frac{B^2n^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(b*d) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b) + (
B*(b*c - a*d)^2*n*(a + b*x)^2*(3*A + B*n + 3*B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*n*(a + b*x)*(6*A + 5*B*n + 6*B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*n*Log[(b*c - a*d)
/(b*(c + d*x))]*(6*A + 11*B*n + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(12*
b*d^4) - (B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2
*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^4\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^5}dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} - \text{Subst}\left(\frac{\left(B(bc-ad)^4 n\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} + \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} + \text{Subst}\left(\frac{\left(B(bc-ad)^4 n\right)\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} + \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} + \frac{B(bc-ad)^2 n(a+bx)^2 \left(3A+Bn+3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2} - \text{Subst}\left(\frac{\left(B(bc-ad)^4 n\right)\text{Subst}\left(\int \frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{12bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} + \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} + \frac{B(bc-ad)^2n(a+bx)^2 \left(3A+Bn+3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2} - \frac{B(bc-ad)^3n(a+bx) \left(6A+5Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^3} + \text{Subst}\left(\frac{\left(B(bc-ad)^4n\right)\text{Subst}\left(\int \frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{12bd^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} + \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} + \frac{B(bc-ad)^2n(a+bx)^2 \left(3A+Bn+3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2} - \frac{B(bc-ad)^3n(a+bx) \left(6A+5Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^3} - \frac{B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^4} + \text{Subst}\left(\frac{\left(B^2(bc-ad)^4n^2\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^3 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}{6bd} + \frac{(a+bx)^4 (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{4b} + \frac{B(bc-ad)^2n(a+bx)^2 (3A+Bn+3B\log(e(a+bx)^n(c+dx)^{-n}))}{12bd^2} - \frac{B(bc-ad)^3n(a+bx) (6A+5Bn+6B\log(e(a+bx)^n(c+dx)^{-n}))}{12bd^3} - \frac{B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right) (6A+11Bn+6B\log(e(a+bx)^n(c+dx)^{-n}))}{12bd^4} - \frac{B^2(bc-ad)^4n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1709 vs. $2(322) = 644$.

Time = 0.99 (sec) , antiderivative size = 1709, normalized size of antiderivative = 5.31

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$

$$= \frac{-24a^4ABd^4n + 6ab^3B^2c^3dn^2 - 24a^2b^2B^2c^2d^2n^2 + 36a^3bB^2cd^3n^2 - 24a^4B^2d^4n^2 + 12a^3A^2bd^4x - 6Ab^4Bc^3}{}$$

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-24*a^4*A*B*d^4*n + 6*a*b^3*B^2*c^3*d*n^2 - 24*a^2*b^2*B^2*c^2*d^2*n^2 + 3
6*a^3*b*B^2*c*d^3*n^2 - 24*a^4*B^2*d^4*n^2 + 12*a^3*A^2*b*d^4*x - 6*A*b^4*B
*c^3*d*n*x + 24*a*A*b^3*B*c^2*d^2*n*x - 36*a^2*A*b^2*B*c*d^3*n*x + 18*a^3*A
*b*B*d^4*n*x - 5*b^4*B^2*c^3*d*n^2*x + 17*a*b^3*B^2*c^2*d^2*n^2*x - 19*a^2*
b^2*B^2*c*d^3*n^2*x + 7*a^3*b*B^2*d^4*n^2*x + 18*a^2*A^2*b^2*d^4*x^2 + 3*A*
b^4*B*c^2*d^2*n*x^2 - 12*a*A*b^3*B*c*d^3*n*x^2 + 9*a^2*A*b^2*B*d^4*n*x^2 +
b^4*B^2*c^2*d^2*n^2*x^2 - 2*a*b^3*B^2*c*d^3*n^2*x^2 + a^2*b^2*B^2*d^4*n^2*x
^2 + 12*a*A^2*b^3*d^4*x^3 - 2*A*b^4*B*c*d^3*n*x^3 + 2*a*A*b^3*B*d^4*n*x^3 +
 3*A^2*b^4*d^4*x^4 - 3*a^4*B^2*d^4*n^2*Log[a + b*x]^2 + 6*A*b^4*B*c^4*n*Log
[c + d*x] - 24*a*A*b^3*B*c^3*d*n*Log[c + d*x] + 36*a^2*A*b^2*B*c^2*d^2*n*Lo
g[c + d*x] - 24*a^3*A*b*B*c*d^3*n*Log[c + d*x] + 11*b^4*B^2*c^4*n^2*Log[c +
 d*x] - 38*a*b^3*B^2*c^3*d*n^2*Log[c + d*x] + 45*a^2*b^2*B^2*c^2*d^2*n^2*Lo
g[c + d*x] - 18*a^3*b*B^2*c*d^3*n^2*Log[c + d*x] - 24*a^4*B^2*d^4*n^2*Log[c
 + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3*d*n^2*Log[c +
 d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3*b*B^2*c*d^3*n^
2*Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a
^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^2*c^3*d*n*x*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Log[(e*(a + b*x)^n)/
```

```
(c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*A*b^2*B*d^4*
x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*b^4*B^2*c^2*d^2*n*x^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n] - 12*a*b^3*B^2*c*d^3*n*x^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n] + 9*a^2*b^2*B^2*d^4*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*A*b
^3*B*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b^4*B^2*c*d^3*n*x^3*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 2*a*b^3*B^2*d^4*n*x^3*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 6*A*b^4*B*d^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^4*B^2*
c^4*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a*b^3*B^2*c^3*d*n*
Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*b^2*B^2*c^2*d^2*n*Lo
g[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a^3*b*B^2*c*d^3*n*Log[c +
d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a^3*b*B^2*d^4*x*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2 + 18*a^2*b^2*B^2*d^4*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + 12*a*b^3*B^2*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^4*B^2*d
^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Log[a + b*x]*(-6*b*B*c*(b^3
*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*n*Log[c + d*x] + 6*B*(b*c
 - a*d)^4*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-6*b^3*B*c^3*n + 21*a*b^2
*B*c^2*d*n - 26*a^2*b*B*c*d^2*n + a^3*d^3*(6*A + 35*B*n) + 6*a^3*B*d^3*Log[
(e*(a + b*x)^n)/(c + d*x)^n])) + 6*B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(12*b*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 269.19 (sec) , antiderivative size = 10586, normalized size of antiderivative = 32.88

method	result	size
risch	Expression too large to display	10586

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (bx+a)^3\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b^3*x^3 + 3*A^2*a*b^2*x^2 + 3*A^2*a^2*b*x + A^2*a^3 + (B^2*b^3
*x^3 + 3*B^2*a*b^2*x^2 + 3*B^2*a^2*b*x + B^2*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^2 + 2*(A*B*b^3*x^3 + 3*A*B*a*b^2*x^2 + 3*A*B*a^2*b*x + A*B*a^3)*log((
b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(309) = 618$.

Time = 0.73 (sec) , antiderivative size = 1871, normalized size of antiderivative = 5.81

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
[Out] 1/2*A*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^2*b^3*x^4 + 2*A*B*a*
b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^2*a^2*b*x^2 + 2*A*B*a^3*x*log((b*x + a
)^n*e/(d*x + c)^n) + A^2*a^3*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x +
c)/d)*A*B*a^3/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a^2*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*
c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*
n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*a*b^2/e - 1/12*(6*a^4*e*n*log(b*x + a)/b
^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 -
3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b
^3*d^3))*A*B*b^3/e + 1/12*((11*n^2 + 6*n*log(e))*b^3*c^4 - 2*(19*n^2 + 12*n
*log(e))*a*b^2*c^3*d + 9*(5*n^2 + 4*n*log(e))*a^2*b*c^2*d^2 - 6*(3*n^2 + 4*
n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 - 4*a*b^3*c^3*
d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n^2)*(log(b*x +
 a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))
*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*x^4*log(e)^2 - 3*B^2*a^4*d^4*n^2*log(b*x
 + a)^2 - 2*(b^4*c*d^3*n*log(e) - (n*log(e) + 6*log(e)^2)*a*b^3*d^4)*B^2*x^
3 + ((n^2 + 3*n*log(e))*b^4*c^2*d^2 - 2*(n^2 + 6*n*log(e))*a*b^3*c*d^3 + (n
```

```
^2 + 9*n*log(e) + 18*log(e)^2)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^4*n^2 - 4*a*
b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^2*log(b*x + a)
*log(d*x + c) + 3*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2
- 4*a^3*b*c*d^3*n^2)*B^2*log(d*x + c)^2 - ((5*n^2 + 6*n*log(e))*b^4*c^3*d -
 (17*n^2 + 24*n*log(e))*a*b^3*c^2*d^2 + (19*n^2 + 36*n*log(e))*a^2*b^2*c*d^
3 - (7*n^2 + 18*n*log(e) + 12*log(e)^2)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*n
^2 - 21*a^2*b^2*c^2*d^2*n^2 + 26*a^3*b*c*d^3*n^2 - (11*n^2 + 6*n*log(e))*a^
4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*
a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*x^
4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((d
*x + c)^n)^2 + (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n*log(b*x + a) + 2
*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*(n +
4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n +
4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*
(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*l
og(d*x + c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n
*log(b*x + a) + 2*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*
a^2*b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(
a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*
d^3*n)*B^2*x + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3
*b*c*d^3*n)*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6
*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n
))/(b*d^4)
```

Giac [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (bx+a)^3\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)^3\, dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3, x)
```

3.157 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)n(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd} \\
&\quad + \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} \\
&\quad + \frac{B(bc-ad)^2n(a+bx)\left(2A+Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^2} \\
&\quad + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

```
[Out] -1/3*B*(-a*d+b*c)*n*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/3*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/3*B*(-a*d+b*c)^2*n*(b*x+a)*(
2*A+B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+1/3*B*(-a*d+b*c)^3*n*ln((-a*
d+b*c)/b/(d*x+c))*(2*A+3*B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2/3*B^2
*(-a*d+b*c)^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2549, 2381, 2384, 2354, 2438}

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \frac{Bn(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2A+3Bn\right)}{3bd^3} + \frac{Bn(a+bx)(bc-ad)^2\left(2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2A+Bn\right)}{3bd^2} - \frac{Bn(a+bx)^2(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3bd} + \frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3b} + \frac{2B^2n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*n*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(b*d) + ((a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*b) + (
B*(b*c - a*d)^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]))/(3*b*d^2) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3
*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) + (2*B^2*(b*c - a*d
)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^3\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^4} dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} - \text{Subst}\left(\frac{(2B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd} + \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} + \text{Subst}\left(\frac{(B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd} + \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} + \frac{B(bc-ad)^2n(a+bx)\left(2A+Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^2} - \text{Subst}\left(\frac{(B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd} + \frac{(a+bx)^3 (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3b} + \frac{B(bc-ad)^2n(a+bx)(2A+Bn+2B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd^2} + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)(2A+3Bn+2B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd^3} - \text{Subst}\left(\frac{(2B^2(bc-ad)^3n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd} + \frac{(a+bx)^3 (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3b} + \frac{B(bc-ad)^2n(a+bx)(2A+Bn+2B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd^2} + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)(2A+3Bn+2B\log(e(a+bx)^n(c+dx)^{-n}))}{3bd^3} + \frac{2B^2(bc-ad)^3n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1149 vs. $2(263) = 526$.

Time = 0.64 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.37

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \frac{-6a^3ABd^3n - 2ab^2B^2c^2dn^2 + 6a^2bB^2cd^2n^2 - 6a^3B^2d^3n^2 + 3a^2A^2bd^3x + 2Ab^3Bc^2dnx - 6aAb^2Bcd^2nx + \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-6*a^3*A*B*d^3*n - 2*a*b^2*B^2*c^2*d*n^2 + 6*a^2*b*B^2*c*d^2*n^2 - 6*a^3*B
^2*d^3*n^2 + 3*a^2*A^2*b*d^3*x + 2*A*b^3*B*c^2*d*n*x - 6*a*A*b^2*B*c*d^2*n*
```

```
x + 4*a^2*A*b*B*d^3*n*x + b^3*B^2*c^2*d*n^2*x - 2*a*b^2*B^2*c*d^2*n^2*x + a
^2*b*B^2*d^3*n^2*x + 3*a*A^2*b^2*d^3*x^2 - A*b^3*B*c*d^2*n*x^2 + a*A*b^2*B*
d^3*n*x^2 + A^2*b^3*d^3*x^3 - a^3*B^2*d^3*n^2*Log[a + b*x]^2 - 2*A*b^3*B*c^
3*n*Log[c + d*x] + 6*a*A*b^2*B*c^2*d*n*Log[c + d*x] - 6*a^2*A*b*B*c*d^2*n*L
og[c + d*x] - 3*b^3*B^2*c^3*n^2*Log[c + d*x] + 7*a*b^2*B^2*c^2*d*n^2*Log[c
+ d*x] - 4*a^2*b*B^2*c*d^2*n^2*Log[c + d*x] - 6*a^3*B^2*d^3*n^2*Log[c + d*x
] - b^3*B^2*c^3*n^2*Log[c + d*x]^2 + 3*a*b^2*B^2*c^2*d*n^2*Log[c + d*x]^2 -
 3*a^2*b*B^2*c*d^2*n^2*Log[c + d*x]^2 - 6*a^3*B^2*d^3*n*Log[(e*(a + b*x)^n)
/(c + d*x)^n] + 6*a^2*A*b*B*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^3*
B^2*c^2*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B^2*c*d^2*n*x*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*b*B^2*d^3*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] + 6*a*A*b^2*B*d^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b^3*B^2*c
*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*b^2*B^2*d^3*n*x^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 2*A*b^3*B*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 2*b^3*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*b^2
*B^2*c^2*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*b*B^2*c*
d^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*a^2*b*B^2*d^3*x*Log
[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b^2*B^2*d^3*x^2*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2 + b^3*B^2*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Lo
g[a + b*x]*(2*b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*Log[c + d*x] - 2*B*
(b*c - a*d)^3*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b^2*B*c^2*n - 5*a*b
*B*c*d*n + a^2*d^2*(2*A + 9*B*n) + 2*a^2*B*d^2*Log[(e*(a + b*x)^n)/(c + d*x
)^n])) - 2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/
(3*b*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 107.03 (sec) , antiderivative size = 7208, normalized size of antiderivative = 27.41

method	result	size
risch	Expression too large to display	7208

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b^2*x^2 + 2*A^2*a*b*x + A^2*a^2 + (B^2*b^2*x^2 + 2*B^2*a*b*x +
 B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b^2*x^2 + 2*A*B*a*b*x +
 A*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(252) = 504$.

Time = 0.69 (sec) , antiderivative size = 1284, normalized size of antiderivative = 4.88

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*b^2*x^3 + 2*A*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*A*B*a^2*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A^2*a^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x
+ c)/d)*A*B*a^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
+ (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a*b/e + 1/3*(2*a^3*e*n*log(b*x + a)/b^3
- 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^
2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*b^2/e - 1/3*((3*n^2 + 2*n*log(e))*b^
2*c^3 - (7*n^2 + 6*n*log(e))*a*b*c^2*d + 2*(2*n^2 + 3*n*log(e))*a^2*c*d^2)*
```

```
B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2
*n^2 - a^3*d^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilo
g(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*x^3*log(e)^2
- B^2*a^3*d^3*n^2*log(b*x + a)^2 - (b^3*c*d^2*n*log(e) - (n*log(e) + 3*log(
e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d
^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 +
3*a^2*b*c*d^2*n^2)*B^2*log(d*x + c)^2 + ((n^2 + 2*n*log(e))*b^3*c^2*d - 2*(
n^2 + 3*n*log(e))*a*b^2*c*d^2 + (n^2 + 4*n*log(e) + 3*log(e)^2)*a^2*b*d^3)*
B^2*x + (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 + 2*n*log(e))*a^3*d
^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b
*d^3*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2
*a^2*b*d^3*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*x^3*log(e) + 2*B^2*a^3*d^
3*n*log(b*x + a) + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^
2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^
3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*log(d*x + c))*log((b*x + a)^n)
 - (2*B^2*b^3*d^3*x^3*log(e) + 2*B^2*a^3*d^3*n*log(b*x + a) + (a*b^2*d^3*(n
 + 6*log(e)) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^2*b*d^3*(2*n + 3*log(e)) + b^3*c
^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*
c*d^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^
2*a^2*b*d^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^3)
```

Giac [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)^2\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2, x)
```

3.158 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx &= -\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd}\\ &\quad+\frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b}\\ &\quad-\frac{B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2}\\ &\quad-\frac{B^2(bc-ad)^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/2*(b*x+a)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b-B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*n+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-B^2*(-a*d+b*c)^2*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used

$= \{2573, 2549, 2381, 2384, 2354, 2438\}$

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= -\frac{Bn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A+Bn\right)}{bd^2} - \frac{Bn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd} + \frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b} - \frac{B^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d
)) + ((a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b) - (B*(b
*c - a*d)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*n + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/(b*d^2) - (B^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^2\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} - \text{Subst}\left(\frac{(B(bc-ad)^2n)\,\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} + \text{Subst}\left(\frac{(B(bc-ad)^2n)\,\text{Subst}\left(\int \frac{A+Bn+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} - \frac{B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} + \text{Subst}\left(\frac{(B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} - \frac{B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} - \frac{B^2(bc-ad)^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 656 vs. 2(195) = 390.

Time = 0.52 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.36

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=-\frac{2a^2ABn}{b}-\frac{2a^2B^2n^2}{b}+\frac{aB^2cn^2}{d}+aA^2x+aABnx-\frac{AbBcnx}{d}+\frac{1}{2}A^2bx^2-\frac{a^2B^2n^2\log^2(a+bx)}{2b}+\frac{AbBc^2n\log(c+dx)}{d^2}-\frac{2aABcn\log(c+dx)}{d}-\frac{2a^2B^2n^2\log(c+dx)}{b}+\frac{bB^2c^2n^2\log(c+dx)}{d^2}-\frac{aB^2cn^2\log(c+dx)}{d}+\frac{bB^2c^2n^2\log^2(c+dx)}{2d^2}-\frac{aB^2cn^2\log^2(c+dx)}{d}-\frac{2a^2B^2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+2aABx\log\left(e(a+bx)^n(c+dx)^{-n}\right)+aB^2nx\log\left(e(a+bx)^n(c+dx)^{-n}\right)-\frac{bB^2cnx\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+AbBx^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{bB^2c^2n\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d^2}-\frac{2aB^2cn\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+aB^2x\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{1}{2}bB^2x^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{Bn\log(a+bx)\left(bBc(-bc+2ad)n\log(c+dx)+B(bc-ad)^2n\log\left(\frac{b(c+dx)}{bc-ad}\right)+ad(-bBcn+ad(A+3B\right.}{bd^2}+\frac{B^2(bc-ad)^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bd^2}$$

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n*
x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*Log[a + b*x]^2)/(2*b) +
(A*b*B*c^2*n*Log[c + d*x])/d^2 - (2*a*A*B*c*n*Log[c + d*x])/d - (2*a^2*B^2*
n^2*Log[c + d*x])/b + (b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (a*B^2*c*n^2*Log[c
 + d*x])/d + (b*B^2*c^2*n^2*Log[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*Log[c +
d*x]^2)/d - (2*a^2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*B*x*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 - (b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/d^2 - (2*a*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/
d + a*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2)/2 + (B*n*Log[a + b*x]*(b*B*c*(-(b*c) + 2*a*d)*n*Log[c
+ d*x] + B*(b*c - a*d)^2*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-(b*B*c*n)
 + a*d*(A + 3*B*n) + a*B*d*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*d^2) + (B
^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 37.57 (sec) , antiderivative size = 4394, normalized size of antiderivative = 22.53

method	result	size
risch	Expression too large to display	4394

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B^2*x*(b*x+2*a)*ln((d*x+c)^n)^2+5/4/d^2*n^2*B^2*b*c^2-3/2/d*n^2*B^2*a*c
+1/4*B^2*a^2*n^2/b+B^2*n*ln((b*x+a)^n)*x*a-1/2*B^2/b*n^2*a^2*ln(b*x+a)^2+(-
B^2*x*(b*x+2*a)*ln((b*x+a)^n)-1/2*B*(2*A*b^2*d^2*x^2+2*B*a*b*d^2*n*x-2*B*b^
2*c*d*n*x+2*B*ln(d*x+c)*b^2*c^2*n+4*B*ln(e)*a*b*d^2*x+4*A*a*b*d^2*x+2*B*a^2
*n*ln(b*x+a)*d^2+2*B*ln(e)*b^2*d^2*x^2-4*B*ln(d*x+c)*a*b*c*d*n+I*B*Pi*b^2*d
^2*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*d^2*x
^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi
*a*b*d^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)-2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*
x+a)^n/((d*x+c)^n))-I*B*Pi*b^2*d^2*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a*b*d^2*x*csgn(I*e)*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*d^2*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)^2+I*B*Pi*b^2*d^2*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^
2-2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a*b*d^2*x*csg
n(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*d^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)
^n))^3-I*B*Pi*b^2*d^2*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*I*B*Pi*a*b*d^
2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*a*b*d^2*x*cs
gn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*a*b*d^2*x*csgn(I
*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b^2*d^2*x^
2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n))/
b/d^2)*ln((d*x+c)^n)+1/4*(-I*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+
c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I*(b*x+a)^n/((d
*x+c)^n))^3+I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)^2-I*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)+2*A)^2*(1/2*b*x^
2+a*x)-1/d*n^2*B^2*c*ln(d*x+c)*a+1/d^2*n^2*B^2*c^2*ln(d*x+c)*b-1/d*n^2*B^2*
c*ln(d*x+c)^2*a+1/2/d^2*n^2*B^2*c^2*ln(d*x+c)^2*b-1/2*B^2*a^2*n^2/b*ln(b*x+
a)+B^2/b*n*ln((b*x+a)^n)*a^2*ln(b*x+a)+(-I*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I*e)*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b
*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I
*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d
```

x+c)^n)(b*x+a)^n)^2-I*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)+2*
A)*B*(1/2*ln((b*x+a)^n)*x^2*b+ln((b*x+a)^n)*x*a-1/2*b*n*(1/b*(1/2*b*x^2+a*x
)-a^2/b^2*ln(b*x+a)))+2*n*B*A*a*x+1/2*n*B*A*b*x^2+I/d*n*B^2*c*ln(d*x+c)*Pi*
a*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I/d*n*B^2*c*ln(d*x+c)*Pi*a*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^3+1/2*B^2*ln((b*x+a)^n)^2*x^2*b+B^2*ln((b*x+a)^n)^2*x*a-1
/d*n^2*B^2*a*ln(a*d-c*b+b*(d*x+c))*c+2/d*n^2*B^2*c*dilog((a*d-c*b+b*(d*x+c)
)/(a*d-b*c))*a-1/d^2*n^2*B^2*b*c^2*dilog((a*d-c*b+b*(d*x+c))/(a*d-b*c))+n^2
*B^2/b*a^2*ln(b*x+a)*ln((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))+I/d*n*B^2*c*ln(d*x
+c)*Pi*a*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)
)+I/d*n*B^2*c*ln(d*x+c)*Pi*a*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I/d*n*B^2*Pi*b*c*x*csgn(I*e)*csgn(I*(b*x+a)^n
/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I/d*n*B^2*Pi*b*c*x*csgn(I
*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I/d^2*n*B
^2*c^2*ln(d*x+c)*Pi*b*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^
n/((d*x+c)^n))-1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I*e)*csgn(I*(b*x+a)^
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*n*B^2*ln(e)*b*x^2+n^2*B^
2/b*a^2*dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))+3/2*n^2*B^2/b*a^2*ln(a*d-c*b
+b*(d*x+c))+2*n*B^2*ln(e)*a*x+I*n*B^2*Pi*a*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)
*(b*x+a)^n)^2+I*n*B^2*Pi*a*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^2+I*n*B^2*Pi*a*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c
)^n))^2+I*n*B^2*Pi*a*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+
1/4*I*n*B^2*Pi*b*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/
4*I*n*B^2*Pi*b*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)^2+1/4*I*n*B^2*Pi*b*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/
4*I*n*B^2*Pi*b*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/
d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I/
d*n*B^2*c*ln(d*x+c)*Pi*a*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^
2-1/2*I/d*n*B^2*Pi*b*c*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^
2-1/2*I/d*n*B^2*Pi*b*c*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)
*(b*x+a)^n)^2-1/2*I/d*n*B^2*Pi*b*c*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d
*x+c)^n))^2-1/2*I/d*n*B^2*Pi*b*c*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2+1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2-I/d*n*B^2*c*ln(d*x+c)*Pi*a*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)
^n/((d*x+c)^n))^2+1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I*(b*x+a)^n)*csgn
(I*(b*x+a)^n/((d*x+c)^n))^2-I/d*n*B^2*c*ln(d*x+c)*Pi*a*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*P
i*b*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I/d*n*B
^2*c*ln(d*x+c)*Pi*a*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/d^2*n*B*c
^2*ln(d*x+c)*A*b-2/d*n*B^2*c*ln(d*x+c)*ln(e)*a+1/d^2*n*B^2*c^2*ln(d*x+c)*ln
(e)*b-1/d*n*B^2*ln(e)*b*c*x-1/4*I*n*B^2*Pi*b*x^2*csgn(I*(b*x+a)^n/((d*x+c)^
n))^3-1/4*I*n*B^2*Pi*b*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*n*B^2*Pi*a*x
csgn(I(b*x+a)^n/((d*x+c)^n))^3-I*n*B^2*Pi*a*x*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^3-1/d*n*B*A*b*c*x-1/d*n*B^2*ln((b*x+a)^n)*b*c*x-2/d*n*B^2*ln((b*x+a)^n
)*c*ln(d*x+c)*a+1/d^2*n*B^2*ln((b*x+a)^n)*c^2*ln(d*x+c)*b+2/d*n^2*B^2*c*ln(
d*x+c)*ln((a*d-c*b+b*(d*x+c))/(a*d-b*c))*a-1/d^2*n^2*B^2*b*c^2*ln(d*x+c)*ln

```
((a*d-c*b+b*(d*x+c))/(a*d-b*c))-2/d*n*B*c*ln(d*x+c)*A*a+1/2*I/d*n*B^2*Pi*b*
c*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I/d*n*B^2*Pi*b*c*x*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^3-1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I*(b*x+a)^n/((d*x
+c)^n))^3-1/2*I/d^2*n*B^2*c^2*ln(d*x+c)*Pi*b*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3-I*n*B^2*Pi*a*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)-I*n*B^2*Pi*a*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))-1/4*I*n*B^2*Pi*b*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/4*I*n*B^2*Pi*b*x^2*csgn(I*(b*x+
a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
```

Fricas [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
[Out] integral(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*b*x + A*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(192) = 384$.

Time = 0.69 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.99

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= ABbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2bx^2+2\,ABax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2ax$$
$$+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABa}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)ABb}{e}$$
$$+\frac{((n^2+n\log(e))bc^2-(n^2+2\,n\log(e))acd)B^2\log(dx+c)}{d^2}$$
$$+\frac{(b^2c^2n^2-2\,abcdn^2+a^2d^2n^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$-\frac{B^2a^2d^2n^2\log(bx+a)^2-B^2b^2d^2x^2\log(e)^2+2\,(b^2c^2n^2-2\,abcdn^2)B^2\log(bx+a)\log(dx+c)-(b^2c^2n}{}$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")
```

```
[Out] A*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*A*B*a*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^2*a*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*a/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*b/e + ((n^2 + n*log(e))*b*c^2 - (n^2 + 2*n
*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 + a^2*d
^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*log(b*x + a)^2 - B^2*
b^2*d^2*x^2*log(e)^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(b*x + a)*log
(d*x + c) - (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(d*x + c)^2 + 2*(b^2*c*d*n
*log(e) - (n*log(e) + log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*n^2 - (n^2 + n*
log(e))*a^2*d^2)*B^2*log(b*x + a) - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log
((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((d*x + c)^n)^2 -
2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*b*d^2*(n + 2*lo
g(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log(d*x + c))*log(
(b*x + a)^n) + 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*
b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log
(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b*d^2)
```

Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2(a+b\,x)\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x), x)
```

3.159 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 131

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx &= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\end{aligned}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b+2*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2573, 2549, 2379, 2421, 6724}

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx &= \frac{2Bn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{b}\\ &\quad-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{b}+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (2*B*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/b + (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a +
 b*x))])/b
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\,\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} - \text{Subst}\left(\frac{(2B^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\,\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2B^2n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 269 vs. 2(131) = 262.

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx$$

$$= \frac{-ABn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)-2ABn\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log(e(a-}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] (-(A*B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + A^2*Log[a + b*x] - 2*A*B*n*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Lo
g[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - B^2*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*A*B*n*
PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B^2*n*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))])/b
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}}{bx+a}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
```

```
[Out] Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))**2/(a + b*x), x)
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="maxima
")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/b + A^2*log(b*x + a)/b - integrate(-(B^
2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n)^
2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b*c +
 (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*log(b*x + a)
+ (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^2 + a*
b*c + (b^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{a+b\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x), x)
```

3.160 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 129

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = -\frac{2B^2n^2(c+dx)}{(bc-ad)(a+bx)} - \frac{2Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(
-a*d+b*c)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2573, 2549, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = -\frac{2Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)} - \frac{(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)} - \frac{2B^2n^2(c+dx)}{(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] (-2*B^2*n^2*(c + d*x))/((b*c - a*d)*(a + b*x)) - (2*B*n*(c + d*x)*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)*(a + b*x)) - ((c + d*x)*(A +
B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)} + \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{2B^2n^2(c+dx)}{(bc-ad)(a+bx)} - \frac{2Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = \frac{B^2dn^2(a+bx)\log^2(a+bx)+B^2dn^2(a+bx)\log^2(c+dx)+2Bdn(a+bx)\log(c+dx)\left(A+Bn+B\log\left(e\right.\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] (B^2*d*n^2*(a + b*x)*Log[a + b*x]^2 + B^2*d*n^2*(a + b*x)*Log[c + d*x]^2 +
2*B*d*n*(a + b*x)*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
]) - 2*B*d*n*(a + b*x)*Log[a + b*x]*(A + B*n + B*n*Log[c + d*x] + B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]) - (b*c - a*d)*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(
A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n]^2))/(b*(b*c - a*d)*(a + b*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(129) = 258.

Time = 7.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.32

method	result
parallelrisch	$-\frac{-A^2b^3cdn+2B^2ab^2d^2n^3-2B^2b^3cdn^3+A^2ab^2d^2n+2ABab^2d^2n^2-2ABb^3cdn^2-B^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^3d^2n-2B^2}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-A^2*b^3*c*d*n+2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n+2*A*
B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*d
^2*n-2*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^2-B^2*ln(e*(b*x+a)^n/((d
*x+c)^n))^2*b^3*c*d*n-2*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^2-2*A*B*x
*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-2*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^
3*c*d*n)/(b*x+a)/b^3/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(129) = 258$.

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -\frac{A^2bc-A^2ad+2(B^2bc-B^2ad)n^2+(B^2bdn^2x+B^2bcn^2)\log(bx+a)^2+(B^2bdn^2x+B^2bcn^2)\log(dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*d*n^2*x + B^2*b*c*
n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(d*x + c)^2 + (B^2*b
*c - B^2*a*d)*log(e)^2 + 2*(A*B*b*c - A*B*a*d)*n + 2*(B^2*b*c*n^2 + A*B*b*c
*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log(b*
x + a) - 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*
d*n^2*x + B^2*b*c*n^2)*log(b*x + a) + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log
(d*x + c) + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n)*log(e))/(a*b^2*c
- a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(129) = 258$.

Time = 0.22 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.48

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -B^2\left(\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd} - \frac{den\log(dx+c)}{b^2c-abd} + \frac{en}{b^2x+ab}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e} + \frac{2\,bce^2n^2 - 2\,ade^2n^2 - (bde^2n^2x + ade^2n^2)\,\mathrm{l}\cdots}{\cdots}\right) - \frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{b^2x+ab} - \frac{2\left(\frac{den\log(bx+a)}{b^2c-abd} - \frac{den\log(dx+c)}{b^2c-abd} + \frac{en}{b^2x+ab}\right)AB}{e} - \frac{2\,AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab} - \frac{A^2}{b^2x+ab}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a
*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c*e^2*n^
2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2
*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(
b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*l
og(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))
 - B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 2*(d*e*n*log(b*x +
a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b)
)*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2/(b^2*x +
 a*b)
```

Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right) + A\right)^2}{(bx+a)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(\frac{2\,A\,B}{x\,b^2+a\,b}+\frac{2\,B^2\,n}{x\,b^2+a\,b}\right) - \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\,\left(\frac{B^2}{b\,(a+b\,x)} - \frac{B^2\,d}{b\,(a\,d-b\,c)}\right) - \frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2+a\,b} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B\,n)\,4\mathrm{i}}{b\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(a*b + b^2*x) + (2*B^2*n)/(a*b
+ b^2*x)) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(b*(a + b*x)) - (B^2*d)
/(b*(a*d - b*c))) - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(a*b + b^2*x) - (B*d*n*atan
((((b^2*c + a*b*d)/b + 2*b*d*x)*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*(a*d - b*
c))
```

3.161 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3} dx$

Optimal result

Integrand size = 33, antiderivative size = 274

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3} dx &= \frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} \\ &+ \frac{2Bdn(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^2(a+bx)} \\ &- \frac{bBn(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{2(bc-ad)^2(a+bx)^2} \\ &+ \frac{d(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^2(a+bx)} \\ &- \frac{b(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2(bc-ad)^2(a+bx)^2}\end{aligned}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+
a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+a)
^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx$$
$$= -\frac{bBn(c+dx)^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{2(a+bx)^2(bc-ad)^2}$$
$$+\frac{2Bdn(c+dx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)^2}$$
$$-\frac{b(c+dx)^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{2(a+bx)^2(bc-ad)^2}$$
$$+\frac{d(c+dx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(bc-ad)^2}$$
$$-\frac{bB^2n^2(c+dx)^2}{4(a+bx)^2(bc-ad)^2}+\frac{2B^2dn^2(c+dx)}{(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] (2*B^2*d*n^2*(c + d*x))/((b*c - a*d)^2*(a + b*x)) - (b*B^2*n^2*(c + d*x)^2)
/(4*(b*c - a*d)^2*(a + b*x)^2) + (2*B*d*n*(c + d*x)*(A + B*Log[(e*(a + b*x)
^n)/(c + d*x)^n]))/((b*c - a*d)^2*(a + b*x)) - (b*B*n*(c + d*x)^2*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^2*(a + b*x)^2) + (d*(c + d*
x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^2*(a + b*x)) -
(b*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^2
*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
```

```
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{x^3}-\frac{d(A+B\log(ex^n))^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{b\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^2(a+bx)^2} + \text{Subst}\left(\frac{(bBn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(2Bdn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} + \frac{2Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} - \frac{bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^2(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^2(a+bx)^2}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.21

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx = -\frac{2B^2d^2n^2(a+bx)^2\log^2(a+bx)+2B^2d^2n^2(a+bx)^2\log^2(c+dx)+2Bd^2n(a+bx)^2\log(c+dx)\left(2A+3\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2*
Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A + 3
*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + (b*c -
a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b*c -
3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x))*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d
*x)^n]^2))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. 2(268) = 536.

Time = 21.60 (sec) , antiderivative size = 871, normalized size of antiderivative = 3.18

method	result
parallelrisch	$-\frac{-6B^2\ln(bx+a)x^2b^5d^3n^2+6B^2\ln(dx+c)x^2b^5d^3n^2-6B^2\ln(bx+a)a^2b^3d^3n^2+6B^2\ln(dx+c)a^2b^3d^3n^2+6B^2xa\,b^4d^3n^2-4A^2a\,b}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-6*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2+6*B^2*ln(d*x+c)*x^2*b^5*d^3*n^2-6*B^
2*ln(b*x+a)*a^2*b^3*d^3*n^2+6*B^2*ln(d*x+c)*a^2*b^3*d^3*n^2-4*B^2*x*ln(e*(b
*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3+6*B^2*x*a*b^4*d^3*n^2-4*A^2*a*b^4*c*d^2-6*
B^2*x*b^5*c*d^2*n^2-4*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2+6*B^2*l
n(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3*n+2*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*
b^5*c^2*d*n+4*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3+4*A*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*b^5*c^2*d-8*B^2*a*b^4*c*d^2*n^2+6*A*B*a^2*b^3*d^3*n+2*A*B*
b^5*c^2*d*n-8*A*B*a*b^4*c*d^2*n-8*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d
^2*n-8*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+4*B^2*x*ln(e*(b*x+a)^n/(
(d*x+c)^n))*a*b^4*d^3*n-4*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n+4*A
*B*x*a*b^4*d^3*n-4*A*B*x*b^5*c*d^2*n-4*A*B*ln(b*x+a)*x^2*b^5*d^3*n+4*A*B*ln
(d*x+c)*x^2*b^5*d^3*n-12*B^2*ln(b*x+a)*x*a*b^4*d^3*n^2+12*B^2*ln(d*x+c)*x*a
*b^4*d^3*n^2-4*A*B*ln(b*x+a)*a^2*b^3*d^3*n+4*A*B*ln(d*x+c)*a^2*b^3*d^3*n+2*
A^2*a^2*b^3*d^3+2*A^2*b^5*c^2*d+7*B^2*a^2*b^3*d^3*n^2+B^2*b^5*c^2*d*n^2-8*A
*B*ln(b*x+a)*x*a*b^4*d^3*n+8*A*B*ln(d*x+c)*x*a*b^4*d^3*n-2*B^2*x^2*ln(e*(b*
x+a)^n/((d*x+c)^n))^2*b^5*d^3+2*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*c^2*d
)/(b*x+a)^2/b^4/d/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(268) = 536.

Time = 0.30 (sec) , antiderivative size = 919, normalized size of antiderivative = 3.35

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx =$$
$$-\frac{2A^2b^2c^2-4A^2abcd+2A^2a^2d^2+(B^2b^2c^2-8B^2abcd+7B^2a^2d^2)n^2-2(B^2b^2d^2n^2x^2+2B^2abd^2n^2x-}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*
a*b*c*d + 7*B^2*a^2*d^2)*n^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x
```

```
 - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a)^2 - 2*(B^2*b^2*d^2*n^2*x
^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(d*x + c)^
2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 + 2*(A*B*b^2*c^2
 - 4*A*B*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2
+ 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2
 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c
*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*
b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n)*log(e)
)*log(b*x + a) - 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2
+ 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d
^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^
2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a) - 2*(B^2*
b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n)*log(e)
)*log(d*x + c) + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2 - 2*(B^2*
b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d + 3*B^2*a^2*d^2)*
n)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*
d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(268) = 536$.

Time = 0.24 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.28

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx$$
$$= \frac{1}{4}B^2\left(\frac{2\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2} - \frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2} + \frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e} - \frac{b^2c^2e^2}{}\right.$$
$$- \frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$
$$+ \frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2} - \frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2} + \frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)AB}{2\,e}$$
$$- \frac{AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^3x^2+2\,ab^2x+a^2b} - \frac{A^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B^2*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*
d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b
*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3
*c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2 - 8
*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e
^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b
*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^
2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^
2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*
n^2)*log(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
+ (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*x)*e^2)) - 1/2*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^3*x^2
 + 2*a*b^2*x + a^2*b) + 1/2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) +
 (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*
d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A*B/e - A*B*log((b*x + a)^n*e/(d*x + c
)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx + a)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="giac
")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.62

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^3}\,dx$$
$$= -\ln\left(\frac{e\,(a + b\,x)^n}{(c + d\,x)^n}\right)^2\left(\frac{B^2}{2\,b\,(a^2 + 2\,a\,b\,x + b^2\,x^2)} - \frac{B^2\,d^2}{2\,b\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}\right)$$
$$- \frac{\frac{2\,A^2\,a\,d - 2\,A^2\,b\,c + 7\,B^2\,a\,d\,n^2 - B^2\,b\,c\,n^2 + 6\,A\,B\,a\,d\,n - 2\,A\,B\,b\,c\,n}{2\,(a\,d - b\,c)} + \frac{d\,x\,(3\,b\,B^2\,n^2 + 2\,A\,b\,B\,n)}{a\,d - b\,c}}{2\,a^2\,b + 4\,a\,b^2\,x + 2\,b^3\,x^2}$$
$$- \ln\left(\frac{e\,(a + b\,x)^n}{(c + d\,x)^n}\right)\left(\frac{A\,B}{a^2\,b + 2\,a\,b^2\,x + b^3\,x^2} + \frac{B^2\,d^2\left(\frac{b\,n\,(a\,d - b\,c)\,(2\,a\,d - b\,c)}{2\,d^2} + \frac{b^2\,n\,x\,(a\,d - b\,c)}{d} + \frac{a\,b\,n\,(a\,d - b\,c)}{2\,d}\right)}{b\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)\,(a^2\,b + 2\,a\,b^2\,x + b^3\,x^2)}\right)$$
$$- \frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x - \frac{2\,b^3\,c^2 - 2\,a^2\,b\,d^2}{2\,b\,(a\,d - b\,c)}\right)\,1\mathrm{i}}{a\,d - b\,c}\right)\,(2\,A + 3\,B\,n)\,1\mathrm{i}}{b\,(a\,d - b\,c)^2}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^3,x)
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^2*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c
 + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c))
 + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*
b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)*((A*B)/(a^2*b + b^3*x^2 + 2*a*b^2
*x) + (B^2*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (b^2*n*x*(a*d - b
*c))/d + (a*b*n*(a*d - b*c))/(2*d)))/(b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^
2*b + b^3*x^2 + 2*a*b^2*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2 - 2*a^2
*b*d^2)/(2*b*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*(a*d - b*c
)^2)
```

3.162 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 427

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx \\
&= -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3(a+bx)} + \frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3(a+bx)^2} - \frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3(a+bx)^3} \\
&\quad - \frac{2Bd^2n(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{bBdn(c+dx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{2b^2Bn(c+dx)^3\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{9(bc-ad)^3(a+bx)^3} \\
&\quad - \frac{d^2(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{bd(c+dx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{b^2(c+dx)^3\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3(bc-ad)^3(a+bx)^3}
\end{aligned}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-2*B*d^2
*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+b*B*d*n*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^2-2/9*b^2*B
*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)+b*d*(d*x+c)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*(d*x+
c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx$$
$$= -\frac{b^2(c+dx)^3(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{3(a+bx)^3(bc-ad)^3}$$
$$-\frac{2b^2Bn(c+dx)^3(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{9(a+bx)^3(bc-ad)^3}$$
$$-\frac{d^2(c+dx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(bc-ad)^3}$$
$$-\frac{2Bd^2n(c+dx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)^3}$$
$$+\frac{bd(c+dx)^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^2(bc-ad)^3}$$
$$+\frac{bBdn(c+dx)^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)^2(bc-ad)^3}$$
$$-\frac{2b^2B^2n^2(c+dx)^3}{27(a+bx)^3(bc-ad)^3}-\frac{2B^2d^2n^2(c+dx)}{(a+bx)(bc-ad)^3}+\frac{bB^2dn^2(c+dx)^2}{2(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] (-2*B^2*d^2*n^2*(c + d*x))/((b*c - a*d)^3*(a + b*x)) + (b*B^2*d*n^2*(c + d*
x)^2)/(2*(b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(b*c
- a*d)^3*(a + b*x)^3) - (2*B*d^2*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]))/((b*c - a*d)^3*(a + b*x)) + (b*B*d*n*(c + d*x)^2*(A + B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(9*(b*c - a*d)^3*(a + b*x)^
3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d
)^3*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2)/((b*c - a*d)^3*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(3*(b*c - a*d)^3*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{x^4}-\frac{2bd(A+B\log(ex^n))^2}{x^3}+\frac{d^2(A+B\log(ex^n))^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)}$$

$$+ \frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)^2}$$

$$- \frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3}$$

$$+ \text{Subst}\left(\frac{(2b^2Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$- \text{Subst}\left(\frac{(2bBdn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+ \text{Subst}\left(\frac{(2Bd^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3(a+bx)} + \frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3(a+bx)^2} - \frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3(a+bx)^3} - \frac{2Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)} + \frac{bBdn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)^2} - \frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{9(bc-ad)^3(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.01

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \frac{18B^2d^3n^2(a+bx)^3\log^2(a+bx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)+6Bd^3n(a+bx)^3\log(c+dx)\,(6A+11}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] (18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*x]^2 + 18*B^2*d^3*n^2*(a + b*x)^3*Log
[c + d*x]^2 + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e
*(a + b*x)^n)/(c + d*x)^n]) - 6*B*d^3*n*(a + b*x)^3*Log[a + b*x]*(6*A + 11*
B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c - a
*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b
^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 14
7*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)^2 + B*
n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))
*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*Log[(e*(a + b*x)^n
)/(c + d*x)^n]^2))/(54*b*(b*c - a*d)^3*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. 2(419) = 838.

Time = 57.75 (sec) , antiderivative size = 1400, normalized size of antiderivative = 3.28

method	result	size
parallelrisch	Expression too large to display	1400
risch	Expression too large to display	25057

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-66*B^2*ln(b*x+a)*x^3*b^7*d^4*n^2+66*B^2*ln(d*x+c)*x^3*b^7*d^4*n^2-6
6*B^2*ln(b*x+a)*a^3*b^4*d^4*n^2+66*B^2*ln(d*x+c)*a^3*b^4*d^4*n^2-36*A*B*ln(
b*x+a)*x^3*b^7*d^4*n+36*A*B*ln(d*x+c)*x^3*b^7*d^4*n-198*B^2*ln(b*x+a)*x^2*a
*b^6*d^4*n^2+198*B^2*ln(d*x+c)*x^2*a*b^6*d^4*n^2-198*B^2*ln(b*x+a)*x*a^2*b^
5*d^4*n^2+198*B^2*ln(d*x+c)*x*a^2*b^5*d^4*n^2-36*A*B*ln(b*x+a)*a^3*b^4*d^4*
n+36*A*B*ln(d*x+c)*a^3*b^4*d^4*n-108*A*B*x*a*b^6*c*d^3*n-66*B^2*x^2*b^7*c*d
^3*n^2-54*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*d^4+147*B^2*x*a^2*b^5
*d^4*n^2+15*B^2*x*b^7*c^2*d^2*n^2-54*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*
b^5*c*d^3+54*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*c^2*d^2+66*B^2*ln(e*(b
*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4*n-12*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c
^3*d*n+36*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4-36*A*B*ln(e*(b*x+a)^n
/((d*x+c)^n))*b^7*c^3*d-54*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4+
66*B^2*x^2*a*b^6*d^4*n^2-108*A*B*ln(b*x+a)*x^2*a*b^6*d^4*n+108*A*B*ln(d*x+c
)*x^2*a*b^6*d^4*n-108*A*B*ln(b*x+a)*x*a^2*b^5*d^4*n+108*A*B*ln(d*x+c)*x*a^2
*b^5*d^4*n-18*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*d^4-18*B^2*ln(e*(b*
x+a)^n/((d*x+c)^n))^2*b^7*c^3*d+27*B^2*a*b^6*c^2*d^2*n^2+66*A*B*a^3*b^4*d^4
*n-12*A*B*b^7*c^3*d*n-54*A^2*a^2*b^5*c*d^3+54*A^2*a*b^6*c^2*d^2+36*B^2*x^2*
ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*d^4*n-36*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^
n))*b^7*c*d^3*n+36*A*B*x^2*a*b^6*d^4*n-36*A*B*x^2*b^7*c*d^3*n+90*B^2*x*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n+18*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*
b^7*c^2*d^2*n-162*B^2*x*a*b^6*c*d^3*n^2+90*A*B*x*a^2*b^5*d^4*n+18*A*B*x*b^7
*c^2*d^2*n-108*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3*n+54*B^2*ln(e*
(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2*n-108*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*
a^2*b^5*c*d^3+108*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2-108*B^2*a^2
*b^5*c*d^3*n^2-108*A*B*a^2*b^5*c*d^3*n+54*A*B*a*b^6*c^2*d^2*n+85*B^2*a^3*b^
4*d^4*n^2-4*B^2*b^7*c^3*d*n^2+18*A^2*a^3*b^4*d^4-18*A^2*b^7*c^3*d-108*B^2*x
*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n)/(b*x+a)^3/(a^3*d^3-3*a^2*b*c*d^
2+3*a*b^2*c^2*d-b^3*c^3)/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1635 vs. 2(419) = 838.

Time = 0.33 (sec) , antiderivative size = 1635, normalized size of antiderivative = 3.83

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^
2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*
c*d^2)*n^2)*log(b*x + a)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*
x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2)*n^2)*log(d*x + c)^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d
+ 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d - 54*B^2*a*b
^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2*c*d^2 + 5
*A*B*a^2*b*d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B
^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d
^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*
B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*
d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x + 6*(B^2*b^3*d^3*n*x^3 + 3*
B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*
d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(b*x + a) - 6*((11*B^2*b^3*d^3*n^2 + 6
*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d
^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x
^2 + 6*(A*B*b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a
^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x +
 6*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x +
 (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n^2)*log(b*x + a) +
6*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b
^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(d*x + c) + 6
*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 +
 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c
*d^2 + 5*B^2*a^2*b*d^3)*n*x + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a
^2*b*c*d^2 - 11*B^2*a^3*d^3)*n)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3
*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^
3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(419) = 838$.

Time = 0.29 (sec) , antiderivative size = 1500, normalized size of antiderivative = 3.51

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/54*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c
*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*
n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^
4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^
5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4*b^3*c^3*e^2*n^2 - 2
7*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^2 + 66*(
b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*
b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a)
^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^
2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2 - 54*a*b^2*c
*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*
d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a) - 6
*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*
x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 +
 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^3
*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b
^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^
2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d
 + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)*e^2)) - 1/3*B^2*log((b*x + a)^n*e/(d*x
 + c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/9*(6*d^3*e*n*l
og(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3
*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) +
```

```
(6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b
^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (
b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a
^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*A*B/e -
 2/3*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*
x + a^3*b) - 1/3*A^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx$$
$$=\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2\,n-42\,A\,B\,a\,b\,c\,d\,n+12\,A\,B\,b^2\,c^2\,n+85\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+4\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}+\frac{x\,(-5\,c\,B\cdots}{\cdots}}{x^3\,\left(9\,b^5\,c-9\,a\,b^4\,d\right)+x\,\left(27\,a^2\,b^3\,c-27\,a^3\,b^2\,d\right)-x^2\,\left(27\,a^2\,b^3\,d-\cdots\right)}$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{3\,b\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)}-\frac{B^2\,d^3}{3\,b\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{2\,A\,B}{3\,\left(a^3\,b+3\,a^2\,b^2\,x+3\,a\,b^3\,x^2+b^4\,x^3\right)}\right.$$
$$+\frac{2\,B^2\,d^3\,\left(a\,\left(\frac{b\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)+x\,\left(b\,\left(\frac{b\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)+\frac{2\,a\,b^2\,n\,(a\,d-b\,c)}{d}+\cdots\right.\right.}{9\,b\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)\,\left(a^3\,b+3\,a^2\,b^2\,x\cdots\right)}$$
$$-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{B\,d^3\,n\,(6\,A+11\,B\,n)\,\left(\frac{a^3\,b\,d^3-a^2\,b^2\,c\,d^2-a\,b^3\,c^2\,d+b^4\,c^3}{a^2\,b\,d^2-2\,a\,b^2\,c\,d+b^3\,c^2}+2\,b\,d\,x\right)\,\left(a^2\,b\,d^2-2\,a\,b^2\,c\,d+b^3\,c^2\right)\,1{}\mathrm{i}}{b\,\left(11\,B^2\,d^3\,n^2+6\,A\,B\,d^3\,n\right)\,{(a\,d-b\,c)}^3}\right)\,(6\,A+11\,B\,n)\,2{}\mathrm{i}}{9\,b\,{(a\,d-b\,c)}^3}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x^3*(9*b^5*c - 9*a*b^4*d) + x
*(27*a^2*b^3*c - 27*a^3*b^2*d) - x^2*(27*a^2*b^3*d - 27*a*b^4*c) + 9*a^3*b^
2*c - 9*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(3*b*(a^3 + b^3*
x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^2*d^3)/(3*b*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(3*
(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)) + (2*B^2*d^3*(a*((b*n*(a*d -
 b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + x*(b*((b*n*(a*d - b
*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c
))/d + (b^2*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + (b*n*(a*d - b*c)*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*n*x^2*(a*d - b*c))/d))/(9*b*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x +
 3*a*b^3*x^2))) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3 + a^3*b*d
^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) + 2*b
*d*x)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*1i)/(b*(11*B^2*d^3*n^2 + 6*A*B*d^
3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*(a*d - b*c)^3)
```

3.163 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 587

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx \\
&= \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4(a+bx)^4} + \frac{2Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{8(bc-ad)^4(a+bx)^4} \\
&\quad + \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^4}
\end{aligned}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-1/32*
b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+2*B*d^3*n*(d*x+c)*(A+B*ln(e*(b
```

```
*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-3/2*b*B*d^2*n*(d*x+c)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B*d*n*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-1/8*b^3*B*n*(d*x+c)^
4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\begin{aligned}&\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5} \, dx \\ &= -\frac{b^3(c+dx)^4 (B\log (e(a+bx)^n(c+dx)^{-n})+A)^2}{4(a+bx)^4(bc-ad)^4} \\ &\quad -\frac{b^3Bn(c+dx)^4 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}{8(a+bx)^4(bc-ad)^4} \\ &\quad +\frac{b^2d(c+dx)^3 (B\log (e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^3(bc-ad)^4} \\ &\quad +\frac{2b^2Bdn(c+dx)^3 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}{3(a+bx)^3(bc-ad)^4} \\ &\quad +\frac{d^3(c+dx) (B\log (e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(bc-ad)^4} \\ &\quad +\frac{2Bd^3n(c+dx) (B\log (e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)^4} \\ &\quad -\frac{3bd^2(c+dx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)^2}{2(a+bx)^2(bc-ad)^4} \\ &\quad -\frac{3bBd^2n(c+dx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}{2(a+bx)^2(bc-ad)^4}-\frac{b^3B^2n^2(c+dx)^4}{32(a+bx)^4(bc-ad)^4} \\ &\quad +\frac{2b^2B^2dn^2(c+dx)^3}{9(a+bx)^3(bc-ad)^4}+\frac{2B^2d^3n^2(c+dx)}{(a+bx)(bc-ad)^4}-\frac{3bB^2d^2n^2(c+dx)^2}{4(a+bx)^2(bc-ad)^4}\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] (2*B^2*d^3*n^2*(c + d*x))/((b*c - a*d)^4*(a + b*x)) - (3*b*B^2*d^2*n^2*(c +
 d*x)^2)/(4*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(
b*c - a*d)^4*(a + b*x)^3) - (b^3*B^2*n^2*(c + d*x)^4)/(32*(b*c - a*d)^4*(a
+ b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/
```

```
((b*c - a*d)^4*(a + b*x)) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^
3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*(b*c - a*d)^4*(a + b*x)^3) -
 (b^3*B*n*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(8*(b*c - a
*d)^4*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)^2)/((b*c - a*d)^4*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^4*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^4*(a + b*x)^3) - (b
^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*(b*c - a*d)^4
*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

rQ[n]

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))^2}{x^5} - \frac{3b^2d(A+B\log(ex^n))^2}{x^4} + \frac{3bd^2(A+B\log(ex^n))^2}{x^3} - \frac{d^3(A+B\log(ex^n))^2}{x^2}\right)\,dx, x\right.}{(bc-ad)^4}\right.$$

$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^4(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^4} + \text{Subst}\left(\frac{(b^3Bn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(2b^2Bdn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) + \text{Subst}\left(\frac{(3bBd^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(2Bd^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4(a+bx)^3}$$
$$- \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4(a+bx)^4} + \frac{2Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)}$$
$$- \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^4(a+bx)^2}$$
$$+ \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3}$$
$$- \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{8(bc-ad)^4(a+bx)^4}$$
$$+ \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)}$$
$$- \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^4(a+bx)^2}$$
$$+ \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3}$$
$$- \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.72

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx =$$
$$-\frac{72bB^2n^2(-4a^3d^3(c+dx)+6a^2bd^2(c^2-d^2x^2)-4ab^2d(c^3+d^3x^3)+b^3(c^4-d^4x^4))\log^2(a+bx)+72bE}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] -1/288*(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) -
4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^2*
n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3 +
d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*(a +
 b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a
+ b*x)^n)/(c + d*x)^n])) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A + 13*B
*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n])) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*(-(n*Log
[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) - 12*B*d^4
*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Lo
g[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*d^4*n*(a + b*x)^4*Lo
g[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[
```

```
(e*(a + b*x)^n)/(c + d*x)^n])) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*B*n + B^2*n^2
 + 16*A*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n]) + 4*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/
(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x
)^n)/(c + d*x)^n])^2) - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4*B*d*(b*c - a*d)^
2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12*B*d^3*n*(a + b*x)
^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n]))) + 12*B*n*Log[c + d*x]*(4*B*d*(b*c - a*
d)^3*n*(a + b*x) - 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 12*B*d^3*(b*c - a*
d)*n*(a + b*x)^3 - 12*B*(b*c - a*d)^4*n*Log[a + b*x] + 12*B*d^4*n*(a + b*x)
^4*Log[a + b*x] - 3*(b*c - a*d)^4*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*L
og[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))))/(b*(b*c - a*d)^4*(a + b*
x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4945 vs. $2(571) = 1142$.

Time = 142.59 (sec) , antiderivative size = 4946, normalized size of antiderivative = 8.43

method	result	size
parallelrisch	Expression too large to display	4946
risch	Expression too large to display	33370

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/288*(144*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^5*n-1008*B^2*x*a^8*b
*c^2*d^3*n^2+624*B^2*x*a^7*b^2*c^3*d^2*n^2-228*B^2*x*a^6*b^3*c^4*d*n^2+288*
A^2*x*a^5*b^4*c^5-36*B^2*ln(b*x+a)*x^4*a^2*b^7*c^5*n^2+36*B^2*ln(d*x+c)*x^4
*a^2*b^7*c^5*n^2-144*B^2*ln(b*x+a)*x^3*a^3*b^6*c^5*n^2+144*B^2*ln(d*x+c)*x^
3*a^3*b^6*c^5*n^2-216*B^2*ln(b*x+a)*x^2*a^4*b^5*c^5*n^2+216*B^2*ln(d*x+c)*x
^2*a^4*b^5*c^5*n^2-144*B^2*ln(b*x+a)*x*a^5*b^4*c^5*n^2+3456*A*B*ln(b*x+a)*x
^2*a^5*b^4*c^4*d*n-3456*A*B*ln(d*x+c)*x^2*a^7*b^2*c^2*d^3*n+5184*A*B*ln(d*x
+c)*x^2*a^6*b^3*c^3*d^2*n-3456*A*B*ln(d*x+c)*x^2*a^5*b^4*c^4*d*n-576*A*B*ln
(d*x+c)*x^4*a^3*b^6*c^4*d*n+576*A*B*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*n-864*A*B
*ln(b*x+a)*x^4*a^4*b^5*c^3*d^2*n+576*A*B*ln(b*x+a)*x^4*a^3*b^6*c^4*d*n-576*
A*B*ln(d*x+c)*x^4*a^5*b^4*c^2*d^3*n+864*A*B*ln(d*x+c)*x^4*a^4*b^5*c^3*d^2*n
+3456*A*B*ln(d*x+c)*x*a^7*b^2*c^3*d^2*n-2304*A*B*ln(d*x+c)*x*a^6*b^3*c^4*d*
n+2304*A*B*ln(b*x+a)*x*a^8*b*c^2*d^3*n-3456*A*B*ln(b*x+a)*x*a^7*b^2*c^3*d^2
*n+2304*A*B*ln(b*x+a)*x^3*a^6*b^3*c^2*d^3*n-3456*A*B*ln(b*x+a)*x^3*a^5*b^4*
c^3*d^2*n+2304*A*B*ln(b*x+a)*x^3*a^4*b^5*c^4*d*n-2304*A*B*ln(d*x+c)*x^3*a^6
*b^3*c^2*d^3*n+3456*A*B*ln(d*x+c)*x^3*a^5*b^4*c^3*d^2*n-2304*A*B*ln(d*x+c)*
x^3*a^4*b^5*c^4*d*n+3456*A*B*ln(b*x+a)*x^2*a^7*b^2*c^2*d^3*n-5184*A*B*ln(b*
x+a)*x^2*a^6*b^3*c^3*d^2*n+2304*A*B*ln(b*x+a)*x*a^6*b^3*c^4*d*n-2304*A*B*ln
(d*x+c)*x*a^8*b*c^2*d^3*n-2304*B^2*ln(d*x+c)*x*a^8*b*c^2*d^3*n^2+1728*B^2*l
```

n(d*x+c)*x*a^7*b^2*c^3*d^2*n^2-768*B^2*ln(d*x+c)*x*a^6*b^3*c^4*d*n^2-576*A*
B*ln(b*x+a)*x*a^5*b^4*c^5*n+576*A*B*ln(d*x+c)*x*a^5*b^4*c^5*n-2304*A*B*x^3*
ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^4*d+1056*A*B*x^3*a^7*b^2*c*d^4*n-2160
*A*B*x^3*a^6*b^3*c^2*d^3*n+1728*A*B*x^3*a^5*b^4*c^3*d^2*n+5184*A*B*x^2*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^3*d^2-3456*A*B*x^2*ln(e*(b*x+a)^n/((d*x+c
)^n))*a^5*b^4*c^4*d+1296*A*B*x^2*a^8*b*c*d^4*n-2880*A*B*x^2*a^7*b^2*c^2*d^3
*n+2520*A*B*x^2*a^6*b^3*c^3*d^2*n-1152*A*B*x^2*a^5*b^4*c^4*d*n-1440*B^2*x*l
n(e*(b*x+a)^n/((d*x+c)^n))*a^8*b*c^2*d^3*n+1440*B^2*x*ln(e*(b*x+a)^n/((d*x+
c)^n))*a^7*b^2*c^3*d^2*n-720*A*B*x*a^6*b^3*c^4*d*n+864*A*B*x^2*ln(e*(b*x+a)
^n/((d*x+c)^n))*a^8*b*c*d^4-3456*A*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^
2*c^2*d^3+300*B^2*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c*d^4*n-576*B^2*x
^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^2*d^3*n+432*B^2*x^4*ln(e*(b*x+a)^n
/((d*x+c)^n))*a^4*b^5*c^3*d^2*n-192*B^2*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3
*b^6*c^4*d*n+144*A*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c*d^4-576*A*B*
x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^2*d^3+1152*B^2*ln(b*x+a)*x^2*a^5*
b^4*c^4*d*n^2-3456*B^2*ln(d*x+c)*x^2*a^7*b^2*c^2*d^3*n^2+2592*B^2*ln(d*x+c)
*x^2*a^6*b^3*c^3*d^2*n^2-1152*B^2*ln(d*x+c)*x^2*a^5*b^4*c^4*d*n^2-864*A*B*l
n(b*x+a)*x^2*a^4*b^5*c^5*n+864*A*B*ln(d*x+c)*x^2*a^4*b^5*c^5*n+2304*B^2*ln(
b*x+a)*x*a^8*b*c^2*d^3*n^2-1728*B^2*ln(b*x+a)*x*a^7*b^2*c^3*d^2*n^2+768*B^2
*ln(b*x+a)*x*a^6*b^3*c^4*d*n^2-864*A*B*ln(b*x+a)*a^8*b*c^3*d^2*n+576*A*B*ln
(b*x+a)*a^7*b^2*c^4*d*n+864*A*B*ln(d*x+c)*a^8*b*c^3*d^2*n-576*A*B*ln(d*x+c)
*a^7*b^2*c^4*d*n-2304*A*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^8*b*c^2*d^3+3456*
A*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c^3*d^2-2304*A*B*x*ln(e*(b*x+a)^n
/((d*x+c)^n))*a^6*b^3*c^4*d-1440*A*B*x*a^8*b*c^2*d^3*n+1440*A*B*x*a^7*b^2*c
^3*d^2*n-576*A*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^4*d+300*A*B*x^4*
a^6*b^3*c*d^4*n-576*A*B*x^4*a^5*b^4*c^2*d^3*n+432*A*B*x^4*a^4*b^5*c^3*d^2*n
-192*A*B*x^4*a^3*b^6*c^4*d*n+1056*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b
^2*c*d^4*n-2160*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3*n+1728*
B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^3*d^2*n-768*B^2*x^3*ln(e*(b*x
+a)^n/((d*x+c)^n))*a^4*b^5*c^4*d*n+576*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*
a^7*b^2*c*d^4-2304*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3+3456
*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^3*d^2+576*B^2*ln(b*x+a)*a^9*
c^2*d^3*n^2-36*B^2*ln(b*x+a)*a^6*b^3*c^5*n^2-576*B^2*ln(d*x+c)*a^9*c^2*d^3*
n^2+36*B^2*ln(d*x+c)*a^6*b^3*c^5*n^2+36*A*B*x^4*a^2*b^7*c^5*n+288*B^2*x^3*l
n(e*(b*x+a)^n/((d*x+c)^n))^2*a^7*b^2*c*d^4+144*B^2*x^3*ln(e*(b*x+a)^n/((d*x
+c)^n))*a^3*b^6*c^5*n+1360*B^2*x^3*a^7*b^2*c*d^4*n^2-2004*B^2*x^3*a^6*b^3*c
^2*d^3*n^2+864*B^2*x^3*a^5*b^4*c^3*d^2*n^2-256*B^2*x^3*a^4*b^5*c^4*d*n^2+57
6*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^5+144*A*B*x^3*a^3*b^6*c^5*n
+432*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^8*b*c*d^4+216*B^2*x^2*ln(e*(b*
x+a)^n/((d*x+c)^n))*a^4*b^5*c^5*n+1512*B^2*x^2*a^8*b*c*d^4*n^2-2400*B^2*x^2
*a^7*b^2*c^2*d^3*n^2+1218*B^2*x^2*a^6*b^3*c^3*d^2*n^2-384*B^2*x^2*a^5*b^4*c
^4*d*n^2+864*A*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^5+216*A*B*x^2*a^
4*b^5*c^5*n+576*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^9*c*d^4*n+72*A^2*x^4*a^
2*b^7*c^5+288*A^2*x^3*a^3*b^6*c^5+432*A^2*x^2*a^4*b^5*c^5+288*B^2*ln(e*(b*x
+a)^n/((d*x+c)^n))^2*a^9*c^2*d^3-72*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^6*b

```
^3*c^5+288*A^2*x*a^9*c*d^4+144*B^2*ln(d*x+c)*x*a^5*b^4*c^5*n^2-432*B^2*ln(b
*x+a)*a^8*b*c^3*d^2*n^2+192*B^2*ln(b*x+a)*a^7*b^2*c^4*d*n^2+432*B^2*ln(d*x+
c)*a^8*b*c^3*d^2*n^2-192*B^2*ln(d*x+c)*a^7*b^2*c^4*d*n^2+576*A*B*ln(b*x+a)*
a^9*c^2*d^3*n-144*A*B*ln(b*x+a)*a^6*b^3*c^5*n-576*A*B*ln(d*x+c)*a^9*c^2*d^3
*n+144*A*B*ln(d*x+c)*a^6*b^3*c^5*n+72*B^2*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))^2
*a^6*b^3*c*d^4+36*B^2*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^7*c^5*n+415*B^2
*x^4*a^6*b^3*c*d^4*n^2-576*B^2*x^4*a^5*b^4*c^2*d^3*n^2+216*B^2*x^4*a^4*b^5*
c^3*d^2*n^2-64*B^2*x^4*a^3*b^6*c^4*d*n^2+144*A*B*x^4*ln(e*(b*x+a)^n/((d*x+c
)^n))*a^2*b^7*c^5+576*A*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^9*c*d^4+576*A*B*x
*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^5+576*A*B*x*a^9*c*d^4*n+144*A*B*x*a^
5*b^4*c^5*n-720*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^4*d*n+288*A^2*x
^3*a^7*b^2*c*d^4-1152*A^2*x^3*a^6*b^3*c^2*d^3+1728*A^2*x^3*a^5*b^4*c^3*d^2-
1152*A^2*x^3*a^4*b^5*c^4*d+288*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^9*c*d^
4+576*B^2*x*a^9*c*d^4*n^2+36*B^2*x*a^5*b^4*c^5*n^2+432*A^2*x^2*a^8*b*c*d^4-
1728*A^2*x^2*a^7*b^2*c^2*d^3+2592*A^2*x^2*a^6*b^3*c^3*d^2-1728*A^2*x^2*a^5*
b^4*c^4*d-432*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^8*b*c^3*d^2+288*B^2*ln(e*
(b*x+a)^n/((d*x+c)^n))^2*a^7*b^2*c^4*d-1152*A^2*x*a^8*b*c^2*d^3+1728*A^2*x*
a^7*b^2*c^3*d^2-1152*A^2*x*a^6*b^3*c^4*d-432*B^2*ln(b*x+a)*x^4*a^4*b^5*c^3*
d^2*n^2+192*B^2*ln(b*x+a)*x^4*a^3*b^6*c^4*d*n^2-576*B^2*ln(d*x+c)*x^4*a^5*b
^4*c^2*d^3*n^2+432*B^2*ln(d*x+c)*x^4*a^4*b^5*c^3*d^2*n^2-192*B^2*ln(d*x+c)*
x^4*a^3*b^6*c^4*d*n^2-144*A*B*ln(b*x+a)*x^4*a^2*b^7*c^5*n+144*A*B*ln(d*x+c)
*x^4*a^2*b^7*c^5*n+2304*B^2*ln(b*x+a)*x^3*a^6*b^3*c^2*d^3*n^2+1728*B^2*ln(d
*x+c)*x^3*a^5*b^4*c^3*d^2*n^2-768*B^2*ln(d*x+c)*x^3*a^4*b^5*c^4*d*n^2-576*A
*B*ln(b*x+a)*x^3*a^3*b^6*c^5*n+576*A*B*ln(d*x+c)*x^3*a^3*b^6*c^5*n+3456*B^2
*ln(b*x+a)*x^2*a^7*b^2*c^2*d^3*n^2-2592*B^2*ln(b*x+a)*x^2*a^6*b^3*c^3*d^2*n
^2-768*A*B*x^3*a^4*b^5*c^4*d*n+1296*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^8
*b*c*d^4*n-2880*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c^2*d^3*n+2520*
B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^3*d^2*n-1152*B^2*x^2*ln(e*(b*
x+a)^n/((d*x+c)^n))*a^5*b^4*c^4*d*n-1728*B^2*ln(b*x+a)*x^3*a^5*b^4*c^3*d^2*
n^2+768*B^2*ln(b*x+a)*x^3*a^4*b^5*c^4*d*n^2-2304*B^2*ln(d*x+c)*x^3*a^6*b^3*
c^2*d^3*n^2+864*A*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^3*d^2+9*B^2*x
^4*a^2*b^7*c^5*n^2+36*B^2*x^3*a^3*b^6*c^5*n^2+72*A^2*x^4*a^6*b^3*c*d^4-288*
A^2*x^4*a^5*b^4*c^2*d^3+432*A^2*x^4*a^4*b^5*c^3*d^2-288*A^2*x^4*a^3*b^6*c^4
*d+54*B^2*x^2*a^4*b^5*c^5*n^2+576*B^2*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*n^2)/(b
*x+a)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/a^6
/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2458 vs. 2(571) = 1142.

Time = 0.37 (sec) , antiderivative size = 2458, normalized size of antiderivative = 4.19

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*
a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d
^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^
4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^
2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n
^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a
^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a)^2 - 72*(B^2*b^4*d^4*n
^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*
d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^
2*a^3*b*c*d^3)*n^2)*log(d*x + c)^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d +
6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 + 12*(3*A
*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d
^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2*d^2 + 324*B
^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2 + 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3
*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^4)*n)*x - 12*((25*B^2*b^4*
d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n + (3*B^2*b^4*c*d^3
+ 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2
*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B*a^2*b^2*d^4*n - (B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B*b
^4*c^4 - 4*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^2 - 4*A*B*a^3*b*c*d^3)*n +
 4*(12*A*B*a^3*b*d^4*n + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*
b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x + 12*(B^2*b^4*d^4*n*x^4 + 4*B^2*a*b^3*
d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2*b^4*c^4 -
4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n)*log(e))*l
og(b*x + a) + 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a
*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4
 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 +
6*(12*A*B*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2
*b^2*d^4)*n^2)*x^2 - 12*(A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^
2*d^2 - 4*A*B*a^3*b*c*d^3)*n + 4*(12*A*B*a^3*b*d^4*n + (B^2*b^4*c^3*d - 6*B
^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x + 12*(B^
2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4
```

```
*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2
*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a) + 12*(B^2*b^4*d^4*n*x^4 + 4*B^2
*a*b^3*d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2*b^4
*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n)*lo
g(e))*log(d*x + c) + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b
^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*
a*b^3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d
^4)*n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 -
 13*B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b
^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n)*log(e))/(a^4*b^5*c^4 -
 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c
^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4
 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a
^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*
a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5
*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^2}{(a + bx)^5}\, dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. 2(571) = 1142.

Time = 0.34 (sec) , antiderivative size = 2238, normalized size of antiderivative = 3.81

$$\int \frac{\left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^2}{(a + bx)^5}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/288*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^
3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 2
5*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n
- 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d +
```

```
3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
 a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*
b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3
*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^
4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4
*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 +
 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*
x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4
*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*
b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2
- 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*
n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4
*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*(
25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^
2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*
x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2
*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5
*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4
*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(
a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4
*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^
4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c
^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e^2)) - 1/4*B^2*log((b*x + a)^n*
e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4
*b) + 1/24*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d
^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a
^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5
*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*
a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^
3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*
x))*A*B/e - 1/2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 +
 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*
a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 1579, normalized size of antiderivative = 2.69

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*((b^5*c^4 - a^4*b*d^4 + 2*a^3*b^2*c*
d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d
) + 2*b*d*x)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)/(b
*(25*B^2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*
(a*d - b*c)^4) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(4*b*(a^4 + b^4*x^
4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B^2*d^4)/(4*b*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((72*A^2*a^
3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b^3*c^3*n^2 + 216*A^2*
a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3*c^3*n +
55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156*A*B*a*b^2*c^2*d*n -
276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3*n^2 - 13*
B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*d^2*n))/(2*(a*d - b*c
)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n
^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B*a*b^2*c*d^2*n))/(3*(
a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*d^2*n))/(a*d - b*c))/
(x*(96*a^3*b^4*c^2 + 96*a^5*b^2*d^2 - 192*a^4*b^3*c*d) + x^3*(96*a*b^6*c^2
+ 96*a^3*b^4*d^2 - 192*a^2*b^5*c*d) + x^4*(24*b^7*c^2 + 24*a^2*b^5*d^2 - 48
*a*b^6*c*d) + x^2*(144*a^2*b^5*c^2 + 144*a^4*b^3*d^2 - 288*a^3*b^4*c*d) + 2
4*a^6*b*d^2 + 24*a^4*b^3*c^2 - 48*a^5*b^2*c*d) - log((e*(a + b*x)^n)/(c + d
*x)^n)*((A*B)/(2*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x
^2)) + (B^2*d^4*(x^2*(b*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
n*(a*d - b*c))/(2*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d
 - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*c))/(2*d) + (b^3*n*(a*d - b*c)*(4*a
*d - b*c))/(2*d^2)) + a*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
```

```
n*(a*d - b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))
/(6*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d
- b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3
)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/(2
*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2))
 + (b^2*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (b*n*(a
*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4) +
(2*b^4*n*x^3*(a*d - b*c))/d))/(4*b*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4
*x^3 + 6*a^2*b^3*x^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*
d - 4*a^3*b*c*d^3)))
```

3.164 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

Optimal result

Integrand size = 33, antiderivative size = 809

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$= -\frac{B^3(bc-ad)^3n^3x}{4d^3} - \frac{B^3(bc-ad)^4n^3\log\left(\frac{a+bx}{c+dx}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\log(c+dx)}{2bd^4}$$
$$- \frac{7B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3}$$
$$+ \frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4}$$
$$- \frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$- \frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+ \frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$- \frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$- \frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+ \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+ \frac{7B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4}$$
$$- \frac{9B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{3B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{7B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

```
[Out] -1/4*B^3*(-a*d+b*c)^3*n^3*x/d^3-1/4*B^3*(-a*d+b*c)^4*n^3*ln((b*x+a)/(d*x+c)
)/b/d^4+3/2*B^3*(-a*d+b*c)^4*n^3*ln(d*x+c)/b/d^4-7/4*B^2*(-a*d+b*c)^3*n^2*(
b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+1/4*b*B^2*(-a*d+b*c)^2*n^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/d^4-9/2*B^2*(-a*d+b*c)^4*n^2*ln(
(-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^4-9/4*B*(-a*d+b
*c)^3*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+9/8*b*B*(-a*d+b*c
)^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-1/4*b^2*B*(-a*d+b*c
)*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-3/4*B*(-a*d+b*c)^4*n*
```

```
ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^4+1/4*(b*x
+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+7/4*B^2*(-a*d+b*c)^4*n^2*(A+B*l
n(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^4-9/2*B^3*(-a*d+b
*c)^4*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-3/2*B^2*(-a*d+b*c)^4*n^2*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-7/4*B^3
*(-a*d+b*c)^4*n^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b/d^4+3/2*B^3*(-a*d+b*c)^4
*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used

$= \{2573, 2549, 2381, 2395, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724\}$

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= -\frac{3Bn\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)^4}{4bd^4}$$
$$-\frac{B^3n^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{4bd^4}+\frac{3B^3n^3\log(c+dx)(bc-ad)^4}{2bd^4}$$
$$-\frac{9B^2n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(bc-ad)^4}{2bd^4}$$
$$+\frac{7B^2n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(bc-ad)^4}{4bd^4}$$
$$-\frac{9B^3n^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^4}{2bd^4}$$
$$-\frac{3B^2n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^4}{2bd^4}$$
$$-\frac{7B^3n^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)(bc-ad)^4}{4bd^4}+\frac{3B^3n^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^4}{2bd^4}$$
$$-\frac{9Bn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)^3}{4bd^3}-\frac{B^3n^3x(bc-ad)^3}{4d^3}$$
$$-\frac{7B^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(bc-ad)^3}{4bd^3}$$
$$+\frac{9bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)^2}{8d^4}$$
$$+\frac{bB^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(bc-ad)^2}{4d^4}$$
$$-\frac{b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)}{4d^4}$$
$$+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -1/4*(B^3*(b*c - a*d)^3*n^3*x)/d^3 - (B^3*(b*c - a*d)^4*n^3*Log[(a + b*x)/(
c + d*x)])/(4*b*d^4) + (3*B^3*(b*c - a*d)^4*n^3*Log[c + d*x])/(2*b*d^4) - (
7*B^2*(b*c - a*d)^3*n^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(4*b*d^3) + (b*B^2*(b*c - a*d)^2*n^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n]))/(4*d^4) - (9*B^2*(b*c - a*d)^4*n^2*Log[(b*c - a*d)/(b*(c +
 d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^4) - (9*B*(b*c - a
*d)^3*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b*d^3) + (
9*b*B*(b*c - a*d)^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
```

```
2)/(8*d^4) - (b^2*B*(b*c - a*d)*n*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])^2)/(4*d^4) - (3*B*(b*c - a*d)^4*n*Log[(b*c - a*d)/(b*(c + d*x))
]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b*d^4) + ((a + b*x)^4*(A +
 B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(4*b) + (7*B^2*(b*c - a*d)^4*n^2*(A
 + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))]
)/(4*b*d^4) - (9*B^3*(b*c - a*d)^4*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/(2*b*d^4) - (3*B^2*(b*c - a*d)^4*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4) - (7*B^3*(b*c -
a*d)^4*n^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(4*b*d^4) + (3*B^3*(b*c
 - a*d)^4*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^4\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^3}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$-\text{Subst}\left(\frac{(3B(bc-ad)^4n)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{4b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$-\text{Subst}\left(\frac{(3B(bc-ad)^4n)\,\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))^2}{d^3(b-dx)^4} - \frac{3b^2(A+B\log(ex^n))^2}{d^3(b-dx)^3} + \frac{3b(A+B\log(ex^n))^2}{d^3(b-dx)^2} - \frac{(A+B}{d^3(}\right.\right.}{4b}\right.$$
$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$

$$- \text{Subst}\left(\frac{(9B(bc-ad)^4n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$+ \text{Subst}\left(\frac{(3B(bc-ad)^4n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{4bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$+ \text{Subst}\left(\frac{(9bB(bc-ad)^4n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$- \text{Subst}\left(\frac{(3b^2B(bc-ad)^4n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+\frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$-\frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$-\frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+\text{Subst}\left(\frac{(3B^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(9bB^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(b^2B^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(9B^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$-\frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+\frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$-\frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$-\frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$-\frac{3B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$-\operatorname{Subst}\left(\frac{(9B^2(bc-ad)^4n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\operatorname{Subst}\left(\frac{(bB^2(bc-ad)^4n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\operatorname{Subst}\left(\frac{(9B^2(bc-ad)^4n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\operatorname{Subst}\left(\frac{(bB^2(bc-ad)^4n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\operatorname{Subst}\left(\frac{(3B^3(bc-ad)^4n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{9B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3}$$
$$+\frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4}$$
$$-\frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$-\frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+\frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$-\frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$-\frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+\frac{9B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4}$$
$$-\frac{9B^3(bc-ad)^4n^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$-\frac{3B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$+\frac{3B^3(bc-ad)^4n^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$+\text{Subst}\left(\frac{(B^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(B^2(bc-ad)^4n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(9B^3(bc-ad)^4n^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{4bd^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{9B^3(bc-ad)^4n^3\log(c+dx)}{4bd^4} - \frac{7B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3} + \frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4} - \frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4} - \frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3} + \frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4} - \frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4} - \frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4} + \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b} + \frac{7B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} - \frac{9B^3(bc-ad)^4n^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} - \frac{3B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} - \frac{9B^3(bc-ad)^4n^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} + \operatorname{Subst}\left(\frac{\left(B^3(bc-ad)^4n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \operatorname{Subst}\left(\frac{\left(bB^3(bc-ad)^4n^3\right)\operatorname{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{4d^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \operatorname{Subst}\left(\frac{\left(B^3(bc-ad)^4n^3\right)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B^3(bc-ad)^3n^3x}{4d^3} - \frac{B^3(bc-ad)^4n^3\log\left(\frac{a+bx}{c+dx}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\log(c+dx)}{2bd^4}$$
$$- \frac{7B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3}$$
$$+ \frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4}$$
$$- \frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$- \frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+ \frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$- \frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$- \frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+ \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+ \frac{7B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4}$$
$$- \frac{9B^3(bc-ad)^4n^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{3B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{7B^3(bc-ad)^4n^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6885 vs. $2(809) = 1618$.

Time = 2.32 (sec) , antiderivative size = 6885, normalized size of antiderivative = 8.51

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int (bx+a)^3 \left(A + B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
[Out] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (a+bx)^3 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3 dx$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
[Out] integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^3
*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2*a^
3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*x^2
+ 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] 3/4*A^2*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2*
B*a*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^2*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*A^2*B*a^3*x*lo
g((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*
log(d*x + c)/d)*A^2*B*a^3/e - 9/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a^2*b/e + 3/2*(2*a^3*e*n*l
og(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)
*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*a*b^2/e - 1/8*(6*a
^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n -
a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n
 - a^3*d^3*e*n)*x)/(b^3*d^3))*A^2*B*b^3/e - 1/8*(2*(B^3*b^4*d^4*x^4 + 4*B^3
*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((d*x + c)^n
)^3 - (6*B^3*a^4*d^4*n*log(b*x + a) + 6*(B^3*b^4*d^4*log(e) + A*B^2*b^4*d^4
)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^
3*n)*B^3*log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(n + 12*log(e))
- b^4*c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*b^2*d^4*(n + 4*l
og(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*A*B^2*a^3*b*d^4 +
 (a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*
c*d^3*n)*B^3)*x + 6*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*
d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^4)
- integrate(-1/4*(4*B^3*a^3*b*c*d^3*log(e)^3 + 12*A*B^2*a^3*b*c*d^3*log(e)^
2 + 4*(B^3*b^4*d^4*log(e)^3 + 3*A*B^2*b^4*d^4*log(e)^2)*x^4 + 4*(3*(b^4*c*d
^3*log(e)^2 + 3*a*b^3*d^4*log(e)^2)*A*B^2 + (b^4*c*d^3*log(e)^3 + 3*a*b^3*d
^4*log(e)^3)*B^3)*x^3 + 4*(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 +
 3*a*b^3*d^4)*B^3*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*
c*d^3 + a^3*b*d^4)*B^3*x)*log((b*x + a)^n)^3 + 12*(3*(a*b^3*c*d^3*log(e)^2
+ a^2*b^2*d^4*log(e)^2)*A*B^2 + (a*b^3*c*d^3*log(e)^3 + a^2*b^2*d^4*log(e)^
3)*B^3)*x^2 + 12*(B^3*a^3*b*c*d^3*log(e) + A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4
*log(e) + A*B^2*b^4*d^4)*x^4 + ((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (b^4*c*d^
3*log(e) + 3*a*b^3*d^4*log(e))*B^3)*x^3 + 3*((a*b^3*c*d^3 + a^2*b^2*d^4)*A*
B^2 + (a*b^3*c*d^3*log(e) + a^2*b^2*d^4*log(e))*B^3)*x^2 + ((3*a^2*b^2*c*d^
3 + a^3*b*d^4)*A*B^2 + (3*a^2*b^2*c*d^3*log(e) + a^3*b*d^4*log(e))*B^3)*x)*
log((b*x + a)^n)^2 + 4*(3*(3*a^2*b^2*c*d^3*log(e)^2 + a^3*b*d^4*log(e)^2)*A
*B^2 + (3*a^2*b^2*c*d^3*log(e)^3 + a^3*b*d^4*log(e)^3)*B^3)*x + 12*(B^3*a^3
```

```
*b*c*d^3*log(e)^2 + 2*A*B^2*a^3*b*c*d^3*log(e) + (B^3*b^4*d^4*log(e)^2 + 2*
A*B^2*b^4*d^4*log(e))*x^4 + (2*(b^4*c*d^3*log(e) + 3*a*b^3*d^4*log(e))*A*B^
2 + (b^4*c*d^3*log(e)^2 + 3*a*b^3*d^4*log(e)^2)*B^3)*x^3 + 3*(2*(a*b^3*c*d^
3*log(e) + a^2*b^2*d^4*log(e))*A*B^2 + (a*b^3*c*d^3*log(e)^2 + a^2*b^2*d^4*
log(e)^2)*B^3)*x^2 + (2*(3*a^2*b^2*c*d^3*log(e) + a^3*b*d^4*log(e))*A*B^2 +
 (3*a^2*b^2*c*d^3*log(e)^2 + a^3*b*d^4*log(e)^2)*B^3)*x)*log((b*x + a)^n) -
 (6*B^3*a^4*d^4*n^2*log(b*x + a) + 12*B^3*a^3*b*c*d^3*log(e)^2 + 24*A*B^2*a
^3*b*c*d^3*log(e) + 6*((n*log(e) + 2*log(e)^2)*B^3*b^4*d^4 + A*B^2*b^4*d^4*
(n + 4*log(e)))*x^4 + 6*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^
2*n^2 - 4*a^3*b*c*d^3*n^2)*B^3*log(d*x + c) + 2*(12*(a*b^3*d^4*(n + 3*log(e
)) + b^4*c*d^3*log(e))*A*B^2 - ((n^2 - 6*log(e)^2)*b^4*c*d^3 - (n^2 + 12*n*
log(e) + 18*log(e)^2)*a*b^3*d^4)*B^3)*x^3 + 3*(12*(a^2*b^2*d^4*(n + 2*log(e
)) + 2*a*b^3*c*d^3*log(e))*A*B^2 + (b^4*c^2*d^2*n^2 - 4*(n^2 - 3*log(e)^2)*
a*b^3*c*d^3 + 3*(n^2 + 4*n*log(e) + 4*log(e)^2)*a^2*b^2*d^4)*B^3)*x^2 + 12*
(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3*x^3 + 3*
(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3*x)*
log((b*x + a)^n)^2 + 6*(4*(a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*log(e))
*A*B^2 - (b^4*c^3*d*n^2 - 4*a*b^3*c^2*d^2*n^2 + 6*(n^2 - log(e)^2)*a^2*b^2*
c*d^3 - (3*n^2 + 4*n*log(e) + 2*log(e)^2)*a^3*b*d^4)*B^3)*x + 6*(4*B^3*a^3*
b*c*d^3*log(e) + 4*A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4*(n + 4*log(e)) + 4*A*B^
2*b^4*d^4)*x^4 + 4*((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (a*b^3*d^4*(n + 3*log
(e)) + b^4*c*d^3*log(e))*B^3)*x^3 + 6*(2*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2
+ (a^2*b^2*d^4*(n + 2*log(e)) + 2*a*b^3*c*d^3*log(e))*B^3)*x^2 + 4*((3*a^2*
b^2*c*d^3 + a^3*b*d^4)*A*B^2 + (a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*lo
g(e))*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^4*x + b*c*d^3), x)
```

Giac [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 (a+b\,x)^3\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3, x)
```

3.165 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

Optimal result

Integrand size = 33, antiderivative size = 614

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx \\
&= -\frac{B^3(bc-ad)^3n^3\log(c+dx)}{bd^3} \\
&\quad + \frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&\quad + \frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^3} \\
&\quad + \frac{2B(bc-ad)^2n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2} \\
&\quad - \frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3} \\
&\quad + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3} \\
&\quad + \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} \\
&\quad - \frac{B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} \\
&\quad + \frac{4B^3(bc-ad)^3n^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&\quad + \frac{B^3(bc-ad)^3n^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} - \frac{2B^3(bc-ad)^3n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}
\end{aligned}$$

```
[Out] -B^3*(-a*d+b*c)^3*n^3*ln(d*x+c)/b/d^3+B^2*(-a*d+b*c)^2*n^2*(b*x+a)*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+4*B^2*(-a*d+b*c)^3*n^2*ln((-a*d+b*c)/b/(d*x
+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2*B*(-a*d+b*c)^2*n*(b*x+a)*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^2-1/2*b*B*(-a*d+b*c)*n*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^3+B*(-a*d+b*c)^3*n*ln((-a*d+b*c)/b/(d*x+c)
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))^3/b-B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^3+4*B^3*(-a*d+b*c)^3*n^3*polylog(2,d*(b*x+a)
/b/(d*x+c))/b/d^3+2*B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3+B^3*(-a*d+b*c)^3*n^3*polylog(2,b*(d*x+
c)/d/(b*x+a))/b/d^3-2*B^3*(-a*d+b*c)^3*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
/d^3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2573, 2549, 2381, 2395, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \frac{2B^2n^2(bc-ad)^3 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^3}$$
$$+\frac{4B^2n^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^3}$$
$$-\frac{B^2n^2(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^3}$$
$$+\frac{B^2n^2(a+bx)(bc-ad)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}$$
$$+\frac{Bn(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{bd^3}$$
$$-\frac{bBn(c+dx)^2(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2d^3}$$
$$+\frac{2Bn(a+bx)(bc-ad)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{bd^2}$$
$$+\frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{3b}$$
$$+\frac{4B^3n^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}+\frac{B^3n^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3}$$
$$-\frac{2B^3n^3(bc-ad)^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}-\frac{B^3n^3(bc-ad)^3\log(c+dx)}{bd^3}$$

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((B^3*(b*c - a*d)^3*n^3*Log[c + d*x])/(b*d^3)) + (B^2*(b*c - a*d)^2*n^2*(a
 + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d^2) + (4*B^2*(b*c - a
*d)^3*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n]))/(b*d^3) + (2*B*(b*c - a*d)^2*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^2)/(b*d^2) - (b*B*(b*c - a*d)*n*(c + d*x)^2*(A + B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n])^2)/(2*d^3) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b
*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b*d^3) + ((a + b*
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b) - (B^2*(b*c - a*d)^3
*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/(b*d^3) + (4*B^3*(b*c - a*d)^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c
+ d*x))])/(b*d^3) + (2*B^2*(b*c - a*d)^3*n^2*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3) + (B^3*(b*c - a
*d)^3*n^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*d^3) - (2*B^3*(b*c -
a*d)^3*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^3 \text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^3}{(b-dx)^4} dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} - \text{Subst}\left(\frac{(B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} - \text{Subst}\left(\frac{(B(bc-ad)^3n)\,\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{d^2(b-dx)^3} - \frac{2b(A+B\log(ex^n))^2}{d^2(b-dx)^2} + \frac{(A+B\log(ex^n))^2}{d^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{b}, \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} + \text{Subst}\left(\frac{(2B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$- \text{Subst}\left(\frac{(B(bc-ad)^3n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$- \text{Subst}\left(\frac{(bB(bc-ad)^3n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{2B(bc-ad)^2 n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2}$$
$$- \frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3}$$
$$+ \frac{B(bc-ad)^3 n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3}$$
$$+ \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b}$$
$$-\text{Subst}\left(\frac{(2B^2(bc-ad)^3n^2)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(bB^2(bc-ad)^3n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(4B^2(bc-ad)^3n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd^3} + \frac{2B(bc-ad)^2n(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{bd^2} - \frac{bB(bc-ad)n(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2d^3} + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{bd^3} + \frac{(a+bx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3b} + \frac{2B^2(bc-ad)^3n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} + \operatorname{Subst}\left(\frac{(B^2(bc-ad)^3n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}dx,x,\frac{a+bx}{c+dx}\right)}{d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) + \operatorname{Subst}\left(\frac{(B^2(bc-ad)^3n^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}dx,x,\frac{a+bx}{c+dx}\right)}{d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \operatorname{Subst}\left(\frac{(2B^3(bc-ad)^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \operatorname{Subst}\left(\frac{(4B^3(bc-ad)^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} + \frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^3} + \frac{2B(bc-ad)^2n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2} - \frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3} + \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3} + \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} - \frac{B^2(bc-ad)^3n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} + \frac{4B^3(bc-ad)^3n^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} + \frac{2B^2(bc-ad)^3n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} - \frac{2B^3(bc-ad)^3n^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} + \text{Subst}\left(\frac{(B^3(bc-ad)^3n^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(B^3(bc-ad)^3n^3)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= -\frac{B^3(bc-ad)^3n^3\log(c+dx)}{bd^3} \\
&+ \frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&+ \frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^3} \\
&+ \frac{2B(bc-ad)^2n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2} \\
&- \frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3} \\
&+ \frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3} \\
&+ \frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} \\
&- \frac{B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} \\
&+ \frac{4B^3(bc-ad)^3n^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&+ \frac{2B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&+ \frac{B^3(bc-ad)^3n^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} - \frac{2B^3(bc-ad)^3n^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4802 vs. $2(614) = 1228$.

Time = 1.38 (sec) , antiderivative size = 4802, normalized size of antiderivative = 7.82

$$
\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
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```
[Out] (-6*a^3*A*B^2*n^2)/b - (2*a*A*b*B^2*c^2*n^2)/d^2 + (4*a^2*A*B^2*c*n^2)/d -
(4*a^3*B^3*n^3)/b - (3*a*b*B^3*c^2*n^3)/d^2 + (7*a^2*B^3*c*n^3)/d + a^2*A^3
*x + 2*a^2*A^2*B*n*x + (A^2*b^2*B*c^2*n*x)/d^2 - (3*a*A^2*b*B*c*n*x)/d + a^
2*A*B^2*n^2*x + (A*b^2*B^2*c^2*n^2*x)/d^2 - (2*a*A*b*B^2*c*n^2*x)/d + a*A^3
*b*x^2 + (a*A^2*b*B*n*x^2)/2 - (A^2*b^2*B*c*n*x^2)/(2*d) + (A^3*b^2*x^3)/3
+ (a^3*A^2*B*n*Log[a + b*x])/b + (3*a^3*A*B^2*n^2*Log[a + b*x])/b + (2*a*A*
b*B^2*c^2*n^2*Log[a + b*x])/d^2 - (5*a^2*A*B^2*c*n^2*Log[a + b*x])/d + (7*a
^3*B^3*n^3*Log[a + b*x])/b + (3*a*b*B^3*c^2*n^3*Log[a + b*x])/d^2 - (6*a^2*
```

B^3*c*n^3*Log[a + b*x])/d - (a^3*A*B^2*n^2*Log[a + b*x]^2)/b - (3*a^3*B^3*n
^3*Log[a + b*x]^2)/(2*b) - (a*b*B^3*c^2*n^3*Log[a + b*x]^2)/d^2 + (5*a^2*B^
3*c*n^3*Log[a + b*x]^2)/(2*d) + (a^3*B^3*n^3*Log[a + b*x]^3)/(3*b) - (A^2*b
^2*B*c^3*n*Log[c + d*x])/d^3 + (3*a*A^2*b*B*c^2*n*Log[c + d*x])/d^2 - (3*a^
2*A^2*B*c*n*Log[c + d*x])/d - (3*A*b^2*B^2*c^3*n^2*Log[c + d*x])/d^3 + (7*a
*A*b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (4*a^2*A*B^2*c*n^2*Log[c + d*x])/d - (
6*a^3*B^3*n^3*Log[c + d*x])/b - (b^2*B^3*c^3*n^3*Log[c + d*x])/d^3 + (3*a^2
*B^3*c*n^3*Log[c + d*x])/d + (2*a^3*A*B^2*n^2*Log[a + b*x]*Log[c + d*x])/b
+ (2*A*b^2*B^2*c^3*n^2*Log[a + b*x]*Log[c + d*x])/d^3 - (6*a*A*b*B^2*c^2*n^
2*Log[a + b*x]*Log[c + d*x])/d^2 + (6*a^2*A*B^2*c*n^2*Log[a + b*x]*Log[c +
d*x])/d + (3*b^2*B^3*c^3*n^3*Log[a + b*x]*Log[c + d*x])/d^3 - (7*a*b*B^3*c^
2*n^3*Log[a + b*x]*Log[c + d*x])/d^2 + (4*a^2*B^3*c*n^3*Log[a + b*x]*Log[c
+ d*x])/d - (2*a^3*B^3*n^3*Log[a + b*x]^2*Log[c + d*x])/b - (b^2*B^3*c^3*n^
3*Log[a + b*x]^2*Log[c + d*x])/d^3 + (3*a*b*B^3*c^2*n^3*Log[a + b*x]^2*Log[
c + d*x])/d^2 - (3*a^2*B^3*c*n^3*Log[a + b*x]^2*Log[c + d*x])/d - (2*a^3*A*
B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x])/b + (2*a^3*B^3*n^3*
Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x])/b - (A*b^2*B^2
*c^3*n^2*Log[c + d*x]^2)/d^3 + (3*a*A*b*B^2*c^2*n^2*Log[c + d*x]^2)/d^2 - (
3*a^2*A*B^2*c*n^2*Log[c + d*x]^2)/d - (3*b^2*B^3*c^3*n^3*Log[c + d*x]^2)/(2
*d^3) + (7*a*b*B^3*c^2*n^3*Log[c + d*x]^2)/(2*d^2) - (2*a^2*B^3*c*n^3*Log[c
 + d*x]^2)/d + (a^3*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2)/b + (2*b^2*B^3*c^3
*n^3*Log[a + b*x]*Log[c + d*x]^2)/d^3 - (6*a*b*B^3*c^2*n^3*Log[a + b*x]*Log
[c + d*x]^2)/d^2 + (6*a^2*B^3*c*n^3*Log[a + b*x]*Log[c + d*x]^2)/d - (a^3*B
^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2)/b - (b^2*B^3*c^3*n
^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2)/d^3 + (3*a*b*B^3*c^2*n
^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2)/d^2 - (3*a^2*B^3*c*n^3
Log[(d(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2)/d - (b^2*B^3*c^3*n^3*Log
[c + d*x]^3)/(3*d^3) + (a*b*B^3*c^2*n^3*Log[c + d*x]^3)/d^2 - (a^2*B^3*c*n^
3*Log[c + d*x]^3)/d - (2*A*b^2*B^2*c^3*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(
b*c - a*d)])/d^3 + (6*a*A*b*B^2*c^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c
 - a*d)])/d^2 - (6*a^2*A*B^2*c*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/d + (3*a^3*B^3*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/b - (3
*b^2*B^3*c^3*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d^3 + (9*a*b*
B^3*c^2*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d^2 - (9*a^2*B^3*c
*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d + (a^3*B^3*n^3*Log[a +
b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)])/b + (b^2*B^3*c^3*n^3*Log[a + b*x]^2*
Log[(b*(c + d*x))/(b*c - a*d)])/d^3 - (3*a*b*B^3*c^2*n^3*Log[a + b*x]^2*Log
[(b*(c + d*x))/(b*c - a*d)])/d^2 + (3*a^2*B^3*c*n^3*Log[a + b*x]^2*Log[(b*(
c + d*x))/(b*c - a*d)])/d - (2*b^2*B^3*c^3*n^3*Log[a + b*x]*Log[c + d*x]*Lo
g[(b*(c + d*x))/(b*c - a*d)])/d^3 + (6*a*b*B^3*c^2*n^3*Log[a + b*x]*Log[c +
 d*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d^2 - (6*a^2*B^3*c*n^3*Log[a + b*x]*L
og[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d - (6*a^3*B^3*n^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n])/b - (2*a*b*B^3*c^2*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/d^2 + (4*a^2*B^3*c*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + 3*a^2*A^2*
B*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*A*B^2*n*x*Log[(e*(a + b*x)^n)/

(c + d*x)^n] + (2*A*b^2*B^2*c^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (6*a*A*b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + a^2*B^3*n^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + (b^2*B^3*c^2*n^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (2*a*b*B^3*c*n^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + 3*a*A^2*b*B*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*A*b*B^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - (A*b^2*B^2*c*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A^2*b^2*B*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] + (2*a^3*A*B^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + (3*a^3*B^3*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + (2*a*b*B^3*c^2*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (5*a^2*B^3*c*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d - (a^3*B^3*n^2*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b - (2*A*b^2*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^3 + (6*a*A*b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (6*a^2*A*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d - (3*b^2*B^3*c^3*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^3 + (7*a*b*B^3*c^2*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (4*a^2*B^3*c*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + (2*a^3*B^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + (2*b^2*B^3*c^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^3 - (6*a*b*B^3*c^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 + (6*a^2*B^3*c*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d - (2*a^3*B^3*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b - (b^2*B^3*c^3*n^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^3 + (3*a*b*B^3*c^2*n^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (3*a^2*B^3*c*n^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d - (2*b^2*B^3*c^3*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^3 + (6*a*b*B^3*c^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (6*a^2*B^3*c*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + 3*a^2*A*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*a^2*B^3*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b^2*B^3*c^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/d^2 - (3*a*b*B^3*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/d + 3*a*A*b*B^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (a*b*B^3*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/2 - (b^2*B^3*c*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*d) + A*b^2*B^2*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (a^3*B^3*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/b - (b^2*B^3*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/d^3 + (3*a*b*B^3*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/d^2 - (3*a^2*B^3*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/d + a^2*B^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + a*b*B^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + (b^2*B^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/3 - (B^2*n^2*(2*A*b^3*c^3 - 6*a*A*b^2*c^2*d + 6*a^2*A*b*c*d^2 + 3*b^3*B*c^3*n - 9*a*b^2*B*c^2*d*n + 9*a^2*b*B*c*d^2*n - 3*a^3*B*d^3*n - 2*a^3*B*d^3*n*Log[a + b*x] + 2*b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*Log[c + d*x] + 2*b^3*B*c^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B*c^2*d*Log[(e*(a +

```
b*x)^n)/(c + d*x)^n] + 6*a^2*b*B*c*d^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d^3) - (2*B^2*n^2*(-(a^3*B*d^3*n
*Log[a + b*x]) + b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*Log[c + d*x] + a
^3*d^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])/(b*d^3) - (2*a^3*B^3*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*
d)])/b + (2*b^2*B^3*c^3*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)])/d^3 -
 (6*a*b*B^3*c^2*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)])/d^2 + (6*a^2*
B^3*c*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)])/d - (2*a^3*B^3*n^3*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)])/b + (2*b^2*B^3*c^3*n^3*PolyLog[3, (b*(c
+ d*x))/(b*c - a*d)])/d^3 - (6*a*b*B^3*c^2*n^3*PolyLog[3, (b*(c + d*x))/(b*
c - a*d)])/d^2 + (6*a^2*B^3*c*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/d
```

Maple [F]

$$\int (bx+a)^2 \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*b^2*x^2 + 2*A^3*a*b*x + A^3*a^2 + (B^3*b^2*x^2 + 2*B^3*a*b*x +
 B^3*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^2*x^2 + 2*A*B^2*a*b
*x + A*B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^2*x^2 + 2*A^2
*B*a*b*x + A^2*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
[Out] A^2*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*A^2*B*a^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^3*a^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A^2*B*a^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a*b/e + 1/2*(2*a^3*e*n*log(b*x + a
)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*b^2/e - 1/6*(2*(B^3*b^3*d^3*
x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((d*x + c)^n)^3 - 3*(2*B^
3*a^3*d^3*n*log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n
)*B^3*log(d*x + c) + 2*(B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + (6*A*B^2*
a*b^2*d^3 + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^3)*x^2 + 2*(3*A*B^2*
a^2*b*d^3 + (a^2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^
3)*x + 2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((b
*x + a)^n))*log((d*x + c)^n)^2)/(b*d^3) - integrate(-(B^3*a^2*b*c*d^2*log(e
)^3 + 3*A*B^2*a^2*b*c*d^2*log(e)^2 + (B^3*b^3*d^3*log(e)^3 + 3*A*B^2*b^3*d^
3*log(e)^2)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*log((b*x + a)^n)^3 + (3*
(b^3*c*d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*A*B^2 + (b^3*c*d^2*log(e)^3 + 2
*a*b^2*d^3*log(e)^3)*B^3)*x^2 + 3*(B^3*a^2*b*c*d^2*log(e) + A*B^2*a^2*b*c*d
^2 + (B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + ((b^3*c*d^2 + 2*a*b^2*d^3)*
A*B^2 + (b^3*c*d^2*log(e) + 2*a*b^2*d^3*log(e))*B^3)*x^2 + ((2*a*b^2*c*d^2
+ a^2*b*d^3)*A*B^2 + (2*a*b^2*c*d^2*log(e) + a^2*b*d^3*log(e))*B^3)*x)*log(
(b*x + a)^n)^2 + (3*(2*a*b^2*c*d^2*log(e)^2 + a^2*b*d^3*log(e)^2)*A*B^2 + (
```

```
2*a*b^2*c*d^2*log(e)^3 + a^2*b*d^3*log(e)^3)*B^3)*x + 3*(B^3*a^2*b*c*d^2*lo
g(e)^2 + 2*A*B^2*a^2*b*c*d^2*log(e) + (B^3*b^3*d^3*log(e)^2 + 2*A*B^2*b^3*d
^3*log(e))*x^3 + (2*(b^3*c*d^2*log(e) + 2*a*b^2*d^3*log(e))*A*B^2 + (b^3*c*
d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*B^3)*x^2 + (2*(2*a*b^2*c*d^2*log(e) +
a^2*b*d^3*log(e))*A*B^2 + (2*a*b^2*c*d^2*log(e)^2 + a^2*b*d^3*log(e)^2)*B^3
)*x)*log((b*x + a)^n) - (2*B^3*a^3*d^3*n^2*log(b*x + a) + 3*B^3*a^2*b*c*d^2
*log(e)^2 + 6*A*B^2*a^2*b*c*d^2*log(e) - 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2
 + 3*a^2*b*c*d^2*n^2)*B^3*log(d*x + c) + ((2*n*log(e) + 3*log(e)^2)*B^3*b^3
*d^3 + 2*A*B^2*b^3*d^3*(n + 3*log(e)))*x^3 + (6*(a*b^2*d^3*(n + 2*log(e)) +
 b^3*c*d^2*log(e))*A*B^2 - ((n^2 - 3*log(e)^2)*b^3*c*d^2 - (n^2 + 6*n*log(e
) + 6*log(e)^2)*a*b^2*d^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2
+ (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*lo
g((b*x + a)^n)^2 + (6*(a^2*b*d^3*(n + log(e)) + 2*a*b^2*c*d^2*log(e))*A*B^2
 + (2*b^3*c^2*d*n^2 - 6*(n^2 - log(e)^2)*a*b^2*c*d^2 + (4*n^2 + 6*n*log(e)
+ 3*log(e)^2)*a^2*b*d^3)*B^3)*x + 2*(3*B^3*a^2*b*c*d^2*log(e) + 3*A*B^2*a^2
*b*c*d^2 + (B^3*b^3*d^3*(n + 3*log(e)) + 3*A*B^2*b^3*d^3)*x^3 + 3*((b^3*c*d
^2 + 2*a*b^2*d^3)*A*B^2 + (a*b^2*d^3*(n + 2*log(e)) + b^3*c*d^2*log(e))*B^3
)*x^2 + 3*((2*a*b^2*c*d^2 + a^2*b*d^3)*A*B^2 + (a^2*b*d^3*(n + log(e)) + 2*
a*b^2*c*d^2*log(e))*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^3*x +
b*c*d^2), x)
```

Giac [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (bx+a)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,(a+b\,x)^2\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2, x)
```

3.166 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 376

$$\begin{aligned}
&\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx \\
&= -\frac{3B^2(bc-ad)^2n^2\log \left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&\quad -\frac{3B(bc-ad)n(a+bx)\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd} \\
&\quad -\frac{3B(bc-ad)^2n\log \left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^2} \\
&\quad +\frac{(a+bx)^2\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} \\
&\quad -\frac{3B^3(bc-ad)^2n^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad -\frac{3B^2(bc-ad)^2n^2\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad +\frac{3B^3(bc-ad)^2n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}
\end{aligned}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))/b/d^2-3/2*B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
2/b/d-3/2*B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))^2/b/d^2+1/2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b-3*B^
3*(-a*d+b*c)^2*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2-3*B^2*(-a*d+b*c)^2*
n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2+
3*B^3*(-a*d+b*c)^2*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2573, 2549, 2381, 2395, 2355, 2354, 2438, 2421, 6724}

$$\begin{aligned}&\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx\\&=-\frac{3B^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}\\&\quad-\frac{3B^2n^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}\\&\quad-\frac{3Bn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2bd^2}\\&\quad-\frac{3Bn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2bd}\\&\quad+\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2b}\\&\quad-\frac{3B^3n^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{3B^3n^3(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*B^2*(b*c - a*d)^2*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]))/(b*d^2) - (3*B*(b*c - a*d)*n*(a + b*x)*(A + B*Log[(
e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d) - (3*B*(b*c - a*d)^2*n*Log[(b*c - a
*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d^2) +
((a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*b) - (3*B^3*(b*
c - a*d)^2*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^2*(b
*c - a*d)^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/(b*d^2) + (3*B^3*(b*c - a*d)^2*n^3*PolyLog[3, (d*(a
+ b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

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rQ[n]
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Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)^2\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^3}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} - \text{Subst}\left(\frac{(3B(bc-ad)^2n)\,\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} - \text{Subst}\left(\frac{(3B(bc-ad)^2n)\,\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{d(-b+dx)^2} + \frac{(A+B\log(ex^n))^2}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2b}, e\left(\frac{a+bx}{c+dx}\right)^n, e + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} - \text{Subst}\left(\frac{(3B(bc-ad)^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3B(bc-ad)^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd} - \frac{3B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^2} + \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} + \text{Subst}\left(\frac{(3B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B^2(bc-ad)^2n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd^2} - \frac{3B(bc-ad)n(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2bd} - \frac{3B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2bd^2} + \frac{(a+bx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2b} - \frac{3B^2(bc-ad)^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \operatorname{Subst}\left(\frac{(3B^3(bc-ad)^2n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) + \operatorname{Subst}\left(\frac{(3B^3(bc-ad)^2n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B^2(bc-ad)^2n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd^2} - \frac{3B(bc-ad)n(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2bd} - \frac{3B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2bd^2} + \frac{(a+bx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2b} - \frac{3B^3(bc-ad)^2n^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{3B^2(bc-ad)^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{3B^3(bc-ad)^2n^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2984 vs. 2(376) = 752.

Time = 0.74 (sec) , antiderivative size = 2984, normalized size of antiderivative = 7.94

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-12*a^2*A*B^2*d^2*n^2 + 6*a*b*B^3*c*d*n^3 - 6*a^2*B^3*d^2*n^3 + 2*a*A^3*b*
d^2*x - 3*A^2*b^2*B*c*d*n*x + 3*a*A^2*b*B*d^2*n*x + A^3*b^2*d^2*x^2 + 3*a^2
*A^2*B*d^2*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*n^2*Log[a + b*x] + 6*a^2*A*B^2*
d^2*n^2*Log[a + b*x] + 12*a^2*B^3*d^2*n^3*Log[a + b*x] - 3*a^2*A*B^2*d^2*n^
2*Log[a + b*x]^2 + 3*a*b*B^3*c*d*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*n^3*Log
[a + b*x]^2 + a^2*B^3*d^2*n^3*Log[a + b*x]^3 + 3*A^2*b^2*B*c^2*n*Log[c + d*
x] - 6*a*A^2*b*B*c*d*n*Log[c + d*x] + 6*A*b^2*B^2*c^2*n^2*Log[c + d*x] - 6*
a*A*b*B^2*c*d*n^2*Log[c + d*x] - 12*a^2*B^3*d^2*n^3*Log[c + d*x] - 6*A*b^2*
B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x] + 12*a*A*b*B^2*c*d*n^2*Log[a + b*x]*L
og[c + d*x] + 6*a^2*A*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c^2
*n^3*Log[a + b*x]*Log[c + d*x] + 6*a*b*B^3*c*d*n^3*Log[a + b*x]*Log[c + d*x
] + 3*b^2*B^3*c^2*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*a*b*B^3*c*d*n^3*Log[a
 + b*x]^2*Log[c + d*x] - 6*a^2*B^3*d^2*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*
a^2*A*B^2*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 6*a^2*B^
3*d^2*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*A
*b^2*B^2*c^2*n^2*Log[c + d*x]^2 - 6*a*A*b*B^2*c*d*n^2*Log[c + d*x]^2 + 3*b^
2*B^3*c^2*n^3*Log[c + d*x]^2 - 3*a*b*B^3*c*d*n^3*Log[c + d*x]^2 - 6*b^2*B^3
*c^2*n^3*Log[a + b*x]*Log[c + d*x]^2 + 12*a*b*B^3*c*d*n^3*Log[a + b*x]*Log[
c + d*x]^2 + 3*a^2*B^3*d^2*n^3*Log[a + b*x]*Log[c + d*x]^2 + 3*b^2*B^3*c^2*
n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 6*a*b*B^3*c*d*n^3*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 3*a^2*B^3*d^2*n^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + b^2*B^3*c^2*n^3*Log[c + d*x]^3 -
 2*a*b*B^3*c*d*n^3*Log[c + d*x]^3 + 6*A*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[(b
*(c + d*x))/(b*c - a*d)] - 12*a*A*b*B^2*c*d*n^2*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)] + 6*b^2*B^3*c^2*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
a*d)] - 12*a*b*B^3*c*d*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*
a^2*B^3*d^2*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 3*b^2*B^3*c^2
*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*a*b*B^3*c*d*n^3*Log[
a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 3*a^2*B^3*d^2*n^3*Log[a + b*x]^
2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*b^2*B^3*c^2*n^3*Log[a + b*x]*Log[c + d
*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12*a*b*B^3*c*d*n^3*Log[a + b*x]*Log[c
+ d*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12*a^2*B^3*d^2*n^2*Log[(e*(a + b*x)
^n)/(c + d*x)^n] + 6*a*A^2*b*B*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*A
*b^2*B^2*c*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*A*b*B^2*d^2*n*x*Log
[(e*(a + b*x)^n)/(c + d*x)^n] + 3*A^2*b^2*B*d^2*x^2*Log[(e*(a + b*x)^n)/(c
```

+ d*x)^n] + 6*a^2*A*B^2*d^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*c*d*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a^2*B^3*d^2*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 3*a^2*B^3*d^2*n^2*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*A*b^2*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a*A*b*B^2*c*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^2*B^3*c^2*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*c*d*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^2*B^3*c^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a*b*B^3*c*d*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a^2*B^3*d^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*B^3*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*b^2*B^3*c^2*n^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*c*d*n^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^2*B^3*c^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a*b*B^3*c*d*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*A*b*B^2*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 3*b^2*B^3*c*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b*B^3*d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*A*b^2*B^2*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a^2*B^3*d^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^2*B^3*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*a*b*B^3*c*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*a*b*B^3*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + b^2*B^3*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 6*B^2*n^2*(A*b^2*c^2 - 2*a*A*b*c*d + b^2*B*c^2*n - 2*a*b*B*c*d*n + a^2*B*d^2*n + a^2*B*d^2*n*Log[a + b*x] + b*B*c*(b*c - 2*a*d)*n*Log[c + d*x] + b^2*B*c^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*a*b*B*c*d*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*B^2*n^2*(a^2*B*d^2*n*Log[a + b*x] + b*B*c*(b*c - 2*a*d)*n*Log[c + d*x] - a^2*d^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 6*b^2*B^3*c^2*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 12*a*b*B^3*c*d*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*a^2*B^3*d^2*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*b^2*B^3*c^2*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 12*a*b*B^3*c*d*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 6*a^2*B^3*d^2*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/(2*b*d^2)

Maple [F]

$$\int (bx+a)\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3dx$$

[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)

[Out] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)

Fricas [F]

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \int (bx+a)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b*
x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \int (bx+a)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*A^2*B*a*
x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x + 3*(a*e*n*log(b*x + a)/b - c*e*
n*log(d*x + c)/d)*A^2*B*a/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*b/e - 1/2*((B^3*b^2*d^2*x^
2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x + a) + (
b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (B^3*b^2*d^2*log(e) + A*B^2*b^2
*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^3)*x
+ (B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)
```

```
/(b*d^2) - integrate(-(B^3*a*b*c*d*log(e)^3 + 3*A*B^2*a*b*c*d*log(e)^2 + (B
^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n)^
3 + (B^3*b^2*d^2*log(e)^3 + 3*A*B^2*b^2*d^2*log(e)^2)*x^2 + 3*(B^3*a*b*c*d*
log(e) + A*B^2*a*b*c*d + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x^2 + ((b^2*c
*d + a*b*d^2)*A*B^2 + (b^2*c*d*log(e) + a*b*d^2*log(e))*B^3)*x)*log((b*x +
a)^n)^2 + (3*(b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*A*B^2 + (b^2*c*d*log(e)^
3 + a*b*d^2*log(e)^3)*B^3)*x + 3*(B^3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*lo
g(e) + (B^3*b^2*d^2*log(e)^2 + 2*A*B^2*b^2*d^2*log(e))*x^2 + (2*(b^2*c*d*lo
g(e) + a*b*d^2*log(e))*A*B^2 + (b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*B^3)*x
)*log((b*x + a)^n) - 3*(B^3*a^2*d^2*n^2*log(b*x + a) + B^3*a*b*c*d*log(e)^2
 + 2*A*B^2*a*b*c*d*log(e) + (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^3*log(d*x + c)
+ ((n*log(e) + log(e)^2)*B^3*b^2*d^2 + A*B^2*b^2*d^2*(n + 2*log(e)))*x^2 +
(B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n
)^2 + (2*(a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*A*B^2 - ((n^2 - log(e)^2)*
b^2*c*d - (n^2 + 2*n*log(e) + log(e)^2)*a*b*d^2)*B^3)*x + (2*B^3*a*b*c*d*lo
g(e) + 2*A*B^2*a*b*c*d + (B^3*b^2*d^2*(n + 2*log(e)) + 2*A*B^2*b^2*d^2)*x^2
 + 2*((b^2*c*d + a*b*d^2)*A*B^2 + (a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*B
^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^2*x + b*c*d), x)
```

Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,(a+b\,x)\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x), x)
```

3.167 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx &= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\end{aligned}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-b*(d*x+c)/d/(b*x+a))/b+3*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b+6*B^2*n^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,b*(d*x+c)/d/(b*x+a))/b+6*B^3*n^
3*polylog(4,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2573, 2549, 2379, 2421, 2430, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{b} + \frac{3Bn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{b} + \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (3*B*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/b + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b + (6*B^3*n^3*PolyLog[
4, (b*(c + d*x))/(d*(a + b*x))])/b
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{a+bx}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

$$+ \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} - \operatorname{Subst}\left(\frac{(6B^2n^2)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{6B^2n^2(A + B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} - \operatorname{Subst}\left(\frac{(6B^3n^3)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_3\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{6B^2n^2(A + B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{6B^3n^3\operatorname{Li}_4\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2513 vs. 2(186) = 372.

Time = 0.57 (sec) , antiderivative size = 2513, normalized size of antiderivative = 13.51

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

```
[Out] (4*A^3*Log[a + b*x] - 6*A^2*B*n*Log[a + b*x]^2 + 4*A*B^2*n^2*Log[a + b*x]^3
 - B^3*n^3*Log[a + b*x]^4 + B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^4 - 4
*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[-((d*(a + b*x))/(b*(c + d*
x)))] + 6*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(
b*(c + d*x)))]^2 - 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a
+ b*x))/(b*(c + d*x)))]^3 + B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^4 -
 12*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]^2 + 12*B^3*n^3*Log[a + b*x]^2*Log[c
 + d*x]^2 + 12*A*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 8*B^3*n^3*Log[a + b*x]*Log[c + d*x]^3 + 8*B^3*n^3*Log[(d*(a + b*x))/(-(b*c
) + a*d)]*Log[c + d*x]^3 + 12*A^2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)] - 12*A*B^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*B^3
*n^3*Log[a + b*x]^3*Log[(b*(c + d*x))/(b*c - a*d)] + 8*B^3*n^3*Log[(d*(a +
b*x))/(-(b*c) + a*d)]^3*Log[(b*(c + d*x))/(b*c - a*d)] - 12*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Log[(b*(c
+ d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]^2*Log[c + d*x]*Log[(b*(c + d*x)
)/(b*c - a*d)] + 12*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2*Log[(b*(c + d*x))/(
b*c - a*d)] + 6*B^3*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]^2 + 1
2*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/
(b*c - a*d)]^2 - 18*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[(b*(c +
 d*x))/(b*c - a*d)]^2 + 12*A^2*B*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)
^n] - 12*A*B^2*n*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*B^3*n^
2*Log[a + b*x]^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*B^3*n^2*Log[a + b*x]
*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*B^3*n^2*Log[(d*(a + b
*x))/(-(b*c) + a*d)]*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*A
*B^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] - 12*B^3*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*
(a + b*x)^n)/(c + d*x)^n] + 24*B^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c
+ d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*A*B^2*Log[a + b*
x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*B^3*n*Log[a + b*x]^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n]^2 + 12*B^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[a + b*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^3 - 4*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^3*Log[(b
*c - a*d)/(b*c + b*d*x)] + 12*B*n*(A^2 + B^2*n^2*Log[(d*(a + b*x))/(-(b*c)
```

```
+ a*d)]^2 + B^2*n^2*Log[c + d*x]^2 + 2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x
))/(b*c - a*d)] - 2*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*(Log[-((d*(a
+ b*x))/(b*(c + d*x)))] + Log[(b*(c + d*x))/(b*c - a*d)]) + 2*A*B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Lo
g[c + d*x]*(A - B*n*Log[a + b*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))*Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(
c + d*x)))]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + 12*B^3*n^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*
n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a + b*x))/(b*(c + d*x)))]*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(c
 + d*x)))]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[c + d
*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]*Log[c +
 d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[c + d*x]^2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/(b*c - a*d)]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] + 24*B^3*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x
)^n]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*A*B^2*n^2*PolyLog[3, (d*(a
+ b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Po
lyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 24*B^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a
 + b*x))/(b*(c + d*x)))]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 24*A*B^2
*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))
/(b*(c + d*x)))]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^
3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(4*b)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n} (dx+c)^{-n}\right)\right)^{3}}{bx+a}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bx+a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*x + a), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log{(e(a + bx)^n (c + dx)^{-n})})^3}{a + bx}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\log{(e(a + bx)^n (c + dx)^{-n})})^3}{a + bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx + a}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="maxima
")
```

```
[Out] -B^3*log(b*x + a)*log((d*x + c)^n)^3/b + A^3*log(b*x + a)/b + integrate((B^
3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + 3*A^2*B*b*c*log(e) + (B^3*b*d*x + B
^3*b*c)*log((b*x + a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e
) + A*B^2*b*d)*x)*log((b*x + a)^n)^2 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3
*b*d*log(e) + A*B^2*b*d)*x + (B^3*b*d*n*x + B^3*a*d*n)*log(b*x + a) + (B^3*
b*d*x + B^3*b*c)*log((b*x + a)^n))*log((d*x + c)^n)^2 + (B^3*b*d*log(e)^3 +
 3*A*B^2*b*d*log(e)^2 + 3*A^2*B*b*d*log(e))*x + 3*(B^3*b*c*log(e)^2 + 2*A*B
^2*b*c*log(e) + A^2*B*b*c + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e) + A^2*B*
b*d)*x)*log((b*x + a)^n) - 3*(B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + A^2*B
*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (B^3*b*d*log(e)^2 + 2*A*B
^2*b*d*log(e) + A^2*B*b*d)*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log
(e) + A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^2 + a*b*c
+ (b^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{(A + B\log{(e(a + bx)^n(c + dx)^{-n})})^3}{a + bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx + a}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="giac")

[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log{(e(a + bx)^n(c + dx)^{-n})})^3}{a + bx}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{a + b\,x}\,dx$$

[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x),x)

[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x), x)

3.168 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 184

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx &= -\frac{6B^3n^3(c+dx)}{(bc-ad)(a+bx)}-\frac{6B^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)}\\ &\quad-\frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}\\ &\quad-\frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)}\end{aligned}$$

```
[Out] -6*B^3*n^3*(d*x+c)/(-a*d+b*c)/(b*x+a)-6*B^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-3*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^2/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a
*d+b*c)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used

$= \{2573, 2549, 2342, 2341\}$

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx$$
$$= -\frac{6B^2n^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)}$$
$$-\frac{3Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)}$$
$$-\frac{(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)(bc-ad)}-\frac{6B^3n^3(c+dx)}{(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] (-6*B^3*n^3*(c + d*x))/((b*c - a*d)*(a + b*x)) - (6*B^2*n^2*(c + d*x)*(A +
B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)*(a + b*x)) - (3*B*n*(c +
d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)*(a + b*x)) -
((c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*c - a*d)*(a + b*
x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

`rQ[n]`

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)} + \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)} + \text{Subst}\left(\frac{(6B^2n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{6B^3n^3(c+dx)}{(bc-ad)(a+bx)} - \frac{6B^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)} - \frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 524 vs. $2(184) = 368$.

Time = 0.50 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.85

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx$$
$$= \frac{-B^3dn^3(a+bx)\log^3(a+bx) + B^3dn^3(a+bx)\log^3(c+dx) + 3B^2dn^2(a+bx)\log^2(c+dx)\,(A+Bn+B\,\mathrm{l}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] (-(B^3*d*n^3*(a + b*x)*Log[a + b*x]^3) + B^3*d*n^3*(a + b*x)*Log[c + d*x]^3
 + 3*B^2*d*n^2*(a + b*x)*Log[c + d*x]^2*(A + B*n + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]) + 3*B^2*d*n^2*(a + b*x)*Log[a + b*x]^2*(A + B*n + B*n*Log[c + d
*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d*n*(a + b*x)*Log[c + d*x]*
(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2*B*n +
 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*Log[(e*(a + b*x)
^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^3
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d*n*(a + b*x)*Log[a + b*x]*(A^2
+ 2*A*B*n + 2*B^2*n^2 + B^2*n^2*Log[c + d*x]^2 + 2*B*(A + B*n)*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c
 + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)*(a
+ b*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(184) = 368$.

Time = 38.58 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-3AB^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^3d^2n-6AB^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n^2-3A^2Bx\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n-}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-3*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*d^2*n-6*A*B^2*x*ln(e*(b*x+a)
^n/((d*x+c)^n))*b^3*d^2*n^2-3*A^2*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n
-3*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*c*d*n-6*A*B^2*ln(e*(b*x+a)^n/((d
*x+c)^n))*b^3*c*d*n^2-3*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n+6*A*B^2
*a*b^2*d^2*n^3-6*A*B^2*b^3*c*d*n^3+3*A^2*B*a*b^2*d^2*n^2-3*A^2*B*b^3*c*d*n^
2-B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*d^2*n-3*B^3*x*ln(e*(b*x+a)^n/((d*
```

```
x+c)^n))^2*b^3*d^2*n^2-6*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^3-B^3*
ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*c*d*n-3*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2
*b^3*c*d*n^2+6*B^3*a*b^2*d^2*n^4-6*B^3*b^3*c*d*n^4+A^3*a*b^2*d^2*n-A^3*b^3*
c*d*n-6*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^3)/(b*x+a)/b^3/d/n/(a*d-b
*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(184) = 368$.

Time = 0.29 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.48

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = -\frac{A^3bc - A^3ad + 6\,(B^3bc - B^3ad)n^3 + (B^3bdn^3x + B^3bcn^3)\log(bx+a)^3 - (B^3bdn^3x + B^3bcn^3)\log(dx}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(A^3*b*c - A^3*a*d + 6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c*
n^3)*log(b*x + a)^3 - (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (B^3*b
*c - B^3*a*d)*log(e)^3 + 6*(A*B^2*b*c - A*B^2*a*d)*n^2 + 3*(B^3*b*c*n^3 + A
*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n
^2)*log(e))*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3
+ A*B^2*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a) + (B^3*b*d*
n^2*x + B^3*b*c*n^2)*log(e))*log(d*x + c)^2 + 3*(A*B^2*b*c - A*B^2*a*d + (B
^3*b*c - B^3*a*d)*n)*log(e)^2 + 3*(A^2*B*b*c - A^2*B*a*d)*n + 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 +
(2*B^3*b*d*n^3 + 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^2 + A*B^2*
b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(b*x + a) - 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x
 + a)^2 + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 + (2*B^3*b*d*n^3 + 2*A*B^2*b*d
*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B
^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(b*x + a) + 2*(B^3
*b*c*n^2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(d*x + c
) + 3*(A^2*B*b*c - A^2*B*a*d + 2*(B^3*b*c - B^3*a*d)*n^2 + 2*(A*B^2*b*c - A
*B^2*a*d)*n)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(184) = 368$.

Time = 0.26 (sec) , antiderivative size = 1129, normalized size of antiderivative = 6.14

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="maxi
ma")
[Out] -B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^2*x + a*b) - (3*(d*e*n*log(b*x + a
)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))
*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (3*(2*b*c*e^2*n^2 - 2*a*d*e^2*n^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)
*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*d*e^2
*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*log(d*x
+ c))*log((b*x + a)^n*e/(d*x + c)^n)/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d
)*x)*e) + (6*b*c*e^3*n^3 - 6*a*d*e^3*n^3 + (b*d*e^3*n^3*x + a*d*e^3*n^3)*lo
g(b*x + a)^3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(d*x + c)^3 - 3*(b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 3*(b*d*e^3*n^3*x + a*d*e^3*n^3 - (b*d*e
^3*n^3*x + a*d*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b*d*e^3*n^3*x + a
*d*e^3*n^3)*log(b*x + a) - 3*(2*b*d*e^3*n^3*x + 2*a*d*e^3*n^3 + (b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 2*(b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x
 + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))/e)*B^
3 - 3*A*B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^
2*c - a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c
*e^2*n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^
2)*log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2
*n^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x
)*e^2)) - 3*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 3*(d*e*n
*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b
^2*x + a*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b)
 - A^3/(b^2*x + a*b)
```

Giac [F]

$$\int \frac{(A + B\log (e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx + a)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="giac
")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.58

$$\begin{aligned}\int \frac{(A + B\log (e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^2}\,dx &= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{3\,B\,b\,d\,A^2\,x^2+3\,B\,(a\,d+b\,c)\,A^2\,x+3\,B\,a\,c\,A^2}{b\,(a+b\,x)^2\,(c+d\,x)}\right.\\&\left.+\frac{6\,d\,(n\,B^3+A\,B^2)\,\left(b^2\,n\,x^2\,(a\,d-b\,c)+\frac{a\,b\,c\,n\,(a\,d-b\,c)}{d}+\frac{b\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{d}\right)}{b^2\,(a\,d-b\,c)\,(a+b\,x)^2\,(c+d\,x)}\right)\\&-\frac{A^3+3\,A^2\,B\,n+6\,A\,B^2\,n^2+6\,B^3\,n^3}{x\,b^2+a\,b}\\&-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{3\,A\,B^2}{x\,b^2+a\,b}+\frac{3\,B^3\,n}{x\,b^2+a\,b}-\frac{3\,d\,(n\,B^3+A\,B^2)}{b\,(a\,d-b\,c)}\right)\\&-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3\left(\frac{B^3}{b\,(a+b\,x)}-\frac{B^3\,d}{b\,(a\,d-b\,c)}\right)\\&-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{B\,d\,n\,\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,3\mathrm{i}}{(a\,d-b\,c)\,(3\,d\,A^2\,B\,n+6\,d\,A\,B^2\,n^2+6\,d\,B^3\,n^3)}\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,6\mathrm{i}}{b\,(a\,d-b\,c)}\end{aligned}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^2,x)
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*A^2*B*a*c + 3*A^2*B*x*(a*d + b*c) +
3*A^2*B*b*d*x^2)/(b*(a + b*x)^2*(c + d*x)) + (6*d*(A*B^2 + B^3*n)*(b^2*n*x^
2*(a*d - b*c) + (a*b*c*n*(a*d - b*c))/d + (b*n*x*(a*d + b*c)*(a*d - b*c))/d
))/(b^2*(a*d - b*c)*(a + b*x)^2*(c + d*x))) - (A^3 + 6*B^3*n^3 + 6*A*B^2*n^
2 + 3*A^2*B*n)/(a*b + b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2
)/(a*b + b^2*x) + (3*B^3*n)/(a*b + b^2*x) - (3*d*(A*B^2 + B^3*n))/(b*(a*d -
 b*c))) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(b*(a + b*x)) - (B^3*d)/(
b*(a*d - b*c))) - (B*d*n*atan((B*d*n*((b^2*c + a*b*d)/b + 2*b*d*x)*(A^2 + 2
*B^2*n^2 + 2*A*B*n)*3i)/((a*d - b*c)*(6*B^3*d*n^3 + 3*A^2*B*d*n + 6*A*B^2*d
*n^2)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*6i)/(b*(a*d - b*c))
```

3.169 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 390

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx \\
&= \frac{6B^3dn^3(c+dx)}{(bc-ad)^2(a+bx)} - \frac{3bB^3n^3(c+dx)^2}{8(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{6B^2dn^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bB^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{3Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}
\end{aligned}$$

```
[Out] 6*B^3*d*n^3*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-3/8*b*B^3*n^3*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+6*B^2*d*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*
c)^2/(b*x+a)-3/4*b*B^2*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*
d+b*c)^2/(b*x+a)^2+3*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^2/(b*x+a)-3/4*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^2/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*c)
^2/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx\\
&=-\frac{3bB^2n^2(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4(a+bx)^2(bc-ad)^2}\\
&\quad+\frac{6B^2dn^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^2}\\
&\quad-\frac{3bBn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4(a+bx)^2(bc-ad)^2}\\
&\quad+\frac{3Bdn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^2}\\
&\quad-\frac{b(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2(a+bx)^2(bc-ad)^2}\\
&\quad+\frac{d(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)(bc-ad)^2}\\
&\quad-\frac{3bB^3n^3(c+dx)^2}{8(a+bx)^2(bc-ad)^2}+\frac{6B^3dn^3(c+dx)}{(a+bx)(bc-ad)^2}
\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] (6*B^3*d*n^3*(c + d*x))/((b*c - a*d)^2*(a + b*x)) - (3*b*B^3*n^3*(c + d*x)^
2)/(8*(b*c - a*d)^2*(a + b*x)^2) + (6*B^2*d*n^2*(c + d*x)*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]))/((b*c - a*d)^2*(a + b*x)) - (3*b*B^2*n^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*(b*c - a*d)^2*(a + b*x)^2)
+ (3*B*d*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*
d)^2*(a + b*x)) - (3*b*B*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)
^n])^2)/(4*(b*c - a*d)^2*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^3)/((b*c - a*d)^2*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log
[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*(b*c - a*d)^2*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^3}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^3}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^3}{x^3}-\frac{d(A+B\log(ex^n))^3}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{b\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}$$

$$+ \text{Subst}\left(\frac{(3bBn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3Bdn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{3Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} - \frac{3bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^2(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}$$

$$+ \text{Subst}\left(\frac{(3bB^2n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(6B^2dn^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= \frac{6B^3dn^3(c+dx)}{(bc-ad)^2(a+bx)} - \frac{3bB^3n^3(c+dx)^2}{8(bc-ad)^2(a+bx)^2} \\
&+ \frac{6B^2dn^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\
&- \frac{3bB^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^2(a+bx)^2} \\
&+ \frac{3Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\
&- \frac{3bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^2(a+bx)^2} \\
&+ \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} \\
&- \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.78

$$
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx = -\frac{-4B^3d^2n^3(a+bx)^2\log^3(a+bx)+4B^3d^2n^3(a+bx)^2\log^3(c+dx)+6B^2d^2n^2(a+bx)^2\log^2(c+dx)\,(2A}{}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] -1/8*(-4*B^3*d^2*n^3*(a + b*x)^2*Log[a + b*x]^3 + 4*B^3*d^2*n^3*(a + b*x)^2
*Log[c + d*x]^3 + 6*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2*(2*A + 3*B*n + 2
*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*
x]^2*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]) + 6*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B
*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2) + (b*c - a*d)*(4*A^3*(b*c - a*d) + 3*B^3*n^3*(-15*a*d + b*
(c - 14*d*x)) + 6*A*B^2*n^2*(-7*a*d + b*(c - 6*d*x)) + 6*A^2*B*n*(-3*a*d +
b*(c - 2*d*x)) + 6*B*(2*A^2*(b*c - a*d) + B^2*n^2*(-7*a*d + b*(c - 6*d*x))
+ 2*A*B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*B^
2*(2*A*(b*c - a*d) + B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]^2 + 4*B^3*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^
2*n*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*n^2*Log[c
 + d*x]^2 + 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1621 vs. 2(382) = 764.

Time = 48.69 (sec) , antiderivative size = 1622, normalized size of antiderivative = 4.16

method	result	size
parallelrisch	Expression too large to display	1622
risch	Expression too large to display	120138

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(24*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n-24*A*B^2*ln(e*(b*x
+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2+36*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b
^3*d^3*n+12*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d*n-24*A^2*B*ln(e*(b*
x+a)^n/((d*x+c)^n))*a*b^4*c*d^2-48*A*B^2*a*b^4*c*d^2*n^2-24*A^2*B*a*b^4*c*d
^2*n+12*A^2*B*ln(d*x+c)*a^2*b^3*d^3*n-24*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*
a*b^4*c*d^2*n-36*A*B^2*ln(b*x+a)*a^2*b^3*d^3*n^2+36*A*B^2*ln(d*x+c)*a^2*b^3
*d^3*n^2-12*A^2*B*ln(b*x+a)*a^2*b^3*d^3*n+36*A*B^2*x*a*b^4*d^3*n^2-36*A*B^2
*x*b^5*c*d^2*n^2-48*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2*n^2+12*A^2*
B*x*a*b^4*d^3*n-12*A^2*B*x*b^5*c*d^2*n+84*B^3*ln(d*x+c)*x*a*b^4*d^3*n^3-84*
B^3*ln(b*x+a)*x*a*b^4*d^3*n^3-36*A*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2-24*B^3*x*l
n(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3*n-12*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n
))^2*b^5*c*d^2*n+36*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n^2-36*B^3*
x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n^2-24*A*B^2*x*ln(e*(b*x+a)^n/((d*x
+c)^n))^2*a*b^4*d^3-12*A^2*B*ln(b*x+a)*x^2*b^5*d^3*n+12*A^2*B*ln(d*x+c)*x^2
*b^5*d^3*n+36*A*B^2*ln(d*x+c)*x^2*b^5*d^3*n^2+4*A^3*a^2*b^3*d^3+4*A^3*b^5*c
^2*d-8*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*a*b^4*c*d^2+6*B^3*ln(e*(b*x+a)^n/(
(d*x+c)^n))^2*b^5*c^2*d*n+42*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3*n^
2+6*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d*n^2+12*A*B^2*ln(e*(b*x+a)^n/(
(d*x+c)^n))^2*b^5*c^2*d+12*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3+12
*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d-42*B^3*ln(b*x+a)*x^2*b^5*d^3*n
^3+42*B^3*ln(d*x+c)*x^2*b^5*d^3*n^3-42*B^3*ln(b*x+a)*a^2*b^3*d^3*n^3+42*B^3
*ln(d*x+c)*a^2*b^3*d^3*n^3-18*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*d^3
*n-12*A*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*d^3-8*B^3*x*ln(e*(b*x+a)^
n/((d*x+c)^n))^3*a*b^4*d^3+42*B^3*x*a*b^4*d^3*n^3-42*B^3*x*b^5*c*d^2*n^3-48
*B^3*a*b^4*c*d^2*n^3+42*A*B^2*a^2*b^3*d^3*n^2+6*A*B^2*b^5*c^2*d*n^2+18*A^2*
B*a^2*b^3*d^3*n+6*A^2*B*b^5*c^2*d*n+45*B^3*a^2*b^3*d^3*n^3+3*B^3*b^5*c^2*d*
n^3-8*A^3*a*b^4*c*d^2-24*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n-48
*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2*n-72*A*B^2*ln(b*x+a)*x*a*b^4
*d^3*n^2+72*A*B^2*ln(d*x+c)*x*a*b^4*d^3*n^2-24*A^2*B*ln(b*x+a)*x*a*b^4*d^3*
n+24*A^2*B*ln(d*x+c)*x*a*b^4*d^3*n-4*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^3*
b^5*d^3+4*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^5*c^2*d)/(b*x+a)^2/b^4/d/(a^2
*d^2-2*a*b*c*d+b^2*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. 2(382) = 764.

Time = 0.36 (sec) , antiderivative size = 2244, normalized size of antiderivative = 5.75

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/8*(4*A^3*b^2*c^2 - 8*A^3*a*b*c*d + 4*A^3*a^2*d^2 + 3*(B^3*b^2*c^2 - 16*B
^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n
^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^3 + 4*(B^3*b^2*d^2*n
^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(d*x +
 c)^3 + 4*(B^3*b^2*c^2 - 2*B^3*a*b*c*d + B^3*a^2*d^2)*log(e)^3 + 6*(A*B^2*b
^2*c^2 - 8*A*B^2*a*b*c*d + 7*A*B^2*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4*B^3*a
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a)^2 + 6*((B^3*b^2*c^2 - 4*B^3*a
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a) - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*
d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(d*x + c)^2 + 6*(
2*A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 2*A*B^2*a^2*d^2 - 2*(B^3*b^2*c*d - B^3*
a*b*d^2)*n*x + (B^3*b^2*c^2 - 4*B^3*a*b*c*d + 3*B^3*a^2*d^2)*n)*log(e)^2 +
6*(A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 3*A^2*B*a^2*d^2)*n - 6*(7*(B^3*b^2*c*d
 - B^3*a*b*d^2)*n^3 + 6*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n^2 + 2*(A^2*B*b^2*
c*d - A^2*B*a*b*d^2)*n)*x + 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2
*b^2*c^2 - 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2
+ 2*A^2*B*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*d^2*n*x - (B^3*
b^2*c^2 - 2*B^3*a*b*c*d)*n)*log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*A^2*B*a*b*c*d)*
n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*(A*B^2*b
^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^2 - (
3*B^3*b^2*d^2*n^2 + 2*A*B^2*b^2*d^2*n)*x^2 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b
*c*d)*n - 2*(2*A*B^2*a*b*d^2*n + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^2)*x)*log(
e))*log(b*x + a) - 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2
- 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2 + 2*A^2*B
*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*
d^2*n*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n)*log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*
A^2*B*a*b*c*d)*n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n
^3 + 2*(A*B^2*b^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a
```

```
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a) + 2*((B^3*b^2*c^2 - 4*B^3*a*b
*c*d)*n^2 - (3*B^3*b^2*d^2*n^2 + 2*A*B^2*b^2*d^2*n)*x^2 + 2*(A*B^2*b^2*c^2
- 2*A*B^2*a*b*c*d)*n - 2*(2*A*B^2*a*b*d^2*n + (B^3*b^2*c*d + 2*B^3*a*b*d^2)
*n^2)*x)*log(e))*log(d*x + c) + 6*(2*A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 2*A^
2*B*a^2*d^2 + (B^3*b^2*c^2 - 8*B^3*a*b*c*d + 7*B^3*a^2*d^2)*n^2 + 2*(A*B^2*
b^2*c^2 - 4*A*B^2*a*b*c*d + 3*A*B^2*a^2*d^2)*n - 2*(3*(B^3*b^2*c*d - B^3*a*
b*d^2)*n^2 + 2*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n)*x)*log(e))/(a^2*b^3*c^2 -
 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*
(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2246 vs. $2(382) = 764$.

Time = 0.33 (sec) , antiderivative size = 2246, normalized size of antiderivative = 5.76

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1
/8*(6*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e
*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*
n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (6*(b^2*c^2*e^2*n^2 - 8
*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e
^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b
*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^
2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^
```

```
2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*
n^2)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e) + (3*b^2*c^2*e^3*n^3 -
48*a*b*c*d*e^3*n^3 + 45*a^2*d^2*e^3*n^3 - 4*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^
2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^3 + 4*(b^2*d^2*e^3*n^3*x^2 + 2*
a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(d*x + c)^3 + 18*(b^2*d^2*e^3*n^3*x
^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^2 + 6*(3*b^2*d^2*e
^3*n^3*x^2 + 6*a*b*d^2*e^3*n^3*x + 3*a^2*d^2*e^3*n^3 - 2*(b^2*d^2*e^3*n^3*x
^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 -
42*(b^2*c*d*e^3*n^3 - a*b*d^2*e^3*n^3)*x - 42*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*
d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a) + 6*(7*b^2*d^2*e^3*n^3*x^2 +
14*a*b*d^2*e^3*n^3*x + 7*a^2*d^2*e^3*n^3 + 2*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d
^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^2 - 6*(b^2*d^2*e^3*n^3*x^2 + 2
*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a))*log(d*x + c))/((a^2*b^3
*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^
2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2))/e)*B^3 + 3/4*A*B^2
*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n
+ 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2 - 8*a*b*c*
d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*
x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^
2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b*d^2*e^
2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*
log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^2*d^2*e
^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*lo
g(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*
c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2)*x)*e^2)) - 3/2*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^3*x^2 + 2*
a*b^2*x + a^2*b) + 3/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b
*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^
2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A^2*B/e - 3/2*A^2*B*log((b*x + a)^n*e/(d*x
+ c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^3/(b^3*x^2 + 2*a*b^2*x + a^2*
b)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3\left(\frac{B^3}{2\,b\,(a^2+2\,a\,b\,x+b^2\,x^2)}-\frac{B^3\,d^2}{2\,b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)$$

$$-\frac{\frac{4\,A^3\,a\,d-4\,A^3\,b\,c+45\,B^3\,a\,d\,n^3-3\,B^3\,b\,c\,n^3+18\,A^2\,B\,a\,d\,n-6\,A^2\,B\,b\,c\,n+42\,A\,B^2\,a\,d\,n^2-6\,A\,B^2\,b\,c\,n^2}{2\,(a\,d-b\,c)}+\frac{3\,x\,(2\,b\,d\,A^2\,B\,n+6\,b\,d\,A\,B^2\,n^2+7\,b}{a\,d-b\,c}}{4\,a^2\,b+8\,a\,b^2\,x+4\,b^3\,x^2}$$

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{3\,A\,B^2}{2\,(a^2\,b+2\,a\,b^2\,x+b^3\,x^2)}-\frac{3\,d^2\,(3\,n\,B^3+2\,A\,B^2)}{4\,b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right.$$

$$\left.+\frac{3\,B^3\,d^2\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{d^2}+\frac{2\,b^2\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)}{4\,b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(a^2\,b+2\,a\,b^2\,x+b^3\,x^2)}\right)$$

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{3\,B\,b\,d\,(A^2-B^2\,n^2)\,x^2+3\,B\,(a\,d+b\,c)\,(A^2-B^2\,n^2)\,x+3\,B\,a\,c\,(A^2-B^2\,n^2)}{2\,b\,(a+b\,x)^3\,(c+d\,x)}\right.$$

$$+\frac{3\,d^2\,(3\,n\,B^3+2\,A\,B^2)\left(x\left(\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)(a\,d+b\,c)+\frac{2\,a\,b^2\,c\,n\,(a\,d-b\,c)}{d}\right)+x^2\left(b\,d\left(\frac{b\,}{}\right.\right.\right.}{4\,b^2\,(a+b\,x)^3\,(c\,}$$

$$-\frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{B\,d^2\,n\left(2\,b\,d\,x-\frac{b^3\,c^2-a^2\,b\,d^2}{b\,(a\,d-b\,c)}\right)(2\,A^2+6\,A\,B\,n+7\,B^2\,n^2)\,3\mathrm{i}}{(a\,d-b\,c)\,(6\,A^2\,B\,d^2\,n+18\,A\,B^2\,d^2\,n^2+21\,B^3\,d^2\,n^3)}\right)(2\,A^2+6\,A\,B\,n+7\,B^2\,n^2)\,3\mathrm{i}}{2\,b\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^3,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^3*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((4*A^3*a*d - 4*A^3*b*c
 + 45*B^3*a*d*n^3 - 3*B^3*b*c*n^3 + 18*A^2*B*a*d*n - 6*A^2*B*b*c*n + 42*A*B
```

```
^2*a*d*n^2 - 6*A*B^2*b*c*n^2)/(2*(a*d - b*c)) + (3*x*(7*B^3*b*d*n^3 + 2*A^2
*B*b*d*n + 6*A*B^2*b*d*n^2))/(a*d - b*c))/(4*a^2*b + 4*b^3*x^2 + 8*a*b^2*x)
 - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2)/(2*(a^2*b + b^3*x^2 + 2*a*
b^2*x)) - (3*d^2*(2*A*B^2 + 3*B^3*n))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 + (3*B^3*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (2*b^2*n*x*(a*d - b*c)
)/d + (a*b*n*(a*d - b*c))/d))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b +
 b^3*x^2 + 2*a*b^2*x))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*B*a*c*(A^2 -
 B^2*n^2) + 3*B*x*(a*d + b*c)*(A^2 - B^2*n^2) + 3*B*b*d*x^2*(A^2 - B^2*n^2)
)/(2*b*(a + b*x)^3*(c + d*x)) + (3*d^2*(2*A*B^2 + 3*B^3*n)*(x*(((b*n*(a*d -
 b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d)*(a*d + b*c) + (2*a*b^2*c*
n*(a*d - b*c))/d) + x^2*(b*d*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*
(a*d - b*c))/d) + (2*b^2*n*(a*d + b*c)*(a*d - b*c))/d) + a*c*((b*n*(a*d - b
*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d) + 2*b^3*n*x^3*(a*d - b*c)))
/(4*b^2*(a + b*x)^3*(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B*d^2*n*
atan((B*d^2*n*(2*b*d*x - (b^3*c^2 - a^2*b*d^2)/(b*(a*d - b*c)))*(2*A^2 + 7*
B^2*n^2 + 6*A*B*n)*3i)/((a*d - b*c)*(21*B^3*d^2*n^3 + 6*A^2*B*d^2*n + 18*A*
B^2*d^2*n^2)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/(2*b*(a*d - b*c)^2)
```

3.170 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 611

$$
\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx \\
&= -\frac{6B^3d^2n^3(c+dx)}{(bc-ad)^3(a+bx)}+\frac{3bB^3dn^3(c+dx)^2}{4(bc-ad)^3(a+bx)^2}-\frac{2b^2B^3n^3(c+dx)^3}{27(bc-ad)^3(a+bx)^3} \\
&\quad -\frac{6B^2d^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)} \\
&\quad +\frac{3bB^2dn^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^3(a+bx)^2} \\
&\quad -\frac{2b^2B^2n^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{9(bc-ad)^3(a+bx)^3} \\
&\quad -\frac{3Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)} \\
&\quad +\frac{3bBdn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^3(a+bx)^2} \\
&\quad -\frac{b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3} \\
&\quad -\frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)} \\
&\quad +\frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)^2} \\
&\quad -\frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bc-ad)^3(a+bx)^3}
\end{aligned}
$$

```
[Out] -6*B^3*d^2*n^3*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+3/4*b*B^3*d*n^3*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^3*n^3*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-6*B^2*d
^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+3/2*b
*B^2*d*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)
^2-2/9*b^2*B^2*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3
/(b*x+a)^3-3*B*d^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)
^3/(b*x+a)+3/2*b*B*d*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+
b*c)^3/(b*x+a)^2-1/3*b^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/
(-a*d+b*c)^3/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^3/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^3/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^3/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx \\
&= -\frac{2b^2B^2n^2(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{9(a+bx)^3(bc-ad)^3} \\
&\quad -\frac{b^2Bn(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3(a+bx)^3(bc-ad)^3} \\
&\quad -\frac{b^2(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{3(a+bx)^3(bc-ad)^3} \\
&\quad -\frac{6B^2d^2n^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^3} \\
&\quad +\frac{3bB^2dn^2(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2(a+bx)^2(bc-ad)^3} \\
&\quad -\frac{3Bd^2n(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^3} \\
&\quad -\frac{d^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)(bc-ad)^3} \\
&\quad +\frac{3bBdn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2(a+bx)^2(bc-ad)^3} \\
&\quad +\frac{bd(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)^2(bc-ad)^3} \\
&\quad -\frac{2b^2B^3n^3(c+dx)^3}{27(a+bx)^3(bc-ad)^3}-\frac{6B^3d^2n^3(c+dx)}{(a+bx)(bc-ad)^3}+\frac{3bB^3dn^3(c+dx)^2}{4(a+bx)^2(bc-ad)^3}
\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] (-6*B^3*d^2*n^3*(c + d*x))/((b*c - a*d)^3*(a + b*x)) + (3*b*B^3*d*n^3*(c +
d*x)^2)/(4*(b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B^3*n^3*(c + d*x)^3)/(27*(b*
c - a*d)^3*(a + b*x)^3) - (6*B^2*d^2*n^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/((b*c - a*d)^3*(a + b*x)) + (3*b*B^2*d*n^2*(c + d*x)^2*(A
 + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^3*(a + b*x)^2) - (2*
b^2*B^2*n^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(9*(b*c -
 a*d)^3*(a + b*x)^3) - (3*B*d^2*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/((b*c - a*d)^3*(a + b*x)) + (3*b*B*d*n*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^3*(a + b*x)^2) - (b^2*B*n*(
c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*(b*c - a*d)^3*(a
+ b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*
c - a*d)^3*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n])^3)/((b*c - a*d)^3*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])^3)/(3*(b*c - a*d)^3*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^4}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^3}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^3}{x^4}-\frac{2bd(A+B\log(ex^n))^3}{x^3}+\frac{d^2(A+B\log(ex^n))^3}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= -\frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bc-ad)^3(a+bx)^3} \\
&\quad + \text{Subst}\left(\frac{(b^2Bn)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c\right. \\
&\quad \left. +dx)^{-n}\right) - \text{Subst}\left(\frac{(3bBdn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a\right. \\
&\quad \left. +bx)^n(c+dx)^{-n}\right) \\
&\quad + \text{Subst}\left(\frac{(3Bd^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c\right. \\
&\quad \left. +dx)^{-n}\right)
\end{aligned}
$$

$$= -\frac{3Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)}$$
$$+\frac{3bBdn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^3(a+bx)^2}$$
$$-\frac{b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3}$$
$$-\frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)}$$
$$+\frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)^2}$$
$$-\frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bc-ad)^3(a+bx)^3}$$
$$+\text{Subst}\left(\frac{(2b^2B^2n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(3bB^2dn^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(6B^2d^2n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{6B^3d^2n^3(c+dx)}{(bc-ad)^3(a+bx)} + \frac{3bB^3dn^3(c+dx)^2}{4(bc-ad)^3(a+bx)^2} - \frac{2b^2B^3n^3(c+dx)^3}{27(bc-ad)^3(a+bx)^3}$$
$$- \frac{6B^2d^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)}$$
$$+ \frac{3bB^2dn^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^3(a+bx)^2}$$
$$- \frac{2b^2B^2n^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{9(bc-ad)^3(a+bx)^3}$$
$$- \frac{3Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)}$$
$$+ \frac{3bBdn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^3(a+bx)^2}$$
$$- \frac{b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3}$$
$$- \frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)}$$
$$+ \frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^3(a+bx)^2}$$
$$- \frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bc-ad)^3(a+bx)^3}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.64

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$$
$$= \frac{-36B^3d^3n^3(a+bx)^3\log^3(a+bx)+36B^3d^3n^3(a+bx)^3\log^3(c+dx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)\,(6}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] (-36*B^3*d^3*n^3*(a + b*x)^3*Log[a + b*x]^3 + 36*B^3*d^3*n^3*(a + b*x)^3*Lo
g[c + d*x]^3 + 18*B^2*d^3*n^2*(a + b*x)^3*Log[c + d*x]^2*(6*A + 11*B*n + 6*
B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*
x]^2*(6*A + 11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)
^n]) + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 +
 6*B*(6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*d + 36*
a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^2*B*d^2*
n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d^2*n^2 +
8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54*A^2*b^2*B
```

```
*c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*a*A*b*B^2*d
^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*A^2*b^2*B*d^
2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2 + 6*B*(18*A^2
*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 -
 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 147*d*x) + b^
2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B
^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^
2 - 3*c*d*x + 6*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 36*B^3*(b*c
 - a*d)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^3*n*(a + b*x)^3*Log[a
 + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2*Log[c + d*x]^2 + 6*B*(
6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2 + 6*B*n*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])))/(108*b*(b*c - a*d)^3*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2686 vs. 2(597) = 1194.

Time = 95.36 (sec) , antiderivative size = 2687, normalized size of antiderivative = 4.40

method	result	size
parallelrisch	Expression too large to display	2687
risch	Expression too large to display	175812

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*(-648*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n-216*A*B^2*x^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n-324*B^3*x*ln(e*(b*x+a)^n/((d*x+c)
^n))^2*a*b^6*c*d^3*n-972*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n^2+
540*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n+108*A*B^2*x*ln(e*(b*x
+a)^n/((d*x+c)^n))*b^7*c^2*d^2*n-972*A*B^2*x*a*b^6*c*d^3*n^2-324*A^2*B*x*a*
b^6*c*d^3*n-648*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3*n+324*A*B^2
*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2*n-1188*A*B^2*ln(b*x+a)*x^2*a*b^6
*d^4*n^2+1188*A*B^2*ln(d*x+c)*x^2*a*b^6*d^4*n^2-324*A^2*B*ln(b*x+a)*x^2*a*b
^6*d^4*n+324*A^2*B*ln(d*x+c)*x^2*a*b^6*d^4*n-1188*A*B^2*ln(b*x+a)*x*a^2*b^5
*d^4*n^2+1188*A*B^2*ln(d*x+c)*x*a^2*b^5*d^4*n^2-324*A^2*B*ln(b*x+a)*x*a^2*b
^5*d^4*n+324*A^2*B*ln(d*x+c)*x*a^2*b^5*d^4*n+108*A^2*B*ln(d*x+c)*a^3*b^4*d^
4*n-486*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4*n-108*B^3*x^2*ln(e*
(b*x+a)^n/((d*x+c)^n))^2*b^7*c*d^3*n+396*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n)
)*a*b^6*d^4*n^2-396*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n^2-324*A
*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4+396*A*B^2*x^2*a*b^6*d^4*n^
2-396*A*B^2*x^2*b^7*c*d^3*n^2-324*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b
^5*d^4*n-396*A*B^2*ln(b*x+a)*x^3*b^7*d^4*n^2+396*A*B^2*ln(d*x+c)*x^3*b^7*d^
4*n^2+54*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*c^2*d^2*n+882*B^3*x*ln(e*(
b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n^2+90*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*
```

```
b^7*c^2*d^2*n^2-1134*B^3*x*a*b^6*c*d^3*n^3+108*A^2*B*x^2*a*b^6*d^4*n-108*A^
2*B*x^2*b^7*c*d^3*n-324*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*d^4+8
82*A*B^2*x*a^2*b^5*d^4*n^2+90*A*B^2*x*b^7*c^2*d^2*n^2-324*B^3*ln(e*(b*x+a)^
n/((d*x+c)^n))^2*a^2*b^5*c*d^3*n+162*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^
6*c^2*d^2*n-648*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3*n^2+162*B^3*l
n(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2*n^2+270*A^2*B*x*a^2*b^5*d^4*n+54*A
^2*B*x*b^7*c^2*d^2*n-324*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*c*d^3+
324*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*c^2*d^2+396*A*B^2*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^3*b^4*d^4*n-72*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3
*d*n-324*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3+324*A^2*B*ln(e*(b*
x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2+1530*B^3*ln(d*x+c)*x*a^2*b^5*d^4*n^3-396*
A*B^2*ln(b*x+a)*a^3*b^4*d^4*n^2+396*A*B^2*ln(d*x+c)*a^3*b^4*d^4*n^2-108*A^2
*B*ln(b*x+a)*a^3*b^4*d^4*n-108*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^3*a*b^6*
d^4+510*B^3*x^2*a*b^6*d^4*n^3-510*B^3*x^2*b^7*c*d^3*n^3-108*B^3*x*ln(e*(b*x
+a)^n/((d*x+c)^n))^3*a^2*b^5*d^4+1077*B^3*x*a^2*b^5*d^4*n^3+57*B^3*x*b^7*c^
2*d^2*n^3-108*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*a^2*b^5*c*d^3+108*B^3*ln(e*
(b*x+a)^n/((d*x+c)^n))^3*a*b^6*c^2*d^2-36*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2
*b^7*c^3*d*n+510*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4*n^2-24*B^3*ln(
e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d*n^2-108*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n)
)^2*b^7*c^3*d+108*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4-108*A^2*B*l
n(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d+162*A^2*B*a*b^6*c^2*d^2*n-108*A*B^2*x^
3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*d^4-1530*B^3*ln(b*x+a)*x^2*a*b^6*d^4*n^
3+1530*B^3*ln(d*x+c)*x^2*a*b^6*d^4*n^3-108*A^2*B*ln(b*x+a)*x^3*b^7*d^4*n+10
8*A^2*B*ln(d*x+c)*x^3*b^7*d^4*n-1530*B^3*ln(b*x+a)*x*a^2*b^5*d^4*n^3+575*B^
3*a^3*b^4*d^4*n^3-8*B^3*b^7*c^3*d*n^3-108*A^3*a^2*b^5*c*d^3-648*A*B^2*a^2*b
^5*c*d^3*n^2+162*A*B^2*a*b^6*c^2*d^2*n^2-324*A^2*B*a^2*b^5*c*d^3*n-510*B^3*
ln(b*x+a)*x^3*b^7*d^4*n^3+510*B^3*ln(d*x+c)*x^3*b^7*d^4*n^3-510*B^3*ln(b*x+
a)*a^3*b^4*d^4*n^3-648*B^3*a^2*b^5*c*d^3*n^3+81*B^3*a*b^6*c^2*d^2*n^3+510*A
*B^2*a^3*b^4*d^4*n^2-24*A*B^2*b^7*c^3*d*n^2+198*A^2*B*a^3*b^4*d^4*n-36*A^2*
B*b^7*c^3*d*n+510*B^3*ln(d*x+c)*a^3*b^4*d^4*n^3-198*B^3*x^3*ln(e*(b*x+a)^n/
((d*x+c)^n))^2*b^7*d^4*n+108*A^3*a*b^6*c^2*d^2-36*B^3*x^3*ln(e*(b*x+a)^n/((
d*x+c)^n))^3*b^7*d^4-36*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^7*c^3*d+36*A^3*
a^3*b^4*d^4-36*A^3*b^7*c^3*d+216*A*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^
6*d^4*n)/(b*x+a)^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(597) = 1194$.

Time = 0.41 (sec) , antiderivative size = 4008, normalized size of antiderivative = 6.56

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/108*(36*A^3*b^3*c^3 - 108*A^3*a*b^2*c^2*d + 108*A^3*a^2*b*c*d^2 - 36*A^3
*a^3*d^3 + (8*B^3*b^3*c^3 - 81*B^3*a*b^2*c^2*d + 648*B^3*a^2*b*c*d^2 - 575*
B^3*a^3*d^3)*n^3 + 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^
3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n
^3)*log(b*x + a)^3 - 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*
B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)
*n^3)*log(d*x + c)^3 + 36*(B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*
d^2 - B^3*a^3*d^3)*log(e)^3 + 6*(4*A*B^2*b^3*c^3 - 27*A*B^2*a*b^2*c^2*d + 1
08*A*B^2*a^2*b*c*d^2 - 85*A*B^2*a^3*d^3)*n^2 + 6*(85*(B^3*b^3*c*d^2 - B^3*a
*b^2*d^3)*n^3 + 66*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^3)*n^2 + 18*(A^2*B*b^3*
c*d^2 - A^2*B*a*b^2*d^3)*n)*x^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d +
18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 +
6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*
B^2*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B
^2*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^
3)*x + 6*(B^3*b^3*d^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n
^2*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*l
og(b*x + a)^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2
)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 -
 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2
+ (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 -
 (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*
d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^
3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(b*x + a) + 6*(B^3*b^3*d
^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3
 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*log(d*x + c)^2 + 18*
(6*A*B^2*b^3*c^3 - 18*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2 - 6*A*B^2*a^
3*d^3 + 6*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n*x^2 - 3*(B^3*b^3*c^2*d - 6*B^3*
a*b^2*c*d^2 + 5*B^3*a^2*b*d^3)*n*x + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 1
8*B^3*a^2*b*c*d^2 - 11*B^3*a^3*d^3)*n)*log(e)^2 + 18*(2*A^2*B*b^3*c^3 - 9*A
^2*B*a*b^2*c^2*d + 18*A^2*B*a^2*b*c*d^2 - 11*A^2*B*a^3*d^3)*n - 3*((19*B^3*
b^3*c^2*d - 378*B^3*a*b^2*c*d^2 + 359*B^3*a^2*b*d^3)*n^3 + 6*(5*A*B^2*b^3*c
^2*d - 54*A*B^2*a*b^2*c*d^2 + 49*A*B^2*a^2*b*d^3)*n^2 + 18*(A^2*B*b^3*c^2*d
 - 6*A^2*B*a*b^2*c*d^2 + 5*A^2*B*a^2*b*d^3)*n)*x + 6*((4*B^3*b^3*c^3 - 27*B
^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 + 66*A*B^2*
b^3*d^3*n^2 + 18*A^2*B*b^3*d^3*n)*x^3 + 6*(2*A*B^2*b^3*c^3 - 9*A*B^2*a*b^2*
c^2*d + 18*A*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*B*a*b^2*d^3*n + (22*B^3*b^3*c
*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*A*B^2*b^3*c*d^2 + 9*A*B^2*a*b^2*d^3)*n^
2)*x^2 + 18*(B^3*b^3*d^3*n*x^3 + 3*B^3*a*b^2*d^3*n*x^2 + 3*B^3*a^2*b*d^3*n*
x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n)*log(e)^2 + 18*
(A^2*B*b^3*c^3 - 3*A^2*B*a*b^2*c^2*d + 3*A^2*B*a^2*b*c*d^2)*n + 3*(18*A^2*B
*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a^2*b*d^3)*n^
3 - 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 - 6*A*B^2*a^2*b*d^3)*n^2)*x +
6*((11*B^3*b^3*d^3*n^2 + 6*A*B^2*b^3*d^3*n)*x^3 + (2*B^3*b^3*c^3 - 9*B^3*a*
b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n + (2*B^3*b^3*c
```

```
*d^2 + 9*B^3*a*b^2*d^3)*n^2)*x^2 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d +
 3*A*B^2*a^2*b*c*d^2)*n + 3*(6*A*B^2*a^2*b*d^3*n - (B^3*b^3*c^2*d - 6*B^3*a
*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^2)*x)*log(e))*log(b*x + a) - 6*((4*B^3*b^3*
c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 +
 66*A*B^2*b^3*d^3*n^2 + 18*A^2*B*b^3*d^3*n)*x^3 + 6*(2*A*B^2*b^3*c^3 - 9*A*
B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*B*a*b^2*d^3*n + (22
*B^3*b^3*c*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*A*B^2*b^3*c*d^2 + 9*A*B^2*a*b
^2*d^3)*n^2)*x^2 + 18*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^
3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n
^3)*log(b*x + a)^2 + 18*(B^3*b^3*d^3*n*x^3 + 3*B^3*a*b^2*d^3*n*x^2 + 3*B^3*
a^2*b*d^3*n*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n)*lo
g(e)^2 + 18*(A^2*B*b^3*c^3 - 3*A^2*B*a*b^2*c^2*d + 3*A^2*B*a^2*b*c*d^2)*n +
 3*(18*A^2*B*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a
^2*b*d^3)*n^3 - 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 - 6*A*B^2*a^2*b*d^
3)*n^2)*x + 6*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3
 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 - 3*A*
B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2 + (2*
B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 - (B^3
*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n
^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3 - 3
*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*log(b*x + a) + 6*((11*B^
3*b^3*d^3*n^2 + 6*A*B^2*b^3*d^3*n)*x^3 + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d
 + 18*B^3*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n + (2*B^3*b^3*c*d^2 + 9*
B^3*a*b^2*d^3)*n^2)*x^2 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*
a^2*b*c*d^2)*n + 3*(6*A*B^2*a^2*b*d^3*n - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^
2 - 6*B^3*a^2*b*d^3)*n^2)*x)*log(e))*log(d*x + c) + 6*(18*A^2*B*b^3*c^3 - 5
4*A^2*B*a*b^2*c^2*d + 54*A^2*B*a^2*b*c*d^2 - 18*A^2*B*a^3*d^3 + (4*B^3*b^3*
c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2 - 85*B^3*a^3*d^3)*n^2 + 6*(1
1*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n^2 + 6*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^
3)*n)*x^2 + 6*(2*A*B^2*b^3*c^3 - 9*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2
 - 11*A*B^2*a^3*d^3)*n - 3*((5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 + 49*B^3*
a^2*b*d^3)*n^2 + 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 + 5*A*B^2*a^2*b*d
^3)*n)*x)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3630 vs. $2(597) = 1194$.

Time = 0.43 (sec) , antiderivative size = 3630, normalized size of antiderivative = 5.94

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="maxi
ma")
[Out] -1/3*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^
2*x + a^3*b) - 1/108*(18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n -
7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^
3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x
^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a
^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (6*(4*b^
3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d
^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e
^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*
n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3
*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2
*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n
^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)
*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a
^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*
d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d
+ 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
- a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^
2*d^3)*x)*e) + (8*b^3*c^3*e^3*n^3 - 81*a*b^2*c^2*d*e^3*n^3 + 648*a^2*b*c*d^
2*e^3*n^3 - 575*a^3*d^3*e^3*n^3 + 36*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3
```

*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^3 - 36*(b^
3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d
^3*e^3*n^3)*log(d*x + c)^3 + 510*(b^3*c*d^2*e^3*n^3 - a*b^2*d^3*e^3*n^3)*x^
2 - 198*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^
3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^2 - 18*(11*b^3*d^3*e^3*n^3*x^3 + 33*a*b
^2*d^3*e^3*n^3*x^2 + 33*a^2*b*d^3*e^3*n^3*x + 11*a^3*d^3*e^3*n^3 - 6*(b^3*d
^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*
e^3*n^3)*log(b*x + a))*log(d*x + c)^2 - 3*(19*b^3*c^2*d*e^3*n^3 - 378*a*b^2
*c*d^2*e^3*n^3 + 359*a^2*b*d^3*e^3*n^3)*x + 510*(b^3*d^3*e^3*n^3*x^3 + 3*a*
b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)
 - 6*(85*b^3*d^3*e^3*n^3*x^3 + 255*a*b^2*d^3*e^3*n^3*x^2 + 255*a^2*b*d^3*e^
3*n^3*x + 85*a^3*d^3*e^3*n^3 + 18*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^
3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^2 - 66*(b^3*d
^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*
e^3*n^3)*log(b*x + a))*log(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^
5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*
b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d
^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)
*x)*e^2))/e)*B^3 - 1/18*A*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3
*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^
2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)
/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^
4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c
^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4
*b^3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^
3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^
3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e
^2*n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2
 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*
e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^
2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n
^2)*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 3
3*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b
^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))
*log(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^
3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^
6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)*e^2)) - A*B^2*log(
(b*x + a)^n*e/(d*x + c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
 - 1/6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*
a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c
*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 -

```
2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2)*x))*A^2*B/e - A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^
3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x
 + a^3*b)
```

Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx + a)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 2069, normalized size of antiderivative = 3.39

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^4}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^4,x)
```

```
[Out] ((36*A^3*a^2*d^2 + 36*A^3*b^2*c^2 + 575*B^3*a^2*d^2*n^3 + 8*B^3*b^2*c^2*n^3
 + 198*A^2*B*a^2*d^2*n + 36*A^2*B*b^2*c^2*n - 72*A^3*a*b*c*d + 510*A*B^2*a^
2*d^2*n^2 + 24*A*B^2*b^2*c^2*n^2 - 73*B^3*a*b*c*d*n^3 - 126*A^2*B*a*b*c*d*n
 - 138*A*B^2*a*b*c*d*n^2)/(6*(a*d - b*c)) + (x*(359*B^3*a*b*d^2*n^3 - 19*B^
3*b^2*c*d*n^3 + 90*A^2*B*a*b*d^2*n - 18*A^2*B*b^2*c*d*n + 294*A*B^2*a*b*d^2
*n^2 - 30*A*B^2*b^2*c*d*n^2))/(2*(a*d - b*c)) + (x^2*(85*B^3*b^2*d^2*n^3 +
18*A^2*B*b^2*d^2*n + 66*A*B^2*b^2*d^2*n^2))/(a*d - b*c))/(x^3*(18*b^5*c - 1
8*a*b^4*d) + x*(54*a^2*b^3*c - 54*a^3*b^2*d) - x^2*(54*a^2*b^3*d - 54*a*b^4
*c) + 18*a^3*b^2*c - 18*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/
(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^3*d^3)/(3*b*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)
^n)^2*((A*B^2)/(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2) - (d^3*(6*A*B^
2 + 11*B^3*n))/(6*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) +
(B^3*d^3*(a*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/
(3*d)) + x*(b*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c)
)/(3*d)) + (2*a*b^2*n*(a*d - b*c))/(3*d) + (b^2*n*(a*d - b*c)*(3*a*d - b*c)
)/(3*d^2)) + (b*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(3*d^3) +
(b^3*n*x^2*(a*d - b*c))/d))/(b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2))) - log((e*(a + b*x)^
```

```
n)/(c + d*x)^n)*((x*((a*d + b*c)*(3*A^2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2
 + 3*B^3*b*c*n^2) - 3*B^3*a*b*c*d*n^2) + x^2*(b*d*(3*A^2*B*a*d - 3*A^2*B*b*
c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b*d*n^2*(a*d + b*c)) + a*c*(3*A^
2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b^2*d^2*n^2*
x^3)/(3*b*(a*d - b*c)*(a + b*x)^4*(c + d*x)) + (d^3*(6*A*B^2 + 11*B^3*n)*(x
*((a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2))
+ (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3)*(a*d + b*c) +
a*c*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)
) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d))
 + x^2*((a*d + b*c)*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d
 - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a
*d - b*c)^2)/d) + b*d*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a
*d - b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))
/d^3) + (3*a*b^3*c*n*(a*d - b*c)^2)/d) + x^3*(b*d*(b*((a*b*n*(a*d - b*c)^2)
/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a
*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d) + (3*b^3*n*(a*d + b*c)*(a*d -
 b*c)^2)/d) + a*c*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d -
 b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3
) + 3*b^4*n*x^4*(a*d - b*c)^2))/(9*b^2*(a*d - b*c)*(a + b*x)^4*(c + d*x)*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*atan((B*d^3*
n*((b^4*c^3 + a^3*b*d^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2
 - 2*a*b^2*c*d) + 2*b*d*x)*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*(b^3*c^2 + a^2*
b*d^2 - 2*a*b^2*c*d)*1i)/(b*(a*d - b*c)^3*(85*B^3*d^3*n^3 + 18*A^2*B*d^3*n
+ 66*A*B^2*d^3*n^2)))*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*1i)/(9*b*(a*d - b*c)
^3)
```

3.171 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 830

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx$$
$$= \frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)} - \frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2} + \frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3} - \frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4} + \frac{6B^2d^3n^2(c+dx)\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}{(bc-ad)^4(a+bx)} - \frac{9bB^2d^2n^2(c+dx)^2\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}{3(bc-ad)^4(a+bx)^3} - \frac{3b^3B^2n^2(c+dx)^4\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}{32(bc-ad)^4(a+bx)^4} + \frac{3Bd^3n(c+dx)\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(bc-ad)^4(a+bx)} - \frac{9bBd^2n(c+dx)^2\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{4(bc-ad)^4(a+bx)^2} + \frac{b^2Bdn(c+dx)^3\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(bc-ad)^4(a+bx)^3} - \frac{3b^3Bn(c+dx)^4\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{16(bc-ad)^4(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(bc-ad)^4(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{2(bc-ad)^4(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(bc-ad)^4(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{4(bc-ad)^4(a+bx)^4}$$

```
[Out] 6*B^3*d^3*n^3*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-9/8*b*B^3*d^2*n^3*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^3*d*n^3*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-3/128
*b^3*B^3*n^3*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+6*B^2*d^3*n^2*(d*x+c)*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-9/4*b*B^2*d^2*n^2*(d*x+c)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B^2*d*n^2
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-3/32*b^
3*B^2*n^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^
4+3*B*d^3*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a
```

```
)-9/4*b*B*d^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/
(b*x+a)^2+b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c
)^4/(b*x+a)^3-3/16*b^3*B*n*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-
a*d+b*c)^4/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 830, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2573, 2549, 2395, 2342, 2341}

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx \\
&= -\frac{b^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3(c+dx)^4}{4(bc-ad)^4(a+bx)^4} \\
&\quad -\frac{3b^3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(c+dx)^4}{16(bc-ad)^4(a+bx)^4} \\
&\quad -\frac{3b^3B^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(c+dx)^4}{32(bc-ad)^4(a+bx)^4} \\
&\quad -\frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4}+\frac{b^2d\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3(c+dx)^3}{(bc-ad)^4(a+bx)^3} \\
&\quad +\frac{b^2Bdn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(c+dx)^3}{(bc-ad)^4(a+bx)^3} \\
&\quad +\frac{2b^2B^2dn^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(c+dx)^3}{3(bc-ad)^4(a+bx)^3} \\
&\quad +\frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3}-\frac{3bd^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3(c+dx)^2}{2(bc-ad)^4(a+bx)^2} \\
&\quad -\frac{9bBd^2n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2} \\
&\quad -\frac{9bB^2d^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(c+dx)^2}{4(bc-ad)^4(a+bx)^2} \\
&\quad -\frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2}+\frac{d^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3(c+dx)}{(bc-ad)^4(a+bx)} \\
&\quad +\frac{3Bd^3n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(c+dx)}{(bc-ad)^4(a+bx)} \\
&\quad +\frac{6B^2d^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(c+dx)}{(bc-ad)^4(a+bx)}+\frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)}
\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] (6*B^3*d^3*n^3*(c + d*x))/((b*c - a*d)^4*(a + b*x)) - (9*b*B^3*d^2*n^3*(c +
 d*x)^2)/(8*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^3*d*n^3*(c + d*x)^3)/(9*(
b*c - a*d)^4*(a + b*x)^3) - (3*b^3*B^3*n^3*(c + d*x)^4)/(128*(b*c - a*d)^4*
(a + b*x)^4) + (6*B^2*d^3*n^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/((b*c - a*d)^4*(a + b*x)) - (9*b*B^2*d^2*n^2*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]))/(4*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^2*d
*n^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*(b*c - a*d)^4
*(a + b*x)^3) - (3*b^3*B^2*n^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]))/(32*(b*c - a*d)^4*(a + b*x)^4) + (3*B*d^3*n*(c + d*x)*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^4*(a + b*x)) - (9*b*B*d^2*n*(
c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*(b*c - a*d)^4*(a
+ b*x)^2) + (b^2*B*d*n*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])
^2)/((b*c - a*d)^4*(a + b*x)^3) - (3*b^3*B*n*(c + d*x)^4*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])^2)/(16*(b*c - a*d)^4*(a + b*x)^4) + (d^3*(c + d*x)*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*c - a*d)^4*(a + b*x)) - (3*b
*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*(b*c - a*d)
^4*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])^3)/((b*c - a*d)^4*(a + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^3)/(4*(b*c - a*d)^4*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^5}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^3}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))^3}{x^5} - \frac{3b^2d(A+B\log(ex^n))^3}{x^4} + \frac{3bd^2(A+B\log(ex^n))^3}{x^3} - \frac{d^3(A+B\log(ex^n))^3}{x^2}\right) dx, x, \right.}{(bc-ad)^4}\right.$$

$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$- \text{Subst}\left(\frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$+ \text{Subst}\left(\frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$- \text{Subst}\left(\frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^4(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4(bc-ad)^4(a+bx)^4} + \text{Subst}\left(\frac{(3b^3Bn)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{4(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3b^2Bdn)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) + \text{Subst}\left(\frac{(9bBd^2n)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3Bd^3n)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= \frac{3Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{9bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{3b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{16(bc-ad)^4(a+bx)^4} \\
&\quad + \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4(bc-ad)^4(a+bx)^4} \\
&\quad + \text{Subst}\left(\frac{(3b^3B^2n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{8(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) \\
&\quad - \text{Subst}\left(\frac{(2b^2B^2dn^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) \\
&\quad + \text{Subst}\left(\frac{(9bB^2d^2n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) \\
&\quad - \text{Subst}\left(\frac{(6B^2d^3n^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)
\end{aligned}
$$

$$= \frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)} - \frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2} + \frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3} - \frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4} + \frac{6B^2d^3n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)} - \frac{9bB^2d^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3} - \frac{3b^3B^2n^2(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{32(bc-ad)^4(a+bx)^4} + \frac{3Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} - \frac{9bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^2} + \frac{b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} - \frac{3b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{16(bc-ad)^4(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^4(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4(bc-ad)^4(a+bx)^4}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 1370, normalized size of antiderivative = 1.65

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx = -\frac{-288B^3d^4n^3(a+bx)^4\log^3(a+bx)+288B^3d^4n^3(a+bx)^4\log^3(c+dx)+72B^2d^4n^2(a+bx)^4\log^2(c+d}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] -1/1152*(-288*B^3*d^4*n^3*(a + b*x)^4*Log[a + b*x]^3 + 288*B^3*d^4*n^3*(a +
 b*x)^4*Log[c + d*x]^3 + 72*B^2*d^4*n^2*(a + b*x)^4*Log[c + d*x]^2*(12*A +
25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 72*B^2*d^4*n^2*(a + b*x)^
```

```
4*Log[a + b*x]^2*(12*A + 25*B*n + 12*B*n*Log[c + d*x] + 12*B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(72*A^2 + 300*A*B
*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(288*A^3*b^3*c^3 -
 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*A^2*b^3*
B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 1800*a^3*A^2
*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 1932*a^2*A*b
*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 229*a*b^2*B^
3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 - 288*A^2*b^3
*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d^3*n*x - 336*
A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*a^2*A*b*B^2*d^
3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3*x - 16468*a^2*
b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^2*B*d^3*n*x^2 +
936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^2 + 690*b^3*B^3*c
*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*B*d^3*n*x^3 - 3600
*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B*(72*A^2*(b*c - a*d
)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 1084*d*x) + a*b^2*d*(-55*c
^2 + 212*c*d*x - 978*d^2*x^2) + b^3*(9*c^3 - 28*c^2*d*x + 78*c*d^2*x^2 - 30
0*d^3*x^3)) + 12*A*B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(
-13*c^2 + 20*c*d*x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 1
2*d^3*x^3)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*(12*A*(b*c - a*d)^3
+ B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(-13*c^2 + 20*c*d*
x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3)))*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 288*B^3*(b*c - a*d)^3*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^3) - 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(72*A^2 + 300*A*B*n +
 415*B^2*n^2 + 72*B^2*n^2*Log[c + d*x]^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + 72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 12*B*n*
Log[c + d*x]*(12*A + 25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(
b*c - a*d)^4*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8291 vs. $2(810) = 1620$.

Time = 191.85 (sec) , antiderivative size = 8292, normalized size of antiderivative = 9.99

method	result	size
parallelrisch	Expression too large to display	8292
risch	Expression too large to display	236754

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6057 vs. 2(810) = 1620.

Time = 0.53 (sec) , antiderivative size = 6057, normalized size of antiderivative = 7.30

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5280 vs. 2(810) = 1620.

Time = 0.57 (sec) , antiderivative size = 5280, normalized size of antiderivative = 6.36

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] -1/4*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^
3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/1152*(72*(12*d^4*e*n*log(b*x + a)/(b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^
4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*
e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*
e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4
*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b
^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^
2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d
```

+ 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (12 *(9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2 *n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d* x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2* c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b ^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^ 4*d^4*e^2*n^2)*log(b*x + a) + 12*(25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^ 2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4* d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2 *d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*l og(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3 *d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4 *a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e) + (27*b^4*c^4*e^3*n^3 - 256*a*b^3*c^3*d *e^3*n^3 + 1296*a^2*b^2*c^2*d^2*e^3*n^3 - 6912*a^3*b*c*d^3*e^3*n^3 + 5845*a ^4*d^4*e^3*n^3 - 4980*(b^4*c*d^3*e^3*n^3 - a*b^3*d^4*e^3*n^3)*x^3 - 288*(b^ 4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4 *a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a)^3 + 288*(b^4*d^4*e^3*n ^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4* e^3*n^3*x + a^4*d^4*e^3*n^3)*log(d*x + c)^3 + 30*(23*b^4*c^2*d^2*e^3*n^3 - 544*a*b^3*c*d^3*e^3*n^3 + 521*a^2*b^2*d^4*e^3*n^3)*x^2 + 1800*(b^4*d^4*e^3* n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4 *e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a)^2 + 72*(25*b^4*d^4*e^3*n^3*x^4 + 100*a*b^3*d^4*e^3*n^3*x^3 + 150*a^2*b^2*d^4*e^3*n^3*x^2 + 100*a^3*b*d^4*e^ 3*n^3*x + 25*a^4*d^4*e^3*n^3 - 12*(b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^ 3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 - 4*(37*b^4*c^3*d*e^3*n^3 - 456*a*b^3*c^2*d^ 2*e^3*n^3 + 4536*a^2*b^2*c*d^3*e^3*n^3 - 4117*a^3*b*d^4*e^3*n^3)*x - 4980*(b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a) + 12*(415*b^4*d^4*e^ 3*n^3*x^4 + 1660*a*b^3*d^4*e^3*n^3*x^3 + 2490*a^2*b^2*d^4*e^3*n^3*x^2 + 166 0*a^3*b*d^4*e^3*n^3*x + 415*a^4*d^4*e^3*n^3 + 72*(b^4*d^4*e^3*n^3*x^4 + 4*a *b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a)^2 - 300*(b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^ 3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3 *n^3)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b

^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e^2))/e)*B^3 + 1/96*A*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*(25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e^2)) - 3/4*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/16*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3* a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*A^2*B/e - 3/4*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^5}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="giac")

[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^5, x)

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 4257, normalized size of antiderivative = 5.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx=\text{Too large to display}$$

[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^5,x)

[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + a*c*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2)) + x^2*((a*d + b*c)*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2) + b*d*(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*a*b^2*c*d^2*n^2) + x^3*(b*d*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2) + 6*B^3*b^2*d^2*n^2*(a*d + b*c)) + a*c*(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*b^3*d^3*n^2*x^4)/(8*b*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)) - (d^4*(12*A*B^2 + 25*B^3*n)*(x^3*((a*d + b*c)*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d

- b*c))/d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(b*(a*((2*a*b*n*(a*d - b*c
)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^
3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^
3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b
c)^3(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3) + (8*a*b^4*c*n*(a*d - b*c)^3)
/d) + x^2*((a*d + b*c)*(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c
)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4
*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) +
 (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d))/d^3) + a*c*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d -
b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/
d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(a*(a*((2*a*b*n*(a*d - b*c)^3)/d +
 (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2
*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*a^3*d^3 - b
^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)) + x*((a*(a*((2*a*b*n*(a*d -
b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*
c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*
a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)*(a*d + b*c) + a*c*
(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a
(b((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2
)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)
^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3)) +
x^4*(b*d*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b
*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*
(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/d^2 + (6*a*b^3*n*
(a*d - b*c)^3)/d) + (8*b^4*n*(a*d + b*c)*(a*d - b*c)^3)/d) + a*c*(a*(a*((2*
a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*
b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d
 - b*c)^3*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4) + 8*b
^5*n*x^5*(a*d - b*c)^3))/(64*b^2*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)*(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((
e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)) - (B^3*d^4)/(4*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2
*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((e*(a + b*x)^n)/(c + d*x)^n)^
2*((3*A*B^2)/(4*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x^
2)) - (d^4*(12*A*B^2 + 25*B^3*n))/(16*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*
d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*B^3*d^4*(x^2*(b*(b*((b*n*(a*d -
b*c)*(4*a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*
c))/d + (2*b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*
c))/d + (b^3*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + a*(a*((b*n*(a*d - b*c)*(4*
a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2

```
+ b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))
/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 -
 4*a*b*c*d))/(3*d^3)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2) + (a*
b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c))/d + (2*b^2*n*(a*d - b*c)*(4*a
*d - b*c))/(3*d^2)) + (b^2*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))
/d^3) + (b*n*(a*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2))/d^4 + (4*b^4*n*x^3*(a*d - b*c))/d))/(16*b*(a^4*b + b^5*x^4 + 4*a^3*b^2
*x + 4*a*b^4*x^3 + 6*a^2*b^3*x^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((288*A^3*a^3*d^3 - 288*A^3*b^3*c^3 + 58
45*B^3*a^3*d^3*n^3 - 27*B^3*b^3*c^3*n^3 + 1800*A^2*B*a^3*d^3*n - 216*A^2*B*
b^3*c^3*n + 864*A^3*a*b^2*c^2*d - 864*A^3*a^2*b*c*d^2 + 4980*A*B^2*a^3*d^3*
n^2 - 108*A*B^2*b^3*c^3*n^2 + 229*B^3*a*b^2*c^2*d*n^3 - 1067*B^3*a^2*b*c*d^
2*n^3 + 660*A*B^2*a*b^2*c^2*d*n^2 - 1932*A*B^2*a^2*b*c*d^2*n^2 + 936*A^2*B*
a*b^2*c^2*d*n - 1656*A^2*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(2605*B^3
*a*b^2*d^3*n^3 - 115*B^3*b^3*c*d^2*n^3 + 504*A^2*B*a*b^2*d^3*n - 72*A^2*B*b
^3*c*d^2*n + 1956*A*B^2*a*b^2*d^3*n^2 - 156*A*B^2*b^3*c*d^2*n^2))/(2*(a*d -
 b*c)) + (x*(4117*B^3*a^2*b*d^3*n^3 + 37*B^3*b^3*c^2*d*n^3 - 419*B^3*a*b^2*
c*d^2*n^3 + 936*A^2*B*a^2*b*d^3*n + 72*A^2*B*b^3*c^2*d*n + 3252*A*B^2*a^2*b
*d^3*n^2 + 84*A*B^2*b^3*c^2*d*n^2 - 636*A*B^2*a*b^2*c*d^2*n^2 - 360*A^2*B*a
*b^2*c*d^2*n))/(3*(a*d - b*c)) + (x^3*(415*B^3*b^3*d^3*n^3 + 72*A^2*B*b^3*d
^3*n + 300*A*B^2*b^3*d^3*n^2))/(a*d - b*c))/(x*(384*a^3*b^4*c^2 + 384*a^5*b
^2*d^2 - 768*a^4*b^3*c*d) + x^3*(384*a*b^6*c^2 + 384*a^3*b^4*d^2 - 768*a^2*
b^5*c*d) + x^4*(96*b^7*c^2 + 96*a^2*b^5*d^2 - 192*a*b^6*c*d) + x^2*(576*a^2
*b^5*c^2 + 576*a^4*b^3*d^2 - 1152*a^3*b^4*c*d) + 96*a^6*b*d^2 + 96*a^4*b^3*
c^2 - 192*a^5*b^2*c*d) + (B*d^4*n*atan((B*d^4*n*((b^5*c^4 - a^4*b*d^4 + 2*a
^3*b^2*c*d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*
b^3*c^2*d) + 2*b*d*x)*(72*A^2 + 415*B^2*n^2 + 300*A*B*n)*(b^4*c^3 - a^3*b*d
^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)/(b*(a*d - b*c)^4*(415*B^3*d^4*n^3
 + 72*A^2*B*d^4*n + 300*A*B^2*d^4*n^2)))*(72*A^2 + 415*B^2*n^2 + 300*A*B*n)
*1i)/(48*b*(a*d - b*c)^4)
```

3.172 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/
((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2573, 2549, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] (E^(A/(B*n))*(c + d*x)*((e*(a + b*x)^n)/(c + d*x)^n)^n^(-1)*ExpIntegralEi[-
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a
+ b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2n(a+bx)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$= \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2}\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$= \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)} \,d x$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.173 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = -\frac{B(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} - \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-1/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*
B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{2548, 21, 45\}$

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= \frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b} + \frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5} - \frac{Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{Bg^4(a+bx)^4(bc-ad)}{20bd}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -1/5*(B*(b*c - a*d)^4*g^4*x)/d^4 + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*
d^3) - (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a
 + b*x)^4)/(20*b*d) + (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(
a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} + \frac{(B(bc-ad))\int \frac{(ag+bgx)^5}{(a+bx)(c+dx)}\,dx}{5bg}$$
$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} + \frac{(B(bc-ad)g^4)\int \frac{(a+bx)^4}{c+dx}\,dx}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} + \frac{(B(bc-ad)g^4)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}$$

$$= -\frac{B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} - \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g^4\left(-\frac{B(-bc+ad)\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx)\right)}{12d^5}+(a+bx)^5\left(A\right.\right.}{5b}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g^4*(-1/12*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(
b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a +
 b*x)])))/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(168) = 336.

Time = 1.16 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.46

method	result
risch	$\frac{g^4 b^4 A x^5}{5} + g^4 b^3 A a x^4 - \frac{g^4 b^3 B a x^4}{20} + \frac{g^4 b^4 B c x^4}{20d} + 2g^4 b^2 A a^2 x^3 - \frac{4 g^4 b^2 B a^2 x^3}{15} - \frac{g^4 b^4 B c^2 x^3}{15 d^2} + 2g$
parts	$\frac{A g^4 (bx+a)^5}{5b} + B g^4 e^5 (ad-cb)^5 \left(-\frac{1}{20 d e b \left(\left(\frac{de}{b} - \frac{e(ad-cb)}{b(bx+a)}\right) b - de\right)^4} + \frac{1}{15 d^2 e^2 b \left(\left(\frac{de}{b} - \frac{e(ad-cb)}{b(bx+a)}\right) b - de\right)^3} + \right.$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{A b e^4 g^4 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right)}{5\left(\left(\frac{de}{b} - \frac{e(ad-cb)}{b(bx+a)}\right) b - de\right)^5} + B b^2 e^4 g^4 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right)\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{A b e^4 g^4 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right)}{5\left(\left(\frac{de}{b} - \frac{e(ad-cb)}{b(bx+a)}\right) b - de\right)^5} + B b^2 e^4 g^4 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right)\right.}{}$
parallelrisch	$\frac{120 B x a^3 b^2 c d^4 g^4 - 120 B x a^2 b^3 c^2 d^3 g^4 + 60 B x a b^4 c^3 d^2 g^4 + 60 B \ln\left(\frac{e(dx+c)}{bx+a}\right) a^4 bc d^4 g^4 - 120 B \ln\left(\frac{e(dx+c)}{bx+a}\right) a^3 b^2 c^2 d^3 g^4 +}{}$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*g^4*b^4*A*x^5+g^4*b^3*A*a*x^4-1/20*g^4*b^3*B*a*x^4+1/20*g^4/d*b^4*B*c*x
^4+2*g^4*b^2*A*a^2*x^3-4/15*g^4*b^2*B*a^2*x^3-1/15*g^4/d^2*b^4*B*c^2*x^3+2*
g^4*b*A*a^3*x^2-3/5*g^4*b*B*a^3*x^2+1/10*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x-
4/5*g^4*B*a^4*x-1/5*g^4/d^4*b^4*B*c^4*x+1/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+g^4
/d*B*ln(d*x+c)*a^4*c+2*g^4/d*b*B*a^3*c*x-2*g^4/d^2*b^2*B*a^2*c^2*x+g^4/d^3*
b^3*B*a*c^3*x-2*g^4/d^2*b*B*ln(d*x+c)*a^3*c^2+2*g^4/d^3*b^2*B*ln(d*x+c)*a^2
*c^3-g^4/d^4*b^3*B*ln(d*x+c)*a*c^4+1/3*g^4/d*b^3*B*a*c*x^3-1/5*g^4/b*B*ln(d
*x+c)*a^5+g^4/d*b^2*B*a^2*c*x^2-1/2*g^4/d^2*b^3*B*a*c^2*x^2+1/5*(b*x+a)^5*g
^4*B/b*ln(e*(d*x+c)/(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(168) = 336$.

Time = 0.35 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.41

$$\int (ag+bgx)^4 \left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 - 12\,Ba^5d^5g^4\log(bx+a) + 3\,(Bb^5cd^4 + (20\,A-B)ab^4d^5)g^4x^4 - 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 -}{}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
 + (20*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - 2*(
15*A - 2*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 + 2*(10*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - (5*A - 4*B
```

```
)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.78 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$

$$= \frac{Ab^4g^4x^5}{5} - \frac{Ba^5g^4\log\left(x+\frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b}$$
$$+ \frac{Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{6Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba\cdots}{\cdots}\right)}{5d^5}$$
$$+x^4\left(Aab^3g^4-\frac{Bab^3g^4}{20}+\frac{Bb^4cg^4}{20d}\right)+x^3\cdot\left(2Aa^2b^2g^4-\frac{4Ba^2b^2g^4}{15}+\frac{Bab^3cg^4}{3d}-\frac{Bb^4c^2g^4}{15d^2}\right)$$
$$+x^2\cdot\left(2Aa^3bg^4-\frac{3Ba^3bg^4}{5}+\frac{Ba^2b^2cg^4}{d}-\frac{Bab^3c^2g^4}{2d^2}+\frac{Bb^4c^3g^4}{10d^3}\right)$$
$$+x\left(Aa^4g^4-\frac{4Ba^4g^4}{5}+\frac{2Ba^3bcg^4}{d}-\frac{2Ba^2b^2c^2g^4}{d^2}+\frac{Bab^3c^3g^4}{d^3}-\frac{Bb^4c^4g^4}{5d^4}\right)$$
$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**4*g**4*x**5/5 - B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) + B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
b**3*g**4 - B*a*b**3*g**4/20 + B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
```

```
g**4 - 4*B*a**2*b**2*g**4/15 + B*a*b**3*c*g**4/(3*d) - B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 - 3*B*a**3*b*g**4/5 + B*a**2*b**2*c*g**4/d
- B*a*b**3*c**2*g**4/(2*d**2) + B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 - 4*B*a**4*g**4/5 + 2*B*a**3*b*c*g**4/d - 2*B*a**2*b**2*c**2*g**4/d**2 +
B*a*b**3*c**3*g**4/d**3 - B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(c + d*x)/(a + b*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(168) = 336$.

Time = 0.23 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.44

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3 + 2Aa^3bg^4x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^4g^4 + 2\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^3bg^4 + \left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d\ldots}{b^2d^2}\right. + \frac{1}{6}\left(6x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{6a^4\log(bx+a)}{b^4} - \frac{6c^4\log(dx+c)}{d^4} + \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2\ldots}{b^3\ldots}\right. + \frac{1}{60}\left(12x^5\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{12a^5\log(bx+a)}{b^5} + \frac{12c^5\log(dx+c)}{d^5} + \frac{3(b^4cd^3-ab^3d^4)x^4 - 4(\ldots}{\ldots}\right. + Aa^4g^4x$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(
b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*
log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*
a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
*a*b^3*g^4 + 1/60*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 12*a^5*log
(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(168) = 336$.

Time = 0.47 (sec) , antiderivative size = 2030, normalized size of antiderivative = 11.28

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/60*(12*(B*b^6*c^6*e^6*g^4 - 6*B*a*b^5*c^5*d*e^6*g^4 + 15*B*a^2*b^4*c^4*d
^2*e^6*g^4 - 20*B*a^3*b^3*c^3*d^3*e^6*g^4 + 15*B*a^4*b^2*c^2*d^4*e^6*g^4 -
6*B*a^5*b*c*d^5*e^6*g^4 + B*a^6*d^6*e^6*g^4)*log((d*e*x + c*e)/(b*x + a))/(
b*d^5*e^5 - 5*(d*e*x + c*e)*b^2*d^4*e^4/(b*x + a) + 10*(d*e*x + c*e)^2*b^3*
d^3*e^3/(b*x + a)^2 - 10*(d*e*x + c*e)^3*b^4*d^2*e^2/(b*x + a)^3 + 5*(d*e*x
 + c*e)^4*b^5*d*e/(b*x + a)^4 - (d*e*x + c*e)^5*b^6/(b*x + a)^5) + (12*A*b^
6*c^6*d^4*e^6*g^4 - 25*B*b^6*c^6*d^4*e^6*g^4 - 72*A*a*b^5*c^5*d^5*e^6*g^4 +
 150*B*a*b^5*c^5*d^5*e^6*g^4 + 180*A*a^2*b^4*c^4*d^6*e^6*g^4 - 375*B*a^2*b^
4*c^4*d^6*e^6*g^4 - 240*A*a^3*b^3*c^3*d^7*e^6*g^4 + 500*B*a^3*b^3*c^3*d^7*e
^6*g^4 + 180*A*a^4*b^2*c^2*d^8*e^6*g^4 - 375*B*a^4*b^2*c^2*d^8*e^6*g^4 - 72
*A*a^5*b*c*d^9*e^6*g^4 + 150*B*a^5*b*c*d^9*e^6*g^4 + 12*A*a^6*d^10*e^6*g^4
- 25*B*a^6*d^10*e^6*g^4 + 77*(d*e*x + c*e)*B*b^7*c^6*d^3*e^5*g^4/(b*x + a)
- 462*(d*e*x + c*e)*B*a*b^6*c^5*d^4*e^5*g^4/(b*x + a) + 1155*(d*e*x + c*e)*
B*a^2*b^5*c^4*d^5*e^5*g^4/(b*x + a) - 1540*(d*e*x + c*e)*B*a^3*b^4*c^3*d^6*
e^5*g^4/(b*x + a) + 1155*(d*e*x + c*e)*B*a^4*b^3*c^2*d^7*e^5*g^4/(b*x + a)
- 462*(d*e*x + c*e)*B*a^5*b^2*c*d^8*e^5*g^4/(b*x + a) + 77*(d*e*x + c*e)*B*
a^6*b*d^9*e^5*g^4/(b*x + a) - 94*(d*e*x + c*e)^2*B*b^8*c^6*d^2*e^4*g^4/(b*x
 + a)^2 + 564*(d*e*x + c*e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(b*x + a)^2 - 1410*(d
*e*x + c*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(b*x + a)^2 + 1880*(d*e*x + c*e)^2*
B*a^3*b^5*c^3*d^5*e^4*g^4/(b*x + a)^2 - 1410*(d*e*x + c*e)^2*B*a^4*b^4*c^2*
d^6*e^4*g^4/(b*x + a)^2 + 564*(d*e*x + c*e)^2*B*a^5*b^3*c*d^7*e^4*g^4/(b*x
+ a)^2 - 94*(d*e*x + c*e)^2*B*a^6*b^2*d^8*e^4*g^4/(b*x + a)^2 + 54*(d*e*x +
 c*e)^3*B*b^9*c^6*d*e^3*g^4/(b*x + a)^3 - 324*(d*e*x + c*e)^3*B*a*b^8*c^5*d
^2*e^3*g^4/(b*x + a)^3 + 810*(d*e*x + c*e)^3*B*a^2*b^7*c^4*d^3*e^3*g^4/(b*x
 + a)^3 - 1080*(d*e*x + c*e)^3*B*a^3*b^6*c^3*d^4*e^3*g^4/(b*x + a)^3 + 810*
(d*e*x + c*e)^3*B*a^4*b^5*c^2*d^5*e^3*g^4/(b*x + a)^3 - 324*(d*e*x + c*e)^3
*B*a^5*b^4*c*d^6*e^3*g^4/(b*x + a)^3 + 54*(d*e*x + c*e)^3*B*a^6*b^3*d^7*e^3
*g^4/(b*x + a)^3 - 12*(d*e*x + c*e)^4*B*b^10*c^6*e^2*g^4/(b*x + a)^4 + 72*(
d*e*x + c*e)^4*B*a*b^9*c^5*d*e^2*g^4/(b*x + a)^4 - 180*(d*e*x + c*e)^4*B*a^
2*b^8*c^4*d^2*e^2*g^4/(b*x + a)^4 + 240*(d*e*x + c*e)^4*B*a^3*b^7*c^3*d^3*e
^2*g^4/(b*x + a)^4 - 180*(d*e*x + c*e)^4*B*a^4*b^6*c^2*d^4*e^2*g^4/(b*x + a
)^4 + 72*(d*e*x + c*e)^4*B*a^5*b^5*c*d^5*e^2*g^4/(b*x + a)^4 - 12*(d*e*x +
c*e)^4*B*a^6*b^4*d^6*e^2*g^4/(b*x + a)^4)/(b*d^9*e^5 - 5*(d*e*x + c*e)*b^2*
d^8*e^4/(b*x + a) + 10*(d*e*x + c*e)^2*b^3*d^7*e^3/(b*x + a)^2 - 10*(d*e*x
+ c*e)^3*b^4*d^6*e^2/(b*x + a)^3 + 5*(d*e*x + c*e)^4*b^5*d^5*e/(b*x + a)^4
```

```
- (d*e*x + c*e)^5*b^6*d^4/(b*x + a)^5) + 12*(B*b^6*c^6*e*g^4 - 6*B*a*b^5*c^
5*d*e*g^4 + 15*B*a^2*b^4*c^4*d^2*e*g^4 - 20*B*a^3*b^3*c^3*d^3*e*g^4 + 15*B*
a^4*b^2*c^2*d^4*e*g^4 - 6*B*a^5*b*c*d^5*e*g^4 + B*a^6*d^6*e*g^4)*log(-d*e +
 (d*e*x + c*e)*b/(b*x + a))/(b*d^5) - 12*(B*b^6*c^6*e*g^4 - 6*B*a*b^5*c^5*d
*e*g^4 + 15*B*a^2*b^4*c^4*d^2*e*g^4 - 20*B*a^3*b^3*c^3*d^3*e*g^4 + 15*B*a^4
*b^2*c^2*d^4*e*g^4 - 6*B*a^5*b*c*d^5*e*g^4 + B*a^6*d^6*e*g^4)*log((d*e*x +
c*e)/(b*x + a))/(b*d^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e -
 a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.60

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\cdots\right.}{10\,b\,d}+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\cdots\right.}{5\,b\,d}\cdots\right.}{5\,b\,d}\right.$$

$$+\frac{a\,c\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x))),x)

[Out] log((e*(c + d*x))/(a + b*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*a*d + B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
- 2*B*a*d + 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*
a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c -
 B*a*d + B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*
x)*((B*b^4*c^5*g^4)/5 + B*a^4*c*d^4*g^4 - 2*B*a^3*b*c^2*d^3*g^4 + 2*B*a^2*b
^2*c^3*d^2*g^4 - B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (B*a^5*g^4*l
og(a + b*x))/(5*b)
```

3.174 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{B(bc-ad)^3g^3x}{4d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{12bd} - \frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}$$

```
[Out] 1/4*B*(-a*d+b*c)^3*g^3*x/d^3-1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/12*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b} - \frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4} + \frac{Bg^3x(bc-ad)^3}{4d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{Bg^3(a+bx)^3(bc-ad)}{12bd}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(4*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3*Log[c
+ d*x])/(4*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(4*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}+\frac{(B(bc-ad))\int\frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{4bg}$$
$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}+\frac{(B(bc-ad)g^3)\int\frac{(a+bx)^3}{c+dx}\,dx}{4b}$$
$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}+\frac{(B(bc-ad)g^3)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{4b}$$
$$=\frac{B(bc-ad)^3g^3x}{4d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{12bd}-\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx=\frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{4b}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(139) = 278.

Time = 0.93 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

method	result
risch	$\frac{g^3(bx+a)^4 B \ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b} + \frac{g^3 b^3 A x^4}{4} + g^3 b^2 A a x^3 - \frac{g^3 b^2 B a x^3}{12} + \frac{g^3 b^3 B c x^3}{12d} + \frac{3g^3 b A a^2 x^2}{2} - \frac{3g^3 b B a^2 x}{8}$
parts	$\frac{A g^3(bx+a)^4}{4b} - B g^3 e^4 (ad-cb)^4 \left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)} - \frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{4d^4e^4b} - \frac{}{12deb}\right.$
derivativedivides	$\frac{e(ad-cb)\left(\frac{Ab e^3 g^3 (a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{4\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^4} - B b^2 e^3 g^3 (a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)\left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+c}\right.\right.}\right.\right.}{}$
default	$\frac{e(ad-cb)\left(\frac{Ab e^3 g^3 (a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{4\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^4} - B b^2 e^3 g^3 (a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)\left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+c}\right.\right.}\right.\right.}{}$
parallelrisch	$\frac{24B\ln(bx+a)a^3bcd^3g^3 - 9Ba^3bcd^3g^3 - 24Ba^2b^2c^2d^2g^3 + 21Bab^3c^3dg^3 + 36Bxa^2b^2cd^3g^3 - 24Bxab^3c^2d^2g^3 + 24Bx\ln}{}$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*g^3*(b*x+a)^4*B/b*ln(e*(d*x+c)/(b*x+a))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*x
^3-1/12*g^3*b^2*B*a*x^3+1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2-3/8*g^3*
b*B*a^2*x^2+1/2*g^3*b^2/d*B*a*c*x^2-1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*x-1
/4*g^3/b*B*ln(d*x+c)*a^4+g^3/d*B*ln(d*x+c)*a^3*c-3/2*g^3*b/d^2*B*ln(d*x+c)*
a^2*c^2+g^3*b^2/d^3*B*ln(d*x+c)*a*c^3-1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4-3/4*g
^3*B*a^3*x+3/2*g^3*b/d*B*a^2*c*x-g^3*b^2/d^2*B*a*c^2*x+1/4*g^3*b^3/d^3*B*c^
3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.15

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$
$$= \frac{6\,Ab^4d^4g^3x^4 - 6\,Ba^4d^4g^3\log(bx+a) + 2\,(Bb^4cd^3 + (12\,A-B)ab^3d^4)g^3x^3 - 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 - 3}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (12*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(4*
A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 + (4*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(128) = 256.

Time = 2.22 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$

$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)(2}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3}\right.}{4d^4}$$

$$+ x^3\left(Aab^2g^3-\frac{Bab^2g^3}{12}+\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}-\frac{3Ba^2bg^3}{8}+\frac{Bab^2cg^3}{2d}-\frac{Bb^3c^2g^3}{8d^2}\right)$$

$$+ x\left(Aa^3g^3-\frac{3Ba^3g^3}{4}+\frac{3Ba^2bcg^3}{2d}-\frac{Bab^2c^2g^3}{d^2}+\frac{Bb^3c^3g^3}{4d^3}\right)$$

$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/12 + B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*
b*g**3/8 + B*a*b**2*c*g**3/(2*d) - B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 - 3*B*a**3*g**3/4 + 3*B*a**2*b*c*g**3/(2*d) - B*a*b**2*c**2*g**3/d**2 +
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)/(a + b*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(139) = 278$.

Time = 0.20 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.93

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{1}{4}Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}Aa^2bg^3x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^3g^3 + \frac{3}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^2bg^3 + \frac{1}{2}\left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2} \right. + \frac{1}{24}\left(6x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{6a^4\log(bx+a)}{b^4} - \frac{6c^4\log(dx+c)}{d^4} + \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2}{} \right. + Aa^3g^3x$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d*e*x/(b
*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(d*e*x
/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d
^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time = 0.45 (sec) , antiderivative size = 1506, normalized size of antiderivative = 10.11

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/24*(6*(B*b^5*c^5*e^5*g^3 - 5*B*a*b^4*c^4*d*e^5*g^3 + 10*B*a^2*b^3*c^3*d^2
*e^5*g^3 - 10*B*a^3*b^2*c^2*d^3*e^5*g^3 + 5*B*a^4*b*c*d^4*e^5*g^3 - B*a^5*d
^5*e^5*g^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^4*e^4 - 4*(d*e*x + c*e)*b^2*d
```

^3*e^3/(b*x + a) + 6*(d*e*x + c*e)^2*b^3*d^2*e^2/(b*x + a)^2 - 4*(d*e*x + c
*e)^3*b^4*d*e/(b*x + a)^3 + (d*e*x + c*e)^4*b^5/(b*x + a)^4) + (6*A*b^5*c^5
*d^3*e^5*g^3 - 11*B*b^5*c^5*d^3*e^5*g^3 - 30*A*a*b^4*c^4*d^4*e^5*g^3 + 55*B
*a*b^4*c^4*d^4*e^5*g^3 + 60*A*a^2*b^3*c^3*d^5*e^5*g^3 - 110*B*a^2*b^3*c^3*d
^5*e^5*g^3 - 60*A*a^3*b^2*c^2*d^6*e^5*g^3 + 110*B*a^3*b^2*c^2*d^6*e^5*g^3 +
30*A*a^4*b*c*d^7*e^5*g^3 - 55*B*a^4*b*c*d^7*e^5*g^3 - 6*A*a^5*d^8*e^5*g^3
+ 11*B*a^5*d^8*e^5*g^3 + 26*(d*e*x + c*e)*B*b^6*c^5*d^2*e^4*g^3/(b*x + a) -
130*(d*e*x + c*e)*B*a*b^5*c^4*d^3*e^4*g^3/(b*x + a) + 260*(d*e*x + c*e)*B*
a^2*b^4*c^3*d^4*e^4*g^3/(b*x + a) - 260*(d*e*x + c*e)*B*a^3*b^3*c^2*d^5*e^4
*g^3/(b*x + a) + 130*(d*e*x + c*e)*B*a^4*b^2*c*d^6*e^4*g^3/(b*x + a) - 26*(
d*e*x + c*e)*B*a^5*b*d^7*e^4*g^3/(b*x + a) - 21*(d*e*x + c*e)^2*B*b^7*c^5*d
*e^3*g^3/(b*x + a)^2 + 105*(d*e*x + c*e)^2*B*a*b^6*c^4*d^2*e^3*g^3/(b*x + a
)^2 - 210*(d*e*x + c*e)^2*B*a^2*b^5*c^3*d^3*e^3*g^3/(b*x + a)^2 + 210*(d*e*
x + c*e)^2*B*a^3*b^4*c^2*d^4*e^3*g^3/(b*x + a)^2 - 105*(d*e*x + c*e)^2*B*a^
4*b^3*c*d^5*e^3*g^3/(b*x + a)^2 + 21*(d*e*x + c*e)^2*B*a^5*b^2*d^6*e^3*g^3/
(b*x + a)^2 + 6*(d*e*x + c*e)^3*B*b^8*c^5*e^2*g^3/(b*x + a)^3 - 30*(d*e*x +
c*e)^3*B*a*b^7*c^4*d*e^2*g^3/(b*x + a)^3 + 60*(d*e*x + c*e)^3*B*a^2*b^6*c^
3*d^2*e^2*g^3/(b*x + a)^3 - 60*(d*e*x + c*e)^3*B*a^3*b^5*c^2*d^3*e^2*g^3/(b
x + a)^3 + 30(d*e*x + c*e)^3*B*a^4*b^4*c*d^4*e^2*g^3/(b*x + a)^3 - 6*(d*e
*x + c*e)^3*B*a^5*b^3*d^5*e^2*g^3/(b*x + a)^3)/(b*d^7*e^4 - 4*(d*e*x + c*e)
*b^2*d^6*e^3/(b*x + a) + 6*(d*e*x + c*e)^2*b^3*d^5*e^2/(b*x + a)^2 - 4*(d*e
*x + c*e)^3*b^4*d^4*e/(b*x + a)^3 + (d*e*x + c*e)^4*b^5*d^3/(b*x + a)^4) +
6*(B*b^5*c^5*e*g^3 - 5*B*a*b^4*c^4*d*e*g^3 + 10*B*a^2*b^3*c^3*d^2*e*g^3 - 1
0*B*a^3*b^2*c^2*d^3*e*g^3 + 5*B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log(-d
*e + (d*e*x + c*e)*b/(b*x + a))/(b*d^4) - 6*(B*b^5*c^5*e*g^3 - 5*B*a*b^4*c^
4*d*e*g^3 + 10*B*a^2*b^3*c^3*d^2*e*g^3 - 10*B*a^3*b^2*c^2*d^3*e*g^3 + 5*B*a
^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^4))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=x\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+ A\,a \cdots\right.}{4\,b\,d}\right.$$
$$\left.+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{2\,d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$
$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$
$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$
$$-\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{4\,d^4}$$
$$+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c - 3*B*a*d + 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c - B*a*d + B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(c +
```

```
d*x))/(a + b*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.175 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = -\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}+\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*g^2*x)/d^2 + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d)
+ (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Lo
g[(e*(c + d*x))/(a + b*x)]))/(3*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{(B(bc-ad))\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{3bg}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{(B(bc-ad)g^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{(B(bc-ad)g^2)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= -\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}\\
&\quad+\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$$
$$=\frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{3b}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/(2*d^3) + (a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a +
b*x)])))/(3*b)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAax^2-\frac{g^2bBax^2}{6}+\frac{g^2b^2Bcx^2}{6d}+g^2Aa^2x-\frac{g^2B\ln(dx+c)}{3b}$
parts	$\frac{Ag^2(bx+a)^3}{3b}+Bg^2e^3(ad-cb)^3\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}+\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{3d^3e^3b}+\frac{}{3d^2e^2b}\right.$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{Abe^2g^2\left(a^2d^2-2abcd+b^2c^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^3}+Bb^2e^2g^2\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}+\frac{\ln\left(\left(\frac{de}{b}-\frac{e(a}{b(b}\right.\right.}{3d^3}\right.\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{Abe^2g^2\left(a^2d^2-2abcd+b^2c^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^3}+Bb^2e^2g^2\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}+\frac{\ln\left(\left(\frac{de}{b}-\frac{e(a}{b(b}\right.\right.}{3d^3}\right.\right.}{}$
parallelrisch	$\frac{2Bx^3\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3d^3g^2+4Ba^3d^3g^2+6Ax^2ab^2d^3g^2-Bx^2ab^2d^3g^2+Bx^2b^3cd^2g^2+6Axa^2bd^3g^2-4Bxa^2bd^3g^2-2B}{}$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x+a)^3*g^2*B/b*ln(e*(d*x+c)/(b*x+a))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^2
-1/6*g^2*b*B*a*x^2+1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x-1/3*g^2/b*B*ln(d*x+c)*
a^3+g^2/d*B*ln(d*x+c)*a^2*c-g^2*b/d^2*B*ln(d*x+c)*a*c^2+1/3*g^2*b^2/d^3*B*l
n(d*x+c)*c^3-2/3*g^2*B*a^2*x+g^2*b/d*B*a*c*x-1/3*g^2*b^2/d^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{2\,Ab^3d^3g^2x^3 - 2\,Ba^3d^3g^2\log(bx+a) + (Bb^3cd^2 + (6\,A-B)ab^2d^3)g^2x^2 - 2\,(Bb^3c^2d - 3\,Bab^2cd^2 - (3\,A-$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (6
*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*B)
*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.48 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= \frac{Ab^2g^2x^3}{3} - \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$
$$+ \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot(3a^2d^2-3ab}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2 - \frac{Babg^2}{6} + \frac{Bb^2cg^2}{6d}\right) + x\left(Aa^2g^2 - \frac{2Ba^2g^2}{3} + \frac{Babcg^2}{d} - \frac{Bb^2c^2g^2}{3d^2}\right)$$
$$+ \left(Ba^2g^2x + Babg^2x^2 + \frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 + B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
```

```
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 - B*a*b*g**2/6 + B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 - 2*B*a**2*g**2/3 + B*a*b*c*g**2/d - B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(c + d*x)/(a + b*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.

Time = 0.22 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.36

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a\cdots}{b^2d^2}\right. + Aa^2g^2x$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a))
 - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*e*x/(b*x + a
) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) -
2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.40 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = -\frac{1}{6}\left(\frac{2\left(Bb^4c^4e^4g^2 - 4Bab^3c^3de^4g^2 + 6Ba^2b^2c^2d^2e^4g^2 - 4Ba^3bcd^3e^4g^2 + Ba^4d^4e^4g^2\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^3e^3 - \frac{3(dex+ce)b^2d^2e^2}{bx+a} + \frac{3(dex+ce)^2b^3de}{(bx+a)^2} - \frac{(dex+ce)^3b^4}{(bx+a)^3}} + \frac{2A\cdots}{\cdots}\right.$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/6*(2*(B*b^4*c^4*e^4*g^2 - 4*B*a*b^3*c^3*d*e^4*g^2 + 6*B*a^2*b^2*c^2*d^2*
e^4*g^2 - 4*B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2)*log((d*e*x + c*e)/(b
*x + a))/(b*d^3*e^3 - 3*(d*e*x + c*e)*b^2*d^2*e^2/(b*x + a) + 3*(d*e*x + c*
e)^2*b^3*d*e/(b*x + a)^2 - (d*e*x + c*e)^3*b^4/(b*x + a)^3) + (2*A*b^4*c^4*
d^2*e^4*g^2 - 3*B*b^4*c^4*d^2*e^4*g^2 - 8*A*a*b^3*c^3*d^3*e^4*g^2 + 12*B*a*
b^3*c^3*d^3*e^4*g^2 + 12*A*a^2*b^2*c^2*d^4*e^4*g^2 - 18*B*a^2*b^2*c^2*d^4*e
^4*g^2 - 8*A*a^3*b*c*d^5*e^4*g^2 + 12*B*a^3*b*c*d^5*e^4*g^2 + 2*A*a^4*d^6*e
^4*g^2 - 3*B*a^4*d^6*e^4*g^2 + 5*(d*e*x + c*e)*B*b^5*c^4*d*e^3*g^2/(b*x + a
) - 20*(d*e*x + c*e)*B*a*b^4*c^3*d^2*e^3*g^2/(b*x + a) + 30*(d*e*x + c*e)*B
*a^2*b^3*c^2*d^3*e^3*g^2/(b*x + a) - 20*(d*e*x + c*e)*B*a^3*b^2*c*d^4*e^3*g
^2/(b*x + a) + 5*(d*e*x + c*e)*B*a^4*b*d^5*e^3*g^2/(b*x + a) - 2*(d*e*x + c
*e)^2*B*b^6*c^4*e^2*g^2/(b*x + a)^2 + 8*(d*e*x + c*e)^2*B*a*b^5*c^3*d*e^2*g
^2/(b*x + a)^2 - 12*(d*e*x + c*e)^2*B*a^2*b^4*c^2*d^2*e^2*g^2/(b*x + a)^2 +
 8*(d*e*x + c*e)^2*B*a^3*b^3*c*d^3*e^2*g^2/(b*x + a)^2 - 2*(d*e*x + c*e)^2*
B*a^4*b^2*d^4*e^2*g^2/(b*x + a)^2)/(b*d^5*e^3 - 3*(d*e*x + c*e)*b^2*d^4*e^2
/(b*x + a) + 3*(d*e*x + c*e)^2*b^3*d^3*e/(b*x + a)^2 - (d*e*x + c*e)^3*b^4*
d^2/(b*x + a)^3) + 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2
*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log(-d*e + (d*e*x
 + c*e)*b/(b*x + a))/(b*d^3) - 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*g^2 +
 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log((
d*e*x + c*e)/(b*x + a))/(b*d^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/(
(b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,\left(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c - B*a*d + B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c + d*x))/(a + b
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 - (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.176 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx=\frac{B(bc-ad)gx}{2d}-\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}$$

```
[Out] 1/2*B*(-a*d+b*c)*g*x/d-1/2*B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2
*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2548, 21, 45}

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx=\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{Bgx(bc-ad)}{2d}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (B*(b*c - a*d)*g*x)/(2*d) - (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2) + (g
*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}+\frac{(B(bc-ad))\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{2bg}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}+\frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}\,dx}{2b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}+\frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{2b}\\
&= \frac{B(bc-ad)gx}{2d}-\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g\left(\frac{B(bc-ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{2b}$$

[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]

[Out] (g*((B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a + b*x)^2 *(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b)

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{2b}+\frac{gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{2d^2}-\frac{gB}{2}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)ab\,d^2g+2Axab\,d^2g-B\ln(bx+a)a^2d^2g+2B\ln(bx+a)abcdg-B\ln(b}{2b}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-Bg\,e^2(ad-cb)^2\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln(}{}\right.$
derivativedivides	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(ad-cb)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln\left(\frac{de}{b}\right.}{}\right.\right.}{b^2}$
default	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(ad-cb)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln\left(\frac{de}{b}\right.}{}\right.\right.}{b^2}$

[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)

[Out] 1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)/(b*x+a))+1/2*g*b*A*x^2+g*A*a*x-1/2*B*a^2*g /b*ln(b*x+a)+g/d*B*ln(-d*x-c)*a*c-1/2*g*b/d^2*B*ln(-d*x-c)*c^2-1/2*g*B*a*x+ 1/2*g*b/d*B*c*x

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2-Ba^2d^2g\log(bx+a)+(Bb^2cd+(2\,A-B)abd^2)gx-(Bb^2c^2-2\,Babcd)g\log(dx+c)+(Bb^2d}{2\,bd^2}$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")

[Out] 1/2*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*log(b*x + a) + (B*b^2*c*d + (2*A - B)*a* b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{Abgx^2}{2} - \frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b} + \frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2} + x\left(Aag-\frac{Bag}{2}+\frac{Bbcg}{2d}\right) + \left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/(2*d**2) + x*(A*a*g - B*a*g/2 + B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(c + d*x)/(a + b*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.77

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{1}{2}Abgx^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Bag + \frac{1}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Bbg + Aagx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
+ c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) +
 a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(75) = 150$.

Time = 0.46 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.74

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3e^3g-3\,Bab^2c^2de^3g+3\,Ba^2bcd^2e^3g-Ba^3d^3e^3g)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2-\frac{2\,(dex+ce)b^2de}{bx+a}+\frac{(dex+ce)^2b^3}{(bx+a)^2}}+\frac{Ab^3c^3de^3g-Bb^3c^3de^3g-3\,Aa\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*e^3*g - 3*B*a*b^2*c^2*d*e^3*g + 3*B*a^2*b*c*d^2*e^3*g - B*a
^3*d^3*e^3*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2 - 2*(d*e*x + c*e)*b^2
*d*e/(b*x + a) + (d*e*x + c*e)^2*b^3/(b*x + a)^2) + (A*b^3*c^3*d*e^3*g - B*
b^3*c^3*d*e^3*g - 3*A*a*b^2*c^2*d^2*e^3*g + 3*B*a*b^2*c^2*d^2*e^3*g + 3*A*a
^2*b*c*d^3*e^3*g - 3*B*a^2*b*c*d^3*e^3*g - A*a^3*d^4*e^3*g + B*a^3*d^4*e^3*
g + (d*e*x + c*e)*B*b^4*c^3*e^2*g/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3*c^2*d
*e^2*g/(b*x + a) + 3*(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2*g/(b*x + a) - (d*e*x
 + c*e)*B*a^3*b*d^3*e^2*g/(b*x + a))/(b*d^3*e^2 - 2*(d*e*x + c*e)*b^2*d^2*e
/(b*x + a) + (d*e*x + c*e)^2*b^3*d/(b*x + a)^2) + (B*b^3*c^3*e*g - 3*B*a*b^
2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-d*e + (d*e*x + c*e)
*b/(b*x + a))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*
d^2*e*g - B*a^3*d^3*e*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)-\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2
*d)) + log((e*(c + d*x))/(a + b*x))*((B*b*g*x^2)/2 + B*a*g*x) - (log(c + d*
x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 - (B*a^2*g*log(a + b*
x))/(2*b)
```

3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} - \frac{B\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g-B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2544, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg} - \frac{B\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(b*g)) - (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rule steps

$$\text{integral} = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} - \frac{(B(bc-ad))\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} - \frac{(B(bc-ad))\text{Subst}\left(\int \frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx, x, a+bx\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} + \frac{(B(bc-ad))\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} + \frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}\,dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} - \frac{B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$$
$$= \frac{\log(g(a+bx))\left(B\log(g(a+bx))+2\left(A-B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)-2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{2bg}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(B*Log[g*(a + b*x)] + 2*(A - B*Log[(b*(c + d*x))/(b*c - a
*d)] + B*Log[(e*(c + d*x))/(a + b*x)])) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])/(2*b*g)
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.84

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}-\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}\right)}{g}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{gb}-\frac{B\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{gb}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a}\right.}{de}\right.}{b}\right.}{ge(ad-cb)}\right.}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a}\right.}{de}\right.}{b}\right.}{ge(ad-cb)}\right.}{b^2}$

[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)

[Out] A/g*ln(b*x+a)/b+B/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/b-l
n(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/
e)/b)

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="fricas")

[Out] integral((B*log((d*e*x + c*e)/(b*x + a)) + A)/(b*g*x + a*g), x)

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e)
- (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a
*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(80) = 160$.

Time = 37.79 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = -\frac{1}{2}\left(\frac{\left(Bb^3c^3e^3-3\,Bab^2c^2de^3+3\,Ba^2bcd^2e^3-Ba^3d^3e^3\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2g-\frac{2\,(dex+ce)b^2deg}{bx+a}+\frac{(dex+ce)^2b^3g}{(bx+a)^2}} + \frac{Ab^3c^3de^3-Bb^3c^3de^3-3\,Aab^2c^2d}{}\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] -1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d^
3*e^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2*g - 2*(d*e*x + c*e)*b^2*d*e*
g/(b*x + a) + (d*e*x + c*e)^2*b^3*g/(b*x + a)^2) + (A*b^3*c^3*d*e^3 - B*b^3
*c^3*d*e^3 - 3*A*a*b^2*c^2*d^2*e^3 + 3*B*a*b^2*c^2*d^2*e^3 + 3*A*a^2*b*c*d^
3*e^3 - 3*B*a^2*b*c*d^3*e^3 - A*a^3*d^4*e^3 + B*a^3*d^4*e^3 + (d*e*x + c*e)
*B*b^4*c^3*e^2/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3*c^2*d*e^2/(b*x + a) + 3*
```

```
(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2/(b*x + a) - (d*e*x + c*e)*B*a^3*b*d^3*e^2
/(b*x + a))/(b*d^3*e^2*g - 2*(d*e*x + c*e)*b^2*d^2*e*g/(b*x + a) + (d*e*x +
 c*e)^2*b^3*d*g/(b*x + a)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b
*c*d^2*e - B*a^3*d^3*e)*log(-d*e + (d*e*x + c*e)*b/(b*x + a))/(b*d^2*g) - (
B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log((d*e
*x + c*e)/(b*x + a))/(b*d^2*g))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((
b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x), x)
```

3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 64

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A-B}{bg^2(a+bx)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] (-A+B)/b/g^2/(b*x+a)-B*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.55, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2552, 2332}

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A(c+dx)}{g^2(a+bx)(bc-ad)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)} + \frac{B(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) + (B*(c + d*x))/((b*c - a*d)*g
^2*(a + b*x)) - (B*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c - a*d)*g^2
*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\ &= -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{B\text{Subst}\left(\int \log(ex)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\ &= -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = \frac{Bd(a+bx)\log(a+bx) - Bd(a+bx)\log(c+dx) - (bc-ad)\left(A-B+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
[Out] (B*d*(a + b*x)*Log[a + b*x] - B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A -
 B + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2*(a + b*x))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

method	result	size
parts	$-\frac{A}{g^{2}(bx+a)b}+\frac{B\left(\frac{e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}-\frac{e(dx+c)}{bx+a}\right)}{g^{2}e(ad-cb)}$	81
norman	$\frac{\frac{(A-B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{Bdx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$	89
parallelrisch	$-\frac{Aa\,b^{2}d^{2}-A\,b^{3}cd-Ba\,b^{2}d^{2}+B\,b^{3}cd-Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)b^{3}d^{2}-B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^{3}cd}{g^{2}(bx+a)b^{3}d(ad-cb)}$	112
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{b\,g^{2}(bx+a)}-\frac{B\ln(bx+a)bdx-B\ln(-dx-c)bdx+B\ln(bx+a)ad-B\ln(-dx-c)ad+Aad-Abc-Bad+Bbc}{g^{2}(bx+a)b(ad-cb)}$	127
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^{2}e^{2}g^{2}}\right)}{b^{2}}$	171
default	$\frac{e(ad-cb)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^{2}e^{2}g^{2}}\right)}{b^{2}}$	171

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/g^2/(b*x+a)/b+B/g^2/e/(a*d-b*c)*(e*(d*x+c)/(b*x+a)*ln(e*(d*x+c)/(b*x+a))
-e*(d*x+c)/(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{(A-B)bc-(A-B)ad+(Bbdx+Bbc)\log\left(\frac{dex+ce}{bx+a}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*log((d*e*x + c*e)/(b*x + a)
))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(48) = 96$.

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.61

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} - \frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} + \frac{-A+B}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A + B)/(a
*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2} - \frac{1}{b^2g^2x+abg^2} - \frac{d\log(bx+a)}{(b^2c-abd)g^2} + \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^2
*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(dex+ce)B\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dex+ce)(A-B)}{(bx+a)g^2}\right)$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
d*e*x + c*e)*B*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e*x + c*e)
*(A - B)/((b*x + a)*g^2))
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A-B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x))/(a + b*x)))/(b^2*g^2*(x + a/b)) - (A - B)/(b^2*g^2*x +
a*b*g^2)
```

3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{B}{4bg^3(a+bx)^2} - \frac{Bd}{2b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3} + \frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}$$

```
[Out] 1/4*B/b/g^3/(b*x+a)^2-1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)-1/2*B*d^2*ln(b*x+a)/
b/(-a*d+b*c)^2/g^3+1/2*B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d
*x+c)/(b*x+a)))/b/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2} - \frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2} + \frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2} - \frac{Bd}{2bg^3(a+bx)(bc-ad)} + \frac{B}{4bg^3(a+bx)^2}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(4*b*g^3*(a + b*x)^2) - (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Lo
g[a + b*x])/(2*b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)
^2*g^3) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(2*b*g^3*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^2}\,dx}{2bg}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}\\
&\quad -\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{2bg^3}\\
&= \frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}\\
&\quad +\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(2Abc-bBc-2aAd+3aBd+2\cdots\right.}{4b(bc-ad)^2g^3(a+bx)^2}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(2*A*b*c - b*B*c - 2*a*A*d + 3*a*B*d + 2*b*B*d*x + 2*B*(b*c -
a*d)*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.56

method	result
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{Bb\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{4}-\frac{de\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots\right.}{b}\cdots\right.}{g^3e^2(ad-cb)^2}$
norman	$\frac{\frac{Bad^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{2Aabd-2Ab^2c-3Babd+Bb^2c}{4gb^2(ad-cb)}+\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d^2Bbx^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{-2Bxab^4d^3+2Bxb^5cd^2+2B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d-4Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)ab^4d^3-4B\ln\left(\frac{e(dx+c)}{bx+a}\right)ab^4cd^2+2Aa^2b^3d^3+2A\cdots}{4g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2bg^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)abd^2x-4B\ln(-dx-c)abd^2x+2Ba^2\ln\cdots}{4(a^2d^2-2\cdots}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots}{4}\cdots\right.}{(ad-cb)^3e^3g^3}\cdots\right.}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots}{4}\cdots\right.}{(ad-cb)^3e^3g^3}\cdots\right.}{b^2}$

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/g^3/(b*x+a)^2/b-B/g^3*b/e^2/(a*d-b*c)^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(b
*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^
2-d*e/b*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a
*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{(2A-B)b^2c^2-4(A-B)abcd+(2A-3B)a^2d^2+2(Bb^2cd-Babd^2)x-2(Bb^2d^2x^2+2Babd^2x-B}{4((b^5c^2-2ab^4cd+a^2b^3d^2)g^3x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2a^3b^2cd}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^
2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.

Time = 1.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}&\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx \\ &= -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\ &\quad + \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\ &\quad - \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\ &\quad + \frac{-2Aad+2Abc+3Bad-Bbc+2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}\end{aligned}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2}{(b^3c^2}\right. - \frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] -1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/
(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/(
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.62

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(2\left(\frac{(dex+ce)^2Bb}{(bceg^3-adeg^3)(bx+a)^2}-\frac{2\,(dex+ce)Bd}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)+\frac{(dex+ce)^2(2\,Ab-Bb}{(bceg^3-adeg^3)(bx+a}\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*((d*e*x + c*e)^2*B*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(d*e
*x + c*e)*B*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a))
 + (d*e*x + c*e)^2*(2*A*b - B*b)/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 4*
(d*e*x + c*e)*(A*d - B*d)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e - a
*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2} - \frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{2\,A\,a\,d-2\,A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] (B*d^2*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2
*b*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x))/(a + b*x
)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((2*A*a*d - 2*A*b*c - 3*B*a*d + B
*b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2
 + 4*a*b^2*g^3*x)
```

3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4} dx = \frac{B}{9bg^4(a+bx)^3} - \frac{Bd}{6b(bc-ad)g^4(a+bx)^2} + \frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}$$

```
[Out] 1/9*B/b/g^4/(b*x+a)^3-1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+1/3*B*d^2/b/(-a*d+
b*c)^2/g^4/(b*x+a)+1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*B*d^3*ln(d*x+
c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)/(b*x+a)))/b/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4} dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3} + \frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3} - \frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd}{6bg^4(a+bx)^2(bc-ad)} + \frac{B}{9bg^4(a+bx)^3}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] B/(9*b*g^4*(a + b*x)^3) - (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) + (B*d^2)
/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^
3*g^4) - (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c +
d*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^3}\,dx}{3bg}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}\\
&\quad-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)\,dx}{3bg^4}\\
&= \frac{B}{9bg^4(a+bx)^3}-\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}+\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}\\
&\quad+\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-6\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{18bg^4(a+bx)^3}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] ((B*((b*c - a*d)*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x
 + 6*d^2*x^2)) + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c +
 d*x]))/(b*c - a*d)^3 - 6*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(18*b*g^4*(
a + b*x)^3)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.89

method	result
parts	$-\frac{A}{3g^4(bx+a)^3 b}+\frac{B b^2\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{3}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{9}-\frac{2de\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+}\right.}{2}\right.}{b}\right.}{g^4e^3(ad-cb)^3}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b\,g^4(bx+a)^3}-\frac{6B\ln(bx+a)b^3d^3x^3-6B\ln(-dx-c)b^3d^3x^3+18B\ln(bx+a)a\,b^2d^3x^2-18B\ln(-dx-c)a\,b^2d^3x^2+18B}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2+18Bxa\,b^6c\,d^3-15Bx\,a^2b^5d^4-3Bx\,b^7c^2d^2-6B\,x^2a\,b^6d^4+6B\,x^2b^7c\,d^3+6A\,a^3b^4d^4-6A\,b^7}{}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{6A\,a^2b\,d^2-12Aa\,b^2cd+6A\,c^2b^3-9B\,a^2b\,d^2+7Ba\,b^2cd-2}{18g\,b^2\left(a^2d^2-2abcd+b^2c^2\right)}}{g}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{3(ad-cb)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{(ad-cb)^4e^3g^4}+\frac{b^2A\,d^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+}\right.}{3}\right.}{(ad-cb)^4e^4g^4}\right.}{}$
default	$\frac{e(ad-cb)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{3(ad-cb)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{(ad-cb)^4e^3g^4}+\frac{b^2A\,d^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+}\right.}{3}\right.}{(ad-cb)^4e^4g^4}\right.}{}$

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/g^4/(b*x+a)^3/b+B/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+d^2*e^2/b^2*((d*e/b-e*(a*d-b*c)/
b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,(3\,A-B)b^3c^3-9\,(2\,A-B)ab^2c^2d+18\,(A-B)a^2bcd^2-(6\,A-11\,B)a^3d^3-6\,(Bb^3cd^2-Bab^2d^3)x^2+}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*d
^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((d*e*x + c*e)/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^
5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b
*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.

Time = 1.77 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} + \frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} - \frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 +
11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B
*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 + 1
8*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4 +
```

```
18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**4
 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g**
4 + 54*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^4}\,dx$$
$$= \frac{1}{18}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)g\right.$$
$$- \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] 1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6*log
(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(163) = 326$.

Time = 0.44 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.55

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^4}\,dx =$$
$$-\frac{1}{18}\left(6\left(\frac{(dex + ce)^3Bb^2}{(b^2c^2e^2g^4 - 2\,abcde^2g^4 + a^2d^2e^2g^4)(bx + a)^3} - \frac{3\,(dex + ce)^2Bbd}{(b^2c^2eg^4 - 2\,abcdeg^4 + a^2d^2eg^4)(bx + a)^2} + \frac{}{(b^2c}\right.\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(6*((d*e*x + c*e)^3*B*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 + a^2
*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B*b*d/((b^2*c^2*e*g^4 - 2*a*
```

```
b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B*d^2/((b^2*c^2
*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)
) + 2*(3*A*b^2 - B*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g
^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*b*d - B*b*d)*(d*e*x + c*e)^2/((
b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 18*(A*d^2 -
 B*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a
)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{5\,B\,a\,d^2\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{7\,B\,a\,c\,d}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c\,d\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^4,x)
```

```
[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b
*c)^3) - (B*log((e*(c + d*x))/(a + b*x)))/(3*b*g^4*(a + b*x)^3) - (A*b*c^2)
/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*
x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d^2)/(1
8*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a
+ b*x)^3) + (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*c*d)/(
3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b
*x)^3) - (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{B}{16bg^5(a+bx)^4} - \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}$$

```
[Out] 1/16*B/b/g^5/(b*x+a)^4-1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/8*B*d^2/b/(-a*
d+b*c)^2/g^5/(b*x+a)^2-1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/4*B*d^4*ln(b*
x+a)/b/(-a*d+b*c)^4/g^5+1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln
(e*(d*x+c)/(b*x+a)))/b/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4} - \frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4} + \frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4} - \frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3} + \frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} - \frac{Bd}{12bg^5(a+bx)^3(bc-ad)} + \frac{B}{16bg^5(a+bx)^4}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(16*b*g^5*(a + b*x)^4) - (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^
2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a
+ b*x)) - (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x
])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(4*b*g^5*
(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^4}\,dx}{4bg}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}\\
&\quad -\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\right.}{4bg^5}
\end{aligned}$$

$$= \frac{B}{16bg^5(a+bx)^4} - \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{\frac{B(-bc+ad)\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^5}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}}{4bg^5}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(-(b*c) + a*d)*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a
 + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b
*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^5) - (A +
 B*Log[(e*(c + d*x))/(a + b*x)])/(a + b*x)^4)/(4*b*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(195) = 390.

Time = 2.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.12

method	result
parts	$-\frac{A}{4g^5(bx+a)^4 b} - \frac{B b^3 \left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4 \ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4} - \frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{16} - \frac{3de\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3 \ln\left(\frac{de}{b}-\frac{e(ad}{b(bx}\right.}{3}\right.}{b}\right.}{}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b g^5 (bx+a)^4} - \frac{48Bab^3cd^3x^2+72Ba^2b^2cd^3x-24Bab^3c^2d^2x-48Aa^3bcd^3+72Aa^2b^2c^2d^2-48Aab^3c^3d-12Bab}{}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{4(ad-cb)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{(ad-cb)^5e^4g^5}-\frac{3b^3Ad^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^5e^3g^5}+\frac{b^2Ad^3\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^5e^2g^5}-\frac{b^5B\left(\right.}{}\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{4(ad-cb)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{(ad-cb)^5e^4g^5}-\frac{3b^3Ad^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^5e^3g^5}+\frac{b^2Ad^3\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^5e^2g^5}-\frac{b^5B\left(\right.}{}\right.}{}$
parallelrisch	$\frac{48Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)a^9cd^4-72B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^8bc^3d^2+48B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^7b^2c^4d+12Ax^4a^2b^7c^5-3Bx^4a^2b^7c^5+48Ax^3c}{}$
norman	$\frac{Ba^3d^4x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)g}+\frac{ad^4Bb^2x^3\ln\left(\frac{e(dx+c)}{bx+a}\right)}{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)g}+\frac{(4Aa^3d^3-12Aa^2bcd^2+12Aa}{4gc}$

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
[Out] -1/4*A/g^5/(b*x+a)^4/b-B/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*e/b-e*(a*d-b*c)/b/(b*x+a
))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b
*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3*d^2*e^2/b^2*(1/2*(d*e/b-e*(a*
d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/
b/(b*x+a))^2)-d^3*e^3/b^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*
c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(192) = 384$.

Time = 0.27 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.09

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(4\,A-B)b^4c^4-16\,(3\,A-B)ab^3c^3d+36\,(2\,A-B)a^2b^2c^2d^2-48\,(A-B)a^3bcd^3+(12\,A-25\,B)a^4d}{48\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7c^3}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b^
2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((d*e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*
b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*
(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*
b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4
*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.

Time = 2.67 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4a^4bg^5 + 16a^3b^2g^5x + 24a^2b^3g^5x^2 + 16ab^4g^5x^3 + 4b^5g^5x^4} + \frac{Bd^4\log\left(x + \frac{-\frac{Ba^5d^9}{(ad-bc)^4} + \frac{5Ba^4bcd^8}{(ad-bc)^4} - \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} + \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} - \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 + \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} - \frac{Bd^4\log\left(x + \frac{\frac{Ba^5d^9}{(ad-bc)^4} - \frac{5Ba^4bcd^8}{(ad-bc)^4} + \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} - \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} + \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 - \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} + \frac{-12Aa^3d^3 + 36Aa^2bcd^2 - 36Aab^2c^2}{48a^7bd^3g^5 - 144a^6b^2cd^2g^5 + 144a^5b^3c^2dg^5 - 48a^4b^4c^3g^5 + x^4\cdot(48a^3b^5d^3g^5 - 144a^2b^6cd^2g^5 + 144ab^7c^2dg}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x +
 (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) - B*d**4*log(x
 + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
```

```
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 + 25*B*a**
3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B*b**
3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*b*d*
*3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a**6
*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 + x*
*4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*d*
g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5*c
*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(288*
a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g**5
 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*c*
d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(192) = 384$.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}\,B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-2\cdots}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6\cdots}\cdots\right) - \frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] -1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. 2(192) = 384.

Time = 0.83 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.65

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}\left(12\left(\frac{(dex+ce)^4Bb^3}{(b^3c^3e^3g^5 - 3\,ab^2c^2de^3g^5 + 3\,a^2bcd^2e^3g^5 - a^3d^3e^3g^5)(bx+a)^4} - \frac{4\,(dex+}{(b^3c^3e^2g^5 - 3\,ab^2c^2de^2g^5 + 3\,a}\right.\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/48*(12*((d*e*x + c*e)^4*B*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5
+ 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c*e)^3
*B*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5
- a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B*b*d^2/((b^3*c^3*e*g^5
 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2)
- 4*(d*e*x + c*e)*B*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g
^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 3*(4*A*b^3 - B
*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3*a^2*b*c
*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 16*(3*A*b^2*d - B*b^2*d)*(d*
e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*
g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*(2*A*b*d^2 - B*b*d^2)*(d*e*x + c*e
)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e
*g^5)*(b*x + a)^2) - 48*(A*d^3 - B*d^3)*(d*e*x + c*e)/((b^3*c^3*g^5 - 3*a*b
^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*(b*c/((b*c*e -
a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.81

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{\left(a\,d-b\,c\right)}^4} - \frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4} - \frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)} + \frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3 +}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b
^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4) - (B*log
((e*(c + d*x))/(a + b*x)))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*
x^2 + 4*a*b^2*x^3)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2)
```

3.182 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 503

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3} + \frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} \\
&\quad - \frac{5B^2(bc-ad)^5g^4\log(a+bx)}{6bd^5} - \frac{13B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5} \\
&\quad + \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3} \\
&\quad - \frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} \\
&\quad + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} \\
&\quad - \frac{2B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5d^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} \\
&\quad - \frac{2B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&\quad + \frac{2B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/60*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
1/30*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-5/6*B^2*(-a*d+b*c)^5*g^4*ln(b*x+a
)/b/d^5-13/30*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+1/5*B*(-a*d+b*
```

```
c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^3-2/15*B*(-a*d+b*c)^2*g^
4*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)
^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/5*B*(-a*d+b*c)^4*g^4*(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/
5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/
b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= -\frac{2Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^5} - \frac{2Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5d^5} + \frac{Bg^4(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^3} - \frac{2Bg^4(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{15bd^2} + \frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{10bd} + \frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b} + \frac{2B^2g^4(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} - \frac{5B^2g^4(bc-ad)^5\log(a+bx)}{6bd^5} - \frac{13B^2g^4(bc-ad)^5\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5} + \frac{13B^2g^4x(bc-ad)^4}{30d^4} - \frac{7B^2g^4(a+bx)^2(bc-ad)^3}{60bd^3} + \frac{B^2g^4(a+bx)^3(bc-ad)^2}{30bd^2}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^
2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(30*b*d^2) - (5*B^2*(b*
c - a*d)^5*g^4*Log[a + b*x])/(6*b*d^5) - (13*B^2*(b*c - a*d)^5*g^4*Log[(c +
 d*x)/(a + b*x)])/(30*b*d^5) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[
(e*(c + d*x))/(a + b*x)]))/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(
A + B*Log[(e*(c + d*x))/(a + b*x)]))/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b
*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(10*b*d) - (2*B*(b*c - a*d)^4*g
```

```
^4*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*d^5) + (g^4*(a + b*x)
^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(5*b) - (2*B*(b*c - a*d)^5*g^4*(
A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
)/(5*b*d^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\left((bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(d-bx)^6}\,dx, x, \frac{c+dx}{a+bx}\right)\right)\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} + \frac{\left(2B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)}{5b}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b}\\
&\quad + \frac{\left(2B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d}\\
&\quad + \frac{\left(2B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd}\\
&= \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd}\\
&\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b}\\
&\quad + \frac{\left(2B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d^2}\\
&\quad + \frac{\left(2B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd^2}\\
&\quad - \frac{\left(B^2(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{1}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{10bd}
\end{aligned}
$$

$$= -\frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} + \frac{(2B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{5d^3} + \frac{(2B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^3} - \frac{(2B^2(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{1}{x(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{15bd^2} - \frac{(B^2(bc-ad)^5g^4)\operatorname{Subst}\left(\int \left(\frac{1}{d^4x}+\frac{b}{d(d-bx)^4}+\frac{b}{d^2(d-bx)^3}+\frac{b}{d^3(d-bx)^2}+\frac{b}{d^4(d-bx)}\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{10bd}$$

$$= \frac{B^2(bc-ad)^4g^4x}{10d^4} - \frac{B^2(bc-ad)^3g^4(a+bx)^2}{20bd^3} + \frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} - \frac{B^2(bc-ad)^5g^4\log(a+bx)}{10bd^5} - \frac{B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{10bd^5} + \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3} - \frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} + \frac{(2B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5d^4} + \frac{(2B(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^4} - \frac{(B^2(bc-ad)^5g^4)\operatorname{Subst}\left(\int \frac{1}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^3} - \frac{(2B^2(bc-ad)^5g^4)\operatorname{Subst}\left(\int \left(\frac{1}{d^3x}+\frac{b}{d(d-bx)^3}+\frac{b}{d^2(d-bx)^2}+\frac{b}{d^3(d-bx)}\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{15bd^2}$$

$$
\begin{aligned}
&= \frac{7B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3} + \frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} \\
&- \frac{7B^2(bc-ad)^5g^4\log(a+bx)}{30bd^5} - \frac{7B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5} \\
&+ \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3} \\
&- \frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} \\
&+ \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} \\
&- \frac{2B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5d^5} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} \\
&- \frac{2B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&- \frac{(2B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int \frac{1}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d^5} \\
&+ \frac{(2B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd^5} \\
&- \frac{(B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int \left(\frac{1}{d^2x}+\frac{b}{d(d-bx)^2}+\frac{b}{d^2(d-bx)}\right)dx, x, \frac{c+dx}{a+bx}\right)}{5bd^3}
\end{aligned}
$$

$$= \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3} + \frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} - \frac{5B^2(bc-ad)^5g^4\log(a+bx)}{6bd^5} - \frac{13B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5} + \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3} - \frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} - \frac{2B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5d^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{2B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} + \frac{2B^2(bc-ad)^5g^4\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.02

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$

$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 - \frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24B(bc-ad)^4\log(a+bx)-4B(bc-ad)^2\left(2bd(bc-ad)x-d^2(a+\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[a + b*x] - 4*B*(b*c - a*d
)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
- B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 +
 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 24*b*B*(b*c - a*d)^3*(c + d*x)*Log[(e*
(c + d*x))/(a + b*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c +
 d*x))/(a + b*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(c + d*x))
/(a + b*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 24*
(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 12*B*(b*c
```

```
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Maple [F]

$$\int (bgx + ag)^4 \left(A + B\ln\left(\frac{e(dx + c)}{bx + a}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx + ag)^4 \left(B\log\left(\frac{(dx + c)e}{bx + a}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*e*x + c*e)/(b
*x + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x
^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*e*x + c*e)/(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(c + dx)}{a + bx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2395 vs. 2(478) = 956.

Time = 0.31 (sec) , antiderivative size = 2395, normalized size of antiderivative = 4.76

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b +
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x +
 a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(d*e*x/(b*x + a) + c
*e/(b*x + a)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*
x + a)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x + 1/30*((12*g^4*log(e) - 25*g^4)*b^4*c^5 - (60*g^4*log(e) - 113*g^4)*a*b
^3*c^4*d + 4*(30*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(10*g^4*log(e) -
 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) - 4*g^4)*a^4*c*d^4)*B^2*log(d*x +
 c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 1
0*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 6*(b^5*c*d^4*g^4*log(e) + (1
0*g^4*log(e)^2 - g^4*log(e))*a*b^4*d^5)*B^2*x^4 - 2*((4*g^4*log(e) - g^4)*b
^5*c^2*d^3 - 2*(10*g^4*log(e) - g^4)*a*b^4*c*d^4 - (60*g^4*log(e)^2 - 16*g^
4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 + ((12*g^4*log(e) - 7*g^4)*b^5*c^3*d^2
 - 3*(20*g^4*log(e) - 9*g^4)*a*b^4*c^2*d^3 + 3*(40*g^4*log(e) - 11*g^4)*a^2
*b^3*c*d^4 + (120*g^4*log(e)^2 - 72*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^2*x
^2 - 2*((12*g^4*log(e) - 13*g^4)*b^5*c^4*d - (60*g^4*log(e) - 59*g^4)*a*b^4
*c^3*d^2 + 6*(20*g^4*log(e) - 17*g^4)*a^2*b^3*c^2*d^3 - (120*g^4*log(e) - 7
9*g^4)*a^3*b^2*c*d^4 - (30*g^4*log(e)^2 - 48*g^4*log(e) + 23*g^4)*a^4*b*d^5
)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^
3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^
5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*
a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 - 2*(12*B^2*b^
```

```
5*d^5*g^4*x^5*log(e) + 3*(b^5*c*d^4*g^4 + (20*g^4*log(e) - g^4)*a*b^4*d^5)*
B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 2*(15*g^4*log(e) - 2*g^4
)*a^2*b^3*d^5)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*
b^3*c*d^4*g^4 + 2*(10*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 12*(b^5*c^
4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g
^4 - (5*g^4*log(e) - 4*g^4)*a^4*b*d^5)*B^2*x - (12*a*b^4*c^4*d*g^4 - 54*a^2
*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 - (12*g^4*lo
g(e) - 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) + 2*(12*B^2*b^5*d^5*g^4*x^5*log(e
) + 3*(b^5*c*d^4*g^4 + (20*g^4*log(e) - g^4)*a*b^4*d^5)*B^2*x^4 - 4*(b^5*c^
2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 2*(15*g^4*log(e) - 2*g^4)*a^2*b^3*d^5)*B^2*
x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 2*(
10*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 12*(b^5*c^4*d*g^4 - 5*a*b^4*c
^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 - (5*g^4*log(e)
- 4*g^4)*a^4*b*d^5)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.183 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 420

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx\\
&= -\frac{5B^2(bc-ad)^3g^3x}{12d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{12bd^4}\\
&\quad+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4}-\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2}\\
&\quad+\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd}\\
&\quad+\frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2d^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b}\\
&\quad+\frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}\\
&\quad-\frac{B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

```
[Out] -5/12*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+
11/12*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/12*B^2*(-a*d+b*c)^4*g^3*ln((d*
x+c)/(b*x+a))/b/d^4-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b/d^2+1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d+
1/2*B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^4+1/4*g^3*(b*x
+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+1/2*B*(-a*d+b*c)^4*g^3*(A+B*ln(e*(d*x
+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bd^4} + \frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2d^4} - \frac{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4bd^2} + \frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{6bd} + \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b} - \frac{B^2g^3(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} + \frac{11B^2g^3(bc-ad)^4\log(a+bx)}{12bd^4} + \frac{5B^2g^3(bc-ad)^4\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4} - \frac{5B^2g^3x(bc-ad)^3}{12d^3} + \frac{B^2g^3(a+bx)^2(bc-ad)^2}{12bd^2}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (-5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)
/(12*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*Log[a + b*x])/(12*b*d^4) + (5*B^2*(
b*c - a*d)^4*g^3*Log[(c + d*x)/(a + b*x)])/(12*b*d^4) - (B*(b*c - a*d)^2*g^
3*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*b*d^2) + (B*(b*c - a
*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(6*b*d) + (B*(b*c
 - a*d)^3*g^3*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*d^4) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(4*b) + (B*(b*c - a*d
)^4*g^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/(2*b*d^4) - (B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 g^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{2d} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{2bd}$$

$$= \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{2d^2} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{2bd^2} + \frac{\left(B^2(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{1}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{6bd}$$

$$= -\frac{B(bc-ad)^2 g^3 (a+bx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2} + \frac{B(bc-ad) g^3 (a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd} + \frac{g^3 (a+bx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{(B(bc-ad)^4 g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{2d^3} - \frac{(B(bc-ad)^4 g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)}\,dx, x, \frac{c+dx}{a+bx}\right)}{2bd^3} + \frac{(B^2(bc-ad)^4 g^3)\,\text{Subst}\left(\int \frac{1}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{4bd^2} + \frac{(B^2(bc-ad)^4 g^3)\,\text{Subst}\left(\int \left(\frac{1}{d^3x} + \frac{b}{d(d-bx)^3} + \frac{b}{d^2(d-bx)^2} + \frac{b}{d^3(d-bx)}\right) dx, x, \frac{c+dx}{a+bx}\right)}{6bd}$$

$$= -\frac{B^2(bc-ad)^3 g^3 x}{6d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} + \frac{B^2(bc-ad)^4 g^3 \log(a+bx)}{6bd^4} + \frac{B^2(bc-ad)^4 g^3 \log\left(\frac{c+dx}{a+bx}\right)}{6bd^4} - \frac{B(bc-ad)^2 g^3 (a+bx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2} + \frac{B(bc-ad) g^3 (a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd} + \frac{B(bc-ad)^3 g^3 (c+dx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2d^4} + \frac{g^3 (a+bx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} + \frac{B(bc-ad)^4 g^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} + \frac{(B^2(bc-ad)^4 g^3)\,\text{Subst}\left(\int \frac{1}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{2d^4} - \frac{(B^2(bc-ad)^4 g^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{2bd^4} + \frac{(B^2(bc-ad)^4 g^3)\,\text{Subst}\left(\int \left(\frac{1}{d^2x} + \frac{b}{d(d-bx)^2} + \frac{b}{d^2(d-bx)}\right) dx, x, \frac{c+dx}{a+bx}\right)}{4bd^2}$$

$$= -\frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^4g^3\log(a+bx)}{12bd^4}$$
$$+ \frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2}$$
$$+ \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd}$$
$$+ \frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2d^4}$$
$$+ \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b}$$
$$+ \frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{B^2(bc-ad)^4g^3\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.93

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6B(bc-ad)^3\log(a+bx)-B(bc-ad)(2bd(bc-ad)x-d^2(a+bx)\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3*B*
(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)^2*(
c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
 + B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a
 + b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^
3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((d*e*x + c*e)/(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(399) = 798$.

Time = 0.31 (sec) , antiderivative size = 1735, normalized size of antiderivative = 4.13

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b
^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
+ c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x - 1/12*((6*g^3*log(e) - 11*g^3)*b^3*c^4 - 2*(12*g^3*log
(e) - 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) - 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g
^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(b^4*c*d^3
*g^3*log(e) + (6*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - ((3*g^3*lo
g(e) - g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) - g^3)*a*b^3*c*d^3 - (18*g^3*log(
e)^2 - 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 + ((6*g^3*log(e) - 5*g^3)*b
^4*c^3*d - (24*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) - 19*g^3
)*a^2*b^2*c*d^3 + (12*g^3*log(e)^2 - 18*g^3*log(e) + 7*g^3)*a^3*b*d^4)*B^2*
x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^
4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2
*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3
 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - (6*B^2*b^4*d^4*g^3*x^4*log(e) +
 2*(b^4*c*d^3*g^3 + (12*g^3*log(e) - g^3)*a*b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d
^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(4*g^3*log(e) - g^3)*a^2*b^2*d^4)*B^2*x^2 +
6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + (4*g^3*log(e
) - 3*g^3)*a^3*b*d^4)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 +
 26*a^3*b*c*d^3*g^3 + (6*g^3*log(e) - 11*g^3)*a^4*d^4)*B^2)*log(b*x + a) +
(6*B^2*b^4*d^4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (12*g^3*log(e) - g^3)*a*
b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(4*g^3*log(e)
 - g^3)*a^2*b^2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a
^2*b^2*c*d^3*g^3 + (4*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*x - 6*(B^2*b^4*d^4
*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*
b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)

[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)

3.184 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 335

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\
&+ \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} \\
&- \frac{2B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3d^3} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} \\
&- \frac{2B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\
&+ \frac{2B^2(bc-ad)^3g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-1/3*B^2
*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*
(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*ln(e*(d
*x+c)/(b*x+a)))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/3*B
*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d
^3+2/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= -\frac{2Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd^3} - \frac{2Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3d^3} + \frac{Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd} + \frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b} + \frac{2B^2g^2(bc-ad)^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} - \frac{B^2g^2(bc-ad)^3\log(a+bx)}{bd^3} - \frac{B^2g^2(bc-ad)^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} + \frac{B^2g^2x(bc-ad)^2}{3d^2}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B^2*(b*c - a*d)^3*g^2*Log[a + b*x])/(b
*d^3) - (B^2*(b*c - a*d)^3*g^2*Log[(c + d*x)/(a + b*x)])/(3*b*d^3) + (B*(b*
c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*d) - (2
*B*(b*c - a*d)^2*g^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*d^3
) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(3*b) - (2*B*(
b*c - a*d)^3*g^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))
/(b*(c + d*x))])/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(\left((bc-ad)^3 g^2\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)\right)$$

$$= \frac{g^2(a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{3b}$$

$$= \frac{g^2(a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{3d} + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd}$$

$$= \frac{B(bc-ad)g^2(a+bx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{3d^2} + \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)}\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd^2} - \frac{(B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{1}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd}$$

$$= \frac{B(bc-ad)g^2(a+bx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} - \frac{2B(bc-ad)^2g^2(c+dx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3d^3} + \frac{g^2(a+bx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{2B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} - \frac{(2B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{1}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{3d^3} + \frac{(2B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd^3} - \frac{(B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \left(\frac{1}{d^2x}+\frac{b}{d(d-bx)^2}+\frac{b}{d^2(d-bx)}\right)\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} + \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} - \frac{2B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3d^3} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{2B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{2B^2(bc-ad)^3g^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.87

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$

$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abd(bc-ad)x+2B(bc-ad)^2\log(a+bx)-B(bc-ad)(bdx+(-bc+ad)\log(c+dx\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b*d
*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(c +
 d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b
^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((d*e*x + c*e)/(b*x + a)), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. $2(320) = 640$.

Time = 0.31 (sec) , antiderivative size = 1172, normalized size of antiderivative = 3.50

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x
 + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*e*
x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*e*x/(b*x + a) + c*e/(
b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x + 1/3*((2*g^2*log(e) - 3*g^2)*b^2*c^3 - (6*g^2*log(e) - 7*g^2)*a*b*c^2*d
 + 2*(3*g^2*log(e) - 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (b^3*c*d^2*g^2*log(e) + (3*g^2
*log(e)^2 - g^2*log(e))*a*b^2*d^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) - g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - (2*B^2*b^3*d^3*g^2*x^
3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x -
 (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (2*g^2*log(e) - 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) + (2*B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*
log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (
3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x - 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\int (bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.185 $\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 30, antiderivative size = 202

$$\begin{aligned}
&\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2 g\log(a+bx)}{bd^2} + \frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2} \\
&\quad + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} \\
&\quad + \frac{B(bc-ad)^2 g\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad - \frac{B^2(bc-ad)^2 g\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}
\end{aligned}$$

```
[Out] B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c)
/(b*x+a)))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+B*(-a*d+b*c)
^2*g*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-B^2*(-a*d+
b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31}

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\frac{Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bd^2}$$
$$+\frac{Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b}$$
$$-\frac{B^2g(bc-ad)^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{B^2g(bc-ad)^2\log(a+bx)}{bd^2}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g*Log[a + b*x])/(b*d^2) + (B*(b*c - a*d)*g*(c + d*x)*(A
+ B*Log[(e*(c + d*x))/(a + b*x)]))/d^2 + (g*(a + b*x)^2*(A + B*Log[(e*(c +
d*x))/(a + b*x)])^2)/(2*b) + (B*(b*c - a*d)^2*g*(A + B*Log[(e*(c + d*x))/(a
 + b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (B^2*(b*c - a*d)^
2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^2 g\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{\left(B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b}\\
&\quad - \frac{\left(B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{d}\\
&\quad - \frac{\left(B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(d-bx)}\,dx, x, \frac{c+dx}{a+bx}\right)}{bd}
\end{aligned}$$

$$= \frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} + \frac{B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{(B^2(bc-ad)^2g)\,\mathrm{Subst}\left(\int\frac{1}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{d^2} - \frac{(B^2(bc-ad)^2g)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bd^2}$$

$$= \frac{B^2(bc-ad)^2g\log(a+bx)}{bd^2} + \frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} + \frac{B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{B^2(bc-ad)^2g\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$= \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B(bc-ad)\left(2Abdx+2B(bc-ad)\log(a+bx)+2bB(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-2(bc-ad)\log(c+dx)\left(\right.\right.}{}\right.}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*(b*c - a*d)*Log[a + b*x] + 2*b*B*(c + d*x)*Log[(e*(c + d*x))/
(a + b*x)] - 2*(b*c - a*d)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]
) - B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2dx$$

[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)

[Out] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")

[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a)), x)

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2dx=\text{Timed out}$$

[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(199)=398$.

Time = 0.31 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.06

$$\begin{aligned}&\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2dx\\&=\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)ABag\\&\quad+\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)ABbg\\&\quad+A^2agx-\frac{((g\log(e)-g)bc^2-(2\,g\log(e)-g)acd)B^2\log(dx+c)}{d^2}\\&\quad+\frac{(b^2c^2g-2\,abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}\\&\quad+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(b^2cdg\log(e)+\left(g\log(e)^2-g\log(e)\right)abd^2\right)B^2x+\left(B^2b^2d^2gx^2+2\,B^2abd^2gx+\ldots\right.}{}\end{aligned}$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")

[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*b*g + A^2*a*g*x - ((g*log(e) - g)*b*c^2 - (2*g*log(e) - g)*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c*d*g*log(e) + (g*log(e)^2 - g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) - g)*a^2*d^2 + a*b*c*d*g)*B^2)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2\,dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)

[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)

3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} - \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g-2*B*(A+B*ln(e*(d
*x+c)/(b*x+a)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x+c
)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2552, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = -\frac{2B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{bg} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2
)/(b*g)) - (2*B*(A + B*Log[(e*(c + d*x))/(a + b*x)])*PolyLog[2, (b*(c + d*x
))/(d*(a + b*x))])/(b*g) + (2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/
(b*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} + \frac{(2B)\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} - \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{(2B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} - \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.97

$$\int\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right) + A^2\log(a+bx) + 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right) - 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] + 2*A*B*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Log[(-(b*
c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)] - B^2*Log[(-(b*c) + a
*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)]^2 - 2*A*B*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)] - 2*B^2*Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/
(b*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(127) = 254.

Time = 1.42 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.70

method	result
parts	$\frac{A^2\ln(bx+a)}{gb}-\frac{B^2\left(\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\mathrm{Li}_2\left(\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)-2\,\mathrm{L}}{gb}$
risch	$\frac{A^2\ln(bx+a)}{gb}-\frac{B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)}{bg}-\frac{2B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\mathrm{Li}_2\left(\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)}{bg}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{bB^2\left(\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\mathrm{Li}_2\left(\right.\right.}{ge(ad-cb)}\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{bB^2\left(\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\mathrm{Li}_2\left(\right.\right.}{ge(ad-cb)}\right.}{}$

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] A^2/g*ln(b*x+a)/b-B^2/g/b*(ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(1-b/d/e*(d*
e/b-e*(a*d-b*c)/b/(b*x+a)))+2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*polylog(2,b/d
/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))-2*polylog(3,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b
*x+a))))+2*B*A/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/b-ln(d
*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/
b)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x
 + a)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c*e/(a + b*x) + d*e*x/(a +
 b*x))**2/(a + b*x), x) + Integral(2*A*B*log(c*e/(a + b*x) + d*e*x/(a + b*x
))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x +
 a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x - (2*B^2*b*d*x + (b*c + a*d)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag + bgx}\,dx = \int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}{bgx + ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((d*x + c)*e/(b*x + a)) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag + bgx}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}{a\,g + b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x), x)
```

3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx=\frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] 2*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2
*B^2*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2552, 2333, 2332}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx=-\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}+\frac{2AB(c+dx)}{g^2(a+bx)(bc-ad)}+\frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (2*A*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B^2*(c + d*x))/((b*c - a
*d)*g^2*(a + b*x)) + (2*B^2*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c -
 a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(
(b*c - a*d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2} \\
&= -\frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(2B)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2} \\
&= \frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(2B^2)\,\text{Subst}\left(\int \log(ex)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2} \\
&= \frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} \\
&\quad + \frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-2(bc-ad)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2d(a+bx)\log}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(2*B*(b*c - a*d + d*(a + b*x
)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 2*(b*c - a*d)*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]) - 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/
(a + b*x)]) + 2*d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

method	result
norman	$\frac{\frac{\left(A^2-2BA+2B^2\right)x}{ga}+\frac{B^2 c\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{B^2 dx\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{2(-B+A)cB\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{2d(-B+A)Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\left(\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 e(dx+c)}{bx+a}-\frac{2e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}+\frac{2e(dx+c)}{bx+a}\right)}{g^2e(ad-cb)}+\frac{2BA\left(\frac{e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}-\frac{e(dx+}{bx+c}\right.}{g^2e(ad-cb)}$
parallelrisch	$-\frac{-2ABx\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3d^2-2AB\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3cd+A^2ab^2d^2-A^2b^3cd+2B^2ab^2d^2-2B^2b^3cd+2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3cd-2}{g^2(bx+a)b^3d(ad-cb)}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\right.}{}\right.}{b^2}$
default	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\right.}{}\right.}{b^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 dx}{g^2(ad-cb)(bx+a)}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 c}{g^2(ad-cb)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)dx}{g^2(ad-cb)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)c}{g^2(ad-cb)(bx+a)}+\frac{2B}{g^2(ad-}$

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A^2-2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2+B^2*d/g/
(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a))^2+2*(-B+A)*c*B/g/(a*d-b*c)*ln(e*(d*x+c)/(
b*x+a))+2*d*(-B+A)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left((AB-B^2)bdx+(\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((A*B - B^2)*b*d*x + (A*B - B^2)*b*
c)*log((d*e*x + c*e)/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*
b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(128) = 256$.

Time = 1.21 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.81

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{2Bd(A-B)\log\left(x + \frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd-\frac{2Ba^2d^3(A-B)}{ad-bc}+\frac{4Babcd^2(A-B)}{ad-bc}-\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$- \frac{2Bd(A-B)\log\left(x + \frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd+\frac{2Ba^2d^3(A-B)}{ad-bc}-\frac{4Babcd^2(A-B)}{ad-bc}+\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+ \frac{(-2AB+2B^2)\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2+2AB-2B^2}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d - 2*B**2*a*d**2 - 2*B**2*
b*c*d - 2*B*a**2*d**3*(A - B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A - B)/(a*d - b
*c) - 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*d**2 - 4*B**2*b*d**2))/
(b*g**2*(a*d - b*c)) - 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d -
2*B**2*a*d**2 - 2*B**2*b*c*d + 2*B*a**2*d**3*(A - B)/(a*d - b*c) - 4*B*a*b*
c*d**2*(A - B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*
d**2 - 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(e*(c +
d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(c + d
*x)/(a + b*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x)
 + (-A**2 + 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(153) = 306$.

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.72

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \left(2\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right) + \frac{(bdx+ad)\log(bx+a)^2 +}{}\right.$$
$$- 2\,AB\left(\frac{\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^2g^2x+abg^2} - \frac{1}{b^2g^2x+abg^2} - \frac{d\log(bx+a)}{(b^2c-abd)g^2} + \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$- \frac{B^2\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)^2}{b^2g^2x+abg^2} - \frac{A^2}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] (2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + ((b*
d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B
^2 - 2*A*B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/
(b^2*g^2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^2
*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dex+ce)B^2\log\left(\frac{dex+ce}{bx+a}\right)^2}{(bx+a)g^2} + \frac{2\,(dex+ce)(AB-B^2)\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2} + \frac{(dex+ce)(A^2-2\,AB+2\,B^2)}{(bx+a)g^2}\right)($$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -((d*e*x + c*e)*B^2*log((d*e*x + c*e)/(b*x + a))^2/((b*x + a)*g^2) + 2*(d*e
*x + c*e)*(A*B - B^2)*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e*x
 + c*e)*(A^2 - 2*A*B + 2*B^2)/((b*x + a)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{A^2-2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2} + \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((2*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2)))/
(x/d + a/(b*d)) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(b^2*g^2*(x + a/b)) -
 (B^2*d)/(b*g^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(b^2*g^2*x + a*b*g^2)
 + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c))*
(A - B)*4i)/(b*g^2*(a*d - b*c))
```

3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = & -\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} \\ & -\frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}-\frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)^2g^3(a+bx)} \\ & +\frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} \\ & +\frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} \\ & -\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\end{aligned}$$

```
[Out] -2*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-2*B^2*d*(d*x+c)*ln(e
*(d*x+c)/(b*x+a))/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*B*(d*x+c)^2*(A+B*ln(e*(d*x
+c)/(b*x+a)))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a
)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^
2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2552, 2367, 2333, 2332, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2} - \frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{2ABd(c+dx)}{g^3(a+bx)(bc-ad)^2} - \frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^3(a+bx)(bc-ad)^2} - \frac{bB^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2} + \frac{2B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-2*A*B*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) + (2*B^2*d*(c + d*x))/((
b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/(4*(b*c - a*d)^2*g^3*(a +
 b*x)^2) - (2*B^2*d*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c - a*d)^2*
g^3*(a + b*x)) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*
(b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a +
 b*x)])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log[(e*(c
+ d*x))/(a + b*x)])^2)/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (d-bx)(A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= \frac{\text{Subst}\left(\int \left(d(A+B\log(ex))^2-bx(A+B\log(ex))^2\right)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= -\frac{b\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}+\frac{d\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)}-\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\\
&\quad+\frac{(bB)\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&\quad-\frac{(2Bd)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}
\end{aligned}$$

$$= -\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2} - \frac{(2B^2d)\,\text{Subst}\left(\int\log(ex)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)} + \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} - \frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(4*B*d*(a + b*x)*(b*c - a*d
 + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2
+ 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a
+ b*x)^2*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 4*
d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 4*d^2*(
a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 2*B*d^2*(a +
 b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2
*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*((2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*b*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.64

method	result
norman	$\frac{\frac{Bd(2Aad-2Bad-Bbc)x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc-4ABad+2ABbc+4B^2ad-B^2bc\right)x}{2ag(ad-cb)}+\frac{Bc(4Aad-2A}{2g}}{}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2b\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}+\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{4}}{g^3e^2(ad}$
parallelrisch	$-\frac{-4A^2a\,b^4c\,d^2-6AB\,a^2b^3d^3-2AB\,b^5c^2d-8B^2a\,b^4c\,d^2+8ABa\,b^4c\,d^2-2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d+6B^2x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5d^3}{}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (B/g*d*(2*A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/(
b*x+a))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/(b*x+a))^2+1
/2*(2*A^2*a*d-2*A^2*b*c-4*A*B*a*d+2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-b*c
)*x+1/2*B*c*(4*A*a*d-2*A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*l
n(e*(d*x+c)/(b*x+a))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln
(e*(d*x+c)/(b*x+a))^2+1/4*(2*A^2*a*d-2*A^2*b*c-6*A*B*a*d+2*A*B*b*c+7*B^2*a*
d-B^2*b*c)/a^2*b/g/(a*d-b*c)*x^2+1/2*b*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/
g*x^2*ln(e*(d*x+c)/(b*x+a))^2+1/2*b*B/g*d^2*(2*A-3*B)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)*x^2*ln(e*(d*x+c)/(b*x+a)))/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{(2A^2-2AB+B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2B\cdots}{\cdots}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((2*A*B - 3*B^2)
*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2 - (2
*A*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B - B^2
)*a*b*d^2)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*
b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(269) = 538$.

Time = 2.27 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{Bd^2\cdot(2A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2-\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}+\frac{Bb^3c^3\cdots}{(a\cdots}}{4ABbd^3-6B^2bd^3}\cdots\right)}{2bg^3(ad-bc)^2\cdots}$$
$$-\frac{Bd^2\cdot(2A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2+\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}-\frac{Bb^3\cdots}{\cdots}}{4ABbd^3-6B^2bd^3}\cdots\right)}{2bg^3(ad-bc)^2\cdots}$$
$$+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+\cdots}$$
$$+\frac{(-2ABad+2ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2}$$
$$+\frac{-2A^2ad+2A^2bc+6ABad-2ABbc-7B^2ad+B^2bc+x(4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 -
 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d*
*4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c
)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*
b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d*
*3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A -
3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a*
b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A - 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
 d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g
**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g
**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3
*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*lo
g(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b**
2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2)
 + (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B**2*a*d + B**2*b*
c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**
2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g*
*3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(290) = 580$.

Time = 0.24 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(2\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3c^2}\right.\right.$$
$$-\frac{1}{2}\,AB\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{}{(b^3c^2}\right.$$
$$-\frac{B^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
```

```
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2 - 8*
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a
*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2
*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*
d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*
c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B*((2*b*d*x
- b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
 (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^3*g
^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*g^3)) - 1/2*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b
*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(2\left(\frac{(dex+ce)^2B^2b}{(bceg^3-adeg^3)(bx+a)^2}-\frac{2\,(dex+ce)B^2d}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left(\frac{(2\,ABb-B^2b)(de\right.\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*((d*e*x + c*e)^2*B^2*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(d
*e*x + c*e)*B^2*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x +
 a))^2 + 2*((2*A*B*b - B^2*b)*(d*e*x + c*e)^2/((b*c*e*g^3 - a*d*e*g^3)*(b*x
 + a)^2) - 4*(A*B*d - B^2*d)*(d*e*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x + a)))
*log((d*e*x + c*e)/(b*x + a)) + (2*A^2*b - 2*A*B*b + B^2*b)*(d*e*x + c*e)^2
/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 4*(A^2*d - 2*A*B*d + 2*B^2*d)*(d*e
*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(3\,B^2\,b\,d-2\,A\,B\,b\,d)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$
$$-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(2\,A-3\,B)\,1\mathrm{i}}{(a\,d-b\,c)\,(3\,B^2\,d^2-2\,A\,B\,d^2)}\right)\,(2\,A-3\,B)\,1\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*
b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x))/(a +
b*x))^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*
c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(3*B^2*b*d - 2*A*B*b*d))/(a
*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*d^2*atan((B*d
^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A - 3*B
)*1i)/((a*d - b*c)*(3*B^2*d^2 - 2*A*B*d^2)))*(2*A - 3*B)*1i)/(b*g^3*(a*d -
b*c)^2)
```

3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 399

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx &= -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}\\ &\quad-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}\\ &\quad+\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)}\\ &\quad-\frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}\\ &\quad+\frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}\\ &\quad-\frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4}\\ &\quad-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3}\end{aligned}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+1/3*B^2
*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+2*B*d^2*(d*x+c)*(A+B*ln(e*(d*
x+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)-b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^2+2/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-2/3*B*d^3*ln((d*x+c)/(b*x+a))*(A+B*ln(e
```

```
*(d*x+c)/(b*x+a)))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g
^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2552, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bg^4(bc-ad)^3} + \frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^4(a+bx)(bc-ad)^3} - \frac{bBd(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3} - \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3bg^4(a+bx)^3} - \frac{2b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3} + \frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3bg^4(bc-ad)^3} - \frac{2B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3} + \frac{bB^2d(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*(c + d*x)^2
)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) + (B^2*d^3*Log[(c + d*x)/(a + b*x)]^2)/(3*b*(b*c - a*
d)^3*g^4) + (2*B*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/((b*c
- a*d)^3*g^4*(a + b*x)) - (b*B*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]))/((b*c - a*d)^3*g^4*(a + b*x)^2) + (2*b^2*B*(c + d*x)^3*(A + B*Log[(
e*(c + d*x))/(a + b*x)]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3) - (2*B*d^3*Log[
(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*(b*c - a*d)
^3*g^4) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(3*b*g^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (d-bx)^2(A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^3g^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} - \frac{(2B)\text{Subst}\left(\int \frac{(d-bx)^3(A+B\log(ex))}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}$$

$$= \frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} + \frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B^2)\operatorname{Subst}\left(\int\left(-\frac{1}{6}b(18d^2-9bdx+2b^2x^2)+\frac{d^3\log(x)}{x}\right)dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}$$

$$= \frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} + \frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} - \frac{B^2\operatorname{Subst}\left(\int(18d^2-9bdx+2b^2x^2)\,dx, x, \frac{c+dx}{a+bx}\right)}{9(bc-ad)^3g^4} + \frac{(2B^2d^3)\operatorname{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}$$

$$= -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} + \frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4} + \frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} + \frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.46

$$\int\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}dx$$

$$= \frac{-18\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B\left(12A(bc-ad)^3-4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+bx)^2-\right.}{}}{}$$

[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]

[Out] (-18*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(12*A*(b*c - a*d)^3 - 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a + b*x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 36*A*d^3*(a + b*x)^3*Log[c + d*x] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 - 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 12*B*(b*c - a*d)^3*Log[(e*(c + d*x))/(a + b*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(c + d*x))/(a + b*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(c + d*x))/(a + b*x)] - 36*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c + d*x))/(a + b*x)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(54*b*g^4*(a + b*x)^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(387) = 774.

Time = 1.41 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.10

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3b}+\frac{B^2b^2\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{3}-\frac{2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{9}+\frac{2\left(\frac{de}{b}-\frac{e(ad-c\cdots}{b(bx+a\cdots}\right.}{27}\cdots\right.}{\cdots}$
norman	$\frac{\frac{B^2a^2d^3x\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{B^2ab\,d^3x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{18A^2a^2b^2d^2-36A^2a\,b^3cd+18A^2b^4c^2-66AB\,a^2b^2d^2+\cdots}{54g\cdots}}{\cdots}$
parallelrisch	$-\frac{-66AB\,a^3b^4d^4+12AB\,b^7c^3d-108B^2a^2b^5c\,d^3+27B^2a\,b^6c^2d^2-54B^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)^2a^2b^5d^4+108B^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)a^2\cdots}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)

[Out] -1/3*A^2/g^4/(b*x+a)^3/b+B^2/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*

```
x+a))^2-1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+1/b^2*d^2*e^2*((d*e/b-e*(a*d-b*c)/b/(
b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*l
n(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))+2*B*A/g^4*
b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*
c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*
d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/
b/(b*x+a))^2)+d^2*e^2/b^2*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*
c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.70

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2\,(9\,A^2-6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2-2\,AB+B^2)ab^2c^2d+54\,(A^2-2\,AB+2\,B^2)a^2bcd^2-(18\,A^2-}{}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*a^
3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
- 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d*e*x + c*e)/(b*x + a))^2 + 3
*((6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*B - 49
*B^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^2)*b^3*
c^3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*(2*B^2*b
^3*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^
2*c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
- 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3
 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 -
3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(362) = 724$.

Time = 12.82 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**4
 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*b*
c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B)/(
a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b**4*c
**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4))/(9*
b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*
b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11*B)/(a*
d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**2
*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/
(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d*
*4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B
**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3
 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)/(a + b*x))**2/(3*a**6*d**3*g**4 - 9
*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a
**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4
 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3
*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 -
 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g
**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*
A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d + 15*B**2*a*b*d**2*x + 2
*B**2*b**2*c**2 - 3*B**2*b**2*c*d*x + 6*B**2*b**2*d**2*x**2)*log(e*(c + d*x
)/(a + b*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**
2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d
**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a
**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**
3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**2
*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*b*c*d - 12*A*B*b**2*c**2 + 85*B**2
*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(-36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(-90*A*B*a*b*d**2 + 18*A*B*b**2*c*d + 147*B**2*a*b*
d**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 5
4*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4
+ 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g*
*4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d
*g**4 + 162*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(387) = 774.

Time = 0.29 (sec) , antiderivative size = 1420, normalized size of antiderivative = 3.56

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] 1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - (4*b^3*c^3 - 27*a*b^2*c^2
*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^
3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b
^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33
*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^
4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^
4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b
^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x
^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b
^2*d^3*g^4)*x))*B^2 + 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*
a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)
*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4
*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
a^5*b*d^2)*g^4) - 6*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a
*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
- 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log(
d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*
g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 0.71 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{54}\left(18\left(\frac{(dex+ce)^3B^2b^2}{(b^2c^2e^2g^4-2\,abcde^2g^4+a^2d^2e^2g^4)(bx+a)^3}-\frac{3\,(dex+ce)^2B^2bd}{(b^2c^2eg^4-2\,abcdeg^4+a^2d^2eg^4)(bx+a)^2}+\right.\right.$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(18*((d*e*x + c*e)^3*B^2*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 +
a^2*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B^2*b*d/((b^2*c^2*e*g^4 -
 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B^2*d^2/((
b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b
*x + a))^2 + 6*(2*(3*A*B*b^2 - B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 -
 2*a*b*c*d*e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*B*b*d - B^2*b*d
)*(d*e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x +
 a)^2) + 18*(A*B*d^2 - B^2*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4
 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 2*(9*A^2*b^2 - 6
*A*B*b^2 + 2*B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4
 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 27*(2*A^2*b*d - 2*A*B*b*d + B^2*b*d)*(d*
e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2
) + 54*(A^2*d^2 - 2*A*B*d^2 + 2*B^2*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*
b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.67

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$=\frac{\frac{18A^2a^2d^2-36A^2abcd+18A^2b^2c^2-66ABa^2d^2+42ABabcd-12ABb^2c^2+85B^2a^2d^2-23B^2abcd+4B^2b^2c^2}{6(ad-bc)}+\frac{x(-5cB^2b^2d+49aB^2\cdots}{\cdots}}{x\,(27a^2b^3cg^4-27a^3b^2dg^4)-x^2\,(27a^2b^3dg^4-27ab^4cg^4)+x^3\,(9b^5cg^4-9ab^4d\cdots}$$
$$-\ln\left(\frac{e(c+dx)}{a+bx}\right)^2\left(\frac{B^2}{3b^2g^4\,\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)}-\frac{B^2d^3}{3bg^4\,(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}\right)$$
$$-\frac{\ln\left(\frac{e(c+dx)}{a+bx}\right)\left(\frac{2AB}{3b^2dg^4}-\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{6bd^3}+\frac{a(ad-bc)}{3bd^2}\right)+\frac{3a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3}{3bd^4}\right)}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{2B^2d^3x^2\left(\frac{b^2c-a\cdots}{3d^2}\cdots\right.}{3bg^4(a^3d^3-3a^2bc\cdots}\cdots\right.}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}+\frac{3ab\cdots}{d}\cdots}$$
$$-\frac{Bd^3\,\mathrm{atan}\left(\frac{Bd^3\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)(6A-11B)\,(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4)\,1i}{bg^4\,(ad-bc)^3\,(11B^2d^3-6ABd^3)}\right)(6A-11B\cdots}{9bg^4\,(ad-bc)^3}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 - 66*A*B
*a^2*d^2 - 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d + 42*A*B*a*b*c*
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 30*A*B*a*b*d^2 +
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d - 6*A*B*b^2*d))/(a*d
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x))/(a + b*x))*((2*A*B)/(3*b^2
*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) + (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A - 11*B)*(b^3*c^2*g^4 +
```

```
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 - 6*A
*B*d^3)))*(6*A - 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 498

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4b(bc-ad)^4g^5} - \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} - \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-1/4*B^2*d^4*ln((d*x+c)/(b*x+a)
```

```
)^2/b/(-a*d+b*c)^4/g^5-2*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)+3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)^2-2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^3+1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^4+1/2*B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a
)))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}dx = \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4} - \frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4} - \frac{2Bd^3(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^5(a+bx)(bc-ad)^4} + \frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4} - \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4bg^5(a+bx)^4} - \frac{b^3B^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4} + \frac{2b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4bg^5(bc-ad)^4} + \frac{2B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4} - \frac{3bB^2d^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32*(b*c - a*d)^4*g^5*(a
+ b*x)^4) - (B^2*d^4*Log[(c + d*x)/(a + b*x)]^2)/(4*b*(b*c - a*d)^4*g^5) -
(2*B*d^3*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/((b*c - a*d)^4*g^5
*(a + b*x)) + (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(2*(b*c - a*d)^4*g^5*(a + b*x)^2) - (2*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(c
```

```
 + d*x))/(a + b*x)]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) + (b^3*B*(c + d*x)^
4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(8*(b*c - a*d)^4*g^5*(a + b*x)^4) +
 (B*d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b
*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(4*b*g^5*(a +
b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (d-bx)^3(A+B\log(ex))^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^4g^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\text{Subst}\left(\int \frac{(d-bx)^4(A+B\log(ex))}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{2b(bc-ad)^4g^5}$$

$$= -\frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} - \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} - \frac{B^2\text{Subst}\left(\int \left(-4bd^3+3b^2d^2x-\frac{4}{3}b^3dx^2+\frac{b^4x^3}{4}+\frac{d^4\log(x)}{x}\right)dx, x, \frac{c+dx}{a+bx}\right)}{2b(bc-ad)^4g^5}$$

$$= \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} - \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} - \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} - \frac{(B^2d^4)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{2b(bc-ad)^4g^5}$$

$$
\begin{aligned}
&= \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} \\
&+ \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4b(bc-ad)^4g^5} \\
&- \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} \\
&- \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} \\
&+ \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.34

$$
\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx
$$

$$
= \frac{-72\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B\left(36A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+48Ad(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2(a+bx\right.}{}}{}
$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(36*A*(b*c - a*d)^4 - 9*B*
(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 48*A*d*(-(b*c) + a*d)^3*(a
 + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^2*(a +
b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 144*A*d^3*(-(b*c) + a*d)*(a +
b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*Log[a +
 b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 144*A*d^4*(a + b*x)^4*Log[c +
 d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[(d*(
a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[c + d*x]^
2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*
B*(b*c - a*d)^4*Log[(e*(c + d*x))/(a + b*x)] + 48*B*d*(-(b*c) + a*d)^3*(a +
 b*x)*Log[(e*(c + d*x))/(a + b*x)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log
[(e*(c + d*x))/(a + b*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(c
+ d*x))/(a + b*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(c + d*x))/(
a + b*x)] + 144*B*d^4*(a + b*x)^4*Log[c + d*x]*Log[(e*(c + d*x))/(a + b*x)]
 + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 144*B*d
^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(288*
b*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. 2(480) = 960.

Time = 2.38 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.23

method	result	size
parts	Expression too large to display	1112
derivativedivides	Expression too large to display	1422
default	Expression too large to display	1422
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	2601

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*c
)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/8*(d*e/b-e*(a*d-b*c)/b/(
b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/32*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3/b^2*d^2*e^2*(1/2*(d*e/b-e*(a*d-b*c
)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b*c)/b/(
b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))
^2)-1/b^3*d^3*e^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e
*(a*d-b*c)/b/(b*x+a)+2*d*e/b))-2*B*A/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*
(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*e/b-e*(a*d-b
*c)/b/(b*x+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a
*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3*d^2*e^2/b^2*(1/2*
(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-
e*(a*d-b*c)/b/(b*x+a))^2)-d^3*e^3/b^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e
/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(480) = 960.

Time = 0.29 (sec) , antiderivative size = 1045, normalized size of antiderivative = 2.10

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{9\,(8\,A^2-4\,AB+B^2)b^4c^4-32\,(9\,A^2-6\,AB+2\,B^2)ab^3c^3d+216\,(2\,A^2-2\,AB+B^2)a^2b^2c^2d^2-288\,(}{}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25*B^
2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2*d^4)
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a))^2 + 4*((12*A*B - 7*B^2)*
b^4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)*a^2*b^2*
c*d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*b^4*d^4*x^
4 - 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*(2*A*B - B^
2)*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(6
*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*
(2*A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 +
 18*B^2*a^2*b^2*c*d^3 - 12*(A*B - B^2)*a^3*b*d^4)*x)*log((d*e*x + c*e)/(b*x
 + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a
^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^
4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^
5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3
+ a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. 2(480) = 960.

Time = 0.36 (sec) , antiderivative size = 2122, normalized size of antiderivative = 4.26

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
```

```
[Out] -1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d*e*x/
(b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*
d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(
13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a
)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^
2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100
*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)
*log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6
*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a
*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*
g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^
6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5
)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 -
4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/24*A*B*((12*b^3*d^3*x^3
- 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c
^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3
- 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^
```

3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(480) = 960$.

Time = 0.56 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.40

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="giac")

[Out] -1/288*(72*((d*e*x + c*e)^4*B^2*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c*e)^3*B^2*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B^2*b*d^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2) - 4*(d*e*x + c*e)*B^2*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a))^2 + 12*(3*(4*A*B*b^3 - B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 16*(3*A*B*b^2*d - B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*(2*A*B*b*d^2 - B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2) - 48*(A*B*d^3 - B^2*d^3)*(d*e*x + c*e)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 9*(8*A^2*b^3 - 4*A*B*b^3 + B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 32*(9*A^2*b^2*d - 6*A*B*b^2*d + 2*B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 216*(2*A^2*b*d^2 - 2*A*B*b*d^2 + B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2) - 288*(A^2*d^3 - 2*A*B*d^3 + 2*B^2*d^3)*(d*e*x + c*e)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))

Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 1880, normalized size of antiderivative = 3.78

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b
*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*
b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2
*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*b^
2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) +
 (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2
*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*
x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2
*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3
*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/
(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^
2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)
/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d
 - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/
d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c + d*x))/(a + b*x))^2*(B^
2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^
2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3))) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^
2*b^3*c^3 - 300*A*B*a^3*d^3 + 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^
2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 156*A*B*a*b^2*c^
2*d + 276*A*B*a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*
B^2*b^3*c*d^2 - 84*A*B*a*b^2*d^3 + 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*
(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 156*A*B*a^2*b*d
^3 - 12*A*B*b^3*c^2*d + 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B
^2*b^3*d^2 - 12*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*
b^2*d^2*g^5 - 192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2
*g^5 - 192*a^2*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48
*a*b^6*c*d*g^5) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*
b^4*c*d*g^5) + 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
+ (B*d^4*atan((B*d^4*(12*A - 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*
a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2
```

```
*d^4 - 12*A*B*d^4)) + (B*d^5*x*(12*A - 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 -
 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^
4 - 12*A*B*d^4))))*(12*A - 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4)
```

3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*e*x + c*e)/(b*x +
a)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] g**2*(Integral(a**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Inte
gral(b**2*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(
2*a*b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Giac [N/A]

Not integrable

Time = 14.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)} \,d x$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{ag + bgx}{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{bgx + ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right) + A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] g*(Integral(a/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(b
*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*e*x + c*e)/(b*x + a)
)), x)
```

Sympy [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bbx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] Integral(1/(A*a + A*b*x + B*a*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*x*
log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = -\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)eg^2}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)/e/exp(A/B)/g^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2552, 2336, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = -\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^2(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] -(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B*(b*c - a*d)*e*E^
(A/B)*g^2))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\ &= -\frac{\text{Subst}\left(\int \frac{e^x}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)eg^2}\\ &= -\frac{e^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)eg^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(-bc+ad)eg^2}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^(A/
B)*g^2)
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{e^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^2 e(ad-cb) B}$	55
derivativedivides	$-\frac{e^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb) g^2 B}$	69
default	$-\frac{e^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb) g^2 B}$	69

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
[Out] -1/g^2/e/(a*d-b*c)/B*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=-\frac{e^{\left(-\frac{A}{B}\right)}\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{(Bbc-Bad)eg^2}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
[Out] -e^(-A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))/((B*b*c - B*a*d)*e*
g^2)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d
*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x
**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 109

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{de^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)^2eg^3} - \frac{be^{-\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B(bc-ad)^2e^2g^3}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3-b*Ei(2*(A
+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2552, 2367, 2336, 2209, 2346}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{de^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^3(bc-ad)^2} - \frac{be^{-\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2g^3(bc-ad)^2}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] (d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*(b*c - a*d)^2*
e*E^(A/B)*g^3) - (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
B])/(B*(b*c - a*d)^2*e^2*E^((2*A)/B)*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d-bx}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d}{A+B\log(ex)} - \frac{bx}{A+B\log(ex)}\right)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{b\text{Subst}\left(\int \frac{x}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3} + \frac{d\text{Subst}\left(\int \frac{1}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{b\text{Subst}\left(\int \frac{e^{2x}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2e^2g^3} + \frac{d\text{Subst}\left(\int \frac{e^{x}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2eg^3}$$

$$= \frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)^2eg^3} - \frac{be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B(bc-ad)^2e^2g^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$
$$= \frac{e^{-\frac{2A}{B}}\left(dee^{A/B}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right) - b\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)\right)}{B(bc-ad)^2e^2g^3}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] (d*e*E^(A/B)*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]] - b*ExpInteg
ralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*(b*c - a*d)^2*e^2*E^(
(2*A)/B)*g^3)
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$-\frac{-\frac{b\,e^{-\frac{2A}{B}}\,\text{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{d e\,e^{-\frac{A}{B}}\,\text{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(ad-cb)^2g^3}$	126
default	$-\frac{-\frac{b\,e^{-\frac{2A}{B}}\,\text{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{d e\,e^{-\frac{A}{B}}\,\text{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(ad-cb)^2g^3}$	126
risch	$\frac{b\,e^{-\frac{2A}{B}}\,\text{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{g^3(ad-cb)^2e^2B}-\frac{d\,e^{-\frac{A}{B}}\,\text{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{g^3(ad-cb)^2eB}$	139

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
[Out] -1/e^2/(a*d-b*c)^2/g^3*(-b/B*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-2*A/B)+d*e/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\left(dee^{\frac{A}{B}}\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)-b\operatorname{log\_integral}\left(\frac{(d^2e^2x^2+2\,cde^2x+c^2e^2)e^{\left(\frac{2\,A}{B}\right)}}{b^2x^2+2\,abx+a^2}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)e^2g^3}$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
```

```
[Out] (d*e*e^(A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)) - b*log_integral
((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2))
)*e^(-2*A/B)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*e^2*g^3)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^2bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^3x^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}}{g^3}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d*e
*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*
*3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag + bgx)^2}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(ag + bgx)^2}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx + ag)^2}{\left(A + B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag + bgx)^2}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(bgx + ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d*e*x + c*e)/(b*x
+ a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x + a)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 400, normalized size of antiderivative = 12.50

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{-a^3cg^2-a^3dg^2x-3a^2bcg^2x-3a^2bdg^2x^2-3ab^2cg^2x^2-3ab^2dg^2x^3-b^3cg^2x^3-b^3dg^2x^4}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(c+dx)}{a+bx}\right)}$$
$$+\frac{g^2\left(\int\frac{a^3d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{3a^2bc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{3b^3cx^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{4b^3dx^3}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,d\right.}{B(ad-bc)}$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g**
2*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(a + b*x
))) + g**2*(Integral(a**3*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x
) + Integral(3*a**2*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) +
Integral(3*b**3*c*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + I
ntegral(4*b**3*d*x**3/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + In
tegral(6*a*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Inte
gral(9*a*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Int
egral(6*a**2*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a*
d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 9.66

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) -
 a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2
```

```
+ 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*
c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B
- (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e
*x + c*e)/(b*x + a)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 6.98 (sec) , antiderivative size = 275, normalized size of antiderivative = 9.17

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(c+dx)}{a+bx}\right)}$$
$$+\frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2b^2cx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{3b^2dx^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,d\right.}{B\,(ad-bc)}$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(a
+ b*x))) + g*(Integral(a**2*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x) + Integral(2*a*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + I
ntegral(2*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integ
ral(3*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integr
al(4*a*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a*d - b*
c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.70

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a
*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b
*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) -
 (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))
*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.86 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Maple [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(
b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a))), x)
```

Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{-c-dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(c+dx)}{a+bx}\right)} + \frac{d\int \frac{1}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg\,(ad-bc)}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-c - d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(c + d*
x)/(a + b*x))) + d*Integral(1/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x)/(B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.19

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x) - (d
*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c
) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 8.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.199 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 104

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)eg^2} + \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)/e/exp(A/B)/g^2+(d*x+c)/B/
(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2552, 2334, 2336, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)} - \frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^2(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] -(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B^2*(b*c - a*d)*e*
E^(A/B)*g^2)) + (c + d*x)/(B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(c + d
*x))/(a + b*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{(A+B\log(ex))^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\
&= \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)g^2}\\
&= \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)} - \frac{\text{Subst}\left(\int \frac{e^x}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(bc-ad)eg^2}
\end{aligned}$$

$$= -\frac{e^{-\frac{A}{B}}\mathrm{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)eg^2} + \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{\frac{e^{-\frac{A}{B}}\,\mathrm{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e} - \frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(-bc+ad)g^2}$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] (ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(e*E^(A/B)) - (B*(c + d*x))/((a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(B^2*(-(b*c) + a*d)*g^2)
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)} - \frac{\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^2B^2e(ad-cb)}$	107
derivativedivides	$\frac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B^2}$	138
default	$\frac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B^2}$	138

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*d-b*c)/B*(d*x+c)/(b*x+a)/g^2/(A+B*ln(e*(d*x+c)/(b*x+a)))-1/g^2/B^2/e/(a*d-b*c)*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{(Bdex+Bce)e^{\frac{A}{B}} - \left(Abx+Aa+(Bbx+Ba)\log\left(\frac{dex+ce}{bx+a}\right)\right)\text{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{((B^3b^2c-B^3abd)eg^2x+(B^3abc-B^3a^2d)eg^2)e^{\frac{A}{B}}\log\left(\frac{dex+ce}{bx+a}\right)+((AB^2b^2c-AB^2abd)eg^2x+(AB^2abc-$$

[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")

[Out] ((B*d*e*x + B*c*e)*e^(A/B) - (A*b*x + A*a + (B*b*x + B*a)*log((d*e*x + c*e)/(b*x + a)))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B^3*b^2*c - B^3*a*b*d)*e*g^2*x + (B^3*a*b*c - B^3*a^2*d)*e*g^2)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + ((A*B^2*b^2*c - A*B^2*a*b*d)*e*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*e*g^2)*e^(A/B))

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{-c-dx}{ABa^2dg^2-ABabcg^2+ABabdg^2x-ABb^2cg^2x+(B^2a^2dg^2-B^2abcg^2+B^2abdg^2x-B^2b^2cg^2x)\log\left(\frac{e(c}{a}\right.} + \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg^2}$$

[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)

[Out] (-c - d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b**2*c*g**2*x)*log(e*(c + d*x)/(a + b*x))) + Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/(B*g**2)

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log(
e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*log(
e))*B^2)*x - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*
log(b*x + a) + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2
)*log(d*x + c)) + integrate(1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2*b^2*
g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*x - (B
^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{dex+ce}{\left(B^2g^2\log\left(\frac{dex+ce}{bx+a}\right)+ABg^2\right)(bx+a)} - \frac{\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right)}}{B^2g^2}\right)$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((d
*e*x + c*e)/((B^2*g^2*log((d*e*x + c*e)/(b*x + a)) + A*B*g^2)*(b*x + a)) -
Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 159

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} - \frac{2be^{-\frac{2A}{B}}\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2(bc-ad)^2e^2g^3} + \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3-2*b*Ei(
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3+(d*x+c
)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{2552, 2357, 2367, 2336, 2209, 2346\}$

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = -\frac{2be^{-\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2e^2g^3(bc-ad)^2} + \frac{de^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^3(bc-ad)^2} + \frac{c+dx}{Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] (d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B^2*(b*c - a*d)^
2*e*E^(A/B)*g^3) - (2*b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)
]))/B])/(B^2*(b*c - a*d)^2*e^2*E^((2*A)/B)*g^3) + (c + d*x)/(B*(b*c - a*d)*
g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d-bx}{(A+B\log(ex))^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)} + \frac{2\text{Subst}\left(\int \frac{d-bx}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{1}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3}$$

$$= \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)} + \frac{2\text{Subst}\left(\int \left(\frac{d}{A+B\log(ex)} - \frac{bx}{A+B\log(ex)}\right)dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3} - \frac{d\text{Subst}\left(\int \frac{e^x}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(bc-ad)^2eg^3}$$

$$= -\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} + \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)} - \frac{(2b)\text{Subst}\left(\int \frac{x}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3} + \frac{(2d)\text{Subst}\left(\int \frac{1}{A+B\log(ex)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3}$$

$$= -\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} + \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$
$$-\frac{(2b)\text{Subst}\left(\int \frac{e^{2x}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(bc-ad)^2e^2g^3} + \frac{(2d)\text{Subst}\left(\int \frac{e^{x}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(bc-ad)^2eg^3}$$
$$= \frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} - \frac{2be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2(bc-ad)^2e^2g^3}$$
$$+\frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \frac{\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e} - \frac{2be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2} + \frac{B(bc-ad)(c+dx)}{(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(bc-ad)^2g^3}$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
[Out] ((d*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]])/(e*E^(A/B)) - (2*b*E
xpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(e^2*E^((2*A)/B))
 + (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x
)])))/(B^2*(b*c - a*d)^2*g^3)
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.69

method	result
derivativedivides	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}\right.\right.\right.}{B^2}}{e^2(ad-cb)^2g^3}$
default	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}\right.\right.\right.}{B^2}}{e^2(ad-cb)^2g^3}$
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)^2g^3\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{a\,d^2\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e\,g^3B^2(ad-cb)^3}+\frac{bcd\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(\right.\right.}{b(}}{e\,g^3B^2(ad-cb)}$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/(a*d-b*c)^2/g^3*(b/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))+A/B)-2*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-2*A/B))-d*e/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(ln(d*e/b-e*(a*d-b*c)
/b/(b*x+a))+A/B)-exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(157) = 314$.

Time = 0.25 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.67

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

$$=\frac{((Bbcd-Bad^2)e^2x+(Bbc^2-Bacd)e^2)e^{\left(\frac{2A}{B}\right)}-2\left(Ab^3x^2+2\,Aab^2x+Aa^2b+(Bb^3x^2+2\,Bab^2\right.}{((B^3b^4c^2-2\,B^3ab^3cd+B^3a^2b^2d^2)e^2g^3x^2+2\,(B^3ab^3c^2-2\,B^3a^2b^2cd+B^3a^3bd^2)e^2g^3x+(B^3a^2b^2c^2-2\,B^3a}$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fric
as")
```

```
[Out] (((B*b*c*d - B*a*d^2)*e^2*x + (B*b*c^2 - B*a*c*d)*e^2)*e^(2*A/B) - 2*(A*b^3
*x^2 + 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log((d*e
*x + c*e)/(b*x + a)))*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^
(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)) + ((B*b^2*d*e*x^2 + 2*B*a*b*d*e*x + B*a^
2*d*e)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + (A*b^2*d*e*x^2 + 2*A*a*b*d*e*
x + A*a^2*d*e)*e^(A/B))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B
^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(B^3*a*b^3*
c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*e^2*g^3*x + (B^3*a^2*b^2*c^2 - 2*B
^3*a^3*b*c*d + B^3*a^4*d^2)*e^2*g^3)*e^(2*A/B)*log((d*e*x + c*e)/(b*x + a))
 + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*e^2*g^3*x^2 + 2
```

```
*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*e^2*g^3*x + (A*B
^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*e^2*g^3)*e^(2*A/B))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3*
log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*d*
g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3*lo
g(e) - a^2*b*d*g^3*log(e))*B^2)*x - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*
(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x
+ a) + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B
^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-(b*d*x + 2
*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a*b^3*d*g
^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3*log(e) -
 a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3*c*g^
3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3
)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - ((b^4*c*g^3 -
a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2
*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) +
 ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x
^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)
*log(d*x + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.83

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\left(\frac{de\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right)}}{B^2bceg^3-B^2adeg^3}-\frac{2\,b\mathrm{Ei}\left(\frac{2\,A}{B}+2\,\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{B^2bceg^3-B^2adeg^3}-\frac{\frac{(dex+ce)de}{bx+a}}{B^2bceg^3\log\left(\frac{dex+ce}{bx+a}\right)-B^2adeg^3\log\right.}\right.$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (d*e*Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*b*c*e*g^3 - B^2*a
*d*e*g^3) - 2*b*Ei(2*A/B + 2*log((d*e*x + c*e)/(b*x + a)))*e^(-2*A/B)/(B^2*
b*c*e*g^3 - B^2*a*d*e*g^3) - ((d*e*x + c*e)*d*e/(b*x + a) - (d*e*x + c*e)^2
*b/(b*x + a)^2)/(B^2*b*c*e*g^3*log((d*e*x + c*e)/(b*x + a)) - B^2*a*d*e*g^3
*log((d*e*x + c*e)/(b*x + a)) + A*B*b*c*e*g^3 - A*B*a*d*e*g^3))*(b*c/((b*c*
e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.201 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 182

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = -\frac{2B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} - \frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{10bd} + \frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}$$

```
[Out] -2/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-2/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+2/5*B
*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}+\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{2Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}-\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{Bg^4(a+bx)^4(bc-ad)}{10bd}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b
*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4
*(a + b*x)^4)/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (
g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}+\frac{(2B(bc-ad))\int\frac{(ag+bgx)^5}{(a+bx)(c+dx)}\,dx}{5bg}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}+\frac{(2B(bc-ad)g^4)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}\\
&\quad+\frac{(2B(bc-ad)g^4)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{5b}
\end{aligned}$$

$$= -\frac{2B(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} - \frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4}{10bd} + \frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g^4\left(-\frac{B(-bc+ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx))}{6d^5}+(a+bx)^5\left(A\right.\right.}{5b}$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^4*(-1/6*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b
*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2])))/(5*b)
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.63

method	result
derivativedivides	$-\frac{-\frac{g^4 A(bx+a)^5}{5}+g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c^4\right.}{d^5}\right.\right.}{}$
default	$-\frac{-\frac{g^4 A(bx+a)^5}{5}+g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c^4\right.}{d^5}\right.\right.}{}$
parts	$\frac{g^4 A(bx+a)^5}{5b}-\frac{g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c^4\right)}{d^5}\right.\right.}{}$
risch	$\frac{4g^4b^2B\ln(dx+c)a^2c^3}{d^3}-\frac{2g^4b^3B\ln(dx+c)a\,c^4}{d^4}-\frac{g^4b^3Ba\,x^4}{10}-\frac{8g^4B\,a^4x}{5}-\frac{2g^4B\ln(dx+c)a^5}{5b}-\frac{g^4b^3Ba\,c^2x^2}{d^2}+$
parallelrisch	$\frac{120Bx\,a^3b^2c\,d^4g^4-120Bx\,a^2b^3c^2d^3g^4+60Bxa\,b^4c^3d^2g^4+60B\ln(bx+a)a^4bc\,d^4g^4-120B\ln(bx+a)a^3b^2c^2d^3g^4+120B\ln(b}{}$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/5*g^4*A*(b*x+a)^5+g^4*B*(-1/5*(b*x+a)^5*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(-2/5*a*d+2/5*c*b)*(1/d^5*(-a^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2*c
^2*d^2+4*a*b^3*c^3*d-b^4*c^4)*ln(1/(b*x+a))+1/4/d*(b*x+a)^4-1/3*(-a*d+b*c)/
d^2*(b*x+a)^3-(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/d^4*(b*x+a)-1/
2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/d^3*(b*x+a)^2+1/d^5*(a*d-b*c)^4*ln(a*d/(b*x+
a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(170) = 340$.

Time = 0.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.51

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{6\,Ab^5d^5g^4x^5-12\,Ba^5d^5g^4\log(bx+a)+3\,(Bb^5cd^4+(10\,A-B)ab^4d^5)g^4x^4-4\,(Bb^5c^2d^3-5\,Bab^4cd^4-(1}{}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
+ (10*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - (15*
```

```
A - 4*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 + 2*(5*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5*A - 8*B)*
a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^
2 + 2*a*b*x + a^2)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 4.02 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx=\frac{Ab^4g^4x^5}{5}$$
$$-\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}$$
$$+\frac{2Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{12Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-\cdots}{\cdots}\right)}{5\cdots}$$
$$+x^4\left(Aab^3g^4-\frac{Bab^3g^4}{10}+\frac{Bb^4cg^4}{10d}\right)+x^3\cdot\left(2Aa^2b^2g^4-\frac{8Ba^2b^2g^4}{15}+\frac{2Bab^3cg^4}{3d}-\frac{2Bb^4c^2g^4}{15d^2}\right)$$
$$+x^2\cdot\left(2Aa^3bg^4-\frac{6Ba^3bg^4}{5}+\frac{2Ba^2b^2cg^4}{d}-\frac{Bab^3c^2g^4}{d^2}+\frac{Bb^4c^3g^4}{5d^3}\right)$$
$$+x\left(Aa^4g^4-\frac{8Ba^4g^4}{5}+\frac{4Ba^3bcg^4}{d}-\frac{4Ba^2b^2c^2g^4}{d^2}+\frac{2Bab^3c^3g^4}{d^3}-\frac{2Bb^4c^4g^4}{5d^4}\right)$$
$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) + 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
```

```
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/10 + B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 - 8*B*a**2*b**2*g**4/15 + 2*B*a*b**3*c*g
**4/(3*d) - 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 - 6*B*a**
3*b*g**4/5 + 2*B*a**2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/d**2 + B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 - 8*B*a**4*g**4/5 + 4*B*a**3*b*c*g**4/d -
4*B*a**2*b**2*c**2*g**4/d**2 + 2*B*a*b**3*c**3*g**4/d**3 - 2*B*b**4*c**4*g*
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(c + d*x)**2/(a + b*x)
**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(170) = 340$.

Time = 0.24 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.85

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{1}{5} Ab^4g^4x^5 + Aab^3g^4x^4 + 2\,Aa^2b^2g^4x^3 + 2\,Aa^3bg^4x^2$$
$$+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{2\,a\log(bx+a)}{b} + \frac{2\,c\log(dx+\right.$$
$$+2\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) + \frac{2\,a^2\log(bx+a)}{b^2} - \frac{2\,c^2\log(d}{d^2}\right.$$
$$+2\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{2\,a^3\log(bx+a)}{b^3} + \frac{2\,c^3\log(d}{d^3}\right.$$
$$+\frac{1}{3}\left(3\,x^4\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) + \frac{6\,a^4\log(bx+a)}{b^4} - \frac{6\,c^4\log}{}\right.$$
$$+\frac{1}{30}\left(6\,x^5\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{12\,a^5\log(bx+a)}{b^5} + \frac{12\,c^5}{}\right.$$
$$+ Aa^4g^4x$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*
b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) +
2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*
```

```
a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^
3*b*g^4 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)
/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(170) = 340$.

Time = 63.43 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.68

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx \\
&= \frac{1}{5}\,Ab^4g^4x^5 - \frac{2\,Ba^5g^4\log\left(bx+a\right)}{5\,b} + \frac{(Bb^4cg^4+10\,Aab^3dg^4-Bab^3dg^4)x^4}{10\,d} \\
&\quad - \frac{2\,(Bb^4c^2g^4-5\,Bab^3cdg^4-15\,Aa^2b^2d^2g^4+4\,Ba^2b^2d^2g^4)x^3}{15\,d^2} \\
&\quad + \frac{1}{5}\,(Bb^4g^4x^5+5\,Bab^3g^4x^4+10\,Ba^2b^2g^4x^3+10\,Ba^3bg^4x^2+5\,Ba^4g^4x)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) \\
&\quad + \frac{(Bb^4c^3g^4-5\,Bab^3c^2dg^4+10\,Ba^2b^2cd^2g^4+10\,Aa^3bd^3g^4-6\,Ba^3bd^3g^4)x^2}{5\,d^3} \\
&\quad - \frac{(2\,Bb^4c^4g^4-10\,Bab^3c^3dg^4+20\,Ba^2b^2c^2d^2g^4-20\,Ba^3bcd^3g^4-5\,Aa^4d^4g^4+8\,Ba^4d^4g^4)x}{5\,d^4} \\
&\quad + \frac{2\,(Bb^4c^5g^4-5\,Bab^3c^4dg^4+10\,Ba^2b^2c^3d^2g^4-10\,Ba^3bc^2d^3g^4+5\,Ba^4cd^4g^4)\log\left(dx+c\right)}{5\,d^5}
\end{aligned}
$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] 1/5*A*b^4*g^4*x^5 - 2/5*B*a^5*g^4*log(b*x + a)/b + 1/10*(B*b^4*c*g^4 + 10*A
*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d - 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d*g
^4 - 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^4*x
^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*B*a^
4*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + 1
/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 + 10*A*a^3
*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(2*B*b^4*c^4*g^4 - 10*B*a*b^3
```

```
*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 - 5*A*a^4*d^4*
g^4 + 8*B*a^4*d^4*g^4)*x/d^4 + 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*d*g^4 + 1
0*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*g^4)*log(d
*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 1024, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$=x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c}{d}\right.}{10\,b\,d}\right.$$

$$+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c-4\,B\,a\,d+4\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$+\frac{a\,c\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d +
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c - 4*B*a*d + 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*
d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(c + d*x)^2)/(a + b*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*x)*((2*B*b^4*c^5*g^
4)/5 + 2*B*a^4*c*d^4*g^4 - 4*B*a^3*b*c^2*d^3*g^4 + 4*B*a^2*b^2*c^3*d^2*g^4
- 2*B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (2*B*a^5*g^4*log(a + b*x)
)/(5*b)
```

3.202 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 151

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{B(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{6bd} - \frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}$$

```
[Out] 1/2*B*(-a*d+b*c)^3*g^3*x/d^3-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/6*B*(
-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b} - \frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4} + \frac{Bg^3x(bc-ad)^3}{2d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2} + \frac{Bg^3(a+bx)^3(bc-ad)}{6bd}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) - (B*(b*c - a*d)^4*g^3*Log[c +
 d*x])/(2*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/(4*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}+\frac{(B(bc-ad))\int\frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{2bg}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}+\frac{(B(bc-ad)g^3)\int\frac{(a+bx)^3}{c+dx}\,dx}{2b}\\
&= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}\\
&\quad+\frac{(B(bc-ad)g^3)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{2b}\\
&= \frac{B(bc-ad)^3g^3x}{2d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{6bd}\\
&\quad-\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{4b}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(4*b)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^3A(bx+a)^4}{4}+g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abc}{}\right.\right.}{b}$
default	$-\frac{-\frac{g^3A(bx+a)^4}{4}+g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abc}{}\right.\right.}{b}$
parts	$\frac{Ag^3(bx+a)^4}{4b}-\frac{g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abc}{}\right.\right.}{b}$
risch	$\frac{g^3(bx+a)^4B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b}+\frac{g^3b^3Ax^4}{4}+g^3b^2Aax^3-\frac{g^3b^2Bax^3}{6}+\frac{g^3b^3Bcx^3}{6d}+\frac{3g^3bAa^2x^2}{2}-\frac{3g^3bBa^2}{4}$
parallelrisch	$24B\ln(bx+a)a^3bcd^3g^3+3Bx^4\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^4d^4g^3+12Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^3bd^4g^3+12B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^3bcd^3g^3-18.$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/4*g^3*A*(b*x+a)^4+g^3*B*(-1/4*(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(-1/2*a*d+1/2*c*b)*(1/d^4*(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x
+a)-d)-(-a^2*d^2+2*a*b*c*d-b^2*c^2)/d^3*(b*x+a)-1/2*(-a*d+b*c)/d^2*(b*x+a)^
```

```
2+1/3/d*(b*x+a)^3+1/d^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*ln(1
/(b*x+a)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(141) = 282$.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.27

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{3\,Ab^4d^4g^3x^4 - 6\,Ba^4d^4g^3\log(bx+a) + 2\,(Bb^4cd^3 + (6\,A-B)ab^3d^4)g^3x^3 - 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 - 3\,(2}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2*A
 - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.19 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x + \frac{\frac{Ba^5d^4g^3}{b} + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{2b}$$
$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2 - 2abcd + b^2c^2\right)\log\left(x + \frac{5Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3 - Bacg^3\cdot(2ad-bc)(2}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3}\right.}{2d^4}$$
$$+ x^3\left(Aab^2g^3 - \frac{Bab^2g^3}{6} + \frac{Bb^3cg^3}{6d}\right) + x^2\cdot\left(\frac{3Aa^2bg^3}{2} - \frac{3Ba^2bg^3}{4} + \frac{Bab^2cg^3}{d} - \frac{Bb^3c^2g^3}{4d^2}\right)$$
$$+ x\left(Aa^3g^3 - \frac{3Ba^3g^3}{2} + \frac{3Ba^2bcg^3}{d} - \frac{2Bab^2c^2g^3}{d^2} + \frac{Bb^3c^3g^3}{2d^3}\right)$$
$$+ \left(Ba^3g^3x + \frac{3Ba^2bg^3x^2}{2} + Bab^2g^3x^3 + \frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/6 + B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*b*
g**3/4 + B*a*b**2*c*g**3/d - B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 -
3*B*a**3*g**3/2 + 3*B*a**2*b*c*g**3/d - 2*B*a*b**2*c**2*g**3/d**2 + B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**2/(a + b*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(141) = 282$.

Time = 0.23 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.27

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2$$
$$+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log\left(bx+a\right)}{b}+\frac{2\,c\log\left(dx\right.}{d}\right.$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{2\,a^2\log\left(bx+a\right)}{b^2}-\frac{2\,c^2\log}{}\right.$$
$$+\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a^3\log\left(bx+a\right)}{b^3}+\frac{2\,c^3\log\left(d\right.}{d^3}\right.$$
$$+\frac{1}{12}\left(3\,x^4\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{6\,a^4\log\left(bx+a\right)}{b^4}-\frac{6\,c^4\,\mathrm{l}}{}\right.$$
$$+Aa^3g^3x$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2
 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b
^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
```

```
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
 + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e
*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(141) = 282$.

Time = 10.54 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.37

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{1}{4}Ab^3g^3x^4 - \frac{Ba^4g^3\log(bx+a)}{2b} + \frac{(Bb^3cg^3+6Aab^2dg^3-Bab^2dg^3)x^3}{6d} + \frac{1}{4}\left(Bb^3g^3x^4+4Bab^2g^3x^3+6Ba^2bg^3x^2+4Ba^3g^3x\right)\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right) - \frac{(Bb^3c^2g^3-4Bab^2cdg^3-6Aa^2bd^2g^3+3Ba^2bd^2g^3)x^2}{4d^2} + \frac{(Bb^3c^3g^3-4Bab^2c^2dg^3+6Ba^2bcd^2g^3+2Aa^3d^3g^3-3Ba^3d^3g^3)x}{2d^3} - \frac{(Bb^3c^4g^3-4Bab^2c^3dg^3+6Ba^2bc^2d^2g^3-4Ba^3cd^3g^3)\log(-dx-c)}{2d^4}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] 1/4*A*b^3*g^3*x^4 - 1/2*B*a^4*g^3*log(b*x + a)/b + 1/6*(B*b^3*c*g^3 + 6*A*a
*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^3
+ 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b
^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 - 6*A*a^2
*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4*B*a*b^2*c^
2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/d^3 -
1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a^3*
c*d^3*g^3)*log(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+x\left(\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\right.}{2\,b\,d}+\frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right)$$

$$-\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] log((e*(c + d*x)^2)/(a + b*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*
a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
```

```
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c - 3*B*a*d + 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.203 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx &= -\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}\\ &\quad+\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}\end{aligned}$$

```
[Out] -2/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+2/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^
2))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx &= \frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}+\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}\\ &\quad-\frac{2Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d
) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{(2B(bc-ad))\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{3bg}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{(2B(bc-ad)g^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{3b}\\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{(2B(bc-ad)g^2)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b}\\
&= -\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}\\
&\quad+\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{3b}$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/d^3 + (a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])))/(3*b)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\Bigg(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb}{d^2}}{b}$
default	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\Bigg(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb}{d^2}}{b}$
parts	$\frac{Ag^2(bx+a)^3}{3b}-\frac{g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\Bigg(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb)}{d^2}}{b}$
risch	$\frac{(bx+a)^3g^2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAax^2-\frac{g^2bBax^2}{3}+\frac{g^2b^2Bcx^2}{3d}+g^2Aa^2x-\frac{2g^2B\ln(dx+}{3b}$
parallelrisch	$\frac{2Bx^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^3g^2+6Ax^2ab^2d^3g^2-2Bx^2ab^2d^3g^2+2Bx^2b^3cd^2g^2-12B\ln(bx+a)ab^2c^2dg^2-8Bxa^2bd^3g^2+}{}$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/3*g^2*A*(b*x+a)^3+g^2*B*(-1/3*(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(-2/3*a*d+2/3*c*b)*(1/d^3*(-a^2*d^2+2*a*b*c*d-b^2*c^2)*ln(1/(
b*x+a))-(-a*d+b*c)/d^2*(b*x+a)+1/2/d*(b*x+a)^2+1/d^3*(a*d-b*c)^2*ln(a*d/(b*
x+a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(112) = 224$.

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.04

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{Ab^3d^3g^2x^3 - 2\,Ba^3d^3g^2\log(bx+a) + (Bb^3cd^2 + (3\,A-B)ab^2d^3)g^2x^2 - (2\,Bb^3c^2d - 6\,Bab^2cd^2 - (3\,A-4}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*A
 - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)*a
^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.47 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b} + \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3a^2d^2}{d}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3} + x^2\left(Aabg^2 - \frac{Babg^2}{3} + \frac{Bb^2cg^2}{3d}\right) + x\left(Aa^2g^2 - \frac{4Ba^2g^2}{3} + \frac{2Babcg^2}{d} - \frac{2Bb^2c^2g^2}{3d^2}\right) + \left(Ba^2g^2x + Babg^2x^2 + \frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
```

```
*2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 -
B*a*b*g**2/3 + B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 - 4*B*a**2*g**2/3 + 2*
B*a*b*c*g**2/d - 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(112) = 224$.

Time = 0.21 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.63

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) - \frac{2a\log(bx+a)}{b} + \frac{2c\log(dx-}{d} \right. + \left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) + \frac{2a^2\log(bx+a)}{b^2} - \frac{2c^2\log(d}{d^2} \right. + \frac{1}{3}\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log}{c} \right. + Aa^2g^2x$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*e*x^2
/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b
^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2
+ 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*
b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
 + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 1.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{1}{3}Ab^2g^2x^3 - \frac{2Ba^3g^2\log(bx+a)}{3b} + \frac{(Bb^2cg^2 + 3Aabdg^2 - Babdg^2)x^2}{3d} + \frac{1}{3}(Bb^2g^2x^3 + 3Babg^2x^2 + 3Ba^2g^2x)\log\left(\frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2}\right) - \frac{(2Bb^2c^2g^2 - 6Babcdg^2 - 3Aa^2d^2g^2 + 4Ba^2d^2g^2)x}{3d^2} + \frac{2(Bb^2c^3g^2 - 3Babc^2dg^2 + 3Ba^2cd^2g^2)\log(dx+c)}{3d^3}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] 1/3*A*b^2*g^2*x^3 - 2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(B*b^2*c*g^2 + 3*A*a
*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*B*
a^2*g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) -
 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2*g^2
)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*log(d
*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,\left(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a
*d + 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c +
d*x)^2)/(a + b*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 - (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.204 $\int (ag+bgx) \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx) \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \, dx = \frac{B(bc-ad)gx}{d} - \frac{B(bc-ad)^2 g \log(c+dx)}{bd^2} + \frac{g(a+bx)^2 \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}$$

```
[Out] B*(-a*d+b*c)*g*x/d-B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*ln
(e*(d*x+c)^2/(b*x+a)^2))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ag+bgx) \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \, dx = \frac{g(a+bx)^2 \left(B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b} - \frac{Bg(bc-ad)^2 \log(c+dx)}{bd^2} + \frac{Bgx(bc-ad)}{d}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)*g*x)/d - (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) + (g*(a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}+\frac{(B(bc-ad))\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{bg}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}+\frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}\,dx}{b}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}+\frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx}{b}\\
&= \frac{B(bc-ad)gx}{d}-\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g\left(-\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g*((-2*B*(-(b*c) + a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(2*b)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{b}+\frac{2gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{d^2}-$
derivativedivides	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)\ln\left(\frac{1}{bx+a}\right)}{d^2}+\right.\right.}{b}$
default	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)\ln\left(\frac{1}{bx+a}\right)}{d^2}+\right.\right.}{b}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)\ln(}{d^2}\right.\right.}{b}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab\,d^2g+2Axab\,d^2g-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)abcdg-2}{}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*g*b*A*x^2+g*A*a*x-B*a^2*g
/b*ln(b*x+a)+2*g/d*B*ln(-d*x-c)*a*c-g*b/d^2*B*ln(-d*x-c)*c^2-g*B*a*x+g*b/d*
B*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.91

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2-2\,Ba^2d^2g\log(bx+a)+2\,(Bb^2cd+(A-B)abd^2)gx-2\,(Bb^2c^2-2\,Babcd)g\log(dx+c)+(B}{2\,bd^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*log(b*x + a) + 2*(B*b^2*c*d + (A - B)*
a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
2 + 2*B*a*b*d^2*g*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x
 + a^2)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.92 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{Abgx^2}{2} - \frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b} + \frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2} + x\left(Aag-Bag+\frac{Bbcg}{d}\right) + \left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g - B*a*g + B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(c + d*x
)**2/(a + b*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{1}{2}Abgx^2 + \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log(bx+a)}{b}+\frac{2\,c\log(dx-}{d}\cdots\right. + \frac{1}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{2\,a^2\log(bx+a)}{b^2}-\frac{2\,c^2\log}{}\cdots\right. + Aagx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2
*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)
/b + 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x
+ a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a
^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx &= \frac{1}{2}Abgx^2-\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Bbgx^2+2Bagx\right)\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)\\&\quad+\frac{(Bbcg+Aadg-Badg)x}{d}-\frac{(Bbc^2g-2Bacdg)\log(-dx-c)}{d^2}\end{aligned}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] 1/2*A*b*g*x^2 - B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((d
^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g + A*a*d
*g - B*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx &= x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c-B\,a\,d+B\,b\,c)}{d}-\frac{A\,g\,(a\,d+b\,c)}{d}\right)\\&\quad+\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)\\&\quad+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln(a+b\,x)}{b}\\&\quad+\frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}\end{aligned}$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c - B*a*d + B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(c + d*x)^2)/(a + b*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 - (B*a^
2*g*log(a + b*x))/b + (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg}-\frac{2B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g-2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{2B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(b*g)) - (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} - \frac{(2B(bc-ad))\int\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} - \frac{(2B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx, x, a+bx\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} + \frac{(2B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b^2g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} + \frac{(2B(bc-ad))\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}\,dx, x, \frac{1}{a+bx}\right)}{b^2 g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} - \frac{2B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$$
$$= \frac{\log(a+bx)\left(A+B\log(a+bx)-2B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A + B*Log[a + b*x] - 2*B*Log[(b*(c + d*x))/(b*c - a*d)] + B*
Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)-(2ad-2cb)\left(\frac{\text{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right)\right)}{g}}{b}$
default	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)-(2ad-2cb)\left(\frac{\text{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right)\right)}{g}}{b}$
parts	$\frac{A\ln(bx+a)}{gb}-\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)-(2ad-2cb)\left(\frac{\text{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right)\right)}{gb}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bg}+\frac{2B\,\text{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)ad}{bg(ad-cb)}-\frac{2B\,\text{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)c}{g(ad-cb)}+$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/g*A*ln(1/(b*x+a))+1/g*B*(ln(1/(b*x+a))*ln(e*(a*d/(b*x+a)-b*c/(b*x+a
)-d)^2/b^2)-(2*a*d-2*b*c)*(dilog(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)+ln(1
/(b*x+a))*ln(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c))))
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
+ A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x*
*2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e
) - 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A}{bgx + ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{a\,g + b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x), x)
```

3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -A*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-B*(d*x
+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2552, 2332}

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A(c+dx)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)} + \frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) + (2*B*(c + d*x))/((b*c - a*d)
*g^2*(a + b*x)) - (B*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b*c - a*
d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \left(A+B\log\left(ex^2\right)\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\ &= -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{B\text{Subst}\left(\int \log\left(ex^2\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\ &= -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$$
$$=\frac{2Bd(a+bx)\log(a+bx)-2Bd(a+bx)\log(c+dx)-(bc-ad)\left(A-2B+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (2*B*d*(a + b*x)*Log[a + b*x] - 2*B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*
(A - 2*B + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2*(a + b*x
))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{(A-2B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{Bdx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2Aa\,b^2d^2-2A\,b^3cd-4Ba\,b^2d^2+4B\,b^3cd-2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3cd}{2g^2(bx+a)b^3d(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-dx-c)bdx+2B\ln(bx+a)bdx-2B\ln(-dx-c)ad+2B\ln(bx+a)ad+Aad-Abc-2Bad+}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2}}{b}$
default	$-\frac{\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2}}{b}$
parts	$-\frac{A}{g^2(bx+a)b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2b}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A-2*B)/g/a*x+B*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)+1/g*B*d/(a*d-b*c)*
x*ln(e*(d*x+c)^2/(b*x+a)^2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(A-2\,B)bc-(A-2\,B)ad+(Bbdx+Bbc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A - 2*B)*b*c - (A - 2*B)*a*d + (B*b*d*x + B*b*c)*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*
c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(83) = 166$.

Time = 0.70 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x} + \frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} - \frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{-A+2B}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
- 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A + 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2} - \frac{2}{b^2g^2x+abg^2} - \frac{2\,d\log(bx+a)}{(b^2c-abd)g^2} + \frac{2\,d\log(dx+c}{(b^2c-abd)g^2}\right. - \frac{A}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
+ a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*g^
2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x + c)/
((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{(b^2cg^2-abdg^2)(bgx+ag)bg}\right)+\frac{\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)}{(bgx+ag)bg}\right.$$
$$-\frac{A}{(bgx+ag)bg}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
 a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g
^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) + log((d*x + c)^2*e/(b*x + a)^2)/((b*g*
x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A-2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)} + \frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x)^2)/(a + b*x)^2))/(b^2*g^2*(x + a/b)) - (A - 2*B)/(b^2*g^
2*x + a*b*g^2)
```

3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 139

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{B}{2bg^3(a+bx)^2} - \frac{Bd}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3} + \frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}$$

```
[Out] 1/2*B/b/g^3/(b*x+a)^2-B*d/b/(-a*d+b*c)/g^3/(b*x+a)-B*d^2*ln(b*x+a)/b/(-a*d+
b*c)^2/g^3+B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d*x+c)^2/(b*x
+a)^2))/b/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2} - \frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2} + \frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2} - \frac{Bd}{bg^3(a+bx)(bc-ad)} + \frac{B}{2bg^3(a+bx)^2}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(2*b*g^3*(a + b*x)^2) - (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Log[
a + b*x])/(b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3
) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*g^3*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^2}\,dx}{bg} \\
&= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3} \\
&= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2} \\
&\quad -\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{bg^3} \\
&= \frac{B}{2bg^3(a+bx)^2}-\frac{Bd}{b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3} \\
&\quad +\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(Abc-bBc-aAd+3aBd+2bBd\right.}{2b(bc-ad)^2g^3(a+bx)^2}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(A*b*c - b*B*c - a*A*d + 3*a*B*d + 2*b*B*d*x + B*(b*c - a*d)*L
og[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3}}{b}$
default	$-\frac{\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3}}{b}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3b}$
norman	$\frac{\frac{Bdx}{g(ad-cb)}+\frac{Bad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{Aabd-Ab^2c-3Babd+Bb^2c}{2gb^2(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{Bd^2bx^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{-2Bxab^4d^3+2Bxb^5cd^2+Aa^2b^3d^3+Ab^5c^2d-3Ba^2b^3d^3-Bb^5c^2d-2Aab^4cd^2+4Bab^4cd^2-2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab^4cd}{2g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2bg^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)abd^2x-4B\ln(-dx-c)abd^2x+2Ba^2\ln}{2(a^2d^2-2ab}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/2/g^3*A/(b*x+a)^2+1/g^3*B*(1/2/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(a*d-b*c)*(1/(a*d-b*c)^2*(1/2*a*d/(b*x+a)^2-1/2*b*c/(b*x+a)^2
+d/(b*x+a))+d^2/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.73

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{(A-B)b^2c^2 - 2\,(A-2\,B)abcd + (A-3\,B)a^2d^2 + 2\,(Bb^2cd - Babd^2)x - (Bb^2d^2x^2 + 2\,Babd^2x - Bb^2}{2\,((b^5c^2 - 2\,ab^4cd + a^2b^3d^2)g^3x^2 + 2\,(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2\,a^3}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A - B)*b^2*c^2 - 2*(A - 2*B)*a*b*c*d + (A - 3*B)*a^2*d^2 + 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c
^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(122) = 244.

Time = 1.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.01

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} \\
&\quad + \frac{Bd^2\log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2} \\
&\quad - \frac{Bd^2\log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 - \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2} \\
&\quad + \frac{-Aad + Abc + 3Bad - Bbc + 2Bbdx}{2a^3bdg^3 - 2a^2b^2cg^3 + x^2\cdot(2ab^3dg^3 - 2b^4cg^3) + x\,(4a^2b^2dg^3 - 4ab^3cg^3)}
\end{aligned}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(135) = 270$.

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.20

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{2}\,B\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{}{b^2}\right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg}\right.$$

$$-\frac{A}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}+\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} - \frac{2\,Bbdx+Abc-Bbc-Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + B*d^2
*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((
d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2*
a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*B*b*d*x + A*b*c - B*b*c - A*a*d + 3*B*a*d
)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x +
a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{A\,a\,d-A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,(a\,d-b\,c)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] (2*B*d^2*atanh((b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d
*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x)^2)/(a + b*x)^
2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((A*a*d - A*b*c - 3*B*a*d + B*b*c
)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b
^2*g^3*x)
```

3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2B}{9bg^4(a+bx)^3} - \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}$$

```
[Out] 2/9*B/b/g^4/(b*x+a)^3-1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+2/3*B*d^2/b/(-a*d+
b*c)^2/g^4/(b*x+a)+2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-2/3*B*d^3*ln(d*x+
c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3} + \frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3} - \frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3} + \frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd}{3bg^4(a+bx)^2(bc-ad)} + \frac{2B}{9bg^4(a+bx)^3}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] (2*B)/(9*b*g^4*(a + b*x)^3) - (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) + (2*
B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (2*B*d^3*Log[a + b*x])/(3*b*(b*c
 - a*d)^3*g^4) - (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2])/(3*b*g^4*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}-\frac{(2B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^3}\,dx}{3bg}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}-\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}\\
&\quad -\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)}{3bg^4}
\end{aligned}$$

$$= \frac{2B}{9bg^4(a+bx)^3} - \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{\frac{B(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx))}{(bc-ad)^3} - 3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
[Out] ((B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a +
 b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))
/(b*c - a*d)^3 - 3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a +
b*x)^3)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{\frac{A}{3g^4(bx+a)^3}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2ad}{3}-\frac{2cb}{3}\right)\left(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+a}}{(ad-cb)^3}\right.}{g^4}}{b}$
default	$-\frac{\frac{A}{3g^4(bx+a)^3}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2ad}{3}-\frac{2cb}{3}\right)\left(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+a}}{(ad-cb)^3}\right.}{g^4}}{b}$
parts	$-\frac{A}{3g^4(bx+a)^3b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2ad}{3}-\frac{2cb}{3}\right)\left(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}}{(ad-cb)^3}\right.}{g^4b}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-dx-c)b^3d^3x^3+6B\ln(bx+a)b^3d^3x^3-18B\ln(-dx-c)a\,b^2d^3x^2+18B\ln(bx+a)a\,b^2d^3x^2}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-18B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^2b^5d^4-18B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^2b^5c\,d}{}$
norman	$\frac{B\,a^2d^3x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{(3A\,a^2d^2-6Aabcd+3A\,b^2c^2-6B\,a^2d^2+6Babcd-2B\,b^2}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/3/g^4*A/(b*x+a)^3+1/g^4*B*(1/3/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(2/3*a*d-2/3*c*b)*(1/(a*d-b*c)^3*(1/3*a^2*d^2/(b*x+a)^3-2/3*a
*b*c*d/(b*x+a)^3+1/3*b^2*c^2/(b*x+a)^3+1/2*a*d^2/(b*x+a)^2-1/2*b*c*d/(b*x+a
)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(165) = 330$.

Time = 0.28 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.44

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{(3\,A-2\,B)b^3c^3-9\,(A-B)ab^2c^2d+9\,(A-2\,B)a^2bcd^2-(3\,A-11\,B)a^3d^3-6\,(Bb^3cd^2-Bab^2d^3)x^2}{9\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^2
 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*
b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3
*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.

Time = 1.78 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} + \frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\left(ad-bc\right)^3} - \frac{2Bd^3\log\left(x+\frac{\frac{2Ba^4d^7}{(ad-bc)^3}-\frac{8Ba^3bcd^6}{(ad-bc)^3}+\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4+\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\left(ad-bc\right)^3} + \frac{-3Aa^2d^2+6Aabcd-3Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+6Bb^2c}{9a^5bd^2g^4-18a^4b^2cdg^4+9a^3b^3c^2g^4+x^3\cdot(9a^2b^4d^2g^4-18ab^5cdg^4+9b^6c^2g^4)+x^2\cdot(27a^3b^3d^2g^4-54a^2b}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6
*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4 -
 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g**
4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4 -
 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4 -
 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(165) = 330.

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$$
$$=\frac{1}{9}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)x}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)g\right.$$
$$-\frac{A}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5
*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
- 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(165) = 330.

Time = 0.38 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx=\frac{2\,Bd^3\log(bx+a)}{3\,(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4)}$$
$$-\frac{2\,Bd^3\log(dx+c)}{3\,(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4)}$$
$$-\frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$
$$+\frac{6\,Bb^2d^2x^2-3\,Bb^2cdx+15\,Babd^2x-3\,Ab^2c^2+2\,Bb^2c^2+6\,Aabcd-7\,}{9\,(b^6c^2g^4x^3-2\,ab^5cdg^4x^3+a^2b^4d^2g^4x^3+3\,ab^5c^2g^4x^2-6\,a^2b^4cdg^4x^2+3\,a^3b^3d^2g^4x^2+3\,a^2b^4c^2g^4x-6\,}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] 2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g
^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g
^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*e*x^2 + 2*c*d*e*
x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*
d^2*x - 3*A*b^2*c^2 + 2*B*b^2*c^2 + 6*A*a*b*c*d - 7*B*a*b*c*d - 3*A*a^2*d^2
 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x
^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 +
3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c
^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}+\frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}+\frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b
*c)^3) - (B*log((e*(c + d*x)^2)/(a + b*x)^2))/(3*b*g^4*(a + b*x)^3) - (A*b*
c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(
a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d
^2)/(9*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^
2*(a + b*x)^3) + (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a
*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*
(a + b*x)^3) - (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{B}{8bg^5(a+bx)^4}-\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}-\frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)}-\frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5}+\frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}$$

```
[Out] 1/8*B/b/g^5/(b*x+a)^4-1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/4*B*d^2/b/(-a*d+
b*c)^2/g^5/(b*x+a)^2-1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/2*B*d^4*ln(b*x+
a)/b/(-a*d+b*c)^4/g^5+1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e
*(d*x+c)^2/(b*x+a)^2))/b/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}-\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}-\frac{Bd}{6bg^5(a+bx)^3(bc-ad)}+\frac{B}{8bg^5(a+bx)^4}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(8*b*g^5*(a + b*x)^4) - (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^2)
/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a +
b*x)) - (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x])
/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(4*b*g^
5*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4} - \frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ag+bgx)^4}\,dx}{2bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4} - \frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4} - \frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2} + \frac{bd^4}{(bc-ad)^5(a+bx)} - \right.}{2bg^5}$$

$$= \frac{B}{8bg^5(a+bx)^4} - \frac{Bd}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{\frac{B\left(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(a+bx)+12d^4(a+bx)^4\log(c+dx)\right)}{(bc-ad)^4} - 6\Big(}{24bg^5(a+bx)^4}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a
 + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]))/(b*c - a*d)^4 - 6*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(24*b*g^5*(a + b*x)^4)
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad-cb)d^2}{2(bx+a)^2}+}{(ad-cb)^4}\right.\right.}{g^5}}{b}$
default	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad-cb)d^2}{2(bx+a)^2}+}{(ad-cb)^4}\right.\right.}{g^5}}{b}$
parts	$-\frac{A}{4g^5(bx+a)^4b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad-c}{2(bx+}}{(ad-cb)^4}\right.\right.}{g^5b}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{48Ba\,b^3c\,d^3x^2+72B\,a^2b^2c\,d^3x-24Ba\,b^3c^2d^2x-24A\,a^3bc\,d^3+36A\,a^2b^2c^2d^2-24Aa\,b^3c^3d-12Ba\,b}{}$
parallelrisch	$\frac{24Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^9c\,d^4+6B\,x^4\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^6b^3c\,d^4+24B\,x^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^7b^2c\,d^4+36B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^8bc\,d^4}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(2A\,a^3d^3-6A\,a^2bc\,d^2+6Aa\,b^2c^2\right.}{2ga\left(a\right.}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/4/g^5*A/(b*x+a)^4+1/g^5*B*(1/4/(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-(1/2*a*d-1/2*c*b)*(1/(a*d-b*c)^4*(1/4*(a*d-b*c)*(a^2*d^2-2*a*
b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^3+1/2*(a
*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(b*x+
a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(194) = 388$.

Time = 0.28 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.16

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{3\,(2\,A-B)b^4c^4-8\,(3\,A-2\,B)ab^3c^3d+36\,(A-B)a^2b^2c^2d^2-24\,(A-2\,B)a^3bcd^3+(6\,A-25\,B)a^4d^4}{24\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^2
*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*
c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5
*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*
b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4
*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^
4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.

Time = 2.59 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4} + \frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} - \frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} + \frac{-6Aa^3d^3+18Aa^2bcd^2-18Aab^2c^2d+}{24a^7bd^3g^5-72a^6b^2cd^2g^5+72a^5b^3c^2dg^5-24a^4b^4c^3g^5+x^4\cdot(24a^3b^5d^3g^5-72a^2b^6cd^2g^5+72ab^7c^2dg^5-}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*l
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) - B*d**4
```

```
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 + 25*B
*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B
*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*
b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72*a
**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5 +
x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*d*
g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*c*
d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*a*
*5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5 -
 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d**
2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.22 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{24}\,B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-2\cdots}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6\,(\cdots}\right.$$

$$-\frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] -1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b
*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
```

```
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(194) = 388$.

Time = 0.41 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.04

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)} - \frac{Bd^3}{2\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg} + \frac{Bd^2}{4\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2} - \frac{B\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)}{4\,(bgx+ag)^4bg} - \frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2} - \frac{2\,Ab^3g^3-Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] 1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
- 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d
^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
 a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^
2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*e*g^2/(b*g*x + a*g)^2
- 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 + 2*b*c*d
*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/((b*g*x + a*g)
^4*b*g) - 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 -
B*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4}$$
$$-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}$$
$$-\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b
^2*c*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4) - (B*log((
e*(c + d*x)^2)/(a + b*x)^2))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*
b*x^2 + 4*a*b^2*x^3)) - ((6*A*a^3*d^3 - 6*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 18*A*a*b^2*c^2*d - 18*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b
^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)
```

3.210 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 515

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2} \\
&\quad - \frac{10B^2(bc-ad)^5g^4\log(a+bx)}{3bd^5} - \frac{26B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5} \\
&\quad + \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3} \\
&\quad - \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2} \\
&\quad + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd} \\
&\quad - \frac{4B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5d^5} \\
&\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} \\
&\quad - \frac{4B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&\quad + \frac{8B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

```
[Out] 26/15*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/15*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
2/15*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-10/3*B^2*(-a*d+b*c)^5*g^4*ln(b*x+
```

```
a)/b/d^5-26/15*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+2/5*B*(-a*d+b
*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^3-4/15*B*(-a*d+b*c)
^2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^2+1/5*B*(-a*d+b*c)*g^4
*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/5*B*(-a*d+b*c)^4*g^4*(d*x+
c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))^2/b-4/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln
(1-d*(b*x+a)/b/(d*x+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/
b/(d*x+c))/b/d^5
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= -\frac{4Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^5} \\
&\quad -\frac{4Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5d^5} \\
&\quad +\frac{2Bg^4(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^3} \\
&\quad -\frac{4Bg^4(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{15bd^2} \\
&\quad +\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd} \\
&\quad +\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b}+\frac{8B^2g^4(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&\quad -\frac{10B^2g^4(bc-ad)^5\log(a+bx)}{3bd^5}-\frac{26B^2g^4(bc-ad)^5\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5} \\
&\quad +\frac{26B^2g^4x(bc-ad)^4}{15d^4}-\frac{7B^2g^4(a+bx)^2(bc-ad)^3}{15bd^3}+\frac{2B^2g^4(a+bx)^3(bc-ad)^2}{15bd^2}
\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (26*B^2*(b*c - a*d)^4*g^4*x)/(15*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^
2)/(15*b*d^3) + (2*B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (10*B^2*
(b*c - a*d)^5*g^4*Log[a + b*x])/(3*b*d^5) - (26*B^2*(b*c - a*d)^5*g^4*Log[(
c + d*x)/(a + b*x)])/(15*b*d^5) + (2*B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d^3) - (4*B*(b*c - a*d)^2*g^4*(a +
```

```
 b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(15*b*d^2) + (B*(b*c - a*
d)*g^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d) - (4*B
*(b*c - a*d)^4*g^4*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*d
^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(5*b) -
(4*B*(b*c - a*d)^5*g^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*
(a + b*x))/(b*(c + d*x))]))/(5*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
```

```
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\left((bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{(d-bx)^6}\,dx, x, \frac{c+dx}{a+bx}\right)\right)\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} + \frac{\left(4B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)}{5b}\\
&= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b}\\
&\quad + \frac{\left(4B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d}\\
&\quad + \frac{\left(4B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd}\\
&= \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd}\\
&\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b}\\
&\quad + \frac{\left(4B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d^2}\\
&\quad + \frac{\left(4B(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd^2}\\
&\quad - \frac{\left(2B^2(bc-ad)^5g^4\right)\text{Subst}\left(\int \frac{1}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd}
\end{aligned}
$$

$$= -\frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} + \frac{(4B(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{5d^3} + \frac{(4B(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^3} - \frac{(8B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{1}{x(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{15bd^2} - \frac{(2B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\left(\frac{1}{d^4x}+\frac{b}{d(d-bx)^4}+\frac{b}{d^2(d-bx)^3}+\frac{b}{d^3(d-bx)^2}+\frac{b}{d^4(d-bx)}\right)dx,x,\frac{c+dx}{a+bx}\right)}{5bd}$$

$$= \frac{2B^2(bc-ad)^4g^4x}{5d^4} - \frac{B^2(bc-ad)^3g^4(a+bx)^2}{5bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{2B^2(bc-ad)^5g^4\log(a+bx)}{5bd^5} - \frac{2B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{5bd^5} + \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3} - \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} + \frac{(4B(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5d^4} + \frac{(4B(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^4} - \frac{(4B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{1}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{5bd^3} - \frac{(8B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\left(\frac{1}{d^3x}+\frac{b}{d(d-bx)^3}+\frac{b}{d^2(d-bx)^2}+\frac{b}{d^3(d-bx)}\right)dx,x,\frac{c+dx}{a+bx}\right)}{15bd^2}$$

$$= \frac{14B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2}$$
$$- \frac{14B^2(bc-ad)^5g^4\log(a+bx)}{15bd^5} - \frac{14B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5}$$
$$+ \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3}$$
$$- \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2}$$
$$+ \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd}$$
$$- \frac{4B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5d^5}$$
$$+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b}$$
$$- \frac{4B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$
$$- \frac{(8B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{1}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{5d^5}$$
$$+ \frac{(8B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{5bd^5}$$
$$- \frac{(4B^2(bc-ad)^5g^4)\,\text{Subst}\left(\int\left(\frac{1}{d^2x}+\frac{b}{d(d-bx)^2}+\frac{b}{d^2(d-bx)}\right)dx, x, \frac{c+dx}{a+bx}\right)}{5bd^3}$$

$$
\begin{aligned}
&= \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2} \\
&- \frac{10B^2(bc-ad)^5g^4\log(a+bx)}{3bd^5} - \frac{26B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5} \\
&+ \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3} \\
&- \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2} \\
&+ \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd} \\
&- \frac{4B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5d^5} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} \\
&- \frac{4B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&+ \frac{8B^2(bc-ad)^5g^4\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02

$$
\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx
$$

$$
= \frac{g^4\left(\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{B(bc-ad)\left(12Abd(bc-ad)^3x+24B(bc-ad)^4\log(c+dx)-4B(bc-ad)^2\left(2bd(bc-ad)x-d^2\right.\right.}{}\right.\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[c + d*x] - 4*B*(b*c -
 a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3
*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/
```

```
(a + b*x)^2]) - 12*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
 + b*x)^2]) - 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

Maple [F]

$$\int (bgx + ag)^4 \left(A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*e*x^2 + 2*c
*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. 2(490) = 980.

Time = 0.38 (sec) , antiderivative size = 2660, normalized size of antiderivative = 5.17

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*
x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/
b + 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
+ a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x
/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*
c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^
3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(d^2*e*x^2/(b^2
*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^
2 + 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (
3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 +
 A^2*a^4*g^4*x + 2/15*((6*g^4*log(e) - 25*g^4)*b^4*c^5 - (30*g^4*log(e) - 1
13*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(5*g^
4*log(e) - 13*g^4)*a^3*b*c^2*d^3 + 6*(5*g^4*log(e) - 8*g^4)*a^4*c*d^4)*B^2*
log(d*x + c)/d^5 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^
2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^5) + 1/15*(3*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 3*(b^5*c*d^4*g^4*log
(e) + (5*g^4*log(e)^2 - g^4*log(e))*a*b^4*d^5)*B^2*x^4 - 2*((2*g^4*log(e) -
 g^4)*b^5*c^2*d^3 - 2*(5*g^4*log(e) - g^4)*a*b^4*c*d^4 - (15*g^4*log(e)^2 -
 8*g^4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 + ((6*g^4*log(e) - 7*g^4)*b^5*c^3
*d^2 - 3*(10*g^4*log(e) - 9*g^4)*a*b^4*c^2*d^3 + 3*(20*g^4*log(e) - 11*g^4)
*a^2*b^3*c*d^4 + (30*g^4*log(e)^2 - 36*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^
2*x^2 - (2*(6*g^4*log(e) - 13*g^4)*b^5*c^4*d - 2*(30*g^4*log(e) - 59*g^4)*a
*b^4*c^3*d^2 + 12*(10*g^4*log(e) - 17*g^4)*a^2*b^3*c^2*d^3 - 2*(60*g^4*log(
e) - 79*g^4)*a^3*b^2*c*d^4 - (15*g^4*log(e)^2 - 48*g^4*log(e) + 46*g^4)*a^4
*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*
```

```
a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x +
B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5
*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*
a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4
 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 - 2*(6*B
^2*b^5*d^5*g^4*x^5*log(e) + 3*(b^5*c*d^4*g^4 + (10*g^4*log(e) - g^4)*a*b^4*
d^5)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - (15*g^4*log(e) - 4*
g^4)*a^2*b^3*d^5)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a
^2*b^3*c*d^4*g^4 + 2*(5*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 6*(2*b^5
*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d
^4*g^4 - (5*g^4*log(e) - 8*g^4)*a^4*b*d^5)*B^2*x - (12*a*b^4*c^4*d*g^4 - 54
*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 - (6*g^4
*log(e) - 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) + 2*(6*B^2*b^5*d^5*g^4*x^5*log
(e) + 3*(b^5*c*d^4*g^4 + (10*g^4*log(e) - g^4)*a*b^4*d^5)*B^2*x^4 - 4*(b^5*
c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - (15*g^4*log(e) - 4*g^4)*a^2*b^3*d^5)*B^2*
x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 2*(
5*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*
c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 - (5*g^4*log(e)
 - 8*g^4)*a^4*b*d^5)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*
x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b
*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^4\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 422

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= -\frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2} + \frac{11B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4} \\
&\quad + \frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} \\
&\quad + \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad + \frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} \\
&\quad - \frac{2B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}
\end{aligned}$$

```
[Out] -5/3*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+11
/3*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/3*B^2*(-a*d+b*c)^4*g^3*ln((d*x+c)
/(b*x+a))/b/d^4-1/2*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))/b/d^2+1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/
b/d+B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+B*(-a*d+b*c)^4*g^3*(A+B*ln(e*
(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-2*B^2*(-a*d+b*c)^4*g^
3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^4} + \frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^4} - \frac{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2bd^2} + \frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd} + \frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b} - \frac{2B^2g^3(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} + \frac{11B^2g^3(bc-ad)^4\log(a+bx)}{3bd^4} + \frac{5B^2g^3(bc-ad)^4\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4} - \frac{5B^2g^3x(bc-ad)^3}{3d^3} + \frac{B^2g^3(a+bx)^2(bc-ad)^2}{3bd^2}$$

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (-5*B^2*(b*c - a*d)^3*g^3*x)/(3*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/
(3*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*Log[a + b*x])/(3*b*d^4) + (5*B^2*(b*c
 - a*d)^4*g^3*Log[(c + d*x)/(a + b*x)])/(3*b*d^4) - (B*(b*c - a*d)^2*g^3*(a
 + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b*d^2) + (B*(b*c - a
*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d) + (B*
(b*c - a*d)^3*g^3*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/d^4 +
 (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(4*b) + (B*(b
*c - a*d)^4*g^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*
x))/(b*(c + d*x))])/(b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 g^3\right) \text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{(d-bx)^5}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)}{d} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{bd}$$

$$= \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{d^2} - \frac{\left(B(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{bd^2} + \frac{\left(2B^2(bc-ad)^4 g^3\right)\text{Subst}\left(\int \frac{1}{x(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)}{3bd}$$

$$= -\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} + \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{(B(bc-ad)^4g^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex^2)}{(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{d^3} - \frac{(B(bc-ad)^4g^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)}\,dx,x,\frac{c+dx}{a+bx}\right)}{bd^3} + \frac{(B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\frac{1}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{bd^2} + \frac{(2B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\left(\frac{1}{d^3x}+\frac{b}{d(d-bx)^3}+\frac{b}{d^2(d-bx)^2}+\frac{b}{d^3(d-bx)}\right)dx,x,\frac{c+dx}{a+bx}\right)}{3bd}$$

$$= -\frac{2B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2} + \frac{2B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4} + \frac{2B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} + \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} + \frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} + \frac{(2B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\frac{1}{d-bx}\,dx,x,\frac{c+dx}{a+bx}\right)}{d^4} - \frac{(2B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx,x,\frac{c+dx}{a+bx}\right)}{bd^4} + \frac{(B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\left(\frac{1}{d^2x}+\frac{b}{d(d-bx)^2}+\frac{b}{d^2(d-bx)}\right)dx,x,\frac{c+dx}{a+bx}\right)}{bd^2}$$

$$= -\frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2} + \frac{11B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4} + \frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} + \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} + \frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} + \frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} - \frac{2B^2(bc-ad)^4g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.95

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+12B(bc-ad)^3\log(c+dx)-2B(bc-ad)\left(2bd(bc-ad)x-d^2(a\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 12*B*(b*c - a*d)^3*Log[c + d*x] - 2*B*(b*c
- a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x
]) - 6*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*d*(b*c -
 a*d)^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 3*d^2*(-(b*c) + a*d)*(
a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^3*(a + b*x)^3*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)]))/(3*d^4)))/(4*b)
```

Maple [F]

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx + ag)^3 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2
*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*
log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. 2(407) = 814.

Time = 0.35 (sec) , antiderivative size = 1950, normalized size of antiderivative = 4.62

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*
x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a
*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2
*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/3*((3*g^3*log(e) - 11*g^3)*b^3*c^4 - 2
*(6*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) - 5*g^3)*a^2*b*c^2*d
^2 - 6*(2*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*
g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d
^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 4*(
b^4*c*d^3*g^3*log(e) + (3*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - 2
*((3*g^3*log(e) - 2*g^3)*b^4*c^2*d^2 - 4*(3*g^3*log(e) - g^3)*a*b^3*c*d^3 -
 (9*g^3*log(e)^2 - 9*g^3*log(e) + 2*g^3)*a^2*b^2*d^4)*B^2*x^2 + 4*((3*g^3*l
og(e) - 5*g^3)*b^4*c^3*d - (12*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (18*g^3
*log(e) - 19*g^3)*a^2*b^2*c*d^3 + (3*g^3*log(e)^2 - 9*g^3*log(e) + 7*g^3)*a
^3*b*d^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2
*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a
)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4
*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2
*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - 4*(3*B^2*b^4*d^
4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^3*log(e) - g^3)*a*b^3*d^4)*B^2*x
^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(2*g^3*log(e) - g^3)*a^2*b^
2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g
^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*
b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 + (3*g^3*log(e) - 11*g^3)*a^4*d^4)*B^2
```

```
)*log(b*x + a) + 4*(3*B^2*b^4*d^4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^
3*log(e) - g^3)*a*b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
 - 3*(2*g^3*log(e) - g^3)*a^2*b^2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3
*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*
x - 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c)
)/(b*d^4)
```

Giac [F]

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 343

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx &= \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{4B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\ &\quad + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\ &\quad - \frac{4B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3d^3} \\ &\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} \\ &\quad - \frac{4B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\ &\quad + \frac{8B^2(bc-ad)^3g^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-4*B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-4/3*B
^2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+2/3*B*(-a*d+b*c)*g^2*(b*x+a)^
2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)
^2))^2/b-4/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x
+a)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c)
)/b/d^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\begin{aligned}&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx\\&=-\frac{4Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd^3}\\&-\frac{4Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3d^3}\\&+\frac{2Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd}\\&+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b}+\frac{8B^2g^2(bc-ad)^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\\&-\frac{4B^2g^2(bc-ad)^3\log(a+bx)}{bd^3}-\frac{4B^2g^2(bc-ad)^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3}+\frac{4B^2g^2x(bc-ad)^2}{3d^2}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (4*B^2*(b*c - a*d)^3*g^2*Log[a + b*x]
)/(b*d^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(c + d*x)/(a + b*x)])/(3*b*d^3) +
(2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/
(3*b*d) - (4*B*(b*c - a*d)^2*g^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2]))/(3*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])^2)/(3*b) - (4*B*(b*c - a*d)^3*g^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3) + (8*B^2*(b*c - a*d)^3*g
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(\left((bc-ad)^3g^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{(d-bx)^4}\,dx, x, \frac{c+dx}{a+bx}\right)\right)$$

$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b}+\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{3b}$$

$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b}+\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(d-bx)^3}\,dx,x,\frac{c+dx}{a+bx}\right)}{3d}+\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{3bd}$$

$$=\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b}+\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{3d^2}+\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x(d-bx)}\,dx,x,\frac{c+dx}{a+bx}\right)}{3bd^2}-\frac{(4B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{1}{x(d-bx)^2}\,dx,x,\frac{c+dx}{a+bx}\right)}{3bd}$$

$$=\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd}-\frac{4B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3d^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b}-\frac{4B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}-\frac{(8B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{1}{d-bx}\,dx,x,\frac{c+dx}{a+bx}\right)}{3d^3}+\frac{(8B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx,x,\frac{c+dx}{a+bx}\right)}{3bd^3}-\frac{(4B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\left(\frac{1}{d^2x}+\frac{b}{d(d-bx)^2}+\frac{b}{d^2(d-bx)}\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{3bd}$$

$$
\begin{aligned}
&= \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{4B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\
&\quad + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad - \frac{4B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3d^3} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} \\
&\quad - \frac{4B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{8B^2(bc-ad)^3g^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.87

$$
\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx
$$

$$
= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{2B(bc-ad)\left(2Abd(bc-ad)x+4B(bc-ad)^2\log(c+dx)-2B(bc-ad)(bdx+(-bc+ad)\log(c\right.}{}\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*B*(b*c - a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2]) - 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*
x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^
2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. 2(328) = 656.

Time = 0.34 (sec) , antiderivative size = 1333, normalized size of antiderivative = 3.89

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
+ a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x
 + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(d^2*e*x^2/(b^
2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x
^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x + 4/3*((g^2*log(e) - 3*g^2)*b^2*c^3 - (3*g^2*log(e) - 7*g^2)*
a*b*c^2*d + (3*g^2*log(e) - 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 8/3*(b
^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*b^3*c*d^2*g^2*log(e)
 + (3*g^2*log(e)^2 - 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 - (4*(g^2*log(e) - g^
2)*b^3*c^2*d - 4*(3*g^2*log(e) - 2*g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 8*g
^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*
d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(
B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^
3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - 4*
(B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (3*g^2*log(e) - g^2)*a*b^2*d
^3)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 4*g^2)
*a^2*b*d^3)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (g^2*log(e) -
3*g^2)*a^3*d^3)*B^2)*log(b*x + a) + 4*(B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*
d^2*g^2 + (3*g^2*log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*
b^2*c*d^2*g^2 - (3*g^2*log(e) - 4*g^2)*a^2*b*d^3)*B^2*x - 2*(B^2*b^3*d^3*g^
2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*
log(b*x + a))*log(d*x + c))/(b*d^3)
```

Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\int (bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
[Out] integrate((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.213 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 211

$$\begin{aligned}&\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx\\ &=\frac{4B^2(bc-ad)^2g\log(a+bx)}{bd^2}+\frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2}\\ &\quad+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b}\\ &\quad+\frac{2B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\\ &\quad-\frac{4B^2(bc-ad)^2g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

```
[Out] 4*B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+2*B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*
x+c)^2/(b*x+a)^2))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+
2*B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c)
)/b/d^2-4*B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31}

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\frac{2Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^2}$$
$$+\frac{2Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b}$$
$$-\frac{4B^2g(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{4B^2g(bc-ad)^2\log(a+bx)}{bd^2}$$

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g*Log[a + b*x])/(b*d^2) + (2*B*(b*c - a*d)*g*(c + d*x)
*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/d^2 + (g*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*b) + (2*B*(b*c - a*d)^2*g*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) -
 (4*B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^2 g\right) \text{Subst}\left(\int \frac{\left(A+B\log\left(ex^2\right)\right)^2}{(d-bx)^3}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{\left(2B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log\left(ex^2\right)}{x(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{\left(2B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log\left(ex^2\right)}{(d-bx)^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{d} - \frac{\left(2B(bc-ad)^2 g\right)\text{Subst}\left(\int \frac{A+B\log\left(ex^2\right)}{x(d-bx)}\,dx, x, \frac{c+dx}{a+bx}\right)}{bd}$$

$$= \frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} + \frac{2B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{(4B^2(bc-ad)^2g)\operatorname{Subst}\left(\int\frac{1}{d-bx}\,dx,x,\frac{c+dx}{a+bx}\right)}{d^2} - \frac{(4B^2(bc-ad)^2g)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{bx}\right)}{x}\,dx,x,\frac{c+dx}{a+bx}\right)}{bd^2}$$

$$= \frac{4B^2(bc-ad)^2g\log(a+bx)}{bd^2} + \frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} + \frac{2B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{4B^2(bc-ad)^2g\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$

$$= \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{4B(bc-ad)\left(Abdx+B(bc-ad)\log^2(c+dx)+Bd(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-(bc-ad)\log(c+dx)\right.}{d^2}\right.}{2b}$$

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(b*c - a*
d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*x)
^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2*a*B
*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2))/(2*b)
```

Maple [F]

$$\int (bgx + ag)\left(A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d
^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(208) = 416$.

Time = 0.32 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.46

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{1}{2}A^2bgx^2$$
$$+2\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log(bx+a)}{b}+\frac{2\,c\log(dx\right.$$
$$+\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{2\,a^2\log(bx+a)}{b^2}-\frac{2\,c^2\log(dx\right.$$
$$+A^2agx-\frac{2\,((g\log(e)-2\,g)bc^2-2\,(g\log(e)-g)acd)B^2\log(dx+c)}{d^2}$$
$$+\frac{4\,(b^2c^2g-2\,abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(2\,b^2cdg\log(e)+\left(g\log(e)^2-2\,g\log(e)\right)abd^2\right)B^2x+4\,(B^2b^2d^2gx^2+2\,B^2abd^2gx}{}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x
+ a)/b + 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*
x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x +
 a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x - 2*((g*log(e) - 2*g)*b*c^2 - 2*(g*log(e) - g)*a
*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*b^2*c*d*g*log(e)
+ (g*log(e)^2 - 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 4*(B^2*b^2*d^2
*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) -
 2*g)*a^2*d^2 + 2*a*b*c*d*g)*B^2)*log(b*x + a) + 4*(B^2*b^2*d^2*g*x^2*log(e
) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - 2*(B^2*b^2*d^2*g*x^2 + 2
*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
[Out] integrate((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg}-\frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{8B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g-4*B*(A+B*ln(
e*(d*x+c)^2/(b*x+a)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(3,
b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{2552, 2354, 2421, 6724\}$

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{4B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{bg} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])^2)/(b*g)) - (4*B*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))])/(b*g) + (8*B^2*PolyLog[3, (b*(c + d*x))/(d*(a +
b*x))])/(b*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{g}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg} + \frac{(4B)\text{Subst}\left(\int \frac{(A+B\log(ex^2))\log\left(1-\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg} - \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{(8B^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bg}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg} - \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right) + A^2\log(a+bx) + 4AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right) - 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (2*A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] + 4*A*B*Log[(
-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2] - B^2*Log[(-(b*
c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2]^2 - 4*A*B*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)] - 4*B^2*Log[(e*(c + d*x)^2)/(a + b*x)^2]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*x))/(d
*(a + b*x))])/(b*g)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}{bgx+ag}dx$$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
)^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
+ A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2}\right.}{a+bx}}{g}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c**2*e/(a**2 + 2*a*b*x + b
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log(b
*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - 2*(2*B^2*b*d*x + (b*c + a*d)*B^2)*l
og(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x), x)
```

3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 157

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] 4*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+4
*B^2*(d*x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*
ln(e*(d*x+c)^2/(b*x+a)^2))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2552, 2333, 2332}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)} + \frac{4AB(c+dx)}{g^2(a+bx)(bc-ad)} + \frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)} - \frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (4*A*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (8*B^2*(c + d*x))/((b*c - a
*d)*g^2*(a + b*x)) + (4*B^2*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b
*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^
2])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \left(A+B\log\left(ex^2\right)\right)^2\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\
&= -\frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)} + \frac{(4B)\text{Subst}\left(\int \left(A+B\log\left(ex^2\right)\right)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\
&= \frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}\\
&\quad + \frac{\left(4B^2\right)\text{Subst}\left(\int \log\left(ex^2\right)\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}\\
&= \frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)}\\
&\quad + \frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-(bc-ad)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-d(a+bx)}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(2*B*(b*c - a*d + d*(a
 + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - (b*c - a*d)*(A + B*Log[(
e*(c + d*x)^2)/(a + b*x)^2]) - d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{\left(A^2-4BA+8B^2\right)x}{ga}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{2(A-2B)cB\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{2d(A-2B)Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2A^2ab^2d^2-2A^2b^3cd+16B^2ab^2d^2-16B^2b^3cd-2B^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2b^3d^2+8B^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-2B^2\ln\left(\frac{e(dx+c)}{(bx+a)^2}\right.}{2g^2(bx+a)b^3d}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}-\frac{2AB}{}$
derivativedivides	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}-}{}$
default	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}-}{}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}-\frac{2AB}{}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] ((A^2-4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)^2+B^2*
d/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^2)^2+2*(A-2*B)*c*B/g/(a*d-b*c)*ln(e*
(d*x+c)^2/(b*x+a)^2)+2*d*(A-2*B)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^2))
/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{(A^2-4\,AB+8\,B^2)bc-(A^2-4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\left((AB-2\,B\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 - 4*A*B + 8*B^2)*b*c - (A^2 - 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(
(A*B - 2*B^2)*b*d*x + (A*B - 2*B^2)*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e
)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d
)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(134) = 268$.

Time = 1.20 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd-\frac{4Ba^2d^3(A-2B)}{ad-bc}+\frac{8Babcd^2(A-2B)}{ad-bc}-\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)} - \frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd+\frac{4Ba^2d^3(A-2B)}{ad-bc}-\frac{8Babcd^2(A-2B)}{ad-bc}+\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)} + \frac{(-2AB+4B^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2+4AB-8B^2}{abg^2+b^2g^2x}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B**
2*b*c*d - 4*B*a**2*d**3*(A - 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A - 2*B)/(a
*d - b*c) - 4*B*b**2*c**2*d*(A - 2*B)/(a*d - b*c))/(8*A*B*b*d**2 - 16*B**2*
b*d**2))/(b*g**2*(a*d - b*c)) - 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A
*B*b*c*d - 8*B**2*a*d**2 - 8*B**2*b*c*d + 4*B*a**2*d**3*(A - 2*B)/(a*d - b*
c) - 8*B*a*b*c*d**2*(A - 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A - 2*B)/(a*d
- b*c))/(8*A*B*b*d**2 - 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 4
*B**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
+ B**2*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2 +
 a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 + 4*A*B - 8*B**2)/(a*b*g**2 + b**2*
g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(157) = 314.

Time = 0.24 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.65

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$=4\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{}{b^2x^2\cdots}\right.\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log(bx+a)}{(b^2c-abd)g^2}+\frac{2\,d\log(dx\cdots}{(b^2c-abc\cdots}\right.$$
$$-\frac{B^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] 4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2
*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + ((b
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) -
2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(
d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^
2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(157) = 314$.

Time = 0.65 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)^2 - \frac{4\,(ABd-2\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2} - \frac{2\,(AB-2\,B^2)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)}{(bgx+ag)bg} - \frac{A^2-4\,AB+8\,B^2}{(bgx+ag)bg}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log((b^2*c^2*e*g
^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x
 + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/
b^2)^2 - 4*(A*B*d - 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g)
+ d)/(b^2*c*g^2 - a*b*d*g^2) - 2*(A*B - 2*B^2)*log((b^2*c^2*e*g^2/(b*g*x +
a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 +
2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/((b*g*x
 + a*g)*b*g) - (A^2 - 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(\frac{4\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{A^2-4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) + \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-2\,B)\,8\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((4*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2
)))/(x/d + a/(b*d)) - (A^2 + 8*B^2 - 4*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*
(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d
- b*c))) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d
- b*c))*(A - 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 299

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}-\frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

```
[Out] -4*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-4*B^2*d*(d*x+c)*ln(e*(d*
x+c)^2/(b*x+a)^2)/(-a*d+b*c)^2/g^3/(b*x+a)+b*B*(d*x+c)^2*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2552, 2367, 2333, 2332, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2} - \frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{4ABd(c+dx)}{g^3(a+bx)(bc-ad)^2} - \frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^3(a+bx)(bc-ad)^2} - \frac{bB^2(c+dx)^2}{g^3(a+bx)^2(bc-ad)^2} + \frac{8B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-4*A*B*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) + (8*B^2*d*(c + d*x))/((
b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/((b*c - a*d)^2*g^3*(a + b
*x)^2) - (4*B^2*d*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b*c - a*d)^
2*g^3*(a + b*x)) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/((b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(c + d*x)^2*(A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (d-bx)\left(A+B\log\left(ex^2\right)\right)^2\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= \frac{\text{Subst}\left(\int\left(d\left(A+B\log\left(ex^2\right)\right)^2-bx\left(A+B\log\left(ex^2\right)\right)^2\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= -\frac{b\text{Subst}\left(\int x\left(A+B\log\left(ex^2\right)\right)^2\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}+\frac{d\text{Subst}\left(\int\left(A+B\log\left(ex^2\right)\right)^2\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&= \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)}-\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\\
&\quad+\frac{(2bB)\text{Subst}\left(\int x\left(A+B\log\left(ex^2\right)\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}\\
&\quad-\frac{(4Bd)\text{Subst}\left(\int\left(A+B\log\left(ex^2\right)\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}
\end{aligned}
$$

$$= -\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2} - \frac{(4B^2d)\,\mathrm{Subst}\left(\int\log\left(ex^2\right)\,dx,x,\frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)} + \frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2} - \frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{2B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(4*B*d*(a + b*x)*(b
*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c -
 a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2
*d^2*(a + b*x)^2*Log[c + d*x]) + (b*c - a*d)^2*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)/(a +
 b*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2]) + 2*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.64

method	result
norman	$\frac{\frac{\left(A^2ad-A^2bc-4ABad+2ABbc+8B^2ad-2B^2bc\right)x}{ag(ad-cb)}+\frac{Bc(2Aad-Abc-4Bad+Bbc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bBd\ldots}{\ldots}}{\ldots}$
derivativedivides	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(ad-cb)(bx+a)}+\frac{3B^2d^2\ln\left(\ldots\right)}{g^3\left(a\ldots\right)}}{\ldots}$
default	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(ad-cb)(bx+a)}+\frac{3B^2d^2\ln\left(\ldots\right)}{g^3\left(a\ldots\right)}}{\ldots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{(4ad\ldots}{\ldots}}{g^2\ldots}$
parallelrisch	$-\frac{-2A^2ab^4cd^2-6ABa^2b^3d^3-2ABb^5c^2d-16B^2ab^4cd^2-2B^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5c^2d-4ABx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab^4d^3-4AB\ldots}{\ldots}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{(4ad\ldots}{\ldots}}{g^2\ldots}$

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE
)

[Out] ((A^2*a*d-A^2*b*c-4*A*B*a*d+2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x+
B*c*(2*A*a*d-A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d*x+c
)^2/(b*x+a)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)^2/(b*
x+a)^2)^2+b*B/g*d^2*(A-3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(d*x+c)^2/
(b*x+a)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d*x+c)
^2/(b*x+a)^2)^2+1/2*(A^2*a*d-A^2*b*c-6*A*B*a*d+2*A*B*b*c+14*B^2*a*d-2*B^2*b
*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*b*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c
^2)/g*x^2*ln(e*(d*x+c)^2/(b*x+a)^2)^2)/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.38

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(A^2-2\,AB+2\,B^2)b^2c^2-2\,(A^2-4\,AB+8\,B^2)abcd+(A^2-6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2ab}{}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A^
2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x
 + a^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((A*B
- 3*B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d - 2*(B
^2*b^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/
(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*
b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. 2(279) = 558.

Time = 2.19 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.93

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2-\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A-\dots}{(ad-bc)^2}}{4ABbd^3-12B^2bd^3}\right.}{bg^3(ad-bc)^2} - \frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2+\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2\dots}{(ad-\dots}}{4ABbd^3-12B^2bd^3}\right.}{bg^3(ad-bc)^2} + \frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+\dots} + \frac{(-ABad+ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a^3bdg^3-a^2b^2cg^3+2a^2b^2dg^3x-2ab^3cg^3x+ab^3dg^3x^2-b^4cg^3x^2} + \frac{-A^2ad+A^2bc+6ABad-2ABbc-14B^2ad+2B^2bc+x(4ABbd-12B^2bd)}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x(4a^2b^2dg^3-4ab^3cg^3)}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 -
 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d*
*4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2
 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d
**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 +
2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B)/
(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c*
*2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)
**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c
*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c + d*x)**2/(
a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*
x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*
x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**
2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*log(e*(c +
 d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g*
*3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*
a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(4*A*
B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**
3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(297) = 594$.

Time = 0.28 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.35

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\left(\left(\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,d^2}{(b^3c^2 - 2}\right.\right.\right.$$

$$-AB\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{}{b^2}\right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3}\right.$$

$$-\frac{B^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] -((((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)*g^3))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*a*b*c
*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^
2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*
x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g
^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g
^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d*x - b*c + 3
*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2
*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2
*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3
)) - 1/2*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^3*g^3*x^2 + 2*a*b^2
*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="giac")

[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^3, x)

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.69

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right) - \frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,B^2\,b\,d-A\,B\,b\,d)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} - \frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(A-3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2-2\,A\,B\,d^2)}\right)(A-3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^3,x)

[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d - A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*d^2*atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A - 3*B)*2i)/((a*d - b*c)*(6*B^2*d^2 - 2*A*B*d^2)))*(A - 3*B)*4i)/(b*g^3*(a*d - b*c)^2)

3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 407

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =& -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2}\\
&-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}\\
&+\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)}\\
&-\frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}\\
&+\frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}\\
&-\frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4}\\
&-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}\end{aligned}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+4/3*B^2*d
^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+4*B*d^2*(d*x+c)*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)-2*b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2+4/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(
```

```
d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-4/3*B*d^3*ln((d*x+c)/(b*x+a
))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)
^2/(b*x+a)^2))^2/b/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2552, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3} + \frac{4Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3} - \frac{2bBd(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3} - \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3bg^4(a+bx)^3} - \frac{8b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3} + \frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3bg^4(bc-ad)^3} - \frac{8B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3} + \frac{2bB^2d(c+dx)^2}{g^4(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B^2*d*(c + d*x)
^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (8*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) + (4*B^2*d^3*Log[(c + d*x)/(a + b*x)]^2)/(3*b*(b*c -
a*d)^3*g^4) + (4*B*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/
((b*c - a*d)^3*g^4*(a + b*x)) - (2*b*B*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x
)^2)/(a + b*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)^2) + (4*b^2*B*(c + d*x)^3*
(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3)
 - (4*B*d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(3
*b*g^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (d-bx)^2 \left(A+B\log\left(ex^2\right)\right)^2 \, dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^3 g^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} - \frac{(4B)\text{Subst}\left(\int \frac{(d-bx)^3\left(A+B\log\left(ex^2\right)\right)}{x} \, dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3 g^4}$$

$$= \frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}$$
$$+ \frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}$$
$$- \frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}$$
$$+ \frac{(8B^2)\,\mathrm{Subst}\left(\int\left(-\frac{1}{6}b(18d^2-9bdx+2b^2x^2)+\frac{d^3\log(x)}{x}\right)dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}$$

$$= \frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}$$
$$+ \frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}$$
$$- \frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}$$
$$- \frac{(4B^2)\,\mathrm{Subst}\left(\int(18d^2-9bdx+2b^2x^2)\,dx, x, \frac{c+dx}{a+bx}\right)}{9(bc-ad)^3g^4}$$
$$+ \frac{(8B^2d^3)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}$$

$$= -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}$$
$$+ \frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4} + \frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)}$$
$$- \frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} + \frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}$$
$$- \frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{-9\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{2B\left(6A(bc-ad)^3-4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx)^2+\cdots\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]

[Out] (-9*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(6*A*(b*c - a*d)^3 -
4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*(a
 + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a + b
*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a + b*
x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 18*A*d^3*(a + b*x)^3*Log[c + d*x
] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 - 36
*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*B*(b*c -
 a*d)^3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 9*B*d*(b*c - a*d)^2*(a + b*x)*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(c
 + d*x)^2)/(a + b*x)^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(c + d*x
)^2)/(a + b*x)^2] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c + d*x)^2)/(
a + b*x)^2] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
 - 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)
^3)/(27*b*g^4*(a + b*x)^3)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.71

method	result
derivativedivides	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(ad-cb)(bx+a)^2}+\frac{\cdots}{9g^4(\cdots}\cdots}{\cdots}$
default	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(ad-cb)(bx+a)^2}+\frac{\cdots}{9g^4(\cdots}\cdots}{\cdots}$
parts	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display
risch	Expression too large to display

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b*(1/3/g^4*A^2/(b*x+a)^3+8/27/g^4*B^2/(b*x+a)^3-4/9/g^4*B^2/(b*x+a)^3*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/3/g^4*B^2/(b*x+a)^3*ln(e*(a*d/(b*x+
a)-b*c/(b*x+a)-d)^2/b^2)^2+10/9/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2+44/9/g^4*B^2*
d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+22/9/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/3/g^4
*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*
c/(b*x+a)-d)^2/b^2)^2-2/3/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b
*c/(b*x+a)-d)^2/b^2)-4/3/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^4*A*B*(1/3/(b*x+a)^3*ln(e*(a*d/(b
*x+a)-b*c/(b*x+a)-d)^2/b^2)-(2/3*a*d-2/3*c*b)*(1/(a*d-b*c)^3*(1/3*a^2*d^2/(
b*x+a)^3-2/3*a*b*c*d/(b*x+a)^3+1/3*b^2*c^2/(b*x+a)^3+1/2*a*d^2/(b*x+a)^2-1/
2*b*c*d/(b*x+a)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d
))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.77

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{(9\,A^2-12\,AB+8\,B^2)b^3c^3-27\,(A^2-2\,AB+2\,B^2)ab^2c^2d+27\,(A^2-4\,AB+8\,B^2)a^2bcd^2-(9\,A^2-66}{}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 - 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 - 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 - 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 - 66*A*B + 170*B^2)*a^3
*d^3 - 12*((3*A*B - 11*B^2)*b^3*c*d^2 - (3*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)
/(b^2*x^2 + 2*a*b*x + a^2))^2 + 6*((3*A*B - 5*B^2)*b^3*c^2*d - 18*(A*B - 3*
B^2)*a*b^2*c*d^2 + (15*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B - 11*B^2)*b^3
*d^3*x^3 + (3*A*B - 2*B^2)*b^3*c^3 - 9*(A*B - B^2)*a*b^2*c^2*d + 9*(A*B - 2
*B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3
*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 3*(A*B - 2*B^2)*a^2*b*d^3)*x)*log((d^
2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*b^
6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5
*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^
4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c
^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. 2(382) = 764.

Time = 13.02 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.84

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*
d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*B*
a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A -
```

```
 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 -
 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b
*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a
*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a**4*d**7
*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)*
*3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c**
3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d -
 b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*
B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3
*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)**2/(a + b*x)*
*2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*
a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*
x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*
d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*
a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g
**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**
2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*a**2*d**2 - 14*B**2*a*b
*c*d + 30*B**2*a*b*d**2*x + 4*B**2*b**2*c**2 - 6*B**2*b**2*c*d*x + 12*B**2*
b**2*d**2*x**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(9*a**5*b*d**2*g**4 - 18*a
**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a
**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*
x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**
2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a*
*2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*
b*c*d - 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b*
*2*c**2 + x**2*(-36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(-90*A*B*a*b*d*
*2 + 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d*
*2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b*
*4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3
*d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**
2*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(397) = 794$.

Time = 0.32 (sec) , antiderivative size = 1576, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] 2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
```

```
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2
*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 + 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) - 3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3
*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^
3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/
((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
 + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.63

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2-66ABa^2d^2+42ABabcd-12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)}+\frac{2x\left(-5cB^2b^2d+49a\cdots\right.}{\cdots}}{x\left(27a^2b^3cg^4-27a^3b^2dg^4\right)-x^2\left(27a^2b^3dg^4-27ab^4cg^4\right)+x^3\left(9b^5cg^4-9ab^4\cdots\right.}$$
$$-\ln\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2\left(\frac{B^2}{3b^2g^4\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)}-\frac{B^2d^3}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)$$
$$-\frac{\ln\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\left(\frac{2AB}{3b^2dg^4}-\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2\left(3a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3\right)}{3bd^4}\right)}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{2B^2d^3x^2\left(\cdots\right.}{3bg^4\left(a^3d^3\cdots\right.}\right.}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}\cdots}$$
$$-\frac{Bd^3\,\mathrm{atan}\left(\frac{Bd^3\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)(3A-11B)\left(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4\right)4i}{bg^4(ad-bc)^3\left(44B^2d^3-12ABd^3\right)}\right)(3A-11\cdots}{9bg^4(ad-bc)^3}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 - 66*A*B*
a^2*d^2 - 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d + 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 15*A*B*a*b*d^2 +
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d - 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*A*B)
/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
```

```
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
- 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 - 12*A*B*d^3)))*(3*A - 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 501

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =& \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} \\ &+ \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} \\ &- \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{b(bc-ad)^4g^5} \\ &- \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} \\ &+ \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} \\ &- \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} \\ &+ \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} \\ &+ \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4g^5} \\ &- \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4}\end{aligned}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-B^2*d^4*ln((d*x+c)/(b*x+a))^2/b/(
-a*d+b*c)^4/g^5-4*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^
4/g^5/(b*x+a)+3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c
)^4/g^5/(b*x+a)^2-4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a
*d+b*c)^4/g^5/(b*x+a)^3+1/4*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))
/(-a*d+b*c)^4/g^5/(b*x+a)^4+B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(
b*x+a)^2))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g^5/(
b*x+a)^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4} - \frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4} - \frac{4Bd^3(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^5(a+bx)(bc-ad)^4} + \frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^5(a+bx)^2(bc-ad)^4} - \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4bg^5(a+bx)^4} - \frac{b^3B^2(c+dx)^4}{8g^5(a+bx)^4(bc-ad)^4} + \frac{8b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{bg^5(bc-ad)^4} + \frac{8B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4} - \frac{3bB^2d^2(c+dx)^2}{g^5(a+bx)^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (8*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/((b*c - a*d)^4*g^5*(a + b*x)^2) + (8*b^2*B^2*d*(c + d*x)^3)/(9*(b*c -
a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(b*c - a*d)^4*g^5*(a + b
```

```
*x)^4) - (B^2*d^4*Log[(c + d*x)/(a + b*x)]^2)/(b*(b*c - a*d)^4*g^5) - (4*B*
d^3*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/((b*c - a*d)^4*g^5*
(a + b*x)) + (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/((b*c - a*d)^4*g^5*(a + b*x)^2) - (4*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(
c + d*x)^2)/(a + b*x)^2]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) + (b^3*B*(c +
d*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*(b*c - a*d)^4*g^5*(a +
b*x)^4) + (B*d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2]))/(b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2
/(4*b*g^5*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
```

```
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (d-bx)^3 \left(A+B\log\left(ex^2\right)\right)^2 \, dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^4 g^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\text{Subst}\left(\int \frac{(d-bx)^4\left(A+B\log\left(ex^2\right)\right)}{x} \, dx, x, \frac{c+dx}{a+bx}\right)}{b(bc-ad)^4 g^5}$$

$$= -\frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4 g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4 g^5(a+bx)^2} - \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4 g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4 g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4 g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} - \frac{(2B^2)\,\text{Subst}\left(\int \left(-4bd^3+3b^2d^2x-\frac{4}{3}b^3dx^2+\frac{b^4x^3}{4}+\frac{d^4\log(x)}{x}\right) dx, x, \frac{c+dx}{a+bx}\right)}{b(bc-ad)^4 g^5}$$

$$= \frac{8B^2d^3(c+dx)}{(bc-ad)^4 g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4 g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4 g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4 g^5(a+bx)^4} - \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4 g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4 g^5(a+bx)^2} - \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4 g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4 g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4 g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} - \frac{(2B^2d^4)\,\text{Subst}\left(\int \frac{\log(x)}{x} \, dx, x, \frac{c+dx}{a+bx}\right)}{b(bc-ad)^4 g^5}$$

$$
\begin{aligned}
&= \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} \\
&\quad + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{b(bc-ad)^4g^5} \\
&\quad - \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} \\
&\quad - \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} \\
&\quad + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.36

$$
\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx
$$

$$
= \frac{-18\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{B\left(18A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+24Ad(-bc+ad)^3(a+bx)+36Ad^2(bc-ad)^2(a\cdots\right.}{}}{}
$$

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-18*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(18*A*(b*c - a*d)^4 -
9*B*(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 24*A*d*(-(b*c) + a*d)^
3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^2*(
a + b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 72*A*d^3*(-(b*c) + a*d)*(a
 + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*Log[a
 + b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 72*A*d^4*(a + b*x)^4*Log[c
+ d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[(d*
(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[c + d*x]
^2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 18
*B*(b*c - a*d)^4*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 24*B*d*(-(b*c) + a*d)^3
*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 36*B*d^2*(b*c - a*d)^2*(a + b
*x)^2*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^3*(-(b*c) + a*d)*(a + b*x)^
3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[
(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]*Log[(e*(c
+ d*x)^2)/(a + b*x)^2] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
)/(b*c - a*d)^4)/(72*b*g^5*(a + b*x)^4)
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{\frac{A^{2}}{4g^{5}(bx+a)^{4}}+\frac{B^{2}}{8g^{5}(bx+a)^{4}}-\frac{B^{2}\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{4g^{5}(bx+a)^{4}}+\frac{B^{2}\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)^{2}}{4g^{5}(bx+a)^{4}}+\frac{25B^{2}d^{3}}{6g^{5}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-\right.}}{}$
default	$-\frac{\frac{A^{2}}{4g^{5}(bx+a)^{4}}+\frac{B^{2}}{8g^{5}(bx+a)^{4}}-\frac{B^{2}\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{4g^{5}(bx+a)^{4}}+\frac{B^{2}\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)^{2}}{4g^{5}(bx+a)^{4}}+\frac{25B^{2}d^{3}}{6g^{5}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-\right.}}{}$
parts	Expression too large to display
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE
)

[Out] -1/b*(1/4/g^5*A^2/(b*x+a)^4+1/8/g^5*B^2/(b*x+a)^4-1/4/g^5*B^2/(b*x+a)^4*ln(
e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/4/g^5*B^2/(b*x+a)^4*ln(e*(a*d/(b*x+a
)-b*c/(b*x+a)-d)^2/b^2)^2+25/6/g^5*B^2*d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/(b*x+a)+7/18/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3+13/12/g^5*d^2*B^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2+25/12/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-
d)^2/b^2)-1/4/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2-1/g^5*B^2*d^3/(a^3
*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-1/3/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x
+a)-d)^2/b^2)-1/2/g^5*d^2*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2*ln(e*(a
*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^5*A*B*(1/4/(b*x+a)^4*ln(e*(a*d/(b*x+a)
-b*c/(b*x+a)-d)^2/b^2)-(1/2*a*d-1/2*c*b)*(1/(a*d-b*c)^4*(1/4*(a*d-b*c)*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)
^3+1/2*(a*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*ln(a*d/(b*x+a)-
b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(491) = 982.

Time = 0.29 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.17

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{9\,(2\,A^2-2\,AB+B^2)b^4c^4-8\,(9\,A^2-12\,AB+8\,B^2)ab^3c^3d+108\,(A^2-2\,AB+2\,B^2)a^2b^2c^2d^2-72\,(A}{}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^2)*
b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((6*A*B - 13*B^2)*b^4*c^2*d
^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)*x^2
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a
^2))^2 + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c^2*d^2 +
108*(A*B - 3*B^2)*a^2*b^2*c*d^3 - (78*A*B - 271*B^2)*a^3*b*d^4)*x - 6*((6*A
*B - 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 + 8*(3*A*B - 2*B^2)*a*b^
3*c^3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3*b*c*d^3 - 4
*(3*B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2
- 8*B^2*a*b^3*c*d^3 + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d -
 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 6*(A*B - 2*B^2)*a^3*b*d^4)*x)
*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*c^4
- 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^
4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2
 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*
d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4
 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2278 vs. $2(491) = 982$.

Time = 0.40 (sec) , antiderivative size = 2278, normalized size of antiderivative = 4.55

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
[Out] -1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
```

```
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/12*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*
x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2
 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^
2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^
4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 1.10 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$=\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)}\cdots}{}\right.$$

$$-\frac{1}{12}\left(\frac{12\,B^2d^3}{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}-\frac{6\,B^2d^2}{(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg\cdots}\right.$$

$$+\frac{(6\,ABd^4-25\,B^2d^4)\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{6\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}$$
$$-\frac{6\,ABd^3-25\,B^2d^3}{6\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}$$
$$+\frac{6\,ABbd^2-13\,B^2bd^2}{12\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2b^2g^2}$$
$$-\frac{6\,ABb^2dg-7\,B^2b^2dg}{18\,(bgx+ag)^3(bc-ad)b^3g^3}-\frac{2\,A^2b^3g^3-2\,ABb^3g^3+B^2b^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log((b^2*c^2
*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b
*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2
*e)/b^2)^2 - 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c
*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*
c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c -
 a*d)*b*g^2) + 3*(2*A*B*b^3*g^3 - B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*l
og((b^2*c^2*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d
^2*e*g^2/(b*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x +
 a*g) + d^2*e)/b^2) + 1/6*(6*A*B*d^4 - 25*B^2*d^4)*log(-b*c*g/(b*g*x + a*g)
 + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^
2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - 1/6*(6*A*B*d^3 - 25*B^2*
d^3)/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(
b*g*x + a*g)*b*g) + 1/12*(6*A*B*b*d^2 - 13*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c
*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b^2*g^2) - 1/18*(6*A*B*b^2*d*g - 7*B^2*b^
2*d*g)/((b*g*x + a*g)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 - 2*A*B*b
^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.76

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5
*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5
*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b
^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*
b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3
) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3)
+ (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2)
+ (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*
((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2*d)/(2*d^2)
))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*
b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
(a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a
*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)
/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/
d + (4*a*b^2*x^3)/d) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(4*b^2*g^5*(
4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^
5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
 - ((18*A^2*a^3*d^3 - 18*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 - 15
0*A*B*a^3*d^3 + 18*A*B*b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 +
55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 78*A*B*a*b^2*c^2*d + 138*A*B*a^2
*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 - 4
2*A*B*a*b^2*d^3 + 6*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3
 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 78*A*B*a^2*b*d^3 - 6*A*B*b^3*c^2*
d + 30*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 - 6*A*B*b
^3*d^2))/(a*d - b*c))/(x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*
b^3*c*d*g^5) + x^3*(24*a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*
g^5) + x^4*(6*b^7*c^2*g^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36
*a^2*b^5*c^2*g^5 + 36*a^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g
^5 + 6*a^4*b^3*c^2*g^5 - 12*a^5*b^2*c*d*g^5) + (B*d^4*atan((B*d^4*(6*A - 25
*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4*c^3*d*g^5 + 12*a^3*b^2*c*d^
3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)) + (B*d^5*x*(6*A
```

```
- 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2
*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)))*(6*A - 25*B)*1i)/(3
*b*g^5*(a*d - b*c)^4)
```

3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d^2*e*x^2 + 2*c*d*e*
x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*
e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)
)), x) + Integral(b**2*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2))), x))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
[Out] integral((b*g*x + a*g)/(B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*
a*b*x + a^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
[Out] g*(Integral(a/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
 + Integral(b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/
(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
x))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*e*x^2 + 2*c*d*e*x
+ c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bbx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2)) + B*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*
a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 4.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[Out] -1/2*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)/exp(1/2
*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2552, 2337, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] -1/2*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B
)])/(B*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)
^2])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2} \\
&= -\frac{(c+dx)\text{Subst}\left(\int \frac{e^{x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} \\
&= -\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a
*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right.}}{g^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 151

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B(bc-ad)^2eg^3}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3+
1/2*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used

= {2552, 2367, 2337, 2209, 2347}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Beg^3(bc-ad)^2}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] (d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(2*B*(b*c - a*d)^2*E^(A/(2*B))*g^3*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x
)^2]) - (b*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*
(b*c - a*d)^2*e*E^(A/B)*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d-bx}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d}{A+B\log(ex^2)} - \frac{bx}{A+B\log(ex^2)}\right)dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{b\text{Subst}\left(\int \frac{x}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3} + \frac{d\text{Subst}\left(\int \frac{1}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3}$$

$$= -\frac{b\text{Subst}\left(\int \frac{e^x}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2(bc-ad)^2eg^3} + \frac{(d(c+dx))\text{Subst}\left(\int \frac{e^{x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

$$= \frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B(bc-ad)^2eg^3}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((d
^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Ba^2bx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\cdots\right.}}{g^3}$$

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*
x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**2*e
/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d*
*2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(a**2
+ 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*a*b*x +
 b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*
x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.08 (sec) , antiderivative size = 792, normalized size of antiderivative = 23.29

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$= \frac{-a^3cg^2-a^3dg^2x-3a^2bcg^2x-3a^2bdg^2x^2-3ab^2cg^2x^2-3ab^2dg^2x^3-b^3cg^2x^3-b^3dg^2x^4}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}$$
$$+\frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{3a^2bc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right.}{}$$

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g**
2*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c + d*x)*
*2/(a + b*x)**2)) + g**2*(Integral(a**3*d/(A + B*log(c**2*e/(a**2 + 2*a*b*x
 + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2
+ 2*a*b*x + b**2*x**2))), x) + Integral(3*a**2*b*c/(A + B*log(c**2*e/(a**2
+ 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**3*c*x**2/(A + B*log(c
**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2)
 + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*b**3*d*x**3/
(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x
+ b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(6*
a*b**2*c*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2
 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) +
Integral(9*a*b**2*d*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2
*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2))), x) + Integral(6*a**2*b*d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**
2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*
b*x + b**2*x**2))), x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 312, normalized size of antiderivative = 9.18

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*
log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2
+ a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*
g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) -
(b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 9.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \mathrm{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fric
as")
[Out] integral((b*g*x + a*g)/(B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*
a*b*x + a^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 559, normalized size of antiderivative = 17.47

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \ldots\right.}{\ldots}$$

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c +
d*x)**2/(a + b*x)**2)) + g*(Integral(a**2*d/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(c**2*e/(a**2 +
 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**
2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(c**2*
e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d
**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
 B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b*
*2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*a*b*
d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*
b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x))/(2*B*(a*
d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 7.31

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*
d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) -
(b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*d*g
*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*
log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(
e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 11.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*
c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.65

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{-c-dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{d\int\frac{1}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e
*(c + d*x)**2/(a + b*x)**2)) + d*Integral(1/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x
) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B
^2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 147

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

```
[Out] 1/2*(d*x+c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))-1/4*(d
*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)/exp(1/2*A/B)
/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

= {2552, 2334, 2337, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)} - \frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] -1/4*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B
)])/(B^2*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*
x)^2]) + (c + d*x)/(2*B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{(A+B\log(ex^2))^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)g^2}$$

$$= \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{2B(bc-ad)g^2}$$

$$= \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} - \frac{(c+dx)\text{Subst}\left(\int \frac{e^{x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

$$= -\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b
^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$= \frac{-c-dx}{2ABa^2dg^2 - 2ABabcg^2 + 2ABabdg^2x - 2ABb^2cg^2x + (2B^2a^2dg^2 - 2B^2abcg^2 + 2B^2abdg^2x - 2B^2b^2cg^2}$$
$$+ \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b}\right.}}{2Bg^2}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A
*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g
**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(c + d*x)**2/(a + b*x)**2)) + Integral(
1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*a*b*x + b
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2
*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
/(2*B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*
log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*
log(e))*B^2)*x - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)
*B^2)*log(b*x + a) + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*
g^2)*B^2)*log(d*x + c)) + integrate(1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 +
 (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g
^2)*x - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) +
2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 206

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2eg^3} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^
3+1/2*(d*x+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))+1/
4*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2552, 2357, 2367, 2337, 2209, 2347}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}dx$$
$$=\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2eg^3(bc-ad)^2}+\frac{c+dx}{2Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}$$

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] (d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(4*B^2*(b*c - a*d)^2*E^(A/(2*B))*g^3*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b
*x)^2]) - (b*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*
B^2*(b*c - a*d)^2*e*E^(A/B)*g^3) + (c + d*x)/(2*B*(b*c - a*d)*g^3*(a + b*x)
^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{d-bx}{(A+B\log(ex^2))^2}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)^2g^3} \\
&= \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} \\
&\quad+\frac{\text{Subst}\left(\int \frac{d-bx}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3}-\frac{d\text{Subst}\left(\int \frac{1}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{2B(bc-ad)^2g^3} \\
&= \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} \\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{d}{A+B\log(ex^2)}-\frac{bx}{A+B\log(ex^2)}\right)dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3} \\
&\quad-\frac{(d(c+dx))\text{Subst}\left(\int \frac{e^{x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}
\end{aligned}$$

$$= -\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} - \frac{b\text{Subst}\left(\int \frac{x}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3} + \frac{d\text{Subst}\left(\int \frac{1}{A+B\log(ex^2)}\,dx, x, \frac{c+dx}{a+bx}\right)}{B(bc-ad)^2g^3}$$

$$= -\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} - \frac{b\text{Subst}\left(\int \frac{e^x}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2B(bc-ad)^2eg^3} + \frac{(d(c+dx))\text{Subst}\left(\int \frac{e^{x/2}}{A+Bx}\,dx, x, \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

$$= \frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2eg^3} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)^{2}}dx$$

[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)

[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{\left(bgx+ag\right)^{3} \left(B\log\left(\frac{\left(dx+c\right)^{2} e}{\left(bx+a\right)^{2}}\right)+A\right)^{2}}\,dx$$

[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=" fricas")

[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2 *a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2 *a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^ 3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{\left(bgx+ag\right)^{3} \left(B\log\left(\frac{\left(dx+c\right)^{2} e}{\left(bx+a\right)^{2}}\right)+A\right)^{2}}\,dx$$

[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=" maxima")

```
[Out] 1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*
g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^
2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^
3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^
2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*lo
g(b*x + a) + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*
d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-1/
2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e)
 - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*
g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B +
 (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - 2*
((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^
2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*
b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.229 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/
((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2573, 2549, 2347, 2209}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] (E^(A/(B*n))*(c + d*x)*((e*(a + b*x)^n)/(c + d*x)^n)^n^(-1)*ExpIntegralEi[-
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a
+ b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x^2(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2n(a+bx)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$= \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2}\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)}dx$$

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$= \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\,(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))}\,dx$$
$$=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.230 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 355

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2df}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{20bd}-\frac{B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}+\frac{B(df-cg)^5\log(c+dx)}{5d^5g}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2
*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*d*g-
b*c*g+5*b*d*f)*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*x^4/b/d-1/5*B*(-a*g+b*f)^5
*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/5*B*(-c*g+d*
f)^5*ln(d*x+c)/d^5/g
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= -\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{10b^3d^3}$$
$$+\frac{Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-1\right.}{5b^4d^4}$$
$$+\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-\frac{B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$-\frac{Bg^3x^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4x^4(bc-ad)}{20bd}+\frac{B(df-cg)^5\log(c+dx)}{5d^5g}$$

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10
*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*
d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*
d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(10*b^3*d^
3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b
*c - a*d)*g^4*x^4)/(20*b*d) - (B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((
f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) + (B*(d*f - c*g)^5*L
og[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g} - \frac{(B(bc-ad))\int\frac{(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$= \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}$$

$$- \frac{(B(bc-ad))\int\left(\frac{g^2(-a^3d^3g^3+a^2bd^2g^2(5df-cg)-ab^2dg(10d^2f^2-5cdfg+c^2g^2)+b^3(10d^3f^3-10cd^2f^2g+5c^2dfg^2-c^3g^3))}{b^4d^4} + \cdots\right.}{}$$

$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g\cdots}{5b^4d^4}$$

$$- \frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{10b^3d^3}$$

$$- \frac{B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2} - \frac{B(bc-ad)g^4x^4}{20bd} - \frac{B(bf-ag)^5\log(a+bx)}{5b^5g}$$

$$+ \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g} + \frac{B(df-cg)^5\log(c+dx)}{5d^5g}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{\frac{B(-bc+ad)g^2x(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2))+b^3(-12c^3g^3+6c^2dg^2(10f+g\cdots}{12b^4d^4}}{}$$

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (f
 + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(d*f - c*g)^5*Log[c + d
*x])/d^5)/(5*g)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.67

method	result
risch	$\frac{g^3 B a^3 f x}{b^3} - \frac{2 g^2 B a^2 f^2 x}{b^2} + \frac{2 g B a f^3 x}{b} - \frac{g^3 B c^3 f x}{d^3} + \frac{2 g^2 B c^2 f^2 x}{d^2} - \frac{2 g B c f^3 x}{d} + \frac{g^4 A x^5}{5} - \frac{B \ln(dx+c) c f^4}{d}$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*g^3*B*a^3*f*x-2/b^2*g^2*B*a^2*f^2*x+2/b*g*B*a*f^3*x-1/d^3*g^3*B*c^3*f
*x+2/d^2*g^2*B*c^2*f^2*x-2/d*g*B*c*f^3*x+1/5*g^4*A*x^5-1/d*B*ln(d*x+c)*c*f^
4+1/b*B*ln(-b*x-a)*a*f^4+1/5/b^5*g^4*B*ln(-b*x-a)*a^5-1/5/d^5*g^4*B*ln(d*x+
c)*c^5+1/3/b*g^3*B*a*f*x^3-1/3/d*g^3*B*c*f*x^3-1/2/b^2*g^3*B*a^2*f*x^2+1/b*
g^2*B*a*f^2*x^2+1/2/d^2*g^3*B*c^2*f*x^2-1/d*g^2*B*c*f^2*x^2-1/15/b^2*g^4*B*
a^2*x^3+1/15/d^2*g^4*B*c^2*x^3+2*g*A*f^3*x^2+1/10/b^3*g^4*B*a^3*x^2-1/10/d^
3*g^4*B*c^3*x^2+A*f^4*x+1/5*(g*x+f)^5*B/g*ln(e*(b*x+a)/(d*x+c))-1/5/g*B*ln(
-b*x-a)*f^5+1/5/g*B*ln(d*x+c)*f^5-1/5/b^4*g^4*B*a^4*x+1/5/d^4*g^4*B*c^4*x-1
/b^4*g^3*B*ln(-b*x-a)*a^4*f+2/b^3*g^2*B*ln(-b*x-a)*a^3*f^2-2/b^2*g*B*ln(-b*
x-a)*a^2*f^3+1/d^4*g^3*B*ln(d*x+c)*c^4*f-2/d^3*g^2*B*ln(d*x+c)*c^3*f^2+2/d^
2*g*B*ln(d*x+c)*c^2*f^3+g^3*A*f*x^4+1/20/b*g^4*B*a*x^4-1/20/d*g^4*B*c*x^4+2
*g^2*A*f^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.79

$$\int (f+gx)^4 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{12\, A b^5 d^5 g^4 x^5 + 3\,(20\, A b^5 d^5 f g^3 - (B b^5 c d^4 - B a b^4 d^5) g^4) x^4 + 4\,(30\, A b^5 d^5 f^2 g^2 - 5\,(B b^5 c d^4 - B a b^4 d^5) f g^3}{}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*
g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g - 10
*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*
g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 - 10*(
```

```
B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g
^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*b*d^5)*
g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*
f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*c
*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*
d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b^5*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(337) = 674$.

Time = 55.68 (sec) , antiderivative size = 1436, normalized size of antiderivative = 4.05

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Ag^4x^5}{5} + \frac{Ba(a^4g^4-5a^3bfg^3+10a^2b^2f^2g^2-10ab^3f^3g+5b^4f^4)\log\left(x+\frac{Ba^5cd^4g^4-5Ba^4bcd^4fg^3+10Ba^3b^2cd^4f^2g^2-10Ba^2b\cdots}{\cdots}\right)}{\cdots} - \frac{Bc(c^4g^4-5c^3dfg^3+10c^2d^2f^2g^2-10cd^3f^3g+5d^4f^4)\log\left(x+\frac{Ba^5cd^4g^4-5Ba^4bcd^4fg^3+10Ba^3b^2cd^4f^2g^2-10Ba^2b^3\cdots}{\cdots}\right)}{\cdots} + x^4\left(Afg^3+\frac{Bag^4}{20b}-\frac{Bcg^4}{20d}\right) + x^3\cdot\left(2Af^2g^2-\frac{Ba^2g^4}{15b^2}+\frac{Bafg^3}{3b}+\frac{Bc^2g^4}{15d^2}-\frac{Bcfg^3}{3d}\right) + x^2\cdot\left(2Af^3g+\frac{Ba^3g^4}{10b^3}-\frac{Ba^2fg^3}{2b^2}+\frac{Baf^2g^2}{b}-\frac{Bc^3g^4}{10d^3}+\frac{Bc^2fg^3}{2d^2}-\frac{Bcf^2g^2}{d}\right) + x\left(Af^4-\frac{Ba^4g^4}{5b^4}+\frac{Ba^3fg^3}{b^3}-\frac{2Ba^2f^2g^2}{b^2}+\frac{2Baf^3g}{b}+\frac{Bc^4g^4}{5d^4}-\frac{Bc^3fg^3}{d^3}+\frac{2Bc^2f^2g^2}{d^2}-\frac{2Bcf^3g}{d}\right) + \left(Bf^4x+2Bf^3gx^2+2Bf^2g^2x^3+Bfg^3x^4+\frac{Bg^4x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 -
 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10
*a*b**3*f**3*g + 5*b**4*f**4)/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*
g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*
B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b*
*2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
```

```
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*b**5) - B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 -
 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*b**4*c**3*d**2*
f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d**4*f**4 - B*a*b*
*4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3
*g + 5*d**4*f**4) + B*b**5*c**2*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2
*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)/d)/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*d**5) + x**4*(A*f*g**3 + B*a*g**4/(20*b) - B*c*g**4/(20*d)) + x**3*
(2*A*f**2*g**2 - B*a**2*g**4/(15*b**2) + B*a*f*g**3/(3*b) + B*c**2*g**4/(15
*d**2) - B*c*f*g**3/(3*d)) + x**2*(2*A*f**3*g + B*a**3*g**4/(10*b**3) - B*a
**2*f*g**3/(2*b**2) + B*a*f**2*g**2/b - B*c**3*g**4/(10*d**3) + B*c**2*f*g*
*3/(2*d**2) - B*c*f**2*g**2/d) + x*(A*f**4 - B*a**4*g**4/(5*b**4) + B*a**3*
f*g**3/b**3 - 2*B*a**2*f**2*g**2/b**2 + 2*B*a*f**3*g/b + B*c**4*g**4/(5*d**
4) - B*c**3*f*g**3/d**3 + 2*B*c**2*f**2*g**2/d**2 - 2*B*c*f**3*g/d) + (B*f*
*4*x + 2*B*f**3*g*x**2 + 2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5
)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.67

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=\frac{1}{5}Ag^4x^5+Afg^3x^4+2Af^2g^2x^3$$
$$+2Af^3gx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^4$$
$$+2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^3g$$
$$+\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2c}{b^3}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(}{}\right.$$
$$+Af^4x$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4 +
 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*l
og(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 + 1/6*(
6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4
*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d
^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x^5*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x +
 c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^
3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*
g^4 + A*f^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10664 vs. $2(341) = 682$.

Time = 1.23 (sec) , antiderivative size = 10664, normalized size of antiderivative = 30.04

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/60*(12*(5*B*b^6*c^2*d^4*e^6*f^4 - 10*B*a*b^5*c*d^5*e^6*f^4 + 5*B*a^2*b^4*
d^6*e^6*f^4 - 10*B*b^6*c^3*d^3*e^6*f^3*g + 10*B*a*b^5*c^2*d^4*e^6*f^3*g + 1
0*B*a^2*b^4*c*d^5*e^6*f^3*g - 10*B*a^3*b^3*d^6*e^6*f^3*g + 10*B*b^6*c^4*d^2
*e^6*f^2*g^2 - 10*B*a*b^5*c^3*d^3*e^6*f^2*g^2 - 10*B*a^3*b^3*c*d^5*e^6*f^2*
g^2 + 10*B*a^4*b^2*d^6*e^6*f^2*g^2 - 5*B*b^6*c^5*d*e^6*f*g^3 + 5*B*a*b^5*c^
4*d^2*e^6*f*g^3 + 5*B*a^4*b^2*c*d^5*e^6*f*g^3 - 5*B*a^5*b*d^6*e^6*f*g^3 + B
*b^6*c^6*e^6*g^4 - B*a*b^5*c^5*d*e^6*g^4 - B*a^5*b*c*d^5*e^6*g^4 + B*a^6*d^
6*e^6*g^4 - 20*(b*e*x + a*e)*B*b^5*c^2*d^5*e^5*f^4/(d*x + c) + 40*(b*e*x +
a*e)*B*a*b^4*c*d^6*e^5*f^4/(d*x + c) - 20*(b*e*x + a*e)*B*a^2*b^3*d^7*e^5*f
^4/(d*x + c) + 50*(b*e*x + a*e)*B*b^5*c^3*d^4*e^5*f^3*g/(d*x + c) - 70*(b*e
*x + a*e)*B*a*b^4*c^2*d^5*e^5*f^3*g/(d*x + c) - 10*(b*e*x + a*e)*B*a^2*b^3*
c*d^6*e^5*f^3*g/(d*x + c) + 30*(b*e*x + a*e)*B*a^3*b^2*d^7*e^5*f^3*g/(d*x +
 c) - 50*(b*e*x + a*e)*B*b^5*c^4*d^3*e^5*f^2*g^2/(d*x + c) + 50*(b*e*x + a*
e)*B*a*b^4*c^3*d^4*e^5*f^2*g^2/(d*x + c) + 30*(b*e*x + a*e)*B*a^2*b^3*c^2*d
^5*e^5*f^2*g^2/(d*x + c) - 10*(b*e*x + a*e)*B*a^3*b^2*c*d^6*e^5*f^2*g^2/(d*
x + c) - 20*(b*e*x + a*e)*B*a^4*b*d^7*e^5*f^2*g^2/(d*x + c) + 25*(b*e*x + a
*e)*B*b^5*c^5*d^2*e^5*f*g^3/(d*x + c) - 25*(b*e*x + a*e)*B*a*b^4*c^4*d^3*e^
5*f*g^3/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c^2*d^5*e^5*f*g^3/(d*x + c)
+ 15*(b*e*x + a*e)*B*a^4*b*c*d^6*e^5*f*g^3/(d*x + c) + 5*(b*e*x + a*e)*B*a^
5*d^7*e^5*f*g^3/(d*x + c) - 5*(b*e*x + a*e)*B*b^5*c^6*d*e^5*g^4/(d*x + c) +
 5*(b*e*x + a*e)*B*a*b^4*c^5*d^2*e^5*g^4/(d*x + c) + 5*(b*e*x + a*e)*B*a^4*
```

b*c^2*d^5*e^5*g^4/(d*x + c) - 5*(b*e*x + a*e)*B*a^5*c*d^6*e^5*g^4/(d*x + c)
+ 30*(b*e*x + a*e)^2*B*b^4*c^2*d^6*e^4*f^4/(d*x + c)^2 - 60*(b*e*x + a*e)^
2*B*a*b^3*c*d^7*e^4*f^4/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^2*b^2*d^8*e^4*
f^4/(d*x + c)^2 - 90*(b*e*x + a*e)^2*B*b^4*c^3*d^5*e^4*f^3*g/(d*x + c)^2 +
150*(b*e*x + a*e)^2*B*a*b^3*c^2*d^6*e^4*f^3*g/(d*x + c)^2 - 30*(b*e*x + a*e
)^2*B*a^2*b^2*c*d^7*e^4*f^3*g/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a^3*b*d^8*
e^4*f^3*g/(d*x + c)^2 + 100*(b*e*x + a*e)^2*B*b^4*c^4*d^4*e^4*f^2*g^2/(d*x
+ c)^2 - 130*(b*e*x + a*e)^2*B*a*b^3*c^3*d^5*e^4*f^2*g^2/(d*x + c)^2 - 30*(
b*e*x + a*e)^2*B*a^2*b^2*c^2*d^6*e^4*f^2*g^2/(d*x + c)^2 + 50*(b*e*x + a*e)
^2*B*a^3*b*c*d^7*e^4*f^2*g^2/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^4*d^8*e^4
*f^2*g^2/(d*x + c)^2 - 50*(b*e*x + a*e)^2*B*b^4*c^5*d^3*e^4*f*g^3/(d*x + c)
^2 + 50*(b*e*x + a*e)^2*B*a*b^3*c^4*d^4*e^4*f*g^3/(d*x + c)^2 + 30*(b*e*x +
a*e)^2*B*a^2*b^2*c^3*d^5*e^4*f*g^3/(d*x + c)^2 - 10*(b*e*x + a*e)^2*B*a^3*
b*c^2*d^6*e^4*f*g^3/(d*x + c)^2 - 20*(b*e*x + a*e)^2*B*a^4*c*d^7*e^4*f*g^3/
(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*b^4*c^6*d^2*e^4*g^4/(d*x + c)^2 - 10*(b*
e*x + a*e)^2*B*a*b^3*c^5*d^3*e^4*g^4/(d*x + c)^2 - 10*(b*e*x + a*e)^2*B*a^3
*b*c^3*d^5*e^4*g^4/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^4*c^2*d^6*e^4*g^4/(
d*x + c)^2 - 20*(b*e*x + a*e)^3*B*b^3*c^2*d^7*e^3*f^4/(d*x + c)^3 + 40*(b*e
*x + a*e)^3*B*a*b^2*c*d^8*e^3*f^4/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^2*b*
d^9*e^3*f^4/(d*x + c)^3 + 70*(b*e*x + a*e)^3*B*b^3*c^3*d^6*e^3*f^3*g/(d*x +
c)^3 - 130*(b*e*x + a*e)^3*B*a*b^2*c^2*d^7*e^3*f^3*g/(d*x + c)^3 + 50*(b*e
*x + a*e)^3*B*a^2*b*c*d^8*e^3*f^3*g/(d*x + c)^3 + 10*(b*e*x + a*e)^3*B*a^3*
d^9*e^3*f^3*g/(d*x + c)^3 - 90*(b*e*x + a*e)^3*B*b^3*c^4*d^5*e^3*f^2*g^2/(d
x + c)^3 + 150(b*e*x + a*e)^3*B*a*b^2*c^3*d^6*e^3*f^2*g^2/(d*x + c)^3 - 3
0*(b*e*x + a*e)^3*B*a^2*b*c^2*d^7*e^3*f^2*g^2/(d*x + c)^3 - 30*(b*e*x + a*e
)^3*B*a^3*c*d^8*e^3*f^2*g^2/(d*x + c)^3 + 50*(b*e*x + a*e)^3*B*b^3*c^5*d^4*
e^3*f*g^3/(d*x + c)^3 - 70*(b*e*x + a*e)^3*B*a*b^2*c^4*d^5*e^3*f*g^3/(d*x +
c)^3 - 10*(b*e*x + a*e)^3*B*a^2*b*c^3*d^6*e^3*f*g^3/(d*x + c)^3 + 30*(b*e*
x + a*e)^3*B*a^3*c^2*d^7*e^3*f*g^3/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*b^3*c
^6*d^3*e^3*g^4/(d*x + c)^3 + 10*(b*e*x + a*e)^3*B*a*b^2*c^5*d^4*e^3*g^4/(d*
x + c)^3 + 10*(b*e*x + a*e)^3*B*a^2*b*c^4*d^5*e^3*g^4/(d*x + c)^3 - 10*(b*e
*x + a*e)^3*B*a^3*c^3*d^6*e^3*g^4/(d*x + c)^3 + 5*(b*e*x + a*e)^4*B*b^2*c^2
*d^8*e^2*f^4/(d*x + c)^4 - 10*(b*e*x + a*e)^4*B*a*b*c*d^9*e^2*f^4/(d*x + c)
^4 + 5*(b*e*x + a*e)^4*B*a^2*d^10*e^2*f^4/(d*x + c)^4 - 20*(b*e*x + a*e)^4*
B*b^2*c^3*d^7*e^2*f^3*g/(d*x + c)^4 + 40*(b*e*x + a*e)^4*B*a*b*c^2*d^8*e^2*
f^3*g/(d*x + c)^4 - 20*(b*e*x + a*e)^4*B*a^2*c*d^9*e^2*f^3*g/(d*x + c)^4 +
30*(b*e*x + a*e)^4*B*b^2*c^4*d^6*e^2*f^2*g^2/(d*x + c)^4 - 60*(b*e*x + a*e)
^4*B*a*b*c^3*d^7*e^2*f^2*g^2/(d*x + c)^4 + 30*(b*e*x + a*e)^4*B*a^2*c^2*d^8
*e^2*f^2*g^2/(d*x + c)^4 - 20*(b*e*x + a*e)^4*B*b^2*c^5*d^5*e^2*f*g^3/(d*x
+ c)^4 + 40*(b*e*x + a*e)^4*B*a*b*c^4*d^6*e^2*f*g^3/(d*x + c)^4 - 20*(b*e*x
+ a*e)^4*B*a^2*c^3*d^7*e^2*f*g^3/(d*x + c)^4 + 5*(b*e*x + a*e)^4*B*b^2*c^6
*d^4*e^2*g^4/(d*x + c)^4 - 10*(b*e*x + a*e)^4*B*a*b*c^5*d^5*e^2*g^4/(d*x +
c)^4 + 5*(b*e*x + a*e)^4*B*a^2*c^4*d^6*e^2*g^4/(d*x + c)^4)*log((b*e*x + a*
e)/(d*x + c))/(b^5*d^5*e^5 - 5*(b*e*x + a*e)*b^4*d^6*e^4/(d*x + c) + 10*(b*
e*x + a*e)^2*b^3*d^7*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2*d^8*e^2/(d*x

+ c)^3 + 5*(b*e*x + a*e)^4*b*d^9*e/(d*x + c)^4 - (b*e*x + a*e)^5*d^10/(d*x
+ c)^5) + (60*A*b^10*c^2*d^4*e^6*f^4 - 120*A*a*b^9*c*d^5*e^6*f^4 + 60*A*a^2
*b^8*d^6*e^6*f^4 - 120*A*b^10*c^3*d^3*e^6*f^3*g - 120*B*b^10*c^3*d^3*e^6*f^
3*g + 120*A*a*b^9*c^2*d^4*e^6*f^3*g + 360*B*a*b^9*c^2*d^4*e^6*f^3*g + 120*A
*a^2*b^8*c*d^5*e^6*f^3*g - 360*B*a^2*b^8*c*d^5*e^6*f^3*g - 120*A*a^3*b^7*d^
6*e^6*f^3*g + 120*B*a^3*b^7*d^6*e^6*f^3*g + 120*A*b^10*c^4*d^2*e^6*f^2*g^2
+ 180*B*b^10*c^4*d^2*e^6*f^2*g^2 - 120*A*a*b^9*c^3*d^3*e^6*f^2*g^2 - 360*B*
a*b^9*c^3*d^3*e^6*f^2*g^2 - 120*A*a^3*b^7*c*d^5*e^6*f^2*g^2 + 360*B*a^3*b^7
*c*d^5*e^6*f^2*g^2 + 120*A*a^4*b^6*d^6*e^6*f^2*g^2 - 180*B*a^4*b^6*d^6*e^6*
f^2*g^2 - 60*A*b^10*c^5*d*e^6*f*g^3 - 110*B*b^10*c^5*d*e^6*f*g^3 + 60*A*a*b
^9*c^4*d^2*e^6*f*g^3 + 190*B*a*b^9*c^4*d^2*e^6*f*g^3 - 20*B*a^2*b^8*c^3*d^3
*e^6*f*g^3 + 20*B*a^3*b^7*c^2*d^4*e^6*f*g^3 + 60*A*a^4*b^6*c*d^5*e^6*f*g^3
- 190*B*a^4*b^6*c*d^5*e^6*f*g^3 - 60*A*a^5*b^5*d^6*e^6*f*g^3 + 110*B*a^5*b^
5*d^6*e^6*f*g^3 + 12*A*b^10*c^6*e^6*g^4 + 25*B*b^10*c^6*e^6*g^4 - 12*A*a*b^
9*c^5*d*e^6*g^4 - 40*B*a*b^9*c^5*d*e^6*g^4 + 5*B*a^2*b^8*c^4*d^2*e^6*g^4 -
5*B*a^4*b^6*c^2*d^4*e^6*g^4 - 12*A*a^5*b^5*c*d^5*e^6*g^4 + 40*B*a^5*b^5*c*d
^5*e^6*g^4 + 12*A*a^6*b^4*d^6*e^6*g^4 - 25*B*a^6*b^4*d^6*e^6*g^4 - 240*(b*e
*x + a*e)*A*b^9*c^2*d^5*e^5*f^4/(d*x + c) + 480*(b*e*x + a*e)*A*a*b^8*c*d^6
*e^5*f^4/(d*x + c) - 240*(b*e*x + a*e)*A*a^2*b^7*d^7*e^5*f^4/(d*x + c) + 60
0*(b*e*x + a*e)*A*b^9*c^3*d^4*e^5*f^3*g/(d*x + c) + 480*(b*e*x + a*e)*B*b^9
*c^3*d^4*e^5*f^3*g/(d*x + c) - 840*(b*e*x + a*e)*A*a*b^8*c^2*d^5*e^5*f^3*g/
(d*x + c) - 1440*(b*e*x + a*e)*B*a*b^8*c^2*d^5*e^5*f^3*g/(d*x + c) - 120*(b
*e*x + a*e)*A*a^2*b^7*c*d^6*e^5*f^3*g/(d*x + c) + 1440*(b*e*x + a*e)*B*a^2*
b^7*c*d^6*e^5*f^3*g/(d*x + c) + 360*(b*e*x + a*e)*A*a^3*b^6*d^7*e^5*f^3*g/(
d*x + c) - 480*(b*e*x + a*e)*B*a^3*b^6*d^7*e^5*f^3*g/(d*x + c) - 600*(b*e*x
 + a*e)*A*b^9*c^4*d^3*e^5*f^2*g^2/(d*x + c) - 780*(b*e*x + a*e)*B*b^9*c^4*d
^3*e^5*f^2*g^2/(d*x + c) + 600*(b*e*x + a*e)*A*a*b^8*c^3*d^4*e^5*f^2*g^2/(d
x + c) + 1680(b*e*x + a*e)*B*a*b^8*c^3*d^4*e^5*f^2*g^2/(d*x + c) + 360*(b
*e*x + a*e)*A*a^2*b^7*c^2*d^5*e^5*f^2*g^2/(d*x + c) - 360*(b*e*x + a*e)*B*a
^2*b^7*c^2*d^5*e^5*f^2*g^2/(d*x + c) - 120*(b*e*x + a*e)*A*a^3*b^6*c*d^6*e^
5*f^2*g^2/(d*x + c) - 1200*(b*e*x + a*e)*B*a^3*b^6*c*d^6*e^5*f^2*g^2/(d*x +
 c) - 240*(b*e*x + a*e)*A*a^4*b^5*d^7*e^5*f^2*g^2/(d*x + c) + 660*(b*e*x +
a*e)*B*a^4*b^5*d^7*e^5*f^2*g^2/(d*x + c) + 300*(b*e*x + a*e)*A*b^9*c^5*d^2*
e^5*f*g^3/(d*x + c) + 490*(b*e*x + a*e)*B*b^9*c^5*d^2*e^5*f*g^3/(d*x + c) -
 300*(b*e*x + a*e)*A*a*b^8*c^4*d^3*e^5*f*g^3/(d*x + c) - 890*(b*e*x + a*e)*
B*a*b^8*c^4*d^3*e^5*f*g^3/(d*x + c) + 100*(b*e*x + a*e)*B*a^2*b^7*c^3*d^4*e
^5*f*g^3/(d*x + c) - 240*(b*e*x + a*e)*A*a^3*b^6*c^2*d^5*e^5*f*g^3/(d*x + c
) + 140*(b*e*x + a*e)*B*a^3*b^6*c^2*d^5*e^5*f*g^3/(d*x + c) + 180*(b*e*x +
a*e)*A*a^4*b^5*c*d^6*e^5*f*g^3/(d*x + c) + 530*(b*e*x + a*e)*B*a^4*b^5*c*d^
6*e^5*f*g^3/(d*x + c) + 60*(b*e*x + a*e)*A*a^5*b^4*d^7*e^5*f*g^3/(d*x + c)
- 370*(b*e*x + a*e)*B*a^5*b^4*d^7*e^5*f*g^3/(d*x + c) - 60*(b*e*x + a*e)*A*
b^9*c^6*d*e^5*g^4/(d*x + c) - 113*(b*e*x + a*e)*B*b^9*c^6*d*e^5*g^4/(d*x +
c) + 60*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5*g^4/(d*x + c) + 188*(b*e*x + a*e)
*B*a*b^8*c^5*d^2*e^5*g^4/(d*x + c) - 25*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5
*g^4/(d*x + c) + 60*(b*e*x + a*e)*A*a^4*b^5*c^2*d^5*e^5*g^4/(d*x + c) - 35*

(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e^5*g^4/(d*x + c) - 60*(b*e*x + a*e)*A*a^5*b^4*c*d^6*e^5*g^4/(d*x + c) - 92*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^4/(d*x + c) + 77*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^4/(d*x + c) + 360*(b*e*x + a*e)^2*A*b^8*c^2*d^6*e^4*f^4/(d*x + c)^2 - 720*(b*e*x + a*e)^2*A*a*b^7*c*d^7*e^4*f^4/(d*x + c)^2 + 360*(b*e*x + a*e)^2*A*a^2*b^6*d^8*e^4*f^4/(d*x + c)^2 - 1080*(b*e*x + a*e)^2*A*b^8*c^3*d^5*e^4*f^3*g/(d*x + c)^2 - 720*(b*e*x + a*e)^2*B*b^8*c^3*d^5*e^4*f^3*g/(d*x + c)^2 + 1800*(b*e*x + a*e)^2*A*a*b^7*c^2*d^6*e^4*f^3*g/(d*x + c)^2 + 2160*(b*e*x + a*e)^2*B*a*b^7*c^2*d^6*e^4*f^3*g/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a^2*b^6*c*d^7*e^4*f^3*g/(d*x + c)^2 - 2160*(b*e*x + a*e)^2*B*a^2*b^6*c*d^7*e^4*f^3*g/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a^3*b^5*d^8*e^4*f^3*g/(d*x + c)^2 + 720*(b*e*x + a*e)^2*B*a^3*b^5*d^8*e^4*f^3*g/(d*x + c)^2 + 1200*(b*e*x + a*e)^2*A*b^8*c^4*d^4*e^4*f^2*g^2/(d*x + c)^2 + 1260*(b*e*x + a*e)^2*B*b^8*c^4*d^4*e^4*f^2*g^2/(d*x + c)^2 - 1560*(b*e*x + a*e)^2*A*a*b^7*c^3*d^5*e^4*f^2*g^2/(d*x + c)^2 - 2880*(b*e*x + a*e)^2*B*a*b^7*c^3*d^5*e^4*f^2*g^2/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a^2*b^6*c^2*d^6*e^4*f^2*g^2/(d*x + c)^2 + 1080*(b*e*x + a*e)^2*B*a^2*b^6*c^2*d^6*e^4*f^2*g^2/(d*x + c)^2 + 600*(b*e*x + a*e)^2*A*a^3*b^5*c*d^7*e^4*f^2*g^2/(d*x + c)^2 + 1440*(b*e*x + a*e)^2*B*a^3*b^5*c*d^7*e^4*f^2*g^2/(d*x + c)^2 + 120*(b*e*x + a*e)^2*A*a^4*b^4*d^8*e^4*f^2*g^2/(d*x + c)^2 - 900*(b*e*x + a*e)^2*B*a^4*b^4*d^8*e^4*f^2*g^2/(d*x + c)^2 - 600*(b*e*x + a*e)^2*A*b^8*c^5*d^3*e^4*f*g^3/(d*x + c)^2 - 830*(b*e*x + a*e)^2*B*b^8*c^5*d^3*e^4*f*g^3/(d*x + c)^2 + 600*(b*e*x + a*e)^2*A*a*b^7*c^4*d^4*e^4*f*g^3/(d*x + c)^2 + 1630*(b*e*x + a*e)^2*B*a*b^7*c^4*d^4*e^4*f*g^3/(d*x + c)^2 + 360*(b*e*x + a*e)^2*A*a^2*b^6*c^3*d^5*e^4*f*g^3/(d*x + c)^2 - 380*(b*e*x + a*e)^2*B*a^2*b^6*c^3*d^5*e^4*f*g^3/(d*x + c)^2 - 120*(b*e*x + a*e)^2*A*a^3*b^5*c^2*d^6*e^4*f*g^3/(d*x + c)^2 - 340*(b*e*x + a*e)^2*B*a^3*b^5*c^2*d^6*e^4*f*g^3/(d*x + c)^2 - 240*(b*e*x + a*e)^2*A*a^4*b^4*c*d^7*e^4*f*g^3/(d*x + c)^2 - 550*(b*e*x + a*e)^2*B*a^4*b^4*c*d^7*e^4*f*g^3/(d*x + c)^2 + 470*(b*e*x + a*e)^2*B*a^5*b^3*d^8*e^4*f*g^3/(d*x + c)^2 + 120*(b*e*x + a*e)^2*A*b^8*c^6*d^2*e^4*g^4/(d*x + c)^2 + 196*(b*e*x + a*e)^2*B*b^8*c^6*d^2*e^4*g^4/(d*x + c)^2 - 120*(b*e*x + a*e)^2*A*a*b^7*c^5*d^3*e^4*g^4/(d*x + c)^2 - 346*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(d*x + c)^2 + 50*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(d*x + c)^2 - 120*(b*e*x + a*e)^2*A*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 60*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 120*(b*e*x + a*e)^2*A*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 + 40*(b*e*x + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 + 94*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e^4*g^4/(d*x + c)^2 - 94*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^4/(d*x + c)^2 - 240*(b*e*x + a*e)^3*A*b^7*c^2*d^7*e^3*f^4/(d*x + c)^3 + 480*(b*e*x + a*e)^3*A*a*b^6*c*d^8*e^3*f^4/(d*x + c)^3 - 240*(b*e*x + a*e)^3*A*a^2*b^5*d^9*e^3*f^4/(d*x + c)^3 + 840*(b*e*x + a*e)^3*A*b^7*c^3*d^6*e^3*f^3*g/(d*x + c)^3 + 480*(b*e*x + a*e)^3*B*b^7*c^3*d^6*e^3*f^3*g/(d*x + c)^3 - 1560*(b*e*x + a*e)^3*A*a*b^6*c^2*d^7*e^3*f^3*g/(d*x + c)^3 - 1440*(b*e*x + a*e)^3*B*a*b^6*c^2*d^7*e^3*f^3*g/(d*x + c)^3 + 600*(b*e*x + a*e)^3*A*a^2*b^5*c*d^8*e^3*f^3*g/(d*x + c)^3 + 1440*(b*e*x + a*e)^3*B*a^2*b^5*c*d^8*e^3*f^3*g/(d*x + c)^3 + 120*(b*e*x + a*e)^3*A*a^3*b^4*d^9*e^3*f^3*g/(d*x + c)^3 - 480*(b*e*x + a*

e)^3*B*a^3*b^4*d^9*e^3*f^3*g/(d*x + c)^3 - 1080*(b*e*x + a*e)^3*A*b^7*c^4*d
^5*e^3*f^2*g^2/(d*x + c)^3 - 900*(b*e*x + a*e)^3*B*b^7*c^4*d^5*e^3*f^2*g^2/
(d*x + c)^3 + 1800*(b*e*x + a*e)^3*A*a*b^6*c^3*d^6*e^3*f^2*g^2/(d*x + c)^3
+ 2160*(b*e*x + a*e)^3*B*a*b^6*c^3*d^6*e^3*f^2*g^2/(d*x + c)^3 - 360*(b*e*x
 + a*e)^3*A*a^2*b^5*c^2*d^7*e^3*f^2*g^2/(d*x + c)^3 - 1080*(b*e*x + a*e)^3*
B*a^2*b^5*c^2*d^7*e^3*f^2*g^2/(d*x + c)^3 - 360*(b*e*x + a*e)^3*A*a^3*b^4*c
*d^8*e^3*f^2*g^2/(d*x + c)^3 - 720*(b*e*x + a*e)^3*B*a^3*b^4*c*d^8*e^3*f^2*
g^2/(d*x + c)^3 + 540*(b*e*x + a*e)^3*B*a^4*b^3*d^9*e^3*f^2*g^2/(d*x + c)^3
 + 600*(b*e*x + a*e)^3*A*b^7*c^5*d^4*e^3*f*g^3/(d*x + c)^3 + 630*(b*e*x + a
*e)^3*B*b^7*c^5*d^4*e^3*f*g^3/(d*x + c)^3 - 840*(b*e*x + a*e)^3*A*a*b^6*c^4
*d^5*e^3*f*g^3/(d*x + c)^3 - 1350*(b*e*x + a*e)^3*B*a*b^6*c^4*d^5*e^3*f*g^3
/(d*x + c)^3 - 120*(b*e*x + a*e)^3*A*a^2*b^5*c^3*d^6*e^3*f*g^3/(d*x + c)^3
+ 540*(b*e*x + a*e)^3*B*a^2*b^5*c^3*d^6*e^3*f*g^3/(d*x + c)^3 + 360*(b*e*x
+ a*e)^3*A*a^3*b^4*c^2*d^7*e^3*f*g^3/(d*x + c)^3 + 180*(b*e*x + a*e)^3*B*a^
3*b^4*c^2*d^7*e^3*f*g^3/(d*x + c)^3 + 270*(b*e*x + a*e)^3*B*a^4*b^3*c*d^8*e
^3*f*g^3/(d*x + c)^3 - 270*(b*e*x + a*e)^3*B*a^5*b^2*d^9*e^3*f*g^3/(d*x + c
)^3 - 120*(b*e*x + a*e)^3*A*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 - 156*(b*e*x +
a*e)^3*B*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 + 120*(b*e*x + a*e)^3*A*a*b^6*c^5*
d^4*e^3*g^4/(d*x + c)^3 + 306*(b*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^4/(d*x
+ c)^3 + 120*(b*e*x + a*e)^3*A*a^2*b^5*c^4*d^5*e^3*g^4/(d*x + c)^3 - 90*(b*
e*x + a*e)^3*B*a^2*b^5*c^4*d^5*e^3*g^4/(d*x + c)^3 - 120*(b*e*x + a*e)^3*A*
a^3*b^4*c^3*d^6*e^3*g^4/(d*x + c)^3 - 60*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*
e^3*g^4/(d*x + c)^3 - 54*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g^4/(d*x + c)^
3 + 54*(b*e*x + a*e)^3*B*a^6*b*d^9*e^3*g^4/(d*x + c)^3 + 60*(b*e*x + a*e)^4
*A*b^6*c^2*d^8*e^2*f^4/(d*x + c)^4 - 120*(b*e*x + a*e)^4*A*a*b^5*c*d^9*e^2*
f^4/(d*x + c)^4 + 60*(b*e*x + a*e)^4*A*a^2*b^4*d^10*e^2*f^4/(d*x + c)^4 - 2
40*(b*e*x + a*e)^4*A*b^6*c^3*d^7*e^2*f^3*g/(d*x + c)^4 - 120*(b*e*x + a*e)^
4*B*b^6*c^3*d^7*e^2*f^3*g/(d*x + c)^4 + 480*(b*e*x + a*e)^4*A*a*b^5*c^2*d^8
*e^2*f^3*g/(d*x + c)^4 + 360*(b*e*x + a*e)^4*B*a*b^5*c^2*d^8*e^2*f^3*g/(d*x
 + c)^4 - 240*(b*e*x + a*e)^4*A*a^2*b^4*c*d^9*e^2*f^3*g/(d*x + c)^4 - 360*(
b*e*x + a*e)^4*B*a^2*b^4*c*d^9*e^2*f^3*g/(d*x + c)^4 + 120*(b*e*x + a*e)^4*
B*a^3*b^3*d^10*e^2*f^3*g/(d*x + c)^4 + 360*(b*e*x + a*e)^4*A*b^6*c^4*d^6*e^
2*f^2*g^2/(d*x + c)^4 + 240*(b*e*x + a*e)^4*B*b^6*c^4*d^6*e^2*f^2*g^2/(d*x
+ c)^4 - 720*(b*e*x + a*e)^4*A*a*b^5*c^3*d^7*e^2*f^2*g^2/(d*x + c)^4 - 600*
(b*e*x + a*e)^4*B*a*b^5*c^3*d^7*e^2*f^2*g^2/(d*x + c)^4 + 360*(b*e*x + a*e)
^4*A*a^2*b^4*c^2*d^8*e^2*f^2*g^2/(d*x + c)^4 + 360*(b*e*x + a*e)^4*B*a^2*b^
4*c^2*d^8*e^2*f^2*g^2/(d*x + c)^4 + 120*(b*e*x + a*e)^4*B*a^3*b^3*c*d^9*e^2
*f^2*g^2/(d*x + c)^4 - 120*(b*e*x + a*e)^4*B*a^4*b^2*d^10*e^2*f^2*g^2/(d*x
+ c)^4 - 240*(b*e*x + a*e)^4*A*b^6*c^5*d^5*e^2*f*g^3/(d*x + c)^4 - 180*(b*e
*x + a*e)^4*B*b^6*c^5*d^5*e^2*f*g^3/(d*x + c)^4 + 480*(b*e*x + a*e)^4*A*a*b
^5*c^4*d^6*e^2*f*g^3/(d*x + c)^4 + 420*(b*e*x + a*e)^4*B*a*b^5*c^4*d^6*e^2*
f*g^3/(d*x + c)^4 - 240*(b*e*x + a*e)^4*A*a^2*b^4*c^3*d^7*e^2*f*g^3/(d*x +
c)^4 - 240*(b*e*x + a*e)^4*B*a^2*b^4*c^3*d^7*e^2*f*g^3/(d*x + c)^4 - 60*(b*
e*x + a*e)^4*B*a^4*b^2*c*d^9*e^2*f*g^3/(d*x + c)^4 + 60*(b*e*x + a*e)^4*B*a
^5*b*d^10*e^2*f*g^3/(d*x + c)^4 + 60*(b*e*x + a*e)^4*A*b^6*c^6*d^4*e^2*g^4/

```
(d*x + c)^4 + 48*(b*e*x + a*e)^4*B*b^6*c^6*d^4*e^2*g^4/(d*x + c)^4 - 120*(b
*e*x + a*e)^4*A*a*b^5*c^5*d^5*e^2*g^4/(d*x + c)^4 - 108*(b*e*x + a*e)^4*B*a
*b^5*c^5*d^5*e^2*g^4/(d*x + c)^4 + 60*(b*e*x + a*e)^4*A*a^2*b^4*c^4*d^6*e^2
*g^4/(d*x + c)^4 + 60*(b*e*x + a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g^4/(d*x + c)^4
 + 12*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g^4/(d*x + c)^4 - 12*(b*e*x + a*e)^
4*B*a^6*d^10*e^2*g^4/(d*x + c)^4)/(b^9*d^5*e^5 - 5*(b*e*x + a*e)*b^8*d^6*e^
4/(d*x + c) + 10*(b*e*x + a*e)^2*b^7*d^7*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)
^3*b^6*d^8*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b^5*d^9*e/(d*x + c)^4 - (b*e
*x + a*e)^5*b^4*d^10/(d*x + c)^5) + 12*(5*B*b^6*c^2*d^4*e*f^4 - 10*B*a*b^5*
c*d^5*e*f^4 + 5*B*a^2*b^4*d^6*e*f^4 - 10*B*b^6*c^3*d^3*e*f^3*g + 10*B*a*b^5
*c^2*d^4*e*f^3*g + 10*B*a^2*b^4*c*d^5*e*f^3*g - 10*B*a^3*b^3*d^6*e*f^3*g +
10*B*b^6*c^4*d^2*e*f^2*g^2 - 10*B*a*b^5*c^3*d^3*e*f^2*g^2 - 10*B*a^3*b^3*c*
d^5*e*f^2*g^2 + 10*B*a^4*b^2*d^6*e*f^2*g^2 - 5*B*b^6*c^5*d*e*f*g^3 + 5*B*a*
b^5*c^4*d^2*e*f*g^3 + 5*B*a^4*b^2*c*d^5*e*f*g^3 - 5*B*a^5*b*d^6*e*f*g^3 + B
*b^6*c^6*e*g^4 - B*a*b^5*c^5*d*e*g^4 - B*a^5*b*c*d^5*e*g^4 + B*a^6*d^6*e*g^
4)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^5*d^5) - 12*(5*B*b^6*c^2*d^4*e*
f^4 - 10*B*a*b^5*c*d^5*e*f^4 + 5*B*a^2*b^4*d^6*e*f^4 - 10*B*b^6*c^3*d^3*e*f
^3*g + 10*B*a*b^5*c^2*d^4*e*f^3*g + 10*B*a^2*b^4*c*d^5*e*f^3*g - 10*B*a^3*b
^3*d^6*e*f^3*g + 10*B*b^6*c^4*d^2*e*f^2*g^2 - 10*B*a*b^5*c^3*d^3*e*f^2*g^2
- 10*B*a^3*b^3*c*d^5*e*f^2*g^2 + 10*B*a^4*b^2*d^6*e*f^2*g^2 - 5*B*b^6*c^5*d
*e*f*g^3 + 5*B*a*b^5*c^4*d^2*e*f*g^3 + 5*B*a^4*b^2*c*d^5*e*f*g^3 - 5*B*a^5*
b*d^6*e*f*g^3 + B*b^6*c^6*e*g^4 - B*a*b^5*c^5*d*e*g^4 - B*a^5*b*c*d^5*e*g^4
 + B*a^6*d^6*e*g^4)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^5))*(b*c/((b*c*e -
a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.92

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2
 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A
*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) -
(A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 +
20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*
f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d +
 5*b*c))/(5*b*d)))/(2*b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) +
 log((e*(a + b*x))/(c + d*x))*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 +
2*B*f^3*g*x^2 + B*f*g^3*x^4) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*
```

```
f^3*g + 10*B*a*d*f^3*g - 10*B*b*c*f^3*g + 30*A*a*c*f^2*g^2)/(5*b*d) - ((5*a
*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c
*f^2*g^2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(5*b*d) + ((5*a*d + 5*b*c)*
((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*
b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c
*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30
*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4
 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5
*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g
^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*
c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*
A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (
A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*
a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5
*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*
b*d)) + (A*g^4*x^5)/5 + (log(a + b*x)*((B*a^5*g^4)/5 + B*a*b^4*f^4 - 2*B*a^
2*b^3*f^3*g + 2*B*a^3*b^2*f^2*g^2 - B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(B*
c^5*g^4 + 5*B*c*d^4*f^4 - 10*B*c^2*d^3*f^3*g + 10*B*c^3*d^2*f^2*g^2 - 5*B*c
^4*d*f*g^3))/(5*d^5)
```

3.231 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{4b^3d^3} \\
&\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{8b^2d^2}-\frac{B(bc-ad)g^3x^3}{12bd}-\frac{B(bf-ag)^4\log(a+bx)}{4b^4g} \\
&\quad +\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{4d^4g}
\end{aligned}$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/12*B*(-a*d+b*c)*g^3*x^3/b/d-1/4*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(
g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/4*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/g
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used

$= \{2548, 84\}$

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= -\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{4b^3d^3} + \frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g} - \frac{B(bf-ag)^4\log(a+bx)}{4b^4g} - \frac{Bg^2x^2(bc-ad)(-adg-bcg+4bdf)}{8b^2d^2} - \frac{Bg^3x^3(bc-ad)}{12bd} + \frac{B(df-cg)^4\log(c+dx)}{4d^4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g
 - a*d*g)*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(12*b*d) - (B*(b*f - a
*g)^4*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(4*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int \left(\frac{g^2\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)}{b^3d^3} + \frac{g^3(4bdf-bcg-adg)x}{b^2d^2} + \frac{g^4x^2}{bd} + \frac{(bf-ag)^4}{b^3(bc-ad)(a+bx}\right.}{4g}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{4b^3d^3} - \frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{8b^2d^2} - \frac{B(bc-ad)g^3x^3}{12bd} - \frac{B(bf-ag)^4\log(a+bx)}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} + \frac{B(df-cg)^4\log(c+dx)}{4d^4g}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - \frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bdf-}{6b^4d^4}}{4g}$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(6*b*d*(b*c - a*d)*g
^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2
*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b^3*
d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f - c
*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.81

method	result
risch	$\frac{(gx+f)^4B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4g}+\frac{g^3Ax^4}{4}-\frac{B\ln(-dx-c)cf^3}{d}+\frac{g^2Bafx^2}{2b}-\frac{g^2Bcfx^2}{2d}-\frac{g^2Ba^2fx}{b^2}+\frac{3gBaf^2x}{2b}+$
parallelrisch	$\frac{-24Bxa^2b^2d^4fg^2+36Bxab^3d^4f^2g-36B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^2d^2f^2g+36B\ln(bx+a)b^4c^2d^2f^2g+24B\ln(bx+a)a^3bd^4fg^2-}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)/(d*x+c))+1/4*g^3*A*x^4-1/d*B*ln(-d*x-c)*c*f^
3+1/2/b*g^2*B*a*f*x^2-1/2/d*g^2*B*c*f*x^2-1/b^2*g^2*B*a^2*f*x+3/2/b*g*B*a*f
^2*x+1/d^2*g^2*B*c^2*f*x-3/2/d*g*B*c*f^2*x+1/4/g*B*ln(-d*x-c)*f^4-1/4/g*B*l
n(b*x+a)*f^4+1/b*B*ln(b*x+a)*a*f^3+1/4/d^4*g^3*B*ln(-d*x-c)*c^4-1/4/b^4*g^3
```

```
*B*ln(b*x+a)*a^4+1/4/b^3*g^3*B*a^3*x-1/4/d^3*g^3*B*c^3*x-1/d^3*g^2*B*ln(-d*
x-c)*c^3*f+3/2/d^2*g*B*ln(-d*x-c)*c^2*f^2+1/b^3*g^2*B*ln(b*x+a)*a^3*f-3/2/b
^2*g*B*ln(b*x+a)*a^2*f^2+g^2*A*f*x^3+1/12/b*g^3*B*a*x^3-1/12/d*g^3*B*c*x^3+
3/2*g*A*f^2*x^2-1/8/b^2*g^3*B*a^2*x^2+1/8/d^2*g^3*B*c^2*x^2+A*f^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(215) = 430$.

Time = 0.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.96

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{6\,Ab^4d^4g^3x^4+2\,(12\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3)x^3+3\,(12\,Ab^4d^4f^2g-4\,(Bb^4cd^3-Bab^3d^4)fg^2+(\dots}{\dots}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2
 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*b^4
*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*
b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^
2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*
f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x
+ c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2
 + 4*B*b^4*d^4*f^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(207) = 414$.

Time = 8.13 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.40

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ag^3x^4}{4}$$
$$-\frac{Ba(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4(ag-2bf)\left(a^2g^2-2abfg+2b\right.}{b}}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g}\right.}{4b^4}$$
$$+\frac{Bc(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+Bab^3c^4g^3-4Bab^3c^3dfg^2+6Bab^3}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-}\right.}{4d^4}$$
$$+x^3\left(Afg^2+\frac{Bag^3}{12b}-\frac{Bcg^3}{12d}\right)+x^2\cdot\left(\frac{3Af^2g}{2}-\frac{Ba^2g^3}{8b^2}+\frac{Bafg^2}{2b}+\frac{Bc^2g^3}{8d^2}-\frac{Bcfg^2}{2d}\right)$$
$$+x\left(Af^3+\frac{Ba^3g^3}{4b^3}-\frac{Ba^2fg^2}{b^2}+\frac{3Baf^2g}{2b}-\frac{Bc^3g^3}{4d^3}+\frac{Bc^2fg^2}{d^2}-\frac{3Bcf^2g}{2d}\right)$$
$$+\left(Bf^3x+\frac{3Bf^2gx^2}{2}+Bfg^2x^3+\frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(12*b) - B*c*g**3/(12*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(8*b**2) + B
*a*f*g**2/(2*b) + B*c**2*g**3/(8*d**2) - B*c*f*g**2/(2*d)) + x*(A*f**3 + B*
a**3*g**3/(4*b**3) - B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/(2*b) - B*c**3*g**3/
(4*d**3) + B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/(2*d)) + (B*f**3*x + 3*B*f**2*
g*x**2/2 + B*f*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.83

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{4}\,Ag^3x^4 + Afg^2x^3 + \frac{3}{2}\,Af^2gx^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^3 + \frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^2g + \frac{1}{2}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2c}{b^2d^2} + \frac{1}{24}\left(6\,x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log(bx+a)}{b^4}+\frac{6\,c^4\log(dx+c)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d}{b^3} + Af^3x$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^
2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b
*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6073 vs. 2(215) = 430.

Time = 0.79 (sec) , antiderivative size = 6073, normalized size of antiderivative = 26.75

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*e^5*f^3 - 8*B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d^
5*e^5*f^3 - 6*B*b^5*c^3*d^2*e^5*f^2*g + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B*a
^2*b^3*c*d^4*e^5*f^2*g - 6*B*a^3*b^2*d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f*g^
2 - 4*B*a*b^4*c^3*d^2*e^5*f*g^2 - 4*B*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4*b*d
```

^5*e^5*f*g^2 - B*b^5*c^5*e^5*g^3 + B*a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d^4*e^5*g^3 - B*a^5*d^5*e^5*g^3 - 12*(b*e*x + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d*x + c) + 24*(b*e*x + a*e)*B*a*b^3*c*d^5*e^4*f^3/(d*x + c) - 12*(b*e*x + a*e)*B*a^2*b^2*d^6*e^4*f^3/(d*x + c) + 24*(b*e*x + a*e)*B*b^4*c^3*d^3*e^4*f^2*g/(d*x + c) - 36*(b*e*x + a*e)*B*a*b^3*c^2*d^4*e^4*f^2*g/(d*x + c) + 12*(b*e*x + a*e)*B*a^3*b*d^6*e^4*f^2*g/(d*x + c) - 16*(b*e*x + a*e)*B*b^4*c^4*d^2*e^4*f*g^2/(d*x + c) + 16*(b*e*x + a*e)*B*a*b^3*c^3*d^3*e^4*f*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a^2*b^2*c^2*d^4*e^4*f*g^2/(d*x + c) - 8*(b*e*x + a*e)*B*a^3*b*c*d^5*e^4*f*g^2/(d*x + c) - 4*(b*e*x + a*e)*B*a^4*d^6*e^4*f*g^2/(d*x + c) + 4*(b*e*x + a*e)*B*b^4*c^5*d*e^4*g^3/(d*x + c) - 4*(b*e*x + a*e)*B*a*b^3*c^4*d^2*e^4*g^3/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b*c^2*d^4*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*B*a^4*c*d^5*e^4*g^3/(d*x + c) + 12*(b*e*x + a*e)^2*B*b^3*c^2*d^5*e^3*f^3/(d*x + c)^2 - 24*(b*e*x + a*e)^2*B*a*b^2*c*d^6*e^3*f^3/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a^2*b*d^7*e^3*f^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*b^3*c^3*d^4*e^3*f^2*g/(d*x + c)^2 + 54*(b*e*x + a*e)^2*B*a*b^2*c^2*d^5*e^3*f^2*g/(d*x + c)^2 - 18*(b*e*x + a*e)^2*B*a^2*b*c*d^6*e^3*f^2*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^3*d^7*e^3*f^2*g/(d*x + c)^2 + 24*(b*e*x + a*e)^2*B*b^3*c^4*d^3*e^3*f*g^2/(d*x + c)^2 - 36*(b*e*x + a*e)^2*B*a*b^2*c^3*d^4*e^3*f*g^2/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a^3*c*d^6*e^3*f*g^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*b^3*c^5*d^2*e^3*g^3/(d*x + c)^2 + 6*(b*e*x + a*e)^2*B*a*b^2*c^4*d^3*e^3*g^3/(d*x + c)^2 + 6*(b*e*x + a*e)^2*B*a^2*b*c^3*d^4*e^3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^3*c^2*d^5*e^3*g^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B*b^2*c^2*d^6*e^2*f^3/(d*x + c)^3 + 8*(b*e*x + a*e)^3*B*a*b*c*d^7*e^2*f^3/(d*x + c)^3 - 4*(b*e*x + a*e)^3*B*a^2*d^8*e^2*f^3/(d*x + c)^3 + 12*(b*e*x + a*e)^3*B*b^2*c^3*d^5*e^2*f^2*g/(d*x + c)^3 - 24*(b*e*x + a*e)^3*B*a*b*c^2*d^6*e^2*f^2*g/(d*x + c)^3 + 12*(b*e*x + a*e)^3*B*a^2*c*d^7*e^2*f^2*g/(d*x + c)^3 - 12*(b*e*x + a*e)^3*B*b^2*c^4*d^4*e^2*f*g^2/(d*x + c)^3 + 24*(b*e*x + a*e)^3*B*a*b*c^3*d^5*e^2*f*g^2/(d*x + c)^3 - 12*(b*e*x + a*e)^3*B*a^2*c^2*d^6*e^2*f*g^2/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B*b^2*c^5*d^3*e^2*g^3/(d*x + c)^3 - 8*(b*e*x + a*e)^3*B*a*b*c^4*d^4*e^2*g^3/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B*a^2*c^3*d^5*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4*e^4 - 4*(b*e*x + a*e)*b^3*d^5*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^8/(d*x + c)^4) + (24*A*b^8*c^2*d^3*e^5*f^3 - 48*A*a*b^7*c*d^4*e^5*f^3 + 24*A*a^2*b^6*d^5*e^5*f^3 - 36*A*b^8*c^3*d^2*e^5*f^2*g - 36*B*b^8*c^3*d^2*e^5*f^2*g + 36*A*a*b^7*c^2*d^3*e^5*f^2*g + 108*B*a*b^7*c^2*d^3*e^5*f^2*g + 36*A*a^2*b^6*c*d^4*e^5*f^2*g - 108*B*a^2*b^6*c*d^4*e^5*f^2*g - 36*A*a^3*b^5*d^5*e^5*f^2*g + 36*B*a^3*b^5*d^5*e^5*f^2*g + 24*A*b^8*c^4*d*e^5*f*g^2 + 36*B*b^8*c^4*d*e^5*f*g^2 - 24*A*a*b^7*c^3*d^2*e^5*f*g^2 - 72*B*a*b^7*c^3*d^2*e^5*f*g^2 - 24*A*a^3*b^5*c*d^4*e^5*f*g^2 + 72*B*a^3*b^5*c*d^4*e^5*f*g^2 + 24*A*a^4*b^4*d^5*e^5*f*g^2 - 36*B*a^4*b^4*d^5*e^5*f*g^2 - 6*A*b^8*c^5*e^5*g^3 - 11*B*b^8*c^5*e^5*g^3 + 6*A*a*b^7*c^4*d*e^5*g^3 + 19*B*a*b^7*c^4*d*e^5*g^3 - 2*B*a^2*b^6*c^3*d^2*e^5*g^3 + 2*B*a^3*b^5*c^2*d^3*e^5*g^3 + 6*A*a^4*b^4*c*d^4*e^5*g^3 - 19*B*a^4*b^4*c*d^4*e^5*g^3 - 6*A*a^5*b^3*d^5*e^5*g^3 + 11*B*a^5*b^3*d^5*e^5*g^3 - 72*(b*e*x + a*e)*A*b^7*c^2*d

^4*e^4*f^3/(d*x + c) + 144*(b*e*x + a*e)*A*a*b^6*c*d^5*e^4*f^3/(d*x + c) -
72*(b*e*x + a*e)*A*a^2*b^5*d^6*e^4*f^3/(d*x + c) + 144*(b*e*x + a*e)*A*b^7*
c^3*d^3*e^4*f^2*g/(d*x + c) + 108*(b*e*x + a*e)*B*b^7*c^3*d^3*e^4*f^2*g/(d*
x + c) - 216*(b*e*x + a*e)*A*a*b^6*c^2*d^4*e^4*f^2*g/(d*x + c) - 324*(b*e*x
 + a*e)*B*a*b^6*c^2*d^4*e^4*f^2*g/(d*x + c) + 324*(b*e*x + a*e)*B*a^2*b^5*c
*d^5*e^4*f^2*g/(d*x + c) + 72*(b*e*x + a*e)*A*a^3*b^4*d^6*e^4*f^2*g/(d*x +
c) - 108*(b*e*x + a*e)*B*a^3*b^4*d^6*e^4*f^2*g/(d*x + c) - 96*(b*e*x + a*e)
*A*b^7*c^4*d^2*e^4*f*g^2/(d*x + c) - 120*(b*e*x + a*e)*B*b^7*c^4*d^2*e^4*f*
g^2/(d*x + c) + 96*(b*e*x + a*e)*A*a*b^6*c^3*d^3*e^4*f*g^2/(d*x + c) + 264*
(b*e*x + a*e)*B*a*b^6*c^3*d^3*e^4*f*g^2/(d*x + c) + 72*(b*e*x + a*e)*A*a^2*
b^5*c^2*d^4*e^4*f*g^2/(d*x + c) - 72*(b*e*x + a*e)*B*a^2*b^5*c^2*d^4*e^4*f*
g^2/(d*x + c) - 48*(b*e*x + a*e)*A*a^3*b^4*c*d^5*e^4*f*g^2/(d*x + c) - 168*
(b*e*x + a*e)*B*a^3*b^4*c*d^5*e^4*f*g^2/(d*x + c) - 24*(b*e*x + a*e)*A*a^4*
b^3*d^6*e^4*f*g^2/(d*x + c) + 96*(b*e*x + a*e)*B*a^4*b^3*d^6*e^4*f*g^2/(d*x
 + c) + 24*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^3/(d*x + c) + 38*(b*e*x + a*e)*B
*b^7*c^5*d*e^4*g^3/(d*x + c) - 24*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g^3/(d*
x + c) - 70*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) + 8*(b*e*x + a*
e)*B*a^2*b^5*c^3*d^3*e^4*g^3/(d*x + c) - 24*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4
*e^4*g^3/(d*x + c) + 16*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(d*x + c) +
 24*(b*e*x + a*e)*A*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 34*(b*e*x + a*e)*B*a^
4*b^3*c*d^5*e^4*g^3/(d*x + c) - 26*(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*g^3/(d*x
 + c) + 72*(b*e*x + a*e)^2*A*b^6*c^2*d^5*e^3*f^3/(d*x + c)^2 - 144*(b*e*x +
 a*e)^2*A*a*b^5*c*d^6*e^3*f^3/(d*x + c)^2 + 72*(b*e*x + a*e)^2*A*a^2*b^4*d^
7*e^3*f^3/(d*x + c)^2 - 180*(b*e*x + a*e)^2*A*b^6*c^3*d^4*e^3*f^2*g/(d*x +
c)^2 - 108*(b*e*x + a*e)^2*B*b^6*c^3*d^4*e^3*f^2*g/(d*x + c)^2 + 324*(b*e*x
 + a*e)^2*A*a*b^5*c^2*d^5*e^3*f^2*g/(d*x + c)^2 + 324*(b*e*x + a*e)^2*B*a*b
^5*c^2*d^5*e^3*f^2*g/(d*x + c)^2 - 108*(b*e*x + a*e)^2*A*a^2*b^4*c*d^6*e^3*
f^2*g/(d*x + c)^2 - 324*(b*e*x + a*e)^2*B*a^2*b^4*c*d^6*e^3*f^2*g/(d*x + c)
^2 - 36*(b*e*x + a*e)^2*A*a^3*b^3*d^7*e^3*f^2*g/(d*x + c)^2 + 108*(b*e*x +
a*e)^2*B*a^3*b^3*d^7*e^3*f^2*g/(d*x + c)^2 + 144*(b*e*x + a*e)^2*A*b^6*c^4*
d^3*e^3*f*g^2/(d*x + c)^2 + 132*(b*e*x + a*e)^2*B*b^6*c^4*d^3*e^3*f*g^2/(d*
x + c)^2 - 216*(b*e*x + a*e)^2*A*a*b^5*c^3*d^4*e^3*f*g^2/(d*x + c)^2 - 312*
(b*e*x + a*e)^2*B*a*b^5*c^3*d^4*e^3*f*g^2/(d*x + c)^2 + 144*(b*e*x + a*e)^2
*B*a^2*b^4*c^2*d^5*e^3*f*g^2/(d*x + c)^2 + 72*(b*e*x + a*e)^2*A*a^3*b^3*c*d
^6*e^3*f*g^2/(d*x + c)^2 + 120*(b*e*x + a*e)^2*B*a^3*b^3*c*d^6*e^3*f*g^2/(d
x + c)^2 - 84(b*e*x + a*e)^2*B*a^4*b^2*d^7*e^3*f*g^2/(d*x + c)^2 - 36*(b*
e*x + a*e)^2*A*b^6*c^5*d^2*e^3*g^3/(d*x + c)^2 - 45*(b*e*x + a*e)^2*B*b^6*c
^5*d^2*e^3*g^3/(d*x + c)^2 + 36*(b*e*x + a*e)^2*A*a*b^5*c^4*d^3*e^3*g^3/(d*
x + c)^2 + 93*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2 + 36*(b*e
*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a^
2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^3*b^3*c^2*d^5*e^
3*g^3/(d*x + c)^2 - 18*(b*e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^
2 - 21*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*g^3/(d*x + c)^2 + 21*(b*e*x + a*
e)^2*B*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 24*(b*e*x + a*e)^3*A*b^5*c^2*d^6*e^2
*f^3/(d*x + c)^3 + 48*(b*e*x + a*e)^3*A*a*b^4*c*d^7*e^2*f^3/(d*x + c)^3 - 2

```
4*(b*e*x + a*e)^3*A*a^2*b^3*d^8*e^2*f^3/(d*x + c)^3 + 72*(b*e*x + a*e)^3*A*
b^5*c^3*d^5*e^2*f^2*g/(d*x + c)^3 + 36*(b*e*x + a*e)^3*B*b^5*c^3*d^5*e^2*f^
2*g/(d*x + c)^3 - 144*(b*e*x + a*e)^3*A*a*b^4*c^2*d^6*e^2*f^2*g/(d*x + c)^3
 - 108*(b*e*x + a*e)^3*B*a*b^4*c^2*d^6*e^2*f^2*g/(d*x + c)^3 + 72*(b*e*x +
a*e)^3*A*a^2*b^3*c*d^7*e^2*f^2*g/(d*x + c)^3 + 108*(b*e*x + a*e)^3*B*a^2*b^
3*c*d^7*e^2*f^2*g/(d*x + c)^3 - 36*(b*e*x + a*e)^3*B*a^3*b^2*d^8*e^2*f^2*g/
(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*b^5*c^4*d^4*e^2*f*g^2/(d*x + c)^3 - 48*(
b*e*x + a*e)^3*B*b^5*c^4*d^4*e^2*f*g^2/(d*x + c)^3 + 144*(b*e*x + a*e)^3*A*
a*b^4*c^3*d^5*e^2*f*g^2/(d*x + c)^3 + 120*(b*e*x + a*e)^3*B*a*b^4*c^3*d^5*e
^2*f*g^2/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a^2*b^3*c^2*d^6*e^2*f*g^2/(d*x
+ c)^3 - 72*(b*e*x + a*e)^3*B*a^2*b^3*c^2*d^6*e^2*f*g^2/(d*x + c)^3 - 24*(b
*e*x + a*e)^3*B*a^3*b^2*c*d^7*e^2*f*g^2/(d*x + c)^3 + 24*(b*e*x + a*e)^3*B*
a^4*b*d^8*e^2*f*g^2/(d*x + c)^3 + 24*(b*e*x + a*e)^3*A*b^5*c^5*d^3*e^2*g^3/
(d*x + c)^3 + 18*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 - 48*(b*
e*x + a*e)^3*A*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 42*(b*e*x + a*e)^3*B*a*b
^4*c^4*d^4*e^2*g^3/(d*x + c)^3 + 24*(b*e*x + a*e)^3*A*a^2*b^3*c^3*d^5*e^2*g
^3/(d*x + c)^3 + 24*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g^3/(d*x + c)^3 +
 6*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 - 6*(b*e*x + a*e)^3*B*
a^5*d^8*e^2*g^3/(d*x + c)^3)/(b^7*d^4*e^4 - 4*(b*e*x + a*e)*b^6*d^5*e^3/(d*
x + c) + 6*(b*e*x + a*e)^2*b^5*d^6*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b^4*
d^7*e/(d*x + c)^3 + (b*e*x + a*e)^4*b^3*d^8/(d*x + c)^4) + 6*(4*B*b^5*c^2*d
^3*e*f^3 - 8*B*a*b^4*c*d^4*e*f^3 + 4*B*a^2*b^3*d^5*e*f^3 - 6*B*b^5*c^3*d^2*
e*f^2*g + 6*B*a*b^4*c^2*d^3*e*f^2*g + 6*B*a^2*b^3*c*d^4*e*f^2*g - 6*B*a^3*b
^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f*g^2 - 4*B*a*b^4*c^3*d^2*e*f*g^2 - 4*B*a^
3*b^2*c*d^4*e*f*g^2 + 4*B*a^4*b*d^5*e*f*g^2 - B*b^5*c^5*e*g^3 + B*a*b^4*c^4
*d*e*g^3 + B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log(-b*e + (b*e*x + a*e)*
d/(d*x + c))/(b^4*d^4) - 6*(4*B*b^5*c^2*d^3*e*f^3 - 8*B*a*b^4*c*d^4*e*f^3 +
 4*B*a^2*b^3*d^5*e*f^3 - 6*B*b^5*c^3*d^2*e*f^2*g + 6*B*a*b^4*c^2*d^3*e*f^2*
g + 6*B*a^2*b^3*c*d^4*e*f^2*g - 6*B*a^3*b^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f
*g^2 - 4*B*a*b^4*c^3*d^2*e*f*g^2 - 4*B*a^3*b^2*c*d^4*e*f*g^2 + 4*B*a^4*b*d^
5*e*f*g^2 - B*b^5*c^5*e*g^3 + B*a*b^4*c^4*d*e*g^3 + B*a^4*b*c*d^4*e*g^3 - B
*a^5*d^5*e*g^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4))*(b*c/((b*c*e - a*d*
e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.26

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\ldots}{\ldots}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{12\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3\right)}{4\,d^4}$$

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g - 6*B*b*c*f^2*g)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4*A*b
*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d +
4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 +
 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) +
 (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3
```

```
 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d)))/
(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b
*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*
d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*
a*d*f*g^2 - 4*B*b*c*f*g^2)/(8*b*d) + (A*a*c*g^3)/(2*b*d)) + log((e*(a + b*x
))/(c + d*x))*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3) +
 x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/
(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x
)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(4*b^4
) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*
d*f*g^2))/(4*d^4)
```

3.232 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2}-\frac{B(bc-ad)g^2x^2}{6bd}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g}+\frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

```
[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2*x
^2/b/d-1/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)/(
d*x+c)))/g+1/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used

$= \{2548, 84\}$

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g} - \frac{B(bf-ag)^3\log(a+bx)}{3b^3g} - \frac{Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2} - \frac{Bg^2x^2(bc-ad)}{6bd} + \frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) - (B*(b*c - a*
d)*g^2*x^2)/(6*b*d) - (B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) + (B*(d*f - c*g)^3*Log[c + d
*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad)) \int \frac{(f+gx)^3}{(a+bx)(c+dx)} \, dx}{3g}$$

$$= \frac{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad)) \int \left(\frac{g^2(3bdf-bcg-adg)}{b^2d^2} + \frac{g^3x}{bd} + \frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)} + \frac{(df-cg)^3}{d^2(-bc+ad)(c+dx)}\right) dx}{3g}$$

$$= -\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{6bd} - \frac{B(bf-ag)^3\log(a+bx)}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} + \frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - \frac{B(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log}{2b^3d^3}}{3g}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*g
^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f -
 a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/(3*g
)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.77

method	result
risch	$\frac{(gx+f)^3 B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3g}+\frac{g^2Ax^3}{3}+gAf\,x^2+\frac{g^2Ba\,x^2}{6b}-\frac{g^2Bc\,x^2}{6d}+A\,f^2x-\frac{g^2B\ln(dx+c)c^3}{3d^3}+\frac{gB\ln(}{}$
parallelrisch	$\frac{B\,x^2a\,b^2d^3g^2-B\,x^2b^3c\,d^2g^2+6Bxa\,b^2d^3fg-2Bx\,a^2b\,d^3g^2-2B\,c^3g^2b^3+2B\,a^3d^3g^2-6Bx\,b^3c\,d^2fg-Ba\,b^2c^2d\,g^2+6B\,b^3}{}$
parts	$\frac{A(gx+f)^3}{3g}-\frac{B(ad-cb)e\left(-2eg\left(acdg-a\,d^2f-b\,c^2g+bcdf\right)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(A\,d^2\left(\frac{e^2g^2\left(a^2d^2-2abcd+b^2c^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{g^2c^2-2gfdc+f^2d^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{eg\left(acdg-a\,d^2f-b\,c^2g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)+B\,d^2\left(\frac{e^2}{}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(A\,d^2\left(\frac{e^2g^2\left(a^2d^2-2abcd+b^2c^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{g^2c^2-2gfdc+f^2d^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{eg\left(acdg-a\,d^2f-b\,c^2g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)+B\,d^2\left(\frac{e^2}{}\right.\right.}{}$

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*A*x^3+g*A*f*x^2+1/6/b*g^2*B
*a*x^2-1/6/d*g^2*B*c*x^2+A*f^2*x-1/3/d^3*g^2*B*ln(d*x+c)*c^3+1/d^2*g*B*ln(d
*x+c)*c^2*f-1/d*B*ln(d*x+c)*c*f^2+1/3/g*B*ln(d*x+c)*f^3+1/3/b^3*g^2*B*ln(-b
*x-a)*a^3-1/b^2*g*B*ln(-b*x-a)*a^2*f+1/b*B*ln(-b*x-a)*a*f^2-1/3/g*B*ln(-b*x
-a)*f^3-1/3/b^2*g^2*B*a^2*x+1/b*g*B*a*f*x+1/3/d^2*g^2*B*c^2*x-1/d*g*B*c*f*x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{2\,Ab^3d^3g^2x^3+(6\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2)x^2+2\,(3\,Ab^3d^3f^2-3\,(Bb^3cd^2-Bab^2d^3)fg+(Bb^3c^2}{}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g
^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3*c
^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a
```

^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3 *c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b ^3*d^3*f^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(131) = 262$.

Time = 3.12 (sec) , antiderivative size = 658, normalized size of antiderivative = 4.39

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ag^2x^3}{3}$$

$$+\frac{Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+\frac{Ba^2d^3\left(a^2g^2-3abfg+3b^2f^2\right)}{b}+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2f^2}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3Bb^3c}\right.}{3b^3}$$

$$-\frac{Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2f^2-Bab^2c(c^2g^2-3cdfg+3d^2f^2)}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3Bb^3cd}\right.}{3d^3}$$

$$+x^2\left(Afg+\frac{Bag^2}{6b}-\frac{Bcg^2}{6d}\right)+x\left(Af^2-\frac{Ba^2g^2}{3b^2}+\frac{Bafg}{b}+\frac{Bc^2g^2}{3d^2}-\frac{Bcfg}{d}\right)$$

$$+\left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)

[Out] A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*c *d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g + 3 *b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d **2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d**3*g **2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b **3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3*c*d*f *g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a *b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 - B*a*b**2 *c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2 - 3*c*d*f *g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d **3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/ (3*d**3) + x**2*(A*f*g + B*a*g**2/(6*b) - B*c*g**2/(6*d)) + x*(A*f**2 - B*a **2*g**2/(3*b**2) + B*a*f*g/b + B*c**2*g**2/(3*d**2) - B*c*f*g/d) + (B*f**2 *x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.75

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{3}\,Ag^2x^3+Afgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^2 + \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bfg + \frac{1}{6}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a}{b^2d^2} + Af^2x$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log
(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))
*B*f*g + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3076 vs. 2(140) = 280.

Time = 0.61 (sec) , antiderivative size = 3076, normalized size of antiderivative = 20.51

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^4
*e^4*f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^2*
c*d^3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*d*e
^4*g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*b^3*
c^2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*x + c
) - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B*b^3*c
^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/(d*x +
c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*a^
3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x + c) + 3
```

*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*
c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/(d*x + c) +
 3*(b*e*x + a*e)^2*B*b^2*c^2*d^4*e^2*f^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*
a*b*c*d^5*e^2*f^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*d^6*e^2*f^2/(d*x +
c)^2 - 6*(b*e*x + a*e)^2*B*b^2*c^3*d^3*e^2*f*g/(d*x + c)^2 + 12*(b*e*x + a*
e)^2*B*a*b*c^2*d^4*e^2*f*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*c*d^5*e^2*
f*g/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*b^2*c^4*d^2*e^2*g^2/(d*x + c)^2 - 6*(
b*e*x + a*e)^2*B*a*b*c^3*d^3*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*
c^2*d^4*e^2*g^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3 - 3*
(b*e*x + a*e)*b^2*d^4*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e/(d*x + c)^2
 - (b*e*x + a*e)^3*d^6/(d*x + c)^3) + (6*A*b^6*c^2*d^2*e^4*f^2 - 12*A*a*b^5
*c*d^3*e^4*f^2 + 6*A*a^2*b^4*d^4*e^4*f^2 - 6*A*b^6*c^3*d*e^4*f*g - 6*B*b^6*
c^3*d*e^4*f*g + 6*A*a*b^5*c^2*d^2*e^4*f*g + 18*B*a*b^5*c^2*d^2*e^4*f*g + 6*
A*a^2*b^4*c*d^3*e^4*f*g - 18*B*a^2*b^4*c*d^3*e^4*f*g - 6*A*a^3*b^3*d^4*e^4*
f*g + 6*B*a^3*b^3*d^4*e^4*f*g + 2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 -
 2*A*a*b^5*c^3*d*e^4*g^2 - 6*B*a*b^5*c^3*d*e^4*g^2 - 2*A*a^3*b^3*c*d^3*e^4*
g^2 + 6*B*a^3*b^3*c*d^3*e^4*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 - 3*B*a^4*b^2*d^4
*e^4*g^2 - 12*(b*e*x + a*e)*A*b^5*c^2*d^3*e^3*f^2/(d*x + c) + 24*(b*e*x + a
*e)*A*a*b^4*c*d^4*e^3*f^2/(d*x + c) - 12*(b*e*x + a*e)*A*a^2*b^3*d^5*e^3*f^
2/(d*x + c) + 18*(b*e*x + a*e)*A*b^5*c^3*d^2*e^3*f*g/(d*x + c) + 12*(b*e*x
+ a*e)*B*b^5*c^3*d^2*e^3*f*g/(d*x + c) - 30*(b*e*x + a*e)*A*a*b^4*c^2*d^3*e
^3*f*g/(d*x + c) - 36*(b*e*x + a*e)*B*a*b^4*c^2*d^3*e^3*f*g/(d*x + c) + 6*(
b*e*x + a*e)*A*a^2*b^3*c*d^4*e^3*f*g/(d*x + c) + 36*(b*e*x + a*e)*B*a^2*b^3
*c*d^4*e^3*f*g/(d*x + c) + 6*(b*e*x + a*e)*A*a^3*b^2*d^5*e^3*f*g/(d*x + c)
- 12*(b*e*x + a*e)*B*a^3*b^2*d^5*e^3*f*g/(d*x + c) - 6*(b*e*x + a*e)*A*b^5*
c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 6
*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) + 16*(b*e*x + a*e)*B*a*b^4
*c^3*d^2*e^3*g^2/(d*x + c) + 6*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*g^2/(d*x
 + c) - 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) - 6*(b*e*x + a*
e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 8*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*
g^2/(d*x + c) + 5*(b*e*x + a*e)*B*a^4*b*d^5*e^3*g^2/(d*x + c) + 6*(b*e*x +
a*e)^2*A*b^4*c^2*d^4*e^2*f^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a*b^3*c*d^5
*e^2*f^2/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^2*b^2*d^6*e^2*f^2/(d*x + c)^2
- 12*(b*e*x + a*e)^2*A*b^4*c^3*d^3*e^2*f*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*
B*b^4*c^3*d^3*e^2*f*g/(d*x + c)^2 + 24*(b*e*x + a*e)^2*A*a*b^3*c^2*d^4*e^2*
f*g/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*a*b^3*c^2*d^4*e^2*f*g/(d*x + c)^2 -
12*(b*e*x + a*e)^2*A*a^2*b^2*c*d^5*e^2*f*g/(d*x + c)^2 - 18*(b*e*x + a*e)^2
*B*a^2*b^2*c*d^5*e^2*f*g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*B*a^3*b*d^6*e^2*f*
g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2 + 4*(b*
e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a*b^3
*c^3*d^3*e^2*g^2/(d*x + c)^2 - 10*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(
d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 + 6*(b
*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 + 2*(b*e*x + a*e)^2*B*a
^3*b*c*d^5*e^2*g^2/(d*x + c)^2 - 2*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x +
 c)^2)/(b^5*d^3*e^3 - 3*(b*e*x + a*e)*b^4*d^4*e^2/(d*x + c) + 3*(b*e*x + a*

```
e)^2*b^3*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*b^2*d^6/(d*x + c)^3) + 2*(3*B*
b^4*c^2*d^2*e*f^2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*f^2 - 3*B*b^4
*c^3*d*e*f*g + 3*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*f*g - 3*B*a^3*
b*d^4*e*f*g + B*b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*b*c*d^3*e*g^2 +
 B*a^4*d^4*e*g^2)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^3*d^3) - 2*(3*B*
b^4*c^2*d^2*e*f^2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*f^2 - 3*B*b^4
*c^3*d*e*f*g + 3*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*f*g - 3*B*a^3*
b*d^4*e*f*g + B*b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*b*c*d^3*e*g^2 +
 B*a^4*d^4*e*g^2)*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.37

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right.$$
$$\left.-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g-3\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)$$
$$+\frac{\ln(a+b\,x)\,\left(B\,a^3\,g^2-3\,B\,a^2\,b\,f\,g+3\,B\,a\,b^2\,f^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(B\,c^3\,g^2-3\,B\,c^2\,d\,f\,g+3\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(6*b
*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*((B*g
^2*x^3)/3 + B*f^2*x + B*f*g*x^2) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*
g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c))/(3*b*d))*(
3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*f*g + 6*A*b*
c*f*g + 3*B*a*d*f*g - 3*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d)) + (log(a +
b*x)*(B*a^3*g^2 + 3*B*a*b^2*f^2 - 3*B*a^2*b*f*g))/(3*b^3) - (log(c + d*x)*(
B*c^3*g^2 + 3*B*c*d^2*f^2 - 3*B*c^2*d*f*g))/(3*d^3) + (A*g^2*x^3)/3
```

3.233 $\int (f+gx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (f+gx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx = -\frac{B(bc-ad)gx}{2bd} - \frac{B(bf-ag)^2\log(a+bx)}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} + \frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/b/d-1/2*B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/2*B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2548, 84}

$$\int (f+gx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx = \frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g} - \frac{B(bf-ag)^2\log(a+bx)}{2b^2g} - \frac{Bgx(bc-ad)}{2bd} + \frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*x)/(b*d) - (B*(b*f - a*g)^2*Log[a + b*x])/(2*b^2*g) +
 ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*g) + (B*(d*f - c*g)^
2*Log[c + d*x])/(2*d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}-\frac{(B(bc-ad))\int\frac{(f+gx)^2}{(a+bx)(c+dx)}\,dx}{2g}\\
&= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}-\frac{(B(bc-ad))\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(-bc+ad)(c+dx)}\right)dx}{2g}\\
&= -\frac{B(bc-ad)gx}{2bd}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}\\
&\quad+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{-Bd^2(bf-ag)^2\log(a+bx)+b\Big(d(B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)}{c+dx}\right)+b\cdots}{2b^2d^2g}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*d
*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d*f
 - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{A\,x^2 g}{2}+Afx+\frac{B\ln(-dx-c)c^2 g}{2d^2}-\frac{B\ln(-dx-c)cf}{d}-\frac{B\ln(bx+a)a^2 g}{2b^2}+\frac{B\ln(bx+\cdots}{b}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2f+2A\,b^2d^2fx-B\ln(bx+a)a^2d^2g+2B\ln(bx+a)ab\,d^2f+B\ln(\cdots}{\cdots}$
parts	$A\left(\frac{1}{2}g\,x^2+fx\right)-\frac{B(ad-cb)e\left(deg(ad-cb)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}+\frac{\ln\left(\frac{be}{d}+\frac{(a\cdots}{d(\cdots}\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^2\left(\frac{eg(ad-cb)}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{cg-df}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2\left(-\frac{eg(ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)\cdots\right)}{2b^2e^2d}\cdots\right)}{\cdots}\cdots\right)\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(-A\,d^2\left(\frac{eg(ad-cb)}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{cg-df}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2\left(-\frac{eg(ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)\cdots\right)}{2b^2e^2d}\cdots\right)}{\cdots}\cdots\right)\right)}{\cdots}$

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)/(d*x+c))+1/2*A*x^2*g+A*f*x+1/2/d^2*B*ln(-d*x
-c)*c^2*g-1/d*B*ln(-d*x-c)*c*f-1/2/b^2*B*ln(b*x+a)*a^2*g+1/b*B*ln(b*x+a)*a*
f+1/2/b*B*x*a*g-1/2/d*B*x*c*g
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{Ab^2d^2gx^2+(2\,Ab^2d^2f-(Bb^2cd-Babd^2)g)x+(2\,Babd^2f-Ba^2d^2g)\log(bx+a)-(2\,Bb^2cdf-Bb^2c^2g)\cdots}{2\,b^2d^2}$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*B
*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*log(
d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b*e*x + a*e)/(d*x + c))
)/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.

Time = 1.42 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Agx^2}{2}-\frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2b^2}$$
$$+\frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2d^2}$$
$$+x\left(Af+\frac{Bag}{2b}-\frac{Bcg}{2d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*b**2) + B*c*(c*g
 - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c
*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f
+ B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + x*(A*f + B*a*g/(2*b) - B*c*g/
(2*d)) + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}Agx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bg$$
$$+Afx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2
*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*f*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(101) = 202.

Time = 0.49 (sec) , antiderivative size = 1145, normalized size of antiderivative = 10.50

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{\left(2\,Bb^3c^2de^3f-4\,Bab^2cd^2e^3f+2\,Ba^2bd^3e^3f-Bb^3c^3e^3g+Bab^2c^2de^3g+Ba^2bcd^2e^3g-Ba^3d^3e^3g-\right.}{b^2d^2}\right.$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f - B
*b^3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*e^3*
g - 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*a*b*c
*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*(b*e*x
 + a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*e^2*g/(
d*x + c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x + a*e)/(
d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x + a*e)^
2*d^4/(d*x + c)^2) + (2*A*b^4*c^2*d*e^3*f - 4*A*a*b^3*c*d^2*e^3*f + 2*A*a^2
*b^2*d^3*e^3*f - A*b^4*c^3*e^3*g - B*b^4*c^3*e^3*g + A*a*b^3*c^2*d*e^3*g +
3*B*a*b^3*c^2*d*e^3*g + A*a^2*b^2*c*d^2*e^3*g - 3*B*a^2*b^2*c*d^2*e^3*g - A
*a^3*b*d^3*e^3*g + B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*b^3*c^2*d^2*e^2*f/
(d*x + c) + 4*(b*e*x + a*e)*A*a*b^2*c*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)
*A*a^2*b*d^4*e^2*f/(d*x + c) + 2*(b*e*x + a*e)*A*b^3*c^3*d*e^2*g/(d*x + c)
+ (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*A*a*b^2*c^2*d
^2*e^2*g/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 2*(b
*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e
^2*g/(d*x + c) - (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^3*d^2*e^2 - 2*
(b*e*x + a*e)*b^2*d^3*e/(d*x + c) + (b*e*x + a*e)^2*b*d^4/(d*x + c)^2) + (2
*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e*f - B*b^3*c^3*e*g
+ B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-b*e + (b*e*x
+ a*e)*d/(d*x + c))/(b^2*d^2) - (2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f +
2*B*a^2*b*d^3*e*f - B*b^3*c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g -
 B*a^3*d^3*e*g)*log((b*e*x + a*e)/(d*x + c))/(b^2*d^2))*(b*c/((b*c*e - a*d*
e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)$$
$$+x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^2\,g-2\,B\,a\,b\,f\right)}{2\,b^2}+\frac{\ln\left(c+d\,x\right)\,\left(B\,c^2\,g-2\,B\,c\,d\,f\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*(B*f*x + (B*g*x^2)/2) + x*((2*A*a*d*g + 2*A*b*
c*g + 2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d) - (A*g*(2*a*d + 2*b*c))/(2*b*d
)) - (log(a + b*x)*(B*a^2*g - 2*B*a*b*f))/(2*b^2) + (log(c + d*x)*(B*c^2*g
- 2*B*c*d*f))/(2*d^2) + (A*g*x^2)/2
```

3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/b-B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2536, 31}

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[In] Int[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= Ax + B\int \log\left(\frac{e(a+bx)}{c+dx}\right)dx\\ &= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{(B(bc-ad))\int \frac{1}{c+dx}\,dx}{b}\\ &= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{B(bc-ad)\log(c+dx)}{bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{B(bc-ad)\log(c+dx)}{bd}$$

```
[In] Integrate[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result	size
risch	$Ax+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)-\frac{Bc\ln(dx+c)}{d}+\frac{Ba\ln(-bx-a)}{b}$	51
parallelrisch	$\frac{B\left(x\ln\left(\frac{e(bx+a)}{dx+c}\right)bd+\ln(bx+a)ad-\ln(bx+a)bc+\ln\left(\frac{e(bx+a)}{dx+c}\right)bc\right)}{bd}+Ax$	70
default	$Ax-B(ad-cb)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)$	162
parts	$Ax-B(ad-cb)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)$	162
derivativedivides	$-\frac{e(ad-cb)\left(\frac{dA}{be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}+\frac{dB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)}{d^2}$	201

```
[In] int(A+B*ln(e*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
[Out] A*x+B*x*ln(e*(b*x+a)/(d*x+c))-B/d*c*ln(d*x+c)+B/b*a*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Bbdx\log\left(\frac{bex+ae}{dx+c}\right)+Abdx+Bad\log\left(bx+a\right)-Bbc\log\left(dx+c\right)}{bd}$$

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="fricas")
[Out] (B*b*d*x*log((b*e*x + a*e)/(d*x + c)) + A*b*d*x + B*a*d*log(b*x + a) - B*b*
c*log(d*x + c))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=Ax+\frac{Ba\log\left(x+\frac{\frac{Ba^2d}{b}+Bac}{Bad+Bbc}\right)}{b}-\frac{Bc\log\left(x+\frac{Bac+\frac{Bbc^2}{d}}{Bad+Bbc}\right)}{d}+Bx\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate(A+B*ln(e*(b*x+a)/(d*x+c)),x)
```

```
[Out] A*x + B*a*log(x + (B*a**2*d/b + B*a*c)/(B*a*d + B*b*c))/b - B*c*log(x + (B*
a*c + B*b*c**2/d)/(B*a*d + B*b*c))/d + B*x*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\left(x\log\left(\frac{(bx+a)e}{dx+c}\right)+\frac{\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}}{e}\right)B+Ax$$

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] (x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)/e
)*B + A*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(52) = 104.

Time = 0.44 (sec) , antiderivative size = 406, normalized size of antiderivative = 7.81

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=$$

$$-\left((b^2c^2e^2-2\,abcde^2+a^2d^2e^2)\left(\frac{\log\left(\frac{|bex+ae|}{|dx+c|}\right)}{bde}-\frac{\log\left(\left|-be+\frac{(bex+ae)d}{dx+c}\right|\right)}{bde}\right)-\frac{(b^2c^2e^2-2\,abcde^2+a^2d^2\cdots}{\left(\cdots\right.}\right.$$

$$+Ax$$

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] -((b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*(log(abs(b*e*x + a*e)/abs(d*x
 + c))/(b*d*e) - log(abs(-b*e + (b*e*x + a*e)*d/(d*x + c)))/(b*d*e)) - (b^2
*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*log((a - b*(a/(b*c - a*d) - (b*e*x
+ a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/((b*
c*e - a*d*e)*(d*x + c))))*e/(c - d*(a/(b*c - a*d) - (b*e*x + a*e)*c/((b*c*e
 - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/((b*c*e - a*d*e)*(d*
x + c)))))/((b*e - (b*e*x + a*e)*d/(d*x + c))*d))*B*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))) + A*x
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right) + \frac{B\,a\,\ln\left(a+b\,x\right)}{b} - \frac{B\,c\,\ln\left(c+d\,x\right)}{d}$$

```
[In] int(A + B*log((e*(a + b*x))/(c + d*x)),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x))/(c + d*x)) + (B*a*log(a + b*x))/b - (B*c*log(c
+ d*x))/d
```

3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{B\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{B\,\mathrm{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

```
[Out] -B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(g*x
+f)/g+B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*polylog(2,b*(g*x+f)/(-a*g+b
*f))/g+B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{2546, 2441, 2440, 2438\}$

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g} - \frac{B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} - \frac{B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g} + \frac{B\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g} + \frac{B\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

```
[Out] -((B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[f + g*x])/g + (B*Log[-((g*(c + d*x))/(d*f - c*g))]
*Log[f + g*x])/g - (B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B*PolyLog
[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{(bB)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bd)\int\frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + B\int\frac{\log\left(\frac{g(a+bx)}{-bf+ag}\right)}{f+gx}\,dx - B\int\frac{\log\left(\frac{g(c+dx)}{-df+cg}\right)}{f+gx}\,dx$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + \frac{B\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bf+ag}\right)}{x}\,dx,x,f+gx\right)}{g} - \frac{B\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{-df+cg}\right)}{x}\,dx,x,f+gx\right)}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{B\,\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{B\,\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \frac{\left(A-B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+B\,\text{Po}}{g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

```
[Out] ((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
+ B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f +
g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(140) = 280$.

Time = 3.98 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.66

method	result
parts	$\frac{A\ln(gx+f)}{g} - \frac{B(ad-cb)e\left(-\frac{d^2(cg-df)\left(\frac{\operatorname{dilog}\left(\frac{(cg-df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef}{-aeg+bef}\right)}{cg-df}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{(cg-df)\left(\frac{be}{d}+\cdots\right.}{-\cdots}\right.}{cg-df}\cdots\right.}{eg(ad-cb)}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-d^2A\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\left(\cdots\right.\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-d^2A\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\left(\cdots\right.\right.}{\cdots}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(g*x+f)/g-B/d^2*(a*d-b*c)*e*(-d^2*(c*g-d*f)/e/g/(a*d-b*c)*(dilog(((c*g-
d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)+l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f))+d^3/e/g/(a*d-b*c)*(dilog(-((b*e/d+(
a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b
*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")
[Out] integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")
[Out] -B*integrate(-(log(b*x + a) - log(d*x + c) + log(e))/(g*x + f), x) + A*log(
g*x + f)/g
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{f+g\,x}\,dx$$

[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x),x)

[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x), x)

3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*ln((g*x
+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2554, 2351, 31}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)*(f + g*x)) +
(B*(b*c - a*d)*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (bc-ad)\text{Subst}\left(\int \frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)\\ &= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}-\frac{(B(bc-ad))\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{bf-ag}\\ &= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}+\frac{B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log[
a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))/(
(b*f - a*g)*(d*f - c*g)))/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(87) = 174$.

Time = 1.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.78

method	result
parts	$-\frac{A}{(gx+f)g}-B(ad-cb)\,e\left(\frac{\ln\left((cg-df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef\right)}{e(ag-bf)(cg-df)}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\right.}{e(ag-bf)\left(cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-df\left(\frac{be}{d}+\right.\right.}\right.$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(gx+f)}-\frac{-B\ln(-dx-c)ad\,g^{2}x+B\ln(-dx-c)bdfgx+B\ln(gx+f)ad\,g^{2}x-B\ln(gx+f)bc\,g^{2}x+B\ln(-bx-a}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^{2}A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^{2}B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{}{e(ag-}\right.\right.}{d^{2}}$
default	$-\frac{e(ad-cb)\left(-\frac{d^{2}A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^{2}B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{}{e(ag-}\right.\right.}{d^{2}}$
parallelrisch	$\frac{B\ln(bx+a)a^{2}cd\,f^{2}-B\ln(bx+a)ab\,c^{2}f^{2}-B\ln(gx+f)a^{2}cd\,f^{2}+B\ln(gx+f)ab\,c^{2}f^{2}-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^{2}cdfg+Bx\ln\left(\frac{e(b}{d}\right.}{}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/(g*x+f)/g-B*(a*d-b*c)*e*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(87) = 174$.

Time = 3.55 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x)\log(bx+a)+(Bbdf^2-B}{bdf^3g+acfg}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - (B*b*d*f^2 - B*b*c*f*g + (B
*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + (B*b*d*f^2 - B*a*d*f*g + (B*b*d*f*g
 - B*a*d*g^2)*x)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)*f*
g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((b*e*x
+ a*e)/(d*x + c)))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*
g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = B\left(\frac{b\log(bx+a)}{bfg-ag^2}-\frac{d\log(dx+c)}{dfg-cg^2}+\frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{g^2x+fg}\right) - \frac{A}{g^2x+fg}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="maxima")
[Out] B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c -
 a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d*x +
 c) + a*e/(d*x + c))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(87) = 174.

Time = 0.50 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \left(\frac{(Bb^2c^2e-2\,Babcde+Ba^2d^2e)\log\left(-bef+aeg+\frac{(bex+ae)df}{dx+c}-\frac{(bex+ae)cg}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{(Bb^2c^2e^2-2\,\ldots}{bdef^2-bcefg-adefg+a\ldots}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] ((B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*log(-b*e*f + a*e*g + (b*e*x +
a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*
f*g + a*c*g^2) + (B*b^2*c^2*e^2 - 2*B*a*b*c*d*e^2 + B*a^2*d^2*e^2)*log((b*e
*x + a*e)/(d*x + c))/(b*d*e*f^2 - b*c*e*f*g - a*d*e*f*g + a*c*e*g^2 - (b*e*
x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*d*f*g/(d*x + c) - (b*e*x + a
*e)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*log((b
*e*x + a*e)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) + (A*b^2*c^2
*e^2 - 2*A*a*b*c*d*e^2 + A*a^2*d^2*e^2)/(b*d*e*f^2 - b*c*e*f*g - a*d*e*f*g
+ a*c*e*g^2 - (b*e*x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*d*f*g/(d*
x + c) - (b*e*x + a*e)*c^2*g^2/(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \frac{B\,d\,\ln(c+d\,x)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{a\,e+b\,e\,x}{c+d\,x}\right)}{x\,g^2+f\,g}-\frac{B\,b\,\ln(a+b\,x)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{B\,a\,d\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{B\,b\,c\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^2,x)
```

```
[Out] (B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a*e + b*e*x)/(c + d*x)))/(f*g
+ g^2*x) - (B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (B*a*d*lo
g(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (B*b*c*log(f + g*x))/
(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)
```

3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = &-\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}\\ &-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{2g(df-cg)^2}\\ &+\frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{2(bf-ag)^2(df-cg)^2}\end{aligned}$$

```
[Out] -1/2*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*ln(b*x+a)/g/(-a*g
+b*f)^2+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^2-1/2*B*d^2*ln(d*x+c)/g/
(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2
/(-c*g+d*f)^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2} - \frac{Bd^2\log(c+dx)}{2g(df-cg)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*Log[a + b
*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*g*(f + g
*x)^2) - (B*d^2*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f
 - b*c*g - a*d*g)*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad))\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}-\frac{g^2(-2bdf+bc}{(bf-ag)^2(df-cg}\right.}{2g}$$

$$= -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} - \frac{Bd^2\log(c+dx)}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2} + B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)} + \frac{d^2\log(c+dx)}{-bc+ad} - \frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2) + B*(b*c - a*d)*((b^2*
Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f - a*g
)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g + a
*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(176) = 352.

Time = 1.89 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.79

method	result
parts	$-\frac{A}{2(gx+f)^2 g}-\frac{B(ad-cb)e\left(-\frac{g\,d^2(ad-cb)e\left(-\frac{\frac{\ln\left(cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef\right)}{cg-df}+\frac{}{(cg-df)\left(cg\left(\frac{be}{d}+\right.\right.}}{2(ag-bf)^2e^2}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{}{2(cg-df)(-cg+df)\left(aeg-bef-\right.}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{}{2(cg-df)(-cg+df)\left(aeg-bef-\right.}\right.\right.}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/(g*x+f)^2/g-B/d^2*(a*d-b*c)*e*(-g*d^2*(a*d-b*c)*e/(c*g-d*f)*(-1/2/(a
*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^
2/(a*g-b*f)^2/e^2)-d^3/(c*g-d*f)*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*
f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(173) = 346$.

Time = 45.30 (sec) , antiderivative size = 1017, normalized size of antiderivative = 5.56

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{Ab^2d^2f^4 + Aa^2c^2g^4 - ((2\,A - B)b^2cd + (2\,A + B)abd^2)f^3g + ((A - B)b^2c^2 + 4\,Aabcd + (A + B)a^2d^2)f}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - ((2*A - B)*b^2*c*d + (2*A + B)*a*b*d^
2)*f^3*g + ((A - B)*b^2*c^2 + 4*A*a*b*c*d + (A + B)*a^2*d^2)*f^2*g^2 - ((2*
A - B)*a*b*c^2 + (2*A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^
2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - (B*b^2
*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B
*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*
g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*f^3*g
 + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g
^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*x)*lo
g(d*x + c) - (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2
*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*x^2
+ 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*x)*
log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f
^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2
*c*d)*f*g^3)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5
 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3
- 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c
*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^
2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d +
 a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 +
a^2*c*d)*f^2*g^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(173) = 346.

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g - 2\,abfg^2 + a^2g^3} - \frac{d^2\log(dx+c)}{d^2f^2g - 2\,cdfg^2 + c^2g^3} + \frac{(2\,(b^2cd - abd^2)f - (b^2c^2 - \cdots}{b^2d^2f^4 + a^2c^2g^4 - 2\,(b^2cd + abd^2)f^3g + (b^2c^2 + \cdots}\right. - \frac{A}{2\,(g^3x^2 + 2\,fg^2x + f^2g)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))/(
g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2969 vs. 2(173) = 346.

Time = 0.52 (sec) , antiderivative size = 2969, normalized size of antiderivative = 16.22

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e*f - B*b^3*c
^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-b*e*f
+ a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^2*d
^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^
2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4)
+ (2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f - B*b^
3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*e^3*g -
 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*a*b*c*d^
3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*(b*e*x +
a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*e^2*g/(d*x
```

+ c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/(b^2*d^2*e^2*f^4 - 2*b^2*c*d*e^2*f^3*g - 2*a*b*d^2*e^2*f^3*g + b^2*c^2*e^2*f^2*g^2 + 4*a*b*c*d*e^2*f^2*g^2 + a^2*d^2*e^2*f^2*g^2 - 2*a*b*c^2*e^2*f*g^3 - 2*a^2*c*d*e^2*f*g^3 + a^2*c^2*e^2*g^4 - 2*(b*e*x + a*e)*b*d^3*e*f^4/(d*x + c) + 6*(b*e*x + a*e)*b*c*d^2*e*f^3*g/(d*x + c) + 2*(b*e*x + a*e)*a*d^3*e*f^3*g/(d*x + c) - 6*(b*e*x + a*e)*b*c^2*d*e*f^2*g^2/(d*x + c) - 6*(b*e*x + a*e)*a*c*d^2*e*f^2*g^2/(d*x + c) + 2*(b*e*x + a*e)*b*c^3*e*f*g^3/(d*x + c) + 6*(b*e*x + a*e)*a*c^2*d*e*f*g^3/(d*x + c) - 2*(b*e*x + a*e)*a*c^3*e*g^4/(d*x + c) + (b*e*x + a*e)^2*d^4*f^4/(d*x + c)^2 - 4*(b*e*x + a*e)^2*c*d^3*f^3*g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 4*(b*e*x + a*e)^2*c^3*d*f*g^3/(d*x + c)^2 + (b*e*x + a*e)^2*c^4*g^4/(d*x + c)^2) - (2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e*f - B*b^3*c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((b*e*x + a*e)/(d*x + c))/(b^2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*A*b^4*c^2*d*e^3*f^2 - 4*A*a*b^3*c*d^2*e^3*f^2 + 2*A*a^2*b^2*d^3*e^3*f^2 - A*b^4*c^3*e^3*f*g + B*b^4*c^3*e^3*f*g - A*a*b^3*c^2*d*e^3*f*g - 3*B*a*b^3*c^2*d*e^3*f*g + 5*A*a^2*b^2*c*d^2*e^3*f*g + 3*B*a^2*b^2*c*d^2*e^3*f*g - 3*A*a^3*b*d^3*e^3*f*g - B*a^3*b*d^3*e^3*f*g + A*a*b^3*c^3*e^3*g^2 - B*a*b^3*c^3*e^3*g^2 - A*a^2*b^2*c^2*d*e^3*g^2 + 3*B*a^2*b^2*c^2*d*e^3*g^2 - A*a^3*b*c*d^2*e^3*g^2 - 3*B*a^3*b*c*d^2*e^3*g^2 + A*a^4*d^3*e^3*g^2 + B*a^4*d^3*e^3*g^2 - 2*(b*e*x + a*e)*A*b^3*c^2*d^2*e^2*f^2/(d*x + c) + 4*(b*e*x + a*e)*A*a*b^2*c*d^3*e^2*f^2/(d*x + c) - 2*(b*e*x + a*e)*A*a^2*b*d^4*e^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*A*b^3*c^3*d*e^2*f*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*f*g/(d*x + c) - 2*(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2*f*g/(d*x + c) - 2*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*f*g/(d*x + c) + 2*(b*e*x + a*e)*A*a^3*d^4*e^2*f*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*f*g/(d*x + c) + (b*e*x + a*e)*B*b^3*c^4*e^2*g^2/(d*x + c) - 2*(b*e*x + a*e)*A*a*b^2*c^3*d*e^2*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^3*d*e^2*g^2/(d*x + c) + 4*(b*e*x + a*e)*A*a^2*b*c^2*d^2*e^2*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c^2*d^2*e^2*g^2/(d*x + c) - 2*(b*e*x + a*e)*A*a^3*c*d^3*e^2*g^2/(d*x + c) - (b*e*x + a*e)*B*a^3*c*d^3*e^2*g^2/(d*x + c))/(b^3*d^2*e^2*f^5 - 2*b^3*c*d*e^2*f^4*g - 3*a*b^2*d^2*e^2*f^4*g + b^3*c^2*e^2*f^3*g^2 + 6*a*b^2*c*d*e^2*f^3*g^2 + 3*a^2*b*d^2*e^2*f^3*g^2 - 3*a*b^2*c^2*e^2*f^2*g^3 - 6*a^2*b*c*d*e^2*f^2*g^3 - a^3*d^2*e^2*f^2*g^3 + 3*a^2*b*c^2*e^2*f*g^4 + 2*a^3*c*d*e^2*f*g^4 - a^3*c^2*e^2*g^5 - 2*(b*e*x + a*e)*b^2*d^3*e*f^5/(d*x + c) + 6*(b*e*x + a*e)*b^2*c*d^2*e*f^4*g/(d*x + c) + 4*(b*e*x + a*e)*a*b*d^3*e*f^4*g/(d*x + c) - 6*(b*e*x + a*e)*b^2*c^2*d*e*f^3*g^2/(d*x + c) - 12*(b*e*x + a*e)*a*b*c*d^2*e*f^3*g^2/(d*x + c) - 2*(b*e*x + a*e)*a^2*d^3*e*f^3*g^2/(d*x + c) + 2*(b*e*x + a*e)*b^2*c^3*e*f^2*g^3/(d*x + c) + 12*(b*e*x + a*e)*a*b*c^2*d*e*f^2*g^3/(d*x + c) + 6*(b*e*x + a*e)*a^2*c*d^2*e*f^2*g^3/(d*x + c) - 4*(b*e*x + a*e)*a*b*c^3*e*f*g^4/(d*x + c) - 6*(b*e*x + a*e)*a^2*c^2*d*e*f*g^4/(d*x + c) + 2*(b*e*x + a*e)*a^2*c^3*e*g^5/(d*x + c) + (b*e*x + a*e)^2*b*d^4*f^5/(d*x + c)^2 - 4*(b*e*x + a*e)^2*b*c

```
*d^3*f^4*g/(d*x + c)^2 - (b*e*x + a*e)^2*a*d^4*f^4*g/(d*x + c)^2 + 6*(b*e*x
 + a*e)^2*b*c^2*d^2*f^3*g^2/(d*x + c)^2 + 4*(b*e*x + a*e)^2*a*c*d^3*f^3*g^2
/(d*x + c)^2 - 4*(b*e*x + a*e)^2*b*c^3*d*f^2*g^3/(d*x + c)^2 - 6*(b*e*x + a
*e)^2*a*c^2*d^2*f^2*g^3/(d*x + c)^2 + (b*e*x + a*e)^2*b*c^4*f*g^4/(d*x + c)
^2 + 4*(b*e*x + a*e)^2*a*c^3*d*f*g^4/(d*x + c)^2 - (b*e*x + a*e)^2*a*c^4*g^
5/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(
b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$$
$$=\frac{\ln(f+g\,x)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g+B\,b\,c\,f\,g}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,\left(B\,a\,d\,g^2-B\,b\,c\,g^2\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$
$$+\frac{B\,b^2\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}-\frac{B\,d^2\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 - 4*
a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*a*b
*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d*f*g
 + B*b*c*f*g)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2 - B*b
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^2 + 4*
f*g^2*x) + (B*b^2*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g^2) - (
B*log((e*(a + b*x))/(c + d*x)))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) - (B*d^2*lo
g(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 275

$$\begin{aligned}
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx
&= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} - \frac{Bd^3\log(c+dx)}{3g(df-cg)^3} \\
&\quad + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^3-1/3*B*d^3*ln(d*x+c)/g
/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g
^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used

$= \{2548, 84\}$

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \frac{B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3} + \frac{b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{B(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2} - \frac{B(bc-ad)}{6(f+gx)^2(bf-ag)(df-cg)} - \frac{Bd^3\log(c+dx)}{3g(df-cg)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^
3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)]
)/(3*g*(f + g*x)^3) - (B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c -
a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} + \frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} + \frac{(B(bc-ad))\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)(f+gx)^3} - \frac{g^2(-2bdf+bc}{(bf-ag)^2(df-cg}\right.}{3g}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} - \frac{Bd^3\log(c+dx)}{3g(df-cg)^3} + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3} + B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2} + \frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3\log(c+dx)}{(bc-ad)(-df+\ldots}\right.}{3g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3) + B*(b*c - a*d)*(-1/2*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/((
b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)*(b*
f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a^2*d
^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Lo
g[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1503 vs. 2(266) = 532.

Time = 3.05 (sec) , antiderivative size = 1504, normalized size of antiderivative = 5.47

method	result	size
parts	Expression too large to display	1504
derivativedivides	Expression too large to display	1821
default	Expression too large to display	1821
risch	Expression too large to display	2293
parallelrisch	Expression too large to display	2896

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/(g*x+f)^3/g-B/d^2*(a*d-b*c)*e*(2*d^3*e*g*(a*d-b*c)/(c*g-d*f)^2*(-1/2
/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e
```

```
/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*
f)^2/(a*g-b*f)^2/e^2)+d^2*e^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*g^2/(c*g-d*f)^2*(
-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3*(1/2*e^2*(a^2*g^2-2*a*b*f*g+
b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-a*e*g+b*e*f)^2-1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)-e*(a*g-b*f)/(c*g-d*f)/(c*g
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*
f))-1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*a^2*e^2*g^2-6*a*b*e^2*f*g-3*a*c*
e*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*a*d*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))+3*b^2*e^2*f^2+3*b*c*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-3*b*d*e*f^2*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))+c^2*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*c*d*f*g
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+d^2*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)
/e^3)+d^4/(c*g-d*f)^2*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c
)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(263) = 526.

Time = 0.26 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.08

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$$
$$=\frac{1}{6}\left(\frac{2\,b^3\log\left(bx+a\right)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4}-\frac{2\,d^3\log\left(dx+c\right)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4}+\frac{}{b^3d^3f^6+a^3c^3g^6-3\,(b^3}\right.$$
$$-\frac{A}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9339 vs. 2(263) = 526.

Time = 0.78 (sec) , antiderivative size = 9339, normalized size of antiderivative = 33.96

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
```

[Out] 1/6*(2*(3*B*b^4*c^2*d^2*e*f^2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*f^2 - 3*B*b^4*c^3*d*e*f*g + 3*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*f*g - 3*B*a^3*b*d^4*e*f*g + B*b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^4*e^4*f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^2*c*d^3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*d*e^4*g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*b^3*c^2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B*b^3*c^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*a^3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^2*c^2*d^4*e^2*f^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a*b*c*d^5*e^2*f^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*d^6*e^2*f^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*b^2*c^3*d^3*e^2*f*g/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a*b*c^2*d^4*e^2*f*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*c*d^5*e^2*f*g/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*b^2*c^4*d^2*e^2*g^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a*b*c^3*d^3*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*c^2*d^4*e^2*g^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3*f^6 - 3*b^3*c*d^2*e^3*f^5*g - 3*a*b^2*d^3*e^3*f^5*g + 3*b^3*c^2*d*e^3*f^4*g^2 + 9*a*b^2*c*d^2*e^3*f^4*g^2 + 3*a^2*b*d^3*e^3*f^4*g^2 - b^3*c^3*e^3*f^3*g^3 - 9*a*b^2*c^2*d*e^3*f^3*g^3 - 9*a^2*b*c*d^2*e^3*f^3*g^3 - a^3*d^3*e^3*f^3*g^3 + 3*a*b^2*c^3*e^3*f^2*g^4 + 9*a^2*b*c^2*d*e^3*f^2*g^4 + 3*a^3*c*d^2*e^3*f^2*g^4 - 3*a^2*b*c^3*e^3*f*g^5 - 3*a^3*c^2*d*e^3*f*g^5 + a^3*c^3*e^3*g^6 - 3*(b*e*x + a*e)*b^2*d^4*e^2*f^6/(d*x + c) + 12*(b*e*x + a*e)*b^2*c*d^3*e^2*f^5*g/(d*x + c) + 6*(b*e*x + a*e)*a*b*d^4*e^2*f^5*g/(d*x + c) - 18*(b*e*x + a*e)*b^2*c^2*d^2*e^2*f^4*g^2/(d*x + c) - 24*(b*e*x + a*e)*a*b*c*d^3*e^2*f^4*g^2/(d*x + c) - 3*(b*e*x + a*e)*a^2*d^4*e^2*f^4*g^2/(d*x + c) + 12*(b*e*x + a*e)*b^2*c^3*d*e^2*f^3*g^3/(d*x + c) + 36*(b*e*x + a*e)*a*b*c^2*d^2*e^2*f^3*g^3/(d*x + c) + 12*(b*e*x + a*e)*a^2*c*d^3*e^2*f^3*g^3/(d*x + c) - 3*(b*e*x + a*e)*b^2*c^4*e^2*f^2*g^4/(d*x + c) - 24*(b*e*x + a*e)*a*b*c^3*d*e^2*f^2*g^4/(d*x + c) - 18*(b*e*x + a*e)*a^2*c^2*d^2*e^2*f^2*g^4/(d*x + c) + 6*(b*e*x + a*e)*a*b*c^4*e^2*f*g^5/(d*x + c) + 12*(b*e*x + a*e)*a^2*c^3*d*e^2*f*g^5/(d*x + c) - 3*(b*e*x + a*e)*a^2*c^4*e^2*g^6/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e*f^6/(d*x + c)^2 - 15*(b*e*x + a*e)^2*b*c*d^4*e*f^5*g/(d*x + c)^2 - 3*(b*e*x + a*e)^2*a*d^5*e*f^5*g/(d*x + c)^2 + 30*(b*e*x + a*e)^2*b*c^2*d^3*e*f^4*g^2/(d*x + c)^2 + 15*(b*e*x + a*e)^2*a*c*d^4*e*f^4*g^2/(d*x + c)^2 - 30*(b*e*x

+ a*e)^2*b*c^3*d^2*e*f^3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*a*c^2*d^3*e*
f^3*g^3/(d*x + c)^2 + 15*(b*e*x + a*e)^2*b*c^4*d*e*f^2*g^4/(d*x + c)^2 + 30
*(b*e*x + a*e)^2*a*c^3*d^2*e*f^2*g^4/(d*x + c)^2 - 3*(b*e*x + a*e)^2*b*c^5*
e*f*g^5/(d*x + c)^2 - 15*(b*e*x + a*e)^2*a*c^4*d*e*f*g^5/(d*x + c)^2 + 3*(b
*e*x + a*e)^2*a*c^5*e*g^6/(d*x + c)^2 - (b*e*x + a*e)^3*d^6*f^6/(d*x + c)^3
+ 6*(b*e*x + a*e)^3*c*d^5*f^5*g/(d*x + c)^3 - 15*(b*e*x + a*e)^3*c^2*d^4*f
^4*g^2/(d*x + c)^3 + 20*(b*e*x + a*e)^3*c^3*d^3*f^3*g^3/(d*x + c)^3 - 15*(b
*e*x + a*e)^3*c^4*d^2*f^2*g^4/(d*x + c)^3 + 6*(b*e*x + a*e)^3*c^5*d*f*g^5/(
d*x + c)^3 - (b*e*x + a*e)^3*c^6*g^6/(d*x + c)^3) - 2*(3*B*b^4*c^2*d^2*e*f^
2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*f^2 - 3*B*b^4*c^3*d*e*f*g + 3
*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*f*g - 3*B*a^3*b*d^4*e*f*g + B*
b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2
)*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d
^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^
2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d
^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*
g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + (6*A*b^6*c^2*d
^2*e^4*f^4 - 12*A*a*b^5*c*d^3*e^4*f^4 + 6*A*a^2*b^4*d^4*e^4*f^4 - 6*A*b^6*c
^3*d*e^4*f^3*g + 6*B*b^6*c^3*d*e^4*f^3*g - 6*A*a*b^5*c^2*d^2*e^4*f^3*g - 18
*B*a*b^5*c^2*d^2*e^4*f^3*g + 30*A*a^2*b^4*c*d^3*e^4*f^3*g + 18*B*a^2*b^4*c*
d^3*e^4*f^3*g - 18*A*a^3*b^3*d^4*e^4*f^3*g - 6*B*a^3*b^3*d^4*e^4*f^3*g + 2*
A*b^6*c^4*e^4*f^2*g^2 - 3*B*b^6*c^4*e^4*f^2*g^2 + 10*A*a*b^5*c^3*d*e^4*f^2*
g^2 - 6*B*a*b^5*c^3*d*e^4*f^2*g^2 - 6*A*a^2*b^4*c^2*d^2*e^4*f^2*g^2 + 36*B*
a^2*b^4*c^2*d^2*e^4*f^2*g^2 - 26*A*a^3*b^3*c*d^3*e^4*f^2*g^2 - 42*B*a^3*b^3
*c*d^3*e^4*f^2*g^2 + 20*A*a^4*b^2*d^4*e^4*f^2*g^2 + 15*B*a^4*b^2*d^4*e^4*f^
2*g^2 - 4*A*a*b^5*c^4*e^4*f*g^3 + 6*B*a*b^5*c^4*e^4*f*g^3 - 2*A*a^2*b^4*c^3
*d*e^4*f*g^3 - 6*B*a^2*b^4*c^3*d*e^4*f*g^3 + 6*A*a^3*b^3*c^2*d^2*e^4*f*g^3
- 18*B*a^3*b^3*c^2*d^2*e^4*f*g^3 + 10*A*a^4*b^2*c*d^3*e^4*f*g^3 + 30*B*a^4*
b^2*c*d^3*e^4*f*g^3 - 10*A*a^5*b*d^4*e^4*f*g^3 - 12*B*a^5*b*d^4*e^4*f*g^3 +
2*A*a^2*b^4*c^4*e^4*g^4 - 3*B*a^2*b^4*c^4*e^4*g^4 - 2*A*a^3*b^3*c^3*d*e^4*
g^4 + 6*B*a^3*b^3*c^3*d*e^4*g^4 - 2*A*a^5*b*c*d^3*e^4*g^4 - 6*B*a^5*b*c*d^3
*e^4*g^4 + 2*A*a^6*d^4*e^4*g^4 + 3*B*a^6*d^4*e^4*g^4 - 12*(b*e*x + a*e)*A*b
^5*c^2*d^3*e^3*f^4/(d*x + c) + 24*(b*e*x + a*e)*A*a*b^4*c*d^4*e^3*f^4/(d*x
+ c) - 12*(b*e*x + a*e)*A*a^2*b^3*d^5*e^3*f^4/(d*x + c) + 18*(b*e*x + a*e)*
A*b^5*c^3*d^2*e^3*f^3*g/(d*x + c) - 12*(b*e*x + a*e)*B*b^5*c^3*d^2*e^3*f^3*
g/(d*x + c) - 6*(b*e*x + a*e)*A*a*b^4*c^2*d^3*e^3*f^3*g/(d*x + c) + 36*(b*e
*x + a*e)*B*a*b^4*c^2*d^3*e^3*f^3*g/(d*x + c) - 42*(b*e*x + a*e)*A*a^2*b^3*
c*d^4*e^3*f^3*g/(d*x + c) - 36*(b*e*x + a*e)*B*a^2*b^3*c*d^4*e^3*f^3*g/(d*x
+ c) + 30*(b*e*x + a*e)*A*a^3*b^2*d^5*e^3*f^3*g/(d*x + c) + 12*(b*e*x + a*
e)*B*a^3*b^2*d^5*e^3*f^3*g/(d*x + c) - 6*(b*e*x + a*e)*A*b^5*c^4*d*e^3*f^2*
g^2/(d*x + c) + 17*(b*e*x + a*e)*B*b^5*c^4*d*e^3*f^2*g^2/(d*x + c) - 30*(b*
e*x + a*e)*A*a*b^4*c^3*d^2*e^3*f^2*g^2/(d*x + c) - 32*(b*e*x + a*e)*B*a*b^4
*c^3*d^2*e^3*f^2*g^2/(d*x + c) + 54*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*f^2
*g^2/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*f^2*g^2/(d*x + c) +
6*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*f^2*g^2/(d*x + c) + 40*(b*e*x + a*e)*B*

a^3*b^2*c*d^4*e^3*f^2*g^2/(d*x + c) - 24*(b*e*x + a*e)*A*a^4*b*d^5*e^3*f^2*
g^2/(d*x + c) - 19*(b*e*x + a*e)*B*a^4*b*d^5*e^3*f^2*g^2/(d*x + c) - 5*(b*e
*x + a*e)*B*b^5*c^5*e^3*f*g^3/(d*x + c) + 12*(b*e*x + a*e)*A*a*b^4*c^4*d*e^
3*f*g^3/(d*x + c) - 9*(b*e*x + a*e)*B*a*b^4*c^4*d*e^3*f*g^3/(d*x + c) + 6*(
b*e*x + a*e)*A*a^2*b^3*c^3*d^2*e^3*f*g^3/(d*x + c) + 50*(b*e*x + a*e)*B*a^2
*b^3*c^3*d^2*e^3*f*g^3/(d*x + c) - 42*(b*e*x + a*e)*A*a^3*b^2*c^2*d^3*e^3*f
*g^3/(d*x + c) - 46*(b*e*x + a*e)*B*a^3*b^2*c^2*d^3*e^3*f*g^3/(d*x + c) + 1
8*(b*e*x + a*e)*A*a^4*b*c*d^4*e^3*f*g^3/(d*x + c) + 3*(b*e*x + a*e)*B*a^4*b
*c*d^4*e^3*f*g^3/(d*x + c) + 6*(b*e*x + a*e)*A*a^5*d^5*e^3*f*g^3/(d*x + c)
+ 7*(b*e*x + a*e)*B*a^5*d^5*e^3*f*g^3/(d*x + c) + 5*(b*e*x + a*e)*B*a*b^4*c
^5*e^3*g^4/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b^3*c^4*d*e^3*g^4/(d*x + c) -
8*(b*e*x + a*e)*B*a^2*b^3*c^4*d*e^3*g^4/(d*x + c) + 6*(b*e*x + a*e)*A*a^3*b
^2*c^3*d^2*e^3*g^4/(d*x + c) - 6*(b*e*x + a*e)*B*a^3*b^2*c^3*d^2*e^3*g^4/(d
x + c) + 6(b*e*x + a*e)*A*a^4*b*c^2*d^3*e^3*g^4/(d*x + c) + 16*(b*e*x + a
*e)*B*a^4*b*c^2*d^3*e^3*g^4/(d*x + c) - 6*(b*e*x + a*e)*A*a^5*c*d^4*e^3*g^4
/(d*x + c) - 7*(b*e*x + a*e)*B*a^5*c*d^4*e^3*g^4/(d*x + c) + 6*(b*e*x + a*e
)^2*A*b^4*c^2*d^4*e^2*f^4/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a*b^3*c*d^5*e^
2*f^4/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^2*b^2*d^6*e^2*f^4/(d*x + c)^2 - 1
2*(b*e*x + a*e)^2*A*b^4*c^3*d^3*e^2*f^3*g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*B
*b^4*c^3*d^3*e^2*f^3*g/(d*x + c)^2 + 12*(b*e*x + a*e)^2*A*a*b^3*c^2*d^4*e^2
*f^3*g/(d*x + c)^2 - 18*(b*e*x + a*e)^2*B*a*b^3*c^2*d^4*e^2*f^3*g/(d*x + c)
^2 + 12*(b*e*x + a*e)^2*A*a^2*b^2*c*d^5*e^2*f^3*g/(d*x + c)^2 + 18*(b*e*x +
 a*e)^2*B*a^2*b^2*c*d^5*e^2*f^3*g/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a^3*b*
d^6*e^2*f^3*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^3*b*d^6*e^2*f^3*g/(d*x +
c)^2 + 6*(b*e*x + a*e)^2*A*b^4*c^4*d^2*e^2*f^2*g^2/(d*x + c)^2 - 14*(b*e*x
+ a*e)^2*B*b^4*c^4*d^2*e^2*f^2*g^2/(d*x + c)^2 + 12*(b*e*x + a*e)^2*A*a*b^3
*c^3*d^3*e^2*f^2*g^2/(d*x + c)^2 + 38*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*f
^2*g^2/(d*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^2*b^2*c^2*d^4*e^2*f^2*g^2/(d*x
+ c)^2 - 30*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*f^2*g^2/(d*x + c)^2 + 12*
(b*e*x + a*e)^2*A*a^3*b*c*d^5*e^2*f^2*g^2/(d*x + c)^2 + 2*(b*e*x + a*e)^2*B
*a^3*b*c*d^5*e^2*f^2*g^2/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^4*d^6*e^2*f^2*
g^2/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^4*d^6*e^2*f^2*g^2/(d*x + c)^2 + 10*
(b*e*x + a*e)^2*B*b^4*c^5*d*e^2*f*g^3/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a*
b^3*c^4*d^2*e^2*f*g^3/(d*x + c)^2 - 22*(b*e*x + a*e)^2*B*a*b^3*c^4*d^2*e^2*
f*g^3/(d*x + c)^2 + 12*(b*e*x + a*e)^2*A*a^2*b^2*c^3*d^3*e^2*f*g^3/(d*x + c
)^2 + 6*(b*e*x + a*e)^2*B*a^2*b^2*c^3*d^3*e^2*f*g^3/(d*x + c)^2 + 12*(b*e*x
 + a*e)^2*A*a^3*b*c^2*d^4*e^2*f*g^3/(d*x + c)^2 + 14*(b*e*x + a*e)^2*B*a^3*
b*c^2*d^4*e^2*f*g^3/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a^4*c*d^5*e^2*f*g^3/
(d*x + c)^2 - 8*(b*e*x + a*e)^2*B*a^4*c*d^5*e^2*f*g^3/(d*x + c)^2 - 2*(b*e*
x + a*e)^2*B*b^4*c^6*e^2*g^4/(d*x + c)^2 + 2*(b*e*x + a*e)^2*B*a*b^3*c^5*d*
e^2*g^4/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^2*b^2*c^4*d^2*e^2*g^4/(d*x + c)
^2 + 6*(b*e*x + a*e)^2*B*a^2*b^2*c^4*d^2*e^2*g^4/(d*x + c)^2 - 12*(b*e*x +
a*e)^2*A*a^3*b*c^3*d^3*e^2*g^4/(d*x + c)^2 - 10*(b*e*x + a*e)^2*B*a^3*b*c^3
*d^3*e^2*g^4/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^4*c^2*d^4*e^2*g^4/(d*x + c
)^2 + 4*(b*e*x + a*e)^2*B*a^4*c^2*d^4*e^2*g^4/(d*x + c)^2)/(b^5*d^3*e^3*f^8

- 3*b^5*c*d^2*e^3*f^7*g - 5*a*b^4*d^3*e^3*f^7*g + 3*b^5*c^2*d*e^3*f^6*g^2
+ 15*a*b^4*c*d^2*e^3*f^6*g^2 + 10*a^2*b^3*d^3*e^3*f^6*g^2 - b^5*c^3*e^3*f^5
*g^3 - 15*a*b^4*c^2*d*e^3*f^5*g^3 - 30*a^2*b^3*c*d^2*e^3*f^5*g^3 - 10*a^3*b
^2*d^3*e^3*f^5*g^3 + 5*a*b^4*c^3*e^3*f^4*g^4 + 30*a^2*b^3*c^2*d*e^3*f^4*g^4
+ 30*a^3*b^2*c*d^2*e^3*f^4*g^4 + 5*a^4*b*d^3*e^3*f^4*g^4 - 10*a^2*b^3*c^3*
e^3*f^3*g^5 - 30*a^3*b^2*c^2*d*e^3*f^3*g^5 - 15*a^4*b*c*d^2*e^3*f^3*g^5 - a
^5*d^3*e^3*f^3*g^5 + 10*a^3*b^2*c^3*e^3*f^2*g^6 + 15*a^4*b*c^2*d*e^3*f^2*g^
6 + 3*a^5*c*d^2*e^3*f^2*g^6 - 5*a^4*b*c^3*e^3*f*g^7 - 3*a^5*c^2*d*e^3*f*g^7
+ a^5*c^3*e^3*g^8 - 3*(b*e*x + a*e)*b^4*d^4*e^2*f^8/(d*x + c) + 12*(b*e*x
+ a*e)*b^4*c*d^3*e^2*f^7*g/(d*x + c) + 12*(b*e*x + a*e)*a*b^3*d^4*e^2*f^7*g
/(d*x + c) - 18*(b*e*x + a*e)*b^4*c^2*d^2*e^2*f^6*g^2/(d*x + c) - 48*(b*e*x
+ a*e)*a*b^3*c*d^3*e^2*f^6*g^2/(d*x + c) - 18*(b*e*x + a*e)*a^2*b^2*d^4*e^
2*f^6*g^2/(d*x + c) + 12*(b*e*x + a*e)*b^4*c^3*d*e^2*f^5*g^3/(d*x + c) + 72
*(b*e*x + a*e)*a*b^3*c^2*d^2*e^2*f^5*g^3/(d*x + c) + 72*(b*e*x + a*e)*a^2*b
^2*c*d^3*e^2*f^5*g^3/(d*x + c) + 12*(b*e*x + a*e)*a^3*b*d^4*e^2*f^5*g^3/(d*
x + c) - 3*(b*e*x + a*e)*b^4*c^4*e^2*f^4*g^4/(d*x + c) - 48*(b*e*x + a*e)*a
*b^3*c^3*d*e^2*f^4*g^4/(d*x + c) - 108*(b*e*x + a*e)*a^2*b^2*c^2*d^2*e^2*f^
4*g^4/(d*x + c) - 48*(b*e*x + a*e)*a^3*b*c*d^3*e^2*f^4*g^4/(d*x + c) - 3*(b
*e*x + a*e)*a^4*d^4*e^2*f^4*g^4/(d*x + c) + 12*(b*e*x + a*e)*a*b^3*c^4*e^2*
f^3*g^5/(d*x + c) + 72*(b*e*x + a*e)*a^2*b^2*c^3*d*e^2*f^3*g^5/(d*x + c) +
72*(b*e*x + a*e)*a^3*b*c^2*d^2*e^2*f^3*g^5/(d*x + c) + 12*(b*e*x + a*e)*a^4
*c*d^3*e^2*f^3*g^5/(d*x + c) - 18*(b*e*x + a*e)*a^2*b^2*c^4*e^2*f^2*g^6/(d*
x + c) - 48*(b*e*x + a*e)*a^3*b*c^3*d*e^2*f^2*g^6/(d*x + c) - 18*(b*e*x + a
*e)*a^4*c^2*d^2*e^2*f^2*g^6/(d*x + c) + 12*(b*e*x + a*e)*a^3*b*c^4*e^2*f*g^
7/(d*x + c) + 12*(b*e*x + a*e)*a^4*c^3*d*e^2*f*g^7/(d*x + c) - 3*(b*e*x + a
*e)*a^4*c^4*e^2*g^8/(d*x + c) + 3*(b*e*x + a*e)^2*b^3*d^5*e*f^8/(d*x + c)^2
- 15*(b*e*x + a*e)^2*b^3*c*d^4*e*f^7*g/(d*x + c)^2 - 9*(b*e*x + a*e)^2*a*b
^2*d^5*e*f^7*g/(d*x + c)^2 + 30*(b*e*x + a*e)^2*b^3*c^2*d^3*e*f^6*g^2/(d*x
+ c)^2 + 45*(b*e*x + a*e)^2*a*b^2*c*d^4*e*f^6*g^2/(d*x + c)^2 + 9*(b*e*x +
a*e)^2*a^2*b*d^5*e*f^6*g^2/(d*x + c)^2 - 30*(b*e*x + a*e)^2*b^3*c^3*d^2*e*f
^5*g^3/(d*x + c)^2 - 90*(b*e*x + a*e)^2*a*b^2*c^2*d^3*e*f^5*g^3/(d*x + c)^2
- 45*(b*e*x + a*e)^2*a^2*b*c*d^4*e*f^5*g^3/(d*x + c)^2 - 3*(b*e*x + a*e)^2
*a^3*d^5*e*f^5*g^3/(d*x + c)^2 + 15*(b*e*x + a*e)^2*b^3*c^4*d*e*f^4*g^4/(d*
x + c)^2 + 90*(b*e*x + a*e)^2*a*b^2*c^3*d^2*e*f^4*g^4/(d*x + c)^2 + 90*(b*e
*x + a*e)^2*a^2*b*c^2*d^3*e*f^4*g^4/(d*x + c)^2 + 15*(b*e*x + a*e)^2*a^3*c*
d^4*e*f^4*g^4/(d*x + c)^2 - 3*(b*e*x + a*e)^2*b^3*c^5*e*f^3*g^5/(d*x + c)^2
- 45*(b*e*x + a*e)^2*a*b^2*c^4*d*e*f^3*g^5/(d*x + c)^2 - 90*(b*e*x + a*e)^
2*a^2*b*c^3*d^2*e*f^3*g^5/(d*x + c)^2 - 30*(b*e*x + a*e)^2*a^3*c^2*d^3*e*f^
3*g^5/(d*x + c)^2 + 9*(b*e*x + a*e)^2*a*b^2*c^5*e*f^2*g^6/(d*x + c)^2 + 45*
(b*e*x + a*e)^2*a^2*b*c^4*d*e*f^2*g^6/(d*x + c)^2 + 30*(b*e*x + a*e)^2*a^3*
c^3*d^2*e*f^2*g^6/(d*x + c)^2 - 9*(b*e*x + a*e)^2*a^2*b*c^5*e*f*g^7/(d*x +
c)^2 - 15*(b*e*x + a*e)^2*a^3*c^4*d*e*f*g^7/(d*x + c)^2 + 3*(b*e*x + a*e)^2
*a^3*c^5*e*g^8/(d*x + c)^2 - (b*e*x + a*e)^3*b^2*d^6*f^8/(d*x + c)^3 + 6*(b
*e*x + a*e)^3*b^2*c*d^5*f^7*g/(d*x + c)^3 + 2*(b*e*x + a*e)^3*a*b*d^6*f^7*g
/(d*x + c)^3 - 15*(b*e*x + a*e)^3*b^2*c^2*d^4*f^6*g^2/(d*x + c)^3 - 12*(b*e

```
*x + a*e)^3*a*b*c*d^5*f^6*g^2/(d*x + c)^3 - (b*e*x + a*e)^3*a^2*d^6*f^6*g^2
/(d*x + c)^3 + 20*(b*e*x + a*e)^3*b^2*c^3*d^3*f^5*g^3/(d*x + c)^3 + 30*(b*e
*x + a*e)^3*a*b*c^2*d^4*f^5*g^3/(d*x + c)^3 + 6*(b*e*x + a*e)^3*a^2*c*d^5*f
^5*g^3/(d*x + c)^3 - 15*(b*e*x + a*e)^3*b^2*c^4*d^2*f^4*g^4/(d*x + c)^3 - 4
0*(b*e*x + a*e)^3*a*b*c^3*d^3*f^4*g^4/(d*x + c)^3 - 15*(b*e*x + a*e)^3*a^2*
c^2*d^4*f^4*g^4/(d*x + c)^3 + 6*(b*e*x + a*e)^3*b^2*c^5*d*f^3*g^5/(d*x + c)
^3 + 30*(b*e*x + a*e)^3*a*b*c^4*d^2*f^3*g^5/(d*x + c)^3 + 20*(b*e*x + a*e)^
3*a^2*c^3*d^3*f^3*g^5/(d*x + c)^3 - (b*e*x + a*e)^3*b^2*c^6*f^2*g^6/(d*x +
c)^3 - 12*(b*e*x + a*e)^3*a*b*c^5*d*f^2*g^6/(d*x + c)^3 - 15*(b*e*x + a*e)^
3*a^2*c^4*d^2*f^2*g^6/(d*x + c)^3 + 2*(b*e*x + a*e)^3*a*b*c^6*f*g^7/(d*x +
c)^3 + 6*(b*e*x + a*e)^3*a^2*c^5*d*f*g^7/(d*x + c)^3 - (b*e*x + a*e)^3*a^2*
c^6*g^8/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*
d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 1154, normalized size of antiderivative = 4.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$$

$$= \frac{\ln(f+g\,x)\,\left(g\,(3\,B\,a^2\,b\,d^3\,f-\cdots\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,\cdots}$$

$$-\frac{\frac{2\,A\,a^2\,c^2\,g^4+2\,A\,b^2\,d^2\,f^4+2\,A\,a^2\,d^2\,f^2\,g^2+2\,A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2-3\,B\,b^2\,c^2\,f^2\,g^2-4\,A\,a\,b\,c^2\,f\,g^3-4\,A\,a\,b\,d^2\,f^3\,g+B\,a\,b\,c^2\,f\,g^3-4\,A\,a\cdots}{2\,(a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\cdots}}{\cdots}$$

$$-\frac{B\,b^3\,\ln(a+b\,x)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g}$$

$$+\frac{B\,d^3\,\ln(c+d\,x)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,g\,(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^4,x)
```

```
[Out] (log(f + g*x)*(g*(3*B*a^2*b*d^3*f - 3*B*b^3*c^2*d*f) - g^2*(B*a^3*d^3 - B*b
^3*c^3) - 3*B*a*b^2*d^3*f^2 + 3*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*d^
3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*b^2
*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^4 +
9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*b^2*
c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^2*b*
c^2*d*f^2*g^4) - ((2*A*a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2
+ 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 4*A*a*b
*c^2*f*g^3 - 4*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g^3 - 5*B*
a*b*d^2*f^3*g - 4*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 8
*A*a*b*c*d*f^2*g^2)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c
^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*
```

```
d*f^3*g + 4*a*b*c*d*f^2*g^2)) + (x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a
*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2
*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*
c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c
*d*f^2*g^2))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2)))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) -
 (B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*
f*g^3) + (B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 -
9*c^2*d*f*g^3) - (B*log((e*(a + b*x))/(c + d*x)))/(3*g*(f^3 + g^3*x^3 + 3*f
^2*g*x + 3*f*g^2*x^2))
```

3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 379

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{4(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^4B\log(a+bx)}{4g(bf-ag)^4}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}-\frac{Bd^4\log(c+dx)}{4g(df-cg)^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{4(bf-ag)^4(df-cg)^4}$$

```
[Out] -1/12*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a^2*d
^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3
/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*
x+a)/(d*x+c)))/g/(g*x+f)^4-1/4*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/4*B*(-a*d+b
*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*
g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$$
$$= -\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}$$
$$-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+2d^2f^2))\right)}{4(bf-ag)^4(df-cg)^4}$$
$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}+\frac{b^4B\log(a+bx)}{4g(bf-ag)^4}-\frac{B(bc-ad)(-adg-bcg+2bdf)}{8(f+gx)^2(bf-ag)^2(df-cg)^2}$$
$$-\frac{B(bc-ad)}{12(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4\log(c+dx)}{4g(df-cg)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d
)*(2*b*d*f - b*c*g - a*d*g))/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) -
(B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2)))/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a
 + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*g*(f
 + g*x)^4) - (B*d^4*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b
*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d
*f*g + c^2*g^2))*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4} + \frac{(B(bc-ad))\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^4}-\frac{g^2(-2bdf+bc}{(bf-ag)^2(df-cg}\right.}{}$$

$$= -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{4(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^4B\log(a+bx)}{4g(bf-ag)^4}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}-\frac{Bd^4\log(c+dx)}{4g(df-cg)^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{4(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.94

$$\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}+B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+b^2(}{(bf-ag)^3(df-cg)^3}\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4) + B*(b*c - a*d)*(-1/3*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/(2
*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d
*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f - c
*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^4*Lo
g[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)*(-2
*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f +
 g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2849 vs. 2(368) = 736.

Time = 6.65 (sec) , antiderivative size = 2850, normalized size of antiderivative = 7.52

method	result	size
parts	Expression too large to display	2850
derivativedivides	Expression too large to display	3309
default	Expression too large to display	3309
risch	Expression too large to display	4450
parallelrisch	Expression too large to display	5539

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/(g*x+f)^4/g-B/d^2*(a*d-b*c)*e*(-3*d^4*e*(a*d-b*c)*g/(c*g-d*f)^3*(-1/
2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(
a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*
g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e
*f)^2/(a*g-b*f)^2/e^2)-3*d^3*e^2*g^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c*g-d*f)^
3*(-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3*(1/2*e^2*(a^2*g^2-2*a*b*f
*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-a*e*g+b*e*f)^2-1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)-e*(a*g-b*f)/(c*g-d*f)/(
c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b
*e*f))-1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*a^2*e^2*g^2-6*a*b*e^2*f*g-3*a
*c*e*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*a*d*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))+3*b^2*e^2*f^2+3*b*c*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-3*b*d*e*f^2*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))+c^2*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*c*d*
f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+d^2*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f
^2)/e^3)-d^5/(c*g-d*f)^3*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))-d^2*e^3*g^3*(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/(c*g-d*f)^3*(-1/4/(a*g-b*f)^2/(a^2*g^2-2*a*b*f*g+b^2*f
^2)/e^4*(-1/2*e^2*(a^2*g^2-2*a*b*f*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2+1/(c*g-d*f
)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*
e*g+b*e*f)+1/3*e^3*(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(c*g-d*f)/
(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+
```

```
b*e*f)^3+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c^3*g^3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-3*c^
2*d*f*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+3*c*d^2*f^2*g*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3-d^3*f^3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-4*a*c^2*e*g^3*(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^2+8*a*c*d*e*f*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4*a
*d^2*e*f^2*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+4*b*c^2*e*f*g^2*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2-8*b*c*d*e*f^2*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+4*b*d^2*e
*f^3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6*a^2*c*e^2*g^3*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-6*a^2*d*e^2*f*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-12*a*b*c*e^2*f*g^2*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))+12*a*b*d*e^2*f^2*g*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))+6*b^2*c*e^2*f^2*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-6*b^2*d*e^2*f^3*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-4*a^3*e^3*g^3+12*a^2*b*e^3*f*g^2-12*a*b^2*e^3*f^2*g+
4*b^3*e^3*f^3)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))-a*e*g+b*e*f)^4/(a*g-b*f)^2/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^4))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1757 vs. $2(365) = 730$.

Time = 0.33 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f + gx)^5} dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20791 vs. 2(365) = 730.

Time = 0.99 (sec) , antiderivative size = 20791, normalized size of antiderivative = 54.86

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*e*f^3 - 8*B*a*b^4*c*d^4*e*f^3 + 4*B*a^2*b^3*d^5*e*
f^3 - 6*B*b^5*c^3*d^2*e*f^2*g + 6*B*a*b^4*c^2*d^3*e*f^2*g + 6*B*a^2*b^3*c*d
^4*e*f^2*g - 6*B*a^3*b^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f*g^2 - 4*B*a*b^4*c^
3*d^2*e*f*g^2 - 4*B*a^3*b^2*c*d^4*e*f*g^2 + 4*B*a^4*b*d^5*e*f*g^2 - B*b^5*c
^5*e*g^3 + B*a*b^4*c^4*d*e*g^3 + B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log
(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c)
)/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*
g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3
- 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3
 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 +
16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2
*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2
*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4
*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^3*e^5*f^3 - 8*
B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d^5*e^5*f^3 - 6*B*b^5*c^3*d^2*e^5*f^2*g
 + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B*a^2*b^3*c*d^4*e^5*f^2*g - 6*B*a^3*b^2*
d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f*g^2 - 4*B*a*b^4*c^3*d^2*e^5*f*g^2 - 4*B
*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4*b*d^5*e^5*f*g^2 - B*b^5*c^5*e^5*g^3 + B*
a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d^4*e^5*g^3 - B*a^5*d^5*e^5*g^3 - 12*(b*e*x
 + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d*x + c) + 24*(b*e*x + a*e)*B*a*b^3*c*d^5*e^
4*f^3/(d*x + c) - 12*(b*e*x + a*e)*B*a^2*b^2*d^6*e^4*f^3/(d*x + c) + 24*(b*
e*x + a*e)*B*b^4*c^3*d^3*e^4*f^2*g/(d*x + c) - 36*(b*e*x + a*e)*B*a*b^3*c^2
*d^4*e^4*f^2*g/(d*x + c) + 12*(b*e*x + a*e)*B*a^3*b*d^6*e^4*f^2*g/(d*x + c)
 - 16*(b*e*x + a*e)*B*b^4*c^4*d^2*e^4*f*g^2/(d*x + c) + 16*(b*e*x + a*e)*B*
a*b^3*c^3*d^3*e^4*f*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a^2*b^2*c^2*d^4*e^4*
f*g^2/(d*x + c) - 8*(b*e*x + a*e)*B*a^3*b*c*d^5*e^4*f*g^2/(d*x + c) - 4*(b*
e*x + a*e)*B*a^4*d^6*e^4*f*g^2/(d*x + c) + 4*(b*e*x + a*e)*B*b^4*c^5*d*e^4*
g^3/(d*x + c) - 4*(b*e*x + a*e)*B*a*b^3*c^4*d^2*e^4*g^3/(d*x + c) - 4*(b*e*
x + a*e)*B*a^3*b*c^2*d^4*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*B*a^4*c*d^5*e^
4*g^3/(d*x + c) + 12*(b*e*x + a*e)^2*B*b^3*c^2*d^5*e^3*f^3/(d*x + c)^2 - 24
*(b*e*x + a*e)^2*B*a*b^2*c*d^6*e^3*f^3/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a
^2*b*d^7*e^3*f^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*b^3*c^3*d^4*e^3*f^2*g/(
d*x + c)^2 + 54*(b*e*x + a*e)^2*B*a*b^2*c^2*d^5*e^3*f^2*g/(d*x + c)^2 - 18*
(b*e*x + a*e)^2*B*a^2*b*c*d^6*e^3*f^2*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a
```

^3*d^7*e^3*f^2*g/(d*x + c)^2 + 24*(b*e*x + a*e)^2*B*b^3*c^4*d^3*e^3*f*g^2/(
d*x + c)^2 - 36*(b*e*x + a*e)^2*B*a*b^2*c^3*d^4*e^3*f*g^2/(d*x + c)^2 + 12*
(b*e*x + a*e)^2*B*a^3*c*d^6*e^3*f*g^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*b^3
*c^5*d^2*e^3*g^3/(d*x + c)^2 + 6*(b*e*x + a*e)^2*B*a*b^2*c^4*d^3*e^3*g^3/(d
x + c)^2 + 6(b*e*x + a*e)^2*B*a^2*b*c^3*d^4*e^3*g^3/(d*x + c)^2 - 6*(b*e*
x + a*e)^2*B*a^3*c^2*d^5*e^3*g^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B*b^2*c^2*
d^6*e^2*f^3/(d*x + c)^3 + 8*(b*e*x + a*e)^3*B*a*b*c*d^7*e^2*f^3/(d*x + c)^3
 - 4*(b*e*x + a*e)^3*B*a^2*d^8*e^2*f^3/(d*x + c)^3 + 12*(b*e*x + a*e)^3*B*b
^2*c^3*d^5*e^2*f^2*g/(d*x + c)^3 - 24*(b*e*x + a*e)^3*B*a*b*c^2*d^6*e^2*f^2
*g/(d*x + c)^3 + 12*(b*e*x + a*e)^3*B*a^2*c*d^7*e^2*f^2*g/(d*x + c)^3 - 12*
(b*e*x + a*e)^3*B*b^2*c^4*d^4*e^2*f*g^2/(d*x + c)^3 + 24*(b*e*x + a*e)^3*B*
a*b*c^3*d^5*e^2*f*g^2/(d*x + c)^3 - 12*(b*e*x + a*e)^3*B*a^2*c^2*d^6*e^2*f*
g^2/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B*b^2*c^5*d^3*e^2*g^3/(d*x + c)^3 - 8*(
b*e*x + a*e)^3*B*a*b*c^4*d^4*e^2*g^3/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B*a^2*
c^3*d^5*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4*e^4*f^8
- 4*b^4*c*d^3*e^4*f^7*g - 4*a*b^3*d^4*e^4*f^7*g + 6*b^4*c^2*d^2*e^4*f^6*g^2
 + 16*a*b^3*c*d^3*e^4*f^6*g^2 + 6*a^2*b^2*d^4*e^4*f^6*g^2 - 4*b^4*c^3*d*e^4
*f^5*g^3 - 24*a*b^3*c^2*d^2*e^4*f^5*g^3 - 24*a^2*b^2*c*d^3*e^4*f^5*g^3 - 4*
a^3*b*d^4*e^4*f^5*g^3 + b^4*c^4*e^4*f^4*g^4 + 16*a*b^3*c^3*d*e^4*f^4*g^4 +
36*a^2*b^2*c^2*d^2*e^4*f^4*g^4 + 16*a^3*b*c*d^3*e^4*f^4*g^4 + a^4*d^4*e^4*f
^4*g^4 - 4*a*b^3*c^4*e^4*f^3*g^5 - 24*a^2*b^2*c^3*d*e^4*f^3*g^5 - 24*a^3*b*
c^2*d^2*e^4*f^3*g^5 - 4*a^4*c*d^3*e^4*f^3*g^5 + 6*a^2*b^2*c^4*e^4*f^2*g^6 +
 16*a^3*b*c^3*d*e^4*f^2*g^6 + 6*a^4*c^2*d^2*e^4*f^2*g^6 - 4*a^3*b*c^4*e^4*f
*g^7 - 4*a^4*c^3*d*e^4*f*g^7 + a^4*c^4*e^4*g^8 - 4*(b*e*x + a*e)*b^3*d^5*e^
3*f^8/(d*x + c) + 20*(b*e*x + a*e)*b^3*c*d^4*e^3*f^7*g/(d*x + c) + 12*(b*e*
x + a*e)*a*b^2*d^5*e^3*f^7*g/(d*x + c) - 40*(b*e*x + a*e)*b^3*c^2*d^3*e^3*f
^6*g^2/(d*x + c) - 60*(b*e*x + a*e)*a*b^2*c*d^4*e^3*f^6*g^2/(d*x + c) - 12*
(b*e*x + a*e)*a^2*b*d^5*e^3*f^6*g^2/(d*x + c) + 40*(b*e*x + a*e)*b^3*c^3*d^
2*e^3*f^5*g^3/(d*x + c) + 120*(b*e*x + a*e)*a*b^2*c^2*d^3*e^3*f^5*g^3/(d*x
+ c) + 60*(b*e*x + a*e)*a^2*b*c*d^4*e^3*f^5*g^3/(d*x + c) + 4*(b*e*x + a*e)
*a^3*d^5*e^3*f^5*g^3/(d*x + c) - 20*(b*e*x + a*e)*b^3*c^4*d*e^3*f^4*g^4/(d*
x + c) - 120*(b*e*x + a*e)*a*b^2*c^3*d^2*e^3*f^4*g^4/(d*x + c) - 120*(b*e*x
 + a*e)*a^2*b*c^2*d^3*e^3*f^4*g^4/(d*x + c) - 20*(b*e*x + a*e)*a^3*c*d^4*e^
3*f^4*g^4/(d*x + c) + 4*(b*e*x + a*e)*b^3*c^5*e^3*f^3*g^5/(d*x + c) + 60*(b
*e*x + a*e)*a*b^2*c^4*d*e^3*f^3*g^5/(d*x + c) + 120*(b*e*x + a*e)*a^2*b*c^3
*d^2*e^3*f^3*g^5/(d*x + c) + 40*(b*e*x + a*e)*a^3*c^2*d^3*e^3*f^3*g^5/(d*x
+ c) - 12*(b*e*x + a*e)*a*b^2*c^5*e^3*f^2*g^6/(d*x + c) - 60*(b*e*x + a*e)*
a^2*b*c^4*d*e^3*f^2*g^6/(d*x + c) - 40*(b*e*x + a*e)*a^3*c^3*d^2*e^3*f^2*g^
6/(d*x + c) + 12*(b*e*x + a*e)*a^2*b*c^5*e^3*f*g^7/(d*x + c) + 20*(b*e*x +
a*e)*a^3*c^4*d*e^3*f*g^7/(d*x + c) - 4*(b*e*x + a*e)*a^3*c^5*e^3*g^8/(d*x +
 c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^2*f^8/(d*x + c)^2 - 36*(b*e*x + a*e)^2*b^
2*c*d^5*e^2*f^7*g/(d*x + c)^2 - 12*(b*e*x + a*e)^2*a*b*d^6*e^2*f^7*g/(d*x +
 c)^2 + 90*(b*e*x + a*e)^2*b^2*c^2*d^4*e^2*f^6*g^2/(d*x + c)^2 + 72*(b*e*x
+ a*e)^2*a*b*c*d^5*e^2*f^6*g^2/(d*x + c)^2 + 6*(b*e*x + a*e)^2*a^2*d^6*e^2*
f^6*g^2/(d*x + c)^2 - 120*(b*e*x + a*e)^2*b^2*c^3*d^3*e^2*f^5*g^3/(d*x + c)

^2 - 180*(b*e*x + a*e)^2*a*b*c^2*d^4*e^2*f^5*g^3/(d*x + c)^2 - 36*(b*e*x + a*e)^2*a^2*c*d^5*e^2*f^5*g^3/(d*x + c)^2 + 90*(b*e*x + a*e)^2*b^2*c^4*d^2*e ^2*f^4*g^4/(d*x + c)^2 + 240*(b*e*x + a*e)^2*a*b*c^3*d^3*e^2*f^4*g^4/(d*x + c)^2 + 90*(b*e*x + a*e)^2*a^2*c^2*d^4*e^2*f^4*g^4/(d*x + c)^2 - 36*(b*e*x + a*e)^2*b^2*c^5*d*e^2*f^3*g^5/(d*x + c)^2 - 180*(b*e*x + a*e)^2*a*b*c^4*d^ 2*e^2*f^3*g^5/(d*x + c)^2 - 120*(b*e*x + a*e)^2*a^2*c^3*d^3*e^2*f^3*g^5/(d* x + c)^2 + 6*(b*e*x + a*e)^2*b^2*c^6*e^2*f^2*g^6/(d*x + c)^2 + 72*(b*e*x + a*e)^2*a*b*c^5*d*e^2*f^2*g^6/(d*x + c)^2 + 90*(b*e*x + a*e)^2*a^2*c^4*d^2*e ^2*f^2*g^6/(d*x + c)^2 - 12*(b*e*x + a*e)^2*a*b*c^6*e^2*f*g^7/(d*x + c)^2 - 36*(b*e*x + a*e)^2*a^2*c^5*d*e^2*f*g^7/(d*x + c)^2 + 6*(b*e*x + a*e)^2*a^2 *c^6*e^2*g^8/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e*f^8/(d*x + c)^3 + 28*(b*e*x + a*e)^3*b*c*d^6*e*f^7*g/(d*x + c)^3 + 4*(b*e*x + a*e)^3*a*d^7*e*f^7* g/(d*x + c)^3 - 84*(b*e*x + a*e)^3*b*c^2*d^5*e*f^6*g^2/(d*x + c)^3 - 28*(b* e*x + a*e)^3*a*c*d^6*e*f^6*g^2/(d*x + c)^3 + 140*(b*e*x + a*e)^3*b*c^3*d^4* e*f^5*g^3/(d*x + c)^3 + 84*(b*e*x + a*e)^3*a*c^2*d^5*e*f^5*g^3/(d*x + c)^3 - 140*(b*e*x + a*e)^3*b*c^4*d^3*e*f^4*g^4/(d*x + c)^3 - 140*(b*e*x + a*e)^3 *a*c^3*d^4*e*f^4*g^4/(d*x + c)^3 + 84*(b*e*x + a*e)^3*b*c^5*d^2*e*f^3*g^5/(d*x + c)^3 + 140*(b*e*x + a*e)^3*a*c^4*d^3*e*f^3*g^5/(d*x + c)^3 - 28*(b*e* x + a*e)^3*b*c^6*d*e*f^2*g^6/(d*x + c)^3 - 84*(b*e*x + a*e)^3*a*c^5*d^2*e*f ^2*g^6/(d*x + c)^3 + 4*(b*e*x + a*e)^3*b*c^7*e*f*g^7/(d*x + c)^3 + 28*(b*e* x + a*e)^3*a*c^6*d*e*f*g^7/(d*x + c)^3 - 4*(b*e*x + a*e)^3*a*c^7*e*g^8/(d*x + c)^3 + (b*e*x + a*e)^4*d^8*f^8/(d*x + c)^4 - 8*(b*e*x + a*e)^4*c*d^7*f^7 *g/(d*x + c)^4 + 28*(b*e*x + a*e)^4*c^2*d^6*f^6*g^2/(d*x + c)^4 - 56*(b*e*x + a*e)^4*c^3*d^5*f^5*g^3/(d*x + c)^4 + 70*(b*e*x + a*e)^4*c^4*d^4*f^4*g^4/ (d*x + c)^4 - 56*(b*e*x + a*e)^4*c^5*d^3*f^3*g^5/(d*x + c)^4 + 28*(b*e*x + a*e)^4*c^6*d^2*f^2*g^6/(d*x + c)^4 - 8*(b*e*x + a*e)^4*c^7*d*f*g^7/(d*x + c)^4 + (b*e*x + a*e)^4*c^8*g^8/(d*x + c)^4) - 6*(4*B*b^5*c^2*d^3*e*f^3 - 8*B *a*b^4*c*d^4*e*f^3 + 4*B*a^2*b^3*d^5*e*f^3 - 6*B*b^5*c^3*d^2*e*f^2*g + 6*B* a*b^4*c^2*d^3*e*f^2*g + 6*B*a^2*b^3*c*d^4*e*f^2*g - 6*B*a^3*b^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f*g^2 - 4*B*a*b^4*c^3*d^2*e*f*g^2 - 4*B*a^3*b^2*c*d^4*e* f*g^2 + 4*B*a^4*b*d^5*e*f*g^2 - B*b^5*c^5*e*g^3 + B*a*b^4*c^4*d*e*g^3 + B*a ^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4*f ^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b ^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c ^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f ^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d ^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g ^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a ^4*c^3*d*f*g^7 + a^4*c^4*g^8) + (24*A*b^8*c^2*d^3*e^5*f^6 - 48*A*a*b^7*c*d^ 4*e^5*f^6 + 24*A*a^2*b^6*d^5*e^5*f^6 - 36*A*b^8*c^3*d^2*e^5*f^5*g + 36*B*b^ 8*c^3*d^2*e^5*f^5*g - 36*A*a*b^7*c^2*d^3*e^5*f^5*g - 108*B*a*b^7*c^2*d^3*e^ 5*f^5*g + 180*A*a^2*b^6*c*d^4*e^5*f^5*g + 108*B*a^2*b^6*c*d^4*e^5*f^5*g - 1 08*A*a^3*b^5*d^5*e^5*f^5*g - 36*B*a^3*b^5*d^5*e^5*f^5*g + 24*A*b^8*c^4*d*e^ 5*f^4*g^2 - 36*B*b^8*c^4*d*e^5*f^4*g^2 + 84*A*a*b^7*c^3*d^2*e^5*f^4*g^2 - 3

6*B*a*b^7*c^3*d^2*e^5*f^4*g^2 - 36*A*a^2*b^6*c^2*d^3*e^5*f^4*g^2 + 324*B*a^2*b^6*c^2*d^3*e^5*f^4*g^2 - 276*A*a^3*b^5*c*d^4*e^5*f^4*g^2 - 396*B*a^3*b^5*c*d^4*e^5*f^4*g^2 + 204*A*a^4*b^4*d^5*e^5*f^4*g^2 + 144*B*a^4*b^4*d^5*e^5*f^4*g^2 - 6*A*b^8*c^5*e^5*f^3*g^3 + 11*B*b^8*c^5*e^5*f^3*g^3 - 66*A*a*b^7*c^4*d*e^5*f^3*g^3 + 89*B*a*b^7*c^4*d*e^5*f^3*g^3 - 36*A*a^2*b^6*c^3*d^2*e^5*f^3*g^3 - 106*B*a^2*b^6*c^3*d^2*e^5*f^3*g^3 + 84*A*a^3*b^5*c^2*d^3*e^5*f^3*g^3 - 326*B*a^3*b^5*c^2*d^3*e^5*f^3*g^3 + 234*A*a^4*b^4*c*d^4*e^5*f^3*g^3 + 559*B*a^4*b^4*c*d^4*e^5*f^3*g^3 - 210*A*a^5*b^3*d^5*e^5*f^3*g^3 - 227*B*a^5*b^3*d^5*e^5*f^3*g^3 + 18*A*a*b^7*c^5*e^5*f^2*g^4 - 33*B*a*b^7*c^5*e^5*f^2*g^4 + 54*A*a^2*b^6*c^4*d*e^5*f^2*g^4 - 51*B*a^2*b^6*c^4*d*e^5*f^2*g^4 - 36*A*a^3*b^5*c^3*d^2*e^5*f^2*g^4 + 174*B*a^3*b^5*c^3*d^2*e^5*f^2*g^4 - 36*A*a^4*b^4*c^2*d^3*e^5*f^2*g^4 + 114*B*a^4*b^4*c^2*d^3*e^5*f^2*g^4 - 126*A*a^5*b^3*c*d^4*e^5*f^2*g^4 - 381*B*a^5*b^3*c*d^4*e^5*f^2*g^4 + 126*A*a^6*b^2*d^5*e^5*f^2*g^4 + 177*B*a^6*b^2*d^5*e^5*f^2*g^4 - 18*A*a^2*b^6*c^5*e^5*f*g^5 + 33*B*a^2*b^6*c^5*e^5*f*g^5 - 6*A*a^3*b^5*c^4*d*e^5*f*g^5 - 21*B*a^3*b^5*c^4*d*e^5*f*g^5 + 24*A*a^4*b^4*c^3*d^2*e^5*f*g^5 - 66*B*a^4*b^4*c^3*d^2*e^5*f*g^5 - 6*B*a^5*b^3*c^2*d^3*e^5*f*g^5 + 42*A*a^6*b^2*c*d^4*e^5*f*g^5 + 129*B*a^6*b^2*c*d^4*e^5*f*g^5 - 42*A*a^7*b*d^5*e^5*f*g^5 - 69*B*a^7*b*d^5*e^5*f*g^5 + 6*A*a^3*b^5*c^5*e^5*g^6 - 11*B*a^3*b^5*c^5*e^5*g^6 - 6*A*a^4*b^4*c^4*d*e^5*g^6 + 19*B*a^4*b^4*c^4*d*e^5*g^6 - 2*B*a^5*b^3*c^3*d^2*e^5*g^6 + 2*B*a^6*b^2*c^2*d^3*e^5*g^6 - 6*A*a^7*b*c*d^4*e^5*g^6 - 19*B*a^7*b*c*d^4*e^5*g^6 + 6*A*a^8*d^5*e^5*g^6 + 11*B*a^8*d^5*e^5*g^6 - 72*(b*e*x + a*e)*A*b^7*c^2*d^4*e^4*f^6/(d*x + c) + 144*(b*e*x + a*e)*A*a*b^6*c*d^5*e^4*f^6/(d*x + c) - 72*(b*e*x + a*e)*A*a^2*b^5*d^6*e^4*f^6/(d*x + c) + 144*(b*e*x + a*e)*A*b^7*c^3*d^3*e^4*f^5*g/(d*x + c) - 108*(b*e*x + a*e)*B*b^7*c^3*d^3*e^4*f^5*g/(d*x + c) + 324*(b*e*x + a*e)*B*a*b^6*c^2*d^4*e^4*f^5*g/(d*x + c) - 432*(b*e*x + a*e)*A*a^2*b^5*c*d^5*e^4*f^5*g/(d*x + c) - 324*(b*e*x + a*e)*B*a^2*b^5*c*d^5*e^4*f^5*g/(d*x + c) + 288*(b*e*x + a*e)*A*a^3*b^4*d^6*e^4*f^5*g/(d*x + c) + 108*(b*e*x + a*e)*B*a^3*b^4*d^6*e^4*f^5*g/(d*x + c) - 96*(b*e*x + a*e)*A*b^7*c^4*d^2*e^4*f^4*g^2/(d*x + c) + 204*(b*e*x + a*e)*B*b^7*c^4*d^2*e^4*f^4*g^2/(d*x + c) - 336*(b*e*x + a*e)*A*a*b^6*c^3*d^3*e^4*f^4*g^2/(d*x + c) - 276*(b*e*x + a*e)*B*a*b^6*c^3*d^3*e^4*f^4*g^2/(d*x + c) + 504*(b*e*x + a*e)*A*a^2*b^5*c^2*d^4*e^4*f^4*g^2/(d*x + c) - 396*(b*e*x + a*e)*B*a^2*b^5*c^2*d^4*e^4*f^4*g^2/(d*x + c) + 384*(b*e*x + a*e)*A*a^3*b^4*c*d^5*e^4*f^4*g^2/(d*x + c) + 804*(b*e*x + a*e)*B*a^3*b^4*c*d^5*e^4*f^4*g^2/(d*x + c) - 456*(b*e*x + a*e)*A*a^4*b^3*d^6*e^4*f^4*g^2/(d*x + c) - 336*(b*e*x + a*e)*B*a^4*b^3*d^6*e^4*f^4*g^2/(d*x + c) + 24*(b*e*x + a*e)*A*b^7*c^5*d*e^4*f^3*g^3/(d*x + c) - 122*(b*e*x + a*e)*B*b^7*c^5*d*e^4*f^3*g^3/(d*x + c) + 264*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*f^3*g^3/(d*x + c) - 206*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*f^3*g^3/(d*x + c) + 144*(b*e*x + a*e)*A*a^2*b^5*c^3*d^3*e^4*f^3*g^3/(d*x + c) + 964*(b*e*x + a*e)*B*a^2*b^5*c^3*d^3*e^4*f^3*g^3/(d*x + c) - 816*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4*e^4*f^3*g^3/(d*x + c) - 436*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*f^3*g^3/(d*x + c) + 24*(b*e*x + a*e)*A*a^4*b^3*c*d^5*e^4*f^3*g^3/(d*x + c) - 586*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*f^3*g^3/(d*x + c) + 360*(b*e*x + a*e)*A*a^5*b^2*d^6*e^4*f^3*g^3/(d*x + c) + 386*

(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*f^3*g^3/(d*x + c) + 26*(b*e*x + a*e)*B*b^7*
c^6*e^4*f^2*g^4/(d*x + c) - 72*(b*e*x + a*e)*A*a*b^6*c^5*d*e^4*f^2*g^4/(d*x
 + c) + 210*(b*e*x + a*e)*B*a*b^6*c^5*d*e^4*f^2*g^4/(d*x + c) - 216*(b*e*x
+ a*e)*A*a^2*b^5*c^4*d^2*e^4*f^2*g^4/(d*x + c) - 216*(b*e*x + a*e)*B*a^2*b^
5*c^4*d^2*e^4*f^2*g^4/(d*x + c) + 144*(b*e*x + a*e)*A*a^3*b^4*c^3*d^3*e^4*f
^2*g^4/(d*x + c) - 676*(b*e*x + a*e)*B*a^3*b^4*c^3*d^3*e^4*f^2*g^4/(d*x + c
) + 504*(b*e*x + a*e)*A*a^4*b^3*c^2*d^4*e^4*f^2*g^4/(d*x + c) + 834*(b*e*x
+ a*e)*B*a^4*b^3*c^2*d^4*e^4*f^2*g^4/(d*x + c) - 216*(b*e*x + a*e)*A*a^5*b^
2*c*d^5*e^4*f^2*g^4/(d*x + c) + 18*(b*e*x + a*e)*B*a^5*b^2*c*d^5*e^4*f^2*g^
4/(d*x + c) - 144*(b*e*x + a*e)*A*a^6*b*d^6*e^4*f^2*g^4/(d*x + c) - 196*(b*
e*x + a*e)*B*a^6*b*d^6*e^4*f^2*g^4/(d*x + c) - 52*(b*e*x + a*e)*B*a*b^6*c^6
*e^4*f*g^5/(d*x + c) + 72*(b*e*x + a*e)*A*a^2*b^5*c^5*d*e^4*f*g^5/(d*x + c)
 - 54*(b*e*x + a*e)*B*a^2*b^5*c^5*d*e^4*f*g^5/(d*x + c) + 24*(b*e*x + a*e)*
A*a^3*b^4*c^4*d^2*e^4*f*g^5/(d*x + c) + 234*(b*e*x + a*e)*B*a^3*b^4*c^4*d^2
*e^4*f*g^5/(d*x + c) - 96*(b*e*x + a*e)*A*a^4*b^3*c^3*d^3*e^4*f*g^5/(d*x +
c) + 104*(b*e*x + a*e)*B*a^4*b^3*c^3*d^3*e^4*f*g^5/(d*x + c) - 144*(b*e*x +
 a*e)*A*a^5*b^2*c^2*d^4*e^4*f*g^5/(d*x + c) - 396*(b*e*x + a*e)*B*a^5*b^2*c
^2*d^4*e^4*f*g^5/(d*x + c) + 120*(b*e*x + a*e)*A*a^6*b*c*d^5*e^4*f*g^5/(d*x
 + c) + 126*(b*e*x + a*e)*B*a^6*b*c*d^5*e^4*f*g^5/(d*x + c) + 24*(b*e*x + a
*e)*A*a^7*d^6*e^4*f*g^5/(d*x + c) + 38*(b*e*x + a*e)*B*a^7*d^6*e^4*f*g^5/(d
x + c) + 26(b*e*x + a*e)*B*a^2*b^5*c^6*e^4*g^6/(d*x + c) - 24*(b*e*x + a*
e)*A*a^3*b^4*c^5*d*e^4*g^6/(d*x + c) - 34*(b*e*x + a*e)*B*a^3*b^4*c^5*d*e^4
*g^6/(d*x + c) + 24*(b*e*x + a*e)*A*a^4*b^3*c^4*d^2*e^4*g^6/(d*x + c) - 16*
(b*e*x + a*e)*B*a^4*b^3*c^4*d^2*e^4*g^6/(d*x + c) - 8*(b*e*x + a*e)*B*a^5*b
^2*c^3*d^3*e^4*g^6/(d*x + c) + 24*(b*e*x + a*e)*A*a^6*b*c^2*d^4*e^4*g^6/(d*
x + c) + 70*(b*e*x + a*e)*B*a^6*b*c^2*d^4*e^4*g^6/(d*x + c) - 24*(b*e*x + a
*e)*A*a^7*c*d^5*e^4*g^6/(d*x + c) - 38*(b*e*x + a*e)*B*a^7*c*d^5*e^4*g^6/(d
x + c) + 72(b*e*x + a*e)^2*A*b^6*c^2*d^5*e^3*f^6/(d*x + c)^2 - 144*(b*e*x
 + a*e)^2*A*a*b^5*c*d^6*e^3*f^6/(d*x + c)^2 + 72*(b*e*x + a*e)^2*A*a^2*b^4*
d^7*e^3*f^6/(d*x + c)^2 - 180*(b*e*x + a*e)^2*A*b^6*c^3*d^4*e^3*f^5*g/(d*x
+ c)^2 + 108*(b*e*x + a*e)^2*B*b^6*c^3*d^4*e^3*f^5*g/(d*x + c)^2 + 108*(b*e
*x + a*e)^2*A*a*b^5*c^2*d^5*e^3*f^5*g/(d*x + c)^2 - 324*(b*e*x + a*e)^2*B*a
*b^5*c^2*d^5*e^3*f^5*g/(d*x + c)^2 + 324*(b*e*x + a*e)^2*A*a^2*b^4*c*d^6*e^
3*f^5*g/(d*x + c)^2 + 324*(b*e*x + a*e)^2*B*a^2*b^4*c*d^6*e^3*f^5*g/(d*x +
c)^2 - 252*(b*e*x + a*e)^2*A*a^3*b^3*d^7*e^3*f^5*g/(d*x + c)^2 - 108*(b*e*x
 + a*e)^2*B*a^3*b^3*d^7*e^3*f^5*g/(d*x + c)^2 + 144*(b*e*x + a*e)^2*A*b^6*c
^4*d^3*e^3*f^4*g^2/(d*x + c)^2 - 300*(b*e*x + a*e)^2*B*b^6*c^4*d^3*e^3*f^4*
g^2/(d*x + c)^2 + 324*(b*e*x + a*e)^2*A*a*b^5*c^3*d^4*e^3*f^4*g^2/(d*x + c)
^2 + 660*(b*e*x + a*e)^2*B*a*b^5*c^3*d^4*e^3*f^4*g^2/(d*x + c)^2 - 756*(b*e
*x + a*e)^2*A*a^2*b^4*c^2*d^5*e^3*f^4*g^2/(d*x + c)^2 - 180*(b*e*x + a*e)^2
*B*a^2*b^4*c^2*d^5*e^3*f^4*g^2/(d*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^3*b^3*c
*d^6*e^3*f^4*g^2/(d*x + c)^2 - 420*(b*e*x + a*e)^2*B*a^3*b^3*c*d^6*e^3*f^4*
g^2/(d*x + c)^2 + 324*(b*e*x + a*e)^2*A*a^4*b^2*d^7*e^3*f^4*g^2/(d*x + c)^2
 + 240*(b*e*x + a*e)^2*B*a^4*b^2*d^7*e^3*f^4*g^2/(d*x + c)^2 - 36*(b*e*x +
a*e)^2*A*b^6*c^5*d^2*e^3*f^3*g^3/(d*x + c)^2 + 297*(b*e*x + a*e)^2*B*b^6*c^

5*d^2*e^3*f^3*g^3/(d*x + c)^2 - 396*(b*e*x + a*e)^2*A*a*b^5*c^4*d^3*e^3*f^3
*g^3/(d*x + c)^2 - 285*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*f^3*g^3/(d*x + c
)^2 + 144*(b*e*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3*f^3*g^3/(d*x + c)^2 - 750*(
b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*f^3*g^3/(d*x + c)^2 + 864*(b*e*x + a*e
)^2*A*a^3*b^3*c^2*d^5*e^3*f^3*g^3/(d*x + c)^2 + 990*(b*e*x + a*e)^2*B*a^3*b
^3*c^2*d^5*e^3*f^3*g^3/(d*x + c)^2 - 396*(b*e*x + a*e)^2*A*a^4*b^2*c*d^6*e^
3*f^3*g^3/(d*x + c)^2 - 75*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*f^3*g^3/(d*x
 + c)^2 - 180*(b*e*x + a*e)^2*A*a^5*b*d^7*e^3*f^3*g^3/(d*x + c)^2 - 177*(b*
e*x + a*e)^2*B*a^5*b*d^7*e^3*f^3*g^3/(d*x + c)^2 - 126*(b*e*x + a*e)^2*B*b^
6*c^6*d*e^3*f^2*g^4/(d*x + c)^2 + 108*(b*e*x + a*e)^2*A*a*b^5*c^5*d^2*e^3*f
^2*g^4/(d*x + c)^2 - 135*(b*e*x + a*e)^2*B*a*b^5*c^5*d^2*e^3*f^2*g^4/(d*x +
 c)^2 + 324*(b*e*x + a*e)^2*A*a^2*b^4*c^4*d^3*e^3*f^2*g^4/(d*x + c)^2 + 765
*(b*e*x + a*e)^2*B*a^2*b^4*c^4*d^3*e^3*f^2*g^4/(d*x + c)^2 - 576*(b*e*x + a
*e)^2*A*a^3*b^3*c^3*d^4*e^3*f^2*g^4/(d*x + c)^2 - 270*(b*e*x + a*e)^2*B*a^3
*b^3*c^3*d^4*e^3*f^2*g^4/(d*x + c)^2 - 216*(b*e*x + a*e)^2*A*a^4*b^2*c^2*d^
5*e^3*f^2*g^4/(d*x + c)^2 - 540*(b*e*x + a*e)^2*B*a^4*b^2*c^2*d^5*e^3*f^2*g
^4/(d*x + c)^2 + 324*(b*e*x + a*e)^2*A*a^5*b*c*d^6*e^3*f^2*g^4/(d*x + c)^2
 + 261*(b*e*x + a*e)^2*B*a^5*b*c*d^6*e^3*f^2*g^4/(d*x + c)^2 + 36*(b*e*x + a
*e)^2*A*a^6*d^7*e^3*f^2*g^4/(d*x + c)^2 + 45*(b*e*x + a*e)^2*B*a^6*d^7*e^3*
f^2*g^4/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*b^6*c^7*e^3*f*g^5/(d*x + c)^2 +
105*(b*e*x + a*e)^2*B*a*b^5*c^6*d*e^3*f*g^5/(d*x + c)^2 - 108*(b*e*x + a*e)
^2*A*a^2*b^4*c^5*d^2*e^3*f*g^5/(d*x + c)^2 - 180*(b*e*x + a*e)^2*B*a^2*b^4*
c^5*d^2*e^3*f*g^5/(d*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^3*b^3*c^4*d^3*e^3*f*
g^5/(d*x + c)^2 - 210*(b*e*x + a*e)^2*B*a^3*b^3*c^4*d^3*e^3*f*g^5/(d*x + c)
^2 + 324*(b*e*x + a*e)^2*A*a^4*b^2*c^3*d^4*e^3*f*g^5/(d*x + c)^2 + 345*(b*e
*x + a*e)^2*B*a^4*b^2*c^3*d^4*e^3*f*g^5/(d*x + c)^2 - 108*(b*e*x + a*e)^2*A
*a^5*b*c^2*d^5*e^3*f*g^5/(d*x + c)^2 + 9*(b*e*x + a*e)^2*B*a^5*b*c^2*d^5*e^
3*f*g^5/(d*x + c)^2 - 72*(b*e*x + a*e)^2*A*a^6*c*d^6*e^3*f*g^5/(d*x + c)^2
 - 90*(b*e*x + a*e)^2*B*a^6*c*d^6*e^3*f*g^5/(d*x + c)^2 - 21*(b*e*x + a*e)^2
*B*a*b^5*c^7*e^3*g^6/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*a^2*b^4*c^6*d*e^3*g
^6/(d*x + c)^2 + 36*(b*e*x + a*e)^2*A*a^3*b^3*c^5*d^2*e^3*g^6/(d*x + c)^2 +
 18*(b*e*x + a*e)^2*B*a^3*b^3*c^5*d^2*e^3*g^6/(d*x + c)^2 - 36*(b*e*x + a*e
)^2*A*a^4*b^2*c^4*d^3*e^3*g^6/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^2*c^
4*d^3*e^3*g^6/(d*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^5*b*c^3*d^4*e^3*g^6/(d*x
 + c)^2 - 93*(b*e*x + a*e)^2*B*a^5*b*c^3*d^4*e^3*g^6/(d*x + c)^2 + 36*(b*e*
x + a*e)^2*A*a^6*c^2*d^5*e^3*g^6/(d*x + c)^2 + 45*(b*e*x + a*e)^2*B*a^6*c^2
*d^5*e^3*g^6/(d*x + c)^2 - 24*(b*e*x + a*e)^3*A*b^5*c^2*d^6*e^2*f^6/(d*x +
c)^3 + 48*(b*e*x + a*e)^3*A*a*b^4*c*d^7*e^2*f^6/(d*x + c)^3 - 24*(b*e*x + a
*e)^3*A*a^2*b^3*d^8*e^2*f^6/(d*x + c)^3 + 72*(b*e*x + a*e)^3*A*b^5*c^3*d^5*
e^2*f^5*g/(d*x + c)^3 - 36*(b*e*x + a*e)^3*B*b^5*c^3*d^5*e^2*f^5*g/(d*x + c
)^3 - 72*(b*e*x + a*e)^3*A*a*b^4*c^2*d^6*e^2*f^5*g/(d*x + c)^3 + 108*(b*e*x
 + a*e)^3*B*a*b^4*c^2*d^6*e^2*f^5*g/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a^2*
b^3*c*d^7*e^2*f^5*g/(d*x + c)^3 - 108*(b*e*x + a*e)^3*B*a^2*b^3*c*d^7*e^2*f
^5*g/(d*x + c)^3 + 72*(b*e*x + a*e)^3*A*a^3*b^2*d^8*e^2*f^5*g/(d*x + c)^3 +
 36*(b*e*x + a*e)^3*B*a^3*b^2*d^8*e^2*f^5*g/(d*x + c)^3 - 72*(b*e*x + a*e)^

3*A*b^5*c^4*d^4*e^2*f^4*g^2/(d*x + c)^3 + 132*(b*e*x + a*e)^3*B*b^5*c^4*d^4
*e^2*f^4*g^2/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a*b^4*c^3*d^5*e^2*f^4*g^2/(
d*x + c)^3 - 348*(b*e*x + a*e)^3*B*a*b^4*c^3*d^5*e^2*f^4*g^2/(d*x + c)^3 +
288*(b*e*x + a*e)^3*A*a^2*b^3*c^2*d^6*e^2*f^4*g^2/(d*x + c)^3 + 252*(b*e*x
+ a*e)^3*B*a^2*b^3*c^2*d^6*e^2*f^4*g^2/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a
^3*b^2*c*d^7*e^2*f^4*g^2/(d*x + c)^3 + 12*(b*e*x + a*e)^3*B*a^3*b^2*c*d^7*e
^2*f^4*g^2/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a^4*b*d^8*e^2*f^4*g^2/(d*x +
c)^3 - 48*(b*e*x + a*e)^3*B*a^4*b*d^8*e^2*f^4*g^2/(d*x + c)^3 + 24*(b*e*x +
a*e)^3*A*b^5*c^5*d^3*e^2*f^3*g^3/(d*x + c)^3 - 186*(b*e*x + a*e)^3*B*b^5*c
^5*d^3*e^2*f^3*g^3/(d*x + c)^3 + 168*(b*e*x + a*e)^3*A*a*b^4*c^4*d^4*e^2*f^
3*g^3/(d*x + c)^3 + 402*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*f^3*g^3/(d*x +
c)^3 - 192*(b*e*x + a*e)^3*A*a^2*b^3*c^3*d^5*e^2*f^3*g^3/(d*x + c)^3 - 108*
(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*f^3*g^3/(d*x + c)^3 - 192*(b*e*x + a*
e)^3*A*a^3*b^2*c^2*d^6*e^2*f^3*g^3/(d*x + c)^3 - 228*(b*e*x + a*e)^3*B*a^3*
b^2*c^2*d^6*e^2*f^3*g^3/(d*x + c)^3 + 168*(b*e*x + a*e)^3*A*a^4*b*c*d^7*e^2
*f^3*g^3/(d*x + c)^3 + 102*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*f^3*g^3/(d*x +
c)^3 + 24*(b*e*x + a*e)^3*A*a^5*d^8*e^2*f^3*g^3/(d*x + c)^3 + 18*(b*e*x +
a*e)^3*B*a^5*d^8*e^2*f^3*g^3/(d*x + c)^3 + 126*(b*e*x + a*e)^3*B*b^5*c^6*d^
2*e^2*f^2*g^4/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a*b^4*c^5*d^3*e^2*f^2*g^4/
(d*x + c)^3 - 198*(b*e*x + a*e)^3*B*a*b^4*c^5*d^3*e^2*f^2*g^4/(d*x + c)^3 -
72*(b*e*x + a*e)^3*A*a^2*b^3*c^4*d^4*e^2*f^2*g^4/(d*x + c)^3 - 108*(b*e*x
+ a*e)^3*B*a^2*b^3*c^4*d^4*e^2*f^2*g^4/(d*x + c)^3 + 288*(b*e*x + a*e)^3*A*
a^3*b^2*c^3*d^5*e^2*f^2*g^4/(d*x + c)^3 + 252*(b*e*x + a*e)^3*B*a^3*b^2*c^3
*d^5*e^2*f^2*g^4/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a^4*b*c^2*d^6*e^2*f^2*g
^4/(d*x + c)^3 - 18*(b*e*x + a*e)^3*B*a^4*b*c^2*d^6*e^2*f^2*g^4/(d*x + c)^3
- 72*(b*e*x + a*e)^3*A*a^5*c*d^7*e^2*f^2*g^4/(d*x + c)^3 - 54*(b*e*x + a*e
)^3*B*a^5*c*d^7*e^2*f^2*g^4/(d*x + c)^3 - 42*(b*e*x + a*e)^3*B*b^5*c^7*d*e^
2*f*g^5/(d*x + c)^3 + 42*(b*e*x + a*e)^3*B*a*b^4*c^6*d^2*e^2*f*g^5/(d*x + c
)^3 + 72*(b*e*x + a*e)^3*A*a^2*b^3*c^5*d^3*e^2*f*g^5/(d*x + c)^3 + 72*(b*e*
x + a*e)^3*B*a^2*b^3*c^5*d^3*e^2*f*g^5/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a
^3*b^2*c^4*d^4*e^2*f*g^5/(d*x + c)^3 - 48*(b*e*x + a*e)^3*B*a^3*b^2*c^4*d^4
*e^2*f*g^5/(d*x + c)^3 - 72*(b*e*x + a*e)^3*A*a^4*b*c^3*d^5*e^2*f*g^5/(d*x
+ c)^3 - 78*(b*e*x + a*e)^3*B*a^4*b*c^3*d^5*e^2*f*g^5/(d*x + c)^3 + 72*(b*e
*x + a*e)^3*A*a^5*c^2*d^6*e^2*f*g^5/(d*x + c)^3 + 54*(b*e*x + a*e)^3*B*a^5*
c^2*d^6*e^2*f*g^5/(d*x + c)^3 + 6*(b*e*x + a*e)^3*B*b^5*c^8*e^2*g^6/(d*x +
c)^3 - 6*(b*e*x + a*e)^3*B*a*b^4*c^7*d*e^2*g^6/(d*x + c)^3 - 24*(b*e*x + a*
e)^3*A*a^3*b^2*c^5*d^3*e^2*g^6/(d*x + c)^3 - 24*(b*e*x + a*e)^3*B*a^3*b^2*c
^5*d^3*e^2*g^6/(d*x + c)^3 + 48*(b*e*x + a*e)^3*A*a^4*b*c^4*d^4*e^2*g^6/(d*
x + c)^3 + 42*(b*e*x + a*e)^3*B*a^4*b*c^4*d^4*e^2*g^6/(d*x + c)^3 - 24*(b*e
*x + a*e)^3*A*a^5*c^3*d^5*e^2*g^6/(d*x + c)^3 - 18*(b*e*x + a*e)^3*B*a^5*c^
3*d^5*e^2*g^6/(d*x + c)^3)/(b^7*d^4*e^4*f^11 - 4*b^7*c*d^3*e^4*f^10*g - 7*a
*b^6*d^4*e^4*f^10*g + 6*b^7*c^2*d^2*e^4*f^9*g^2 + 28*a*b^6*c*d^3*e^4*f^9*g^
2 + 21*a^2*b^5*d^4*e^4*f^9*g^2 - 4*b^7*c^3*d*e^4*f^8*g^3 - 42*a*b^6*c^2*d^2
*e^4*f^8*g^3 - 84*a^2*b^5*c*d^3*e^4*f^8*g^3 - 35*a^3*b^4*d^4*e^4*f^8*g^3 +
b^7*c^4*e^4*f^7*g^4 + 28*a*b^6*c^3*d*e^4*f^7*g^4 + 126*a^2*b^5*c^2*d^2*e^4*

f^7*g^4 + 140*a^3*b^4*c*d^3*e^4*f^7*g^4 + 35*a^4*b^3*d^4*e^4*f^7*g^4 - 7*a*
b^6*c^4*e^4*f^6*g^5 - 84*a^2*b^5*c^3*d*e^4*f^6*g^5 - 210*a^3*b^4*c^2*d^2*e^
4*f^6*g^5 - 140*a^4*b^3*c*d^3*e^4*f^6*g^5 - 21*a^5*b^2*d^4*e^4*f^6*g^5 + 21
*a^2*b^5*c^4*e^4*f^5*g^6 + 140*a^3*b^4*c^3*d*e^4*f^5*g^6 + 210*a^4*b^3*c^2*
d^2*e^4*f^5*g^6 + 84*a^5*b^2*c*d^3*e^4*f^5*g^6 + 7*a^6*b*d^4*e^4*f^5*g^6 -
35*a^3*b^4*c^4*e^4*f^4*g^7 - 140*a^4*b^3*c^3*d*e^4*f^4*g^7 - 126*a^5*b^2*c^
2*d^2*e^4*f^4*g^7 - 28*a^6*b*c*d^3*e^4*f^4*g^7 - a^7*d^4*e^4*f^4*g^7 + 35*a
^4*b^3*c^4*e^4*f^3*g^8 + 84*a^5*b^2*c^3*d*e^4*f^3*g^8 + 42*a^6*b*c^2*d^2*e^
4*f^3*g^8 + 4*a^7*c*d^3*e^4*f^3*g^8 - 21*a^5*b^2*c^4*e^4*f^2*g^9 - 28*a^6*b
*c^3*d*e^4*f^2*g^9 - 6*a^7*c^2*d^2*e^4*f^2*g^9 + 7*a^6*b*c^4*e^4*f*g^10 + 4
*a^7*c^3*d*e^4*f*g^10 - a^7*c^4*e^4*g^11 - 4*(b*e*x + a*e)*b^6*d^5*e^3*f^11
/(d*x + c) + 20*(b*e*x + a*e)*b^6*c*d^4*e^3*f^10*g/(d*x + c) + 24*(b*e*x +
a*e)*a*b^5*d^5*e^3*f^10*g/(d*x + c) - 40*(b*e*x + a*e)*b^6*c^2*d^3*e^3*f^9*
g^2/(d*x + c) - 120*(b*e*x + a*e)*a*b^5*c*d^4*e^3*f^9*g^2/(d*x + c) - 60*(b
*e*x + a*e)*a^2*b^4*d^5*e^3*f^9*g^2/(d*x + c) + 40*(b*e*x + a*e)*b^6*c^3*d^
2*e^3*f^8*g^3/(d*x + c) + 240*(b*e*x + a*e)*a*b^5*c^2*d^3*e^3*f^8*g^3/(d*x
+ c) + 300*(b*e*x + a*e)*a^2*b^4*c*d^4*e^3*f^8*g^3/(d*x + c) + 80*(b*e*x +
a*e)*a^3*b^3*d^5*e^3*f^8*g^3/(d*x + c) - 20*(b*e*x + a*e)*b^6*c^4*d*e^3*f^7
*g^4/(d*x + c) - 240*(b*e*x + a*e)*a*b^5*c^3*d^2*e^3*f^7*g^4/(d*x + c) - 60
0*(b*e*x + a*e)*a^2*b^4*c^2*d^3*e^3*f^7*g^4/(d*x + c) - 400*(b*e*x + a*e)*a
^3*b^3*c*d^4*e^3*f^7*g^4/(d*x + c) - 60*(b*e*x + a*e)*a^4*b^2*d^5*e^3*f^7*g
^4/(d*x + c) + 4*(b*e*x + a*e)*b^6*c^5*e^3*f^6*g^5/(d*x + c) + 120*(b*e*x +
a*e)*a*b^5*c^4*d*e^3*f^6*g^5/(d*x + c) + 600*(b*e*x + a*e)*a^2*b^4*c^3*d^2
*e^3*f^6*g^5/(d*x + c) + 800*(b*e*x + a*e)*a^3*b^3*c^2*d^3*e^3*f^6*g^5/(d*x
+ c) + 300*(b*e*x + a*e)*a^4*b^2*c*d^4*e^3*f^6*g^5/(d*x + c) + 24*(b*e*x +
a*e)*a^5*b*d^5*e^3*f^6*g^5/(d*x + c) - 24*(b*e*x + a*e)*a*b^5*c^5*e^3*f^5*
g^6/(d*x + c) - 300*(b*e*x + a*e)*a^2*b^4*c^4*d*e^3*f^5*g^6/(d*x + c) - 800
*(b*e*x + a*e)*a^3*b^3*c^3*d^2*e^3*f^5*g^6/(d*x + c) - 600*(b*e*x + a*e)*a^
4*b^2*c^2*d^3*e^3*f^5*g^6/(d*x + c) - 120*(b*e*x + a*e)*a^5*b*c*d^4*e^3*f^5
*g^6/(d*x + c) - 4*(b*e*x + a*e)*a^6*d^5*e^3*f^5*g^6/(d*x + c) + 60*(b*e*x
+ a*e)*a^2*b^4*c^5*e^3*f^4*g^7/(d*x + c) + 400*(b*e*x + a*e)*a^3*b^3*c^4*d*
e^3*f^4*g^7/(d*x + c) + 600*(b*e*x + a*e)*a^4*b^2*c^3*d^2*e^3*f^4*g^7/(d*x
+ c) + 240*(b*e*x + a*e)*a^5*b*c^2*d^3*e^3*f^4*g^7/(d*x + c) + 20*(b*e*x +
a*e)*a^6*c*d^4*e^3*f^4*g^7/(d*x + c) - 80*(b*e*x + a*e)*a^3*b^3*c^5*e^3*f^3
*g^8/(d*x + c) - 300*(b*e*x + a*e)*a^4*b^2*c^4*d*e^3*f^3*g^8/(d*x + c) - 24
0*(b*e*x + a*e)*a^5*b*c^3*d^2*e^3*f^3*g^8/(d*x + c) - 40*(b*e*x + a*e)*a^6*
c^2*d^3*e^3*f^3*g^8/(d*x + c) + 60*(b*e*x + a*e)*a^4*b^2*c^5*e^3*f^2*g^9/(d
x + c) + 120(b*e*x + a*e)*a^5*b*c^4*d*e^3*f^2*g^9/(d*x + c) + 40*(b*e*x +
a*e)*a^6*c^3*d^2*e^3*f^2*g^9/(d*x + c) - 24*(b*e*x + a*e)*a^5*b*c^5*e^3*f*
g^10/(d*x + c) - 20*(b*e*x + a*e)*a^6*c^4*d*e^3*f*g^10/(d*x + c) + 4*(b*e*x
+ a*e)*a^6*c^5*e^3*g^11/(d*x + c) + 6*(b*e*x + a*e)^2*b^5*d^6*e^2*f^11/(d*
x + c)^2 - 36*(b*e*x + a*e)^2*b^5*c*d^5*e^2*f^10*g/(d*x + c)^2 - 30*(b*e*x
+ a*e)^2*a*b^4*d^6*e^2*f^10*g/(d*x + c)^2 + 90*(b*e*x + a*e)^2*b^5*c^2*d^4*
e^2*f^9*g^2/(d*x + c)^2 + 180*(b*e*x + a*e)^2*a*b^4*c*d^5*e^2*f^9*g^2/(d*x
+ c)^2 + 60*(b*e*x + a*e)^2*a^2*b^3*d^6*e^2*f^9*g^2/(d*x + c)^2 - 120*(b*e*

x + a*e)^2*b^5*c^3*d^3*e^2*f^8*g^3/(d*x + c)^2 - 450*(b*e*x + a*e)^2*a*b^4*
c^2*d^4*e^2*f^8*g^3/(d*x + c)^2 - 360*(b*e*x + a*e)^2*a^2*b^3*c*d^5*e^2*f^8
*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*a^3*b^2*d^6*e^2*f^8*g^3/(d*x + c)^2 +
 90*(b*e*x + a*e)^2*b^5*c^4*d^2*e^2*f^7*g^4/(d*x + c)^2 + 600*(b*e*x + a*e)
^2*a*b^4*c^3*d^3*e^2*f^7*g^4/(d*x + c)^2 + 900*(b*e*x + a*e)^2*a^2*b^3*c^2*
d^4*e^2*f^7*g^4/(d*x + c)^2 + 360*(b*e*x + a*e)^2*a^3*b^2*c*d^5*e^2*f^7*g^4
/(d*x + c)^2 + 30*(b*e*x + a*e)^2*a^4*b*d^6*e^2*f^7*g^4/(d*x + c)^2 - 36*(b
*e*x + a*e)^2*b^5*c^5*d*e^2*f^6*g^5/(d*x + c)^2 - 450*(b*e*x + a*e)^2*a*b^4
*c^4*d^2*e^2*f^6*g^5/(d*x + c)^2 - 1200*(b*e*x + a*e)^2*a^2*b^3*c^3*d^3*e^2
*f^6*g^5/(d*x + c)^2 - 900*(b*e*x + a*e)^2*a^3*b^2*c^2*d^4*e^2*f^6*g^5/(d*x
 + c)^2 - 180*(b*e*x + a*e)^2*a^4*b*c*d^5*e^2*f^6*g^5/(d*x + c)^2 - 6*(b*e*
x + a*e)^2*a^5*d^6*e^2*f^6*g^5/(d*x + c)^2 + 6*(b*e*x + a*e)^2*b^5*c^6*e^2*
f^5*g^6/(d*x + c)^2 + 180*(b*e*x + a*e)^2*a*b^4*c^5*d*e^2*f^5*g^6/(d*x + c)
^2 + 900*(b*e*x + a*e)^2*a^2*b^3*c^4*d^2*e^2*f^5*g^6/(d*x + c)^2 + 1200*(b*
e*x + a*e)^2*a^3*b^2*c^3*d^3*e^2*f^5*g^6/(d*x + c)^2 + 450*(b*e*x + a*e)^2*
a^4*b*c^2*d^4*e^2*f^5*g^6/(d*x + c)^2 + 36*(b*e*x + a*e)^2*a^5*c*d^5*e^2*f^
5*g^6/(d*x + c)^2 - 30*(b*e*x + a*e)^2*a*b^4*c^6*e^2*f^4*g^7/(d*x + c)^2 -
360*(b*e*x + a*e)^2*a^2*b^3*c^5*d*e^2*f^4*g^7/(d*x + c)^2 - 900*(b*e*x + a*
e)^2*a^3*b^2*c^4*d^2*e^2*f^4*g^7/(d*x + c)^2 - 600*(b*e*x + a*e)^2*a^4*b*c^
3*d^3*e^2*f^4*g^7/(d*x + c)^2 - 90*(b*e*x + a*e)^2*a^5*c^2*d^4*e^2*f^4*g^7/
(d*x + c)^2 + 60*(b*e*x + a*e)^2*a^2*b^3*c^6*e^2*f^3*g^8/(d*x + c)^2 + 360*
(b*e*x + a*e)^2*a^3*b^2*c^5*d*e^2*f^3*g^8/(d*x + c)^2 + 450*(b*e*x + a*e)^2
*a^4*b*c^4*d^2*e^2*f^3*g^8/(d*x + c)^2 + 120*(b*e*x + a*e)^2*a^5*c^3*d^3*e^
2*f^3*g^8/(d*x + c)^2 - 60*(b*e*x + a*e)^2*a^3*b^2*c^6*e^2*f^2*g^9/(d*x + c
)^2 - 180*(b*e*x + a*e)^2*a^4*b*c^5*d*e^2*f^2*g^9/(d*x + c)^2 - 90*(b*e*x +
 a*e)^2*a^5*c^4*d^2*e^2*f^2*g^9/(d*x + c)^2 + 30*(b*e*x + a*e)^2*a^4*b*c^6*
e^2*f*g^10/(d*x + c)^2 + 36*(b*e*x + a*e)^2*a^5*c^5*d*e^2*f*g^10/(d*x + c)^
2 - 6*(b*e*x + a*e)^2*a^5*c^6*e^2*g^11/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b^4*
d^7*e*f^11/(d*x + c)^3 + 28*(b*e*x + a*e)^3*b^4*c*d^6*e*f^10*g/(d*x + c)^3
+ 16*(b*e*x + a*e)^3*a*b^3*d^7*e*f^10*g/(d*x + c)^3 - 84*(b*e*x + a*e)^3*b^
4*c^2*d^5*e*f^9*g^2/(d*x + c)^3 - 112*(b*e*x + a*e)^3*a*b^3*c*d^6*e*f^9*g^2
/(d*x + c)^3 - 24*(b*e*x + a*e)^3*a^2*b^2*d^7*e*f^9*g^2/(d*x + c)^3 + 140*(
b*e*x + a*e)^3*b^4*c^3*d^4*e*f^8*g^3/(d*x + c)^3 + 336*(b*e*x + a*e)^3*a*b^
3*c^2*d^5*e*f^8*g^3/(d*x + c)^3 + 168*(b*e*x + a*e)^3*a^2*b^2*c*d^6*e*f^8*g
^3/(d*x + c)^3 + 16*(b*e*x + a*e)^3*a^3*b*d^7*e*f^8*g^3/(d*x + c)^3 - 140*(
b*e*x + a*e)^3*b^4*c^4*d^3*e*f^7*g^4/(d*x + c)^3 - 560*(b*e*x + a*e)^3*a*b^
3*c^3*d^4*e*f^7*g^4/(d*x + c)^3 - 504*(b*e*x + a*e)^3*a^2*b^2*c^2*d^5*e*f^7
*g^4/(d*x + c)^3 - 112*(b*e*x + a*e)^3*a^3*b*c*d^6*e*f^7*g^4/(d*x + c)^3 -
4*(b*e*x + a*e)^3*a^4*d^7*e*f^7*g^4/(d*x + c)^3 + 84*(b*e*x + a*e)^3*b^4*c^
5*d^2*e*f^6*g^5/(d*x + c)^3 + 560*(b*e*x + a*e)^3*a*b^3*c^4*d^3*e*f^6*g^5/(
d*x + c)^3 + 840*(b*e*x + a*e)^3*a^2*b^2*c^3*d^4*e*f^6*g^5/(d*x + c)^3 + 33
6*(b*e*x + a*e)^3*a^3*b*c^2*d^5*e*f^6*g^5/(d*x + c)^3 + 28*(b*e*x + a*e)^3*
a^4*c*d^6*e*f^6*g^5/(d*x + c)^3 - 28*(b*e*x + a*e)^3*b^4*c^6*d*e*f^5*g^6/(d
x + c)^3 - 336(b*e*x + a*e)^3*a*b^3*c^5*d^2*e*f^5*g^6/(d*x + c)^3 - 840*(
b*e*x + a*e)^3*a^2*b^2*c^4*d^3*e*f^5*g^6/(d*x + c)^3 - 560*(b*e*x + a*e)^3*

a^3*b*c^3*d^4*e*f^5*g^6/(d*x + c)^3 - 84*(b*e*x + a*e)^3*a^4*c^2*d^5*e*f^5*
g^6/(d*x + c)^3 + 4*(b*e*x + a*e)^3*b^4*c^7*e*f^4*g^7/(d*x + c)^3 + 112*(b*
e*x + a*e)^3*a*b^3*c^6*d*e*f^4*g^7/(d*x + c)^3 + 504*(b*e*x + a*e)^3*a^2*b^
2*c^5*d^2*e*f^4*g^7/(d*x + c)^3 + 560*(b*e*x + a*e)^3*a^3*b*c^4*d^3*e*f^4*g
^7/(d*x + c)^3 + 140*(b*e*x + a*e)^3*a^4*c^3*d^4*e*f^4*g^7/(d*x + c)^3 - 16
*(b*e*x + a*e)^3*a*b^3*c^7*e*f^3*g^8/(d*x + c)^3 - 168*(b*e*x + a*e)^3*a^2*
b^2*c^6*d*e*f^3*g^8/(d*x + c)^3 - 336*(b*e*x + a*e)^3*a^3*b*c^5*d^2*e*f^3*g
^8/(d*x + c)^3 - 140*(b*e*x + a*e)^3*a^4*c^4*d^3*e*f^3*g^8/(d*x + c)^3 + 24
*(b*e*x + a*e)^3*a^2*b^2*c^7*e*f^2*g^9/(d*x + c)^3 + 112*(b*e*x + a*e)^3*a^
3*b*c^6*d*e*f^2*g^9/(d*x + c)^3 + 84*(b*e*x + a*e)^3*a^4*c^5*d^2*e*f^2*g^9/
(d*x + c)^3 - 16*(b*e*x + a*e)^3*a^3*b*c^7*e*f*g^10/(d*x + c)^3 - 28*(b*e*x
 + a*e)^3*a^4*c^6*d*e*f*g^10/(d*x + c)^3 + 4*(b*e*x + a*e)^3*a^4*c^7*e*g^11
/(d*x + c)^3 + (b*e*x + a*e)^4*b^3*d^8*f^11/(d*x + c)^4 - 8*(b*e*x + a*e)^4
*b^3*c*d^7*f^10*g/(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*d^8*f^10*g/(d*x + c
)^4 + 28*(b*e*x + a*e)^4*b^3*c^2*d^6*f^9*g^2/(d*x + c)^4 + 24*(b*e*x + a*e)
^4*a*b^2*c*d^7*f^9*g^2/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*d^8*f^9*g^2/(d
x + c)^4 - 56(b*e*x + a*e)^4*b^3*c^3*d^5*f^8*g^3/(d*x + c)^4 - 84*(b*e*x
+ a*e)^4*a*b^2*c^2*d^6*f^8*g^3/(d*x + c)^4 - 24*(b*e*x + a*e)^4*a^2*b*c*d^7
*f^8*g^3/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^8*f^8*g^3/(d*x + c)^4 + 70*(b*
e*x + a*e)^4*b^3*c^4*d^4*f^7*g^4/(d*x + c)^4 + 168*(b*e*x + a*e)^4*a*b^2*c^
3*d^5*f^7*g^4/(d*x + c)^4 + 84*(b*e*x + a*e)^4*a^2*b*c^2*d^6*f^7*g^4/(d*x +
 c)^4 + 8*(b*e*x + a*e)^4*a^3*c*d^7*f^7*g^4/(d*x + c)^4 - 56*(b*e*x + a*e)^
4*b^3*c^5*d^3*f^6*g^5/(d*x + c)^4 - 210*(b*e*x + a*e)^4*a*b^2*c^4*d^4*f^6*g
^5/(d*x + c)^4 - 168*(b*e*x + a*e)^4*a^2*b*c^3*d^5*f^6*g^5/(d*x + c)^4 - 28
*(b*e*x + a*e)^4*a^3*c^2*d^6*f^6*g^5/(d*x + c)^4 + 28*(b*e*x + a*e)^4*b^3*c
^6*d^2*f^5*g^6/(d*x + c)^4 + 168*(b*e*x + a*e)^4*a*b^2*c^5*d^3*f^5*g^6/(d*x
 + c)^4 + 210*(b*e*x + a*e)^4*a^2*b*c^4*d^4*f^5*g^6/(d*x + c)^4 + 56*(b*e*x
 + a*e)^4*a^3*c^3*d^5*f^5*g^6/(d*x + c)^4 - 8*(b*e*x + a*e)^4*b^3*c^7*d*f^4
*g^7/(d*x + c)^4 - 84*(b*e*x + a*e)^4*a*b^2*c^6*d^2*f^4*g^7/(d*x + c)^4 - 1
68*(b*e*x + a*e)^4*a^2*b*c^5*d^3*f^4*g^7/(d*x + c)^4 - 70*(b*e*x + a*e)^4*a
^3*c^4*d^4*f^4*g^7/(d*x + c)^4 + (b*e*x + a*e)^4*b^3*c^8*f^3*g^8/(d*x + c)^
4 + 24*(b*e*x + a*e)^4*a*b^2*c^7*d*f^3*g^8/(d*x + c)^4 + 84*(b*e*x + a*e)^4
*a^2*b*c^6*d^2*f^3*g^8/(d*x + c)^4 + 56*(b*e*x + a*e)^4*a^3*c^5*d^3*f^3*g^8
/(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^8*f^2*g^9/(d*x + c)^4 - 24*(b*e*x
+ a*e)^4*a^2*b*c^7*d*f^2*g^9/(d*x + c)^4 - 28*(b*e*x + a*e)^4*a^3*c^6*d^2*f
^2*g^9/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c^8*f*g^10/(d*x + c)^4 + 8*(b*
e*x + a*e)^4*a^3*c^7*d*f*g^10/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*c^8*g^11/(d
x + c)^4))(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 2518, normalized size of antiderivative = 6.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^5,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3
*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f
^3 + 4*B*b^4*c*d^3*f^3))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4
 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24
*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b
^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g
^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 6
4*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 6
4*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3
 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^
2*f^4*g^4) - ((6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3*g^3 - 6*
A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 18*A*a*b^
2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 18*A*a^3*c
*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 18*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*
d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^
3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 18*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g
 - 18*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 54*A
*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3
+ 54*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^
3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 -
3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5
*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^
3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c
*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^
2*g^6 + 7*B*a^3*d^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21
*B*a^2*b*d^3*f^2*g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*
B*a*b^2*c^2*d*f*g^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6
 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*
g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d
^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*
g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4
)) + (x*(B*a^2*b*c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^
3*c^3*f^2*g^4 - 34*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*
c*d^2*f^4*g^2 - 38*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^
2*f*g^5 + 12*B*a*b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3
*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
```

- 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f ^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a ^2*b*c^2*d*f^2*g^4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3* f^2*g^4 - 3*B*b^3*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5))/(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b *c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3* a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d *f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^ 3*g^3 + 9*a^2*b*c^2*d*f^2*g^4))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f* g^4*x^3 + 24*f^2*g^3*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*g*(f^4 + g^ 4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) + (B*b^4*log(a + b*x))/(4 *a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2*b^2*f^2*g^3 - 16*a^3*b*f *g^4) - (B*d^4*log(c + d*x))/(4*c^4*g^5 + 4*d^4*f^4*g - 16*c*d^3*f^3*g^2 + 24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)

3.240 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 29, antiderivative size = 874

$$
\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^3g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{4b^3d^3} + \frac{B^2(bc-ad)^2g^3(c+dx)^2}{12b^2d^4} \\
&\quad + \frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4} \\
&\quad - \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4d^3} \\
&\quad - \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4} \\
&\quad - \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4} \\
&\quad - \frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\right.}{2b^4d^4} \\
&\quad - \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} \\
&\quad + \frac{B^2(bc-ad)^4g^3\log(c+dx)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{4b^4d^4} \\
&\quad + \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{2b^4d^4} \\
&\quad - \frac{B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}
\end{aligned}
$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g
+4*b*d*f)*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4+1/6*B^2*(-a
*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-
3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b^4/d^4-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^
2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f
)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*
f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-1/4*(-a*g+b*f)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+
1/6*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g
-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a
*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4-
1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c
^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules

used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{B^2g^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{6b^4d^4}+\frac{B^2g^3\log(c+dx)(bc-ad)^4}{6b^4d^4}$$
$$+\frac{B^2g^3x(bc-ad)^3}{6b^3d^3}+\frac{B^2g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^3}{4b^4d^4}$$
$$+\frac{B^2g^2(4bdf-3bcg-adg)\log(c+dx)(bc-ad)^3}{4b^4d^4}$$
$$+\frac{B^2g^3(c+dx)^2(bc-ad)^2}{12b^2d^4}+\frac{B^2g^2(4bdf-3bcg-adg)x(bc-ad)^2}{4b^3d^3}$$
$$+\frac{B^2g\left(\left(6d^2f^2-8cdgf+3c^2g^2\right)b^2-2adg(2df-cg)b+a^2d^2g^2\right)\log(c+dx)(bc-ad)^2}{2b^4d^4}$$
$$-\frac{Bg^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)}{6bd^4}$$
$$-\frac{Bg^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)}{4b^2d^4}$$
$$-\frac{Bg\left(\left(6d^2f^2-8cdgf+3c^2g^2\right)b^2-2adg(2df-cg)b+a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)}{2b^4d^3}$$
$$-\frac{B(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{2b^4d^4}$$
$$-\frac{B^2(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-a}{2b^4d^4}$$
$$-\frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^3*g^3*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - 3
*b*c*g - a*d*g)*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(12*b^
2*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(6*b^4*d^4) + (B^
2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c + d*x)])/(
4*b^4*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*
(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]))/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c +
 d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b^2*d^4) - (B*(b*c - a*d)*
g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*b*d^4) - (B*(b*c -
 a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f
^2 - 2*c*d*f*g + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(2*b^4*d^4) - ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x))/
```

```
(c + d*x)])^2)/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2)/(4*g) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(6*b^4*d^4) + (B^2*(b*c -
 a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/(4*b^4*d^4) + (B^2*(b
*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c
*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(2*b^4*d^4) - (B^2*(b*c - a*d)*(2*b*d*f
- b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g
+ c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b^4*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^3(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{B\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^4(A+B\log(ex))}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$

$$- \frac{B\text{Subst}\left(\int \left(\frac{(bf-ag)^4(A+B\log(ex))}{b^4x} + \frac{(bc-ad)^4g^4(A+B\log(ex))}{bd^3(b-dx)^4} + \frac{(bc-ad)^3g^3(4bdf-3bcg-adg)(A+B\log(ex))}{b^2d^3(b-dx)^3} + \frac{(bc$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd^3} - \frac{(B(bf-ag)^4)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^4g} - \frac{(B(bc-ad)^3g^2(4bdf-3bcg-adg))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2d^3} + \frac{(B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)))\,\text{Subst}\left(\int \frac{A+B\log}{b-dx}\right.}{2b^4d^3} - \frac{(B(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2)))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, \right.}{2b^3d^3}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{2b^4d^3} - \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4} - \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A\right.}{2b^4d^4} - \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} + \frac{(B^2(bc-ad)^4g^3)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd^4} + \frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg))\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{4b^2d^4} + \frac{(B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)))\,\text{Subst}\left(\int \frac{\log\left(1-\right.}{x}\right.}{2b^4d^4} + \frac{(B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2)))\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+b}{c+d}\right.}{2b^4d^3}$$

$$=$$

$$-\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a\cdots}{c+\cdots}\right.\right.}{2b^4d^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4}$$

$$-\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A\cdots\right.}{2b^4d^4}$$

$$-\frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$

$$+\frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{2b^4d^4}$$

$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}$$

$$+\frac{(B^2(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{6bd^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg))\,\mathrm{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{4b^2d^4}$$

$$= \frac{B^2(bc-ad)^3g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{4b^3d^3} + \frac{B^2(bc-ad)^2g^3(c+dx)^2}{12b^2d^4}$$
$$+ \frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$
$$- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{2b^4d^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4}$$
$$- \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A-\right.}{2b^4d^4}$$
$$- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$
$$+ \frac{B^2(bc-ad)^4g^3\log(c+dx)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{4b^4d^4}$$
$$+ \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{2b^4d^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 733, normalized size of antiderivative = 0.84

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2\left(a^2d^2\right.\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(6*A*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g)
+ b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c + d
*x)] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*
```

```
g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x] - 6*b^4*(d*f - c*g
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*g^4*(
b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log[a + b*x] - 2*b^3*
c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d*g)*(-(a^2*d
^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*B*d^4*(
b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]
) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d*f - c*g)^4*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g)
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x +
c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b*
e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2140 vs. 2(843) = 1686.

Time = 0.32 (sec) , antiderivative size = 2140, normalized size of antiderivative = 2.45

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^3 + 3*(x^
2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x
 + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f^2*g + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
 - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^2 + 1/12*(6*x
^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x - 1/12*(6*a^
3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d^3*f^2*g - 6
*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (24*c*d^3*f^3*log(e) - (6*g^3*log(e) +
11*g^3)*c^4 + 12*(2*f*g^2*log(e) + 3*f*g^2)*c^3*d - 36*(f^2*g*log(e) + f^2*
g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4*f^3 - 6*a^2*
b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^3 - 6*c^2*d^2*
f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*B^
2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(a*b^3*d^4*g^3*log(e) + (6*d^4*f*g^2*log(e)^
2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - ((3*g^3*log(e) - g^3)*a^2*b^2*d^4 - 2*
(6*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (18*d^4*f^2*g*log(e)^2 - 12*c*d^3*
f*g^2*log(e) + (3*g^3*log(e) + g^3)*c^2*d^2)*b^4)*B^2*x^2 + ((6*g^3*log(e)
- 5*g^3)*a^3*b*d^4 + (5*c*d^3*g^3 - 12*(2*f*g^2*log(e) - f*g^2)*d^4)*a^2*b^
2 + (36*d^4*f^2*g*log(e) - 24*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*a*b^3 + (12*d^4*
f^3*log(e)^2 - 36*c*d^3*f^2*g*log(e) - (6*g^3*log(e) + 5*g^3)*c^3*d + 12*(2
*f*g^2*log(e) + f*g^2)*c^2*d^2)*b^4)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2
*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b
^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*lo
g(b*x + a)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4
*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c
^3*d*f*g^2 - c^4*g^3)*B^2*b^4)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(
e) + 2*(a*b^3*d^4*g^3 + (12*d^4*f*g^2*log(e) - c*d^3*g^3)*b^4)*B^2*x^3 + 3*
(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (12*d^4*f^2*g*log(e) - 4*c*d^3*f*g^2
 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b^2*d^4*f*g^2 +
 a^3*b*d^4*g^3 + (4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*
d*g^3)*b^4)*B^2*x - ((6*g^3*log(e) - 11*g^3)*a^4*d^4 + 2*(c*d^3*g^3 - 6*(2*
f*g^2*log(e) - 3*f*g^2)*d^4)*a^3*b - 3*(4*c*d^3*f*g^2 - c^2*d^2*g^3 - 12*(f
```

```
^2*g*log(e) - f^2*g)*d^4)*a^2*b^2 - 6*(4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4
*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*log(b*x + a) - (6*B^2*b^4*d^4*g^3*x
^4*log(e) + 2*(a*b^3*d^4*g^3 + (12*d^4*f*g^2*log(e) - c*d^3*g^3)*b^4)*B^2*x
^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (12*d^4*f^2*g*log(e) - 4*c*d^
3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b^2*d^4*
f*g^2 + a^3*b*d^4*g^3 + (4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2
 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3
 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^3 - 6*a^2
*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a))*log(d*
x + c))/(b^4*d^4)
```

Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (f+g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.241 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 29, antiderivative size = 532

$$\begin{aligned}
\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx &= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2}+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}\\
&-\frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2}\\
&-\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3}\\
&+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^3}\\
&-\frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g}\\
&+\frac{B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3}+\frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}\\
&+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x
+c))/b^3/d^3-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^3/d^2-1/3*B*(-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2
*g^2-3*c*d*f*g+3*d^2*f^2))*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^3/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/3*(g*x+
f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+1/3*B^2*(-a*d+b*c)^3*g^2*ln(d*x+c)/b^3
/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln(d*x+c)/b^3/d^3+2/3*
```

```
B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d
^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$
$$=\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$
$$-\frac{2Bg(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d^2}$$
$$-\frac{(bf-ag)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b^3g}-\frac{Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd^3}$$
$$+\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g}+\frac{2B^2g(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}$$
$$+\frac{B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2g^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2g^2x(bc-ad)^2}{3b^2d^2}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x
)/(c + d*x)])/(3*b^3*d^3) - (2*B*(b*c - a*d)*g*(3*b*d*f - 2*b*c*g - a*d*g)*
(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b^3*d^2) - (B*(b*c - a*d
)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^3) + (2*B*(b
*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g
 + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(3*b^3*d^3) - ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*g) + (
B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*g*(3
*b*d*f - 2*b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)*(a
^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^2(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^3(A+B\log(ex))}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B)\text{Subst}\left(\int \left(\frac{(bf-ag)^3(A+B\log(ex))}{b^3x} + \frac{(bc-ad)^3g^3(A+B\log(ex))}{bd^2(b-dx)^3} + \frac{(bc-ad)^2g^2(3bdf-2bcg-adg)(A+B\log(ex))}{b^2d^2(b-dx)^2} + \right.\right.}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2} - \frac{(2B(bf-ag)^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^3g} - \frac{(2B(bc-ad)^2g(3bdf-2bcg-adg))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2d^2} - \frac{(2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x,\right.}{3b^3d^2}$$

$$= -\frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2} - \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{3b^3d^3} - \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} + \frac{(B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3} + \frac{(2B^2(bc-ad)^2g(3bdf-2bcg-adg))\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3d^2} - \frac{(2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a}{c}\right.}{3b^3d^3}$$

$$= -\frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2} - \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{3b^3d^3} - \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} + \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3} + \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3} + \frac{(B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}$$
$$- \frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2}$$
$$- \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e}{}\right.\right.}{3b^3d^3}$$
$$- \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g}$$
$$+ \frac{B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} + \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.91

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b*c*g
 - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^2*(b*c - a*d)*g^3*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*B*(b*c - a*d)^2*g^2*(-3*b*d*f + b
*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])*Log[c + d*x] - B*(b*c - a*d)*g^3*(a^2*d^2*Log[a + b*x] - b*(d*(-
(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^3*d^3))/(3*g)
```

Maple [F]

$$\int (gx + f)^2 \left(A + B \ln \left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

[Out] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

Fricas [F]

$$\int (f + gx)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \int (gx + f)^2 \left(B \log \left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x + B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b*e*x + a*e)/(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int (f + gx)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \text{Timed out}$$

[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(511) = 1022$.

Time = 0.30 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.44

$$\int (f + gx)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \text{Too large to display}$$

[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")

[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f*g + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^

```
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^
2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2*log(e) + (2*g^2*log(e) + 3
*g^2)*c^3 - 6*(f*g*log(e) + f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2
/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2 - 3*c^2*
d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*l
og(e)^2 + (a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - c*d^2*g^2*log(e))*b
^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*a^2*b*d^3 - 2*(3*d^3*f*g*log(e) - c*d^2
*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 6*c*d^2*f*g*log(e) + (2*g^2*log(e) + g^
2)*c^2*d)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2
*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log
(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*
f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3)*log(d*x + c)^2 + (2*
B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g*log(e) - c*d^2*g^2
)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 3
*c*d^2*f*g + c^2*d*g^2)*b^3)*B^2*x + ((2*g^2*log(e) - 3*g^2)*a^3*d^3 + (c*d
^2*g^2 - 6*(f*g*log(e) - f*g)*d^3)*a^2*b + 2*(3*d^3*f^2*log(e) - 3*c*d^2*f*
g + c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*log(e) + (
a*b^2*d^3*g^2 + (6*d^3*f*g*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^
3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 3*c*d^2*f*g + c^2*d*g^2)*b^3)*B
^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x
 + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a))*log
(d*x + c))/(b^3*d^3)
```

Giac [F]

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 27, antiderivative size = 270

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx\\
&=-\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d}\\
&\quad+\frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d^2}\\
&\quad-\frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g}\\
&\quad+\frac{B^2(bc-ad)^2g\log(c+dx)}{b^2d^2}+\frac{B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d+B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/d^2-1/2*(-a*g+b*f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/2*(g*x+f)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/g+B^2*(-a*d+b*c)^2*g*ln(d*x+c)/b^2/d^2+B^2*(-a*d
+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d^2}$$
$$-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g}$$
$$-\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g}$$
$$+\frac{B^2(bc-ad)(-adg-bcg+2bdf)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{B^2g(bc-ad)^2\log(c+dx)}{b^2d^2}$$

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d))
 + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[(b*c - a*d)/(b*(c + d*x))]*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d^2) - ((b*f - a*g)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(2*g) + (B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) +
(B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{B\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^2(A+B\log(ex))}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{B\text{Subst}\left(\int\left(\frac{(bf-ag)^2(A+B\log(ex))}{b^2x}+\frac{(bc-ad)^2g^2(A+B\log(ex))}{bd(b-dx)^2}+\frac{(bc-ad)g(2bdf-bcg-adg)(A+B\log(ex))}{b^2d(b-dx)}\right)dx,x,}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad)^2g)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd} - \frac{(B(bf-ag)^2)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(B(bc-ad)(2bdf-bcg-adg))\text{Subst}\left(\int\frac{A+B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d}$$

$$= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d} + \frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d^2} - \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} + \frac{(B^2(bc-ad)^2g)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d} - \frac{(B^2(bc-ad)(2bdf-bcg-adg))\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^2}$$

$$= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d} + \frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d^2} - \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} + \frac{B^2(bc-ad)^2g\log(c+dx)}{b^2d^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+B\log\left(\right.\right.\right.}{}}{}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 2
*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*
(b*c - a*d)^2*g^2*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*d
^2))/(2*g)
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c))^2
 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(265) = 530$.

Time = 0.27 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.49

$$\begin{aligned}&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx\\ &= \frac{1}{2}A^2gx^2 + 2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABf\\ &\quad+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABg\\ &\quad+A^2fx-\frac{(acdg+(2\,cdf\log(e)-(g\log(e)+g)c^2)b)B^2\log(dx+c)}{bd^2}\\ &\quad+\frac{(2\,abd^2f-a^2d^2g-(2\,cdf-c^2g)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}\\ &\quad+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(abd^2g\log(e)+\left(d^2f\log(e)^2-cdg\log(e)\right)b^2\right)B^2x+\left(B^2b^2d^2gx^2+2\,B^2b^2d^2fx\right.}{}\end{aligned}$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/
b - c*log(d*x + c)/d)*A*B*f + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a
^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g + A
^2*f*x - (a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + g)*c^2)*b)*B^2*log(d*x +
c)/(b*d^2) + (2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*x + a
)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B
^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(a*b*d^2*g*log(e) + (d^2
*f*log(e)^2 - c*d*g*log(e))*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2
*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 +
 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*log(d*x + c)^2 + 2*(B^2*b^2
*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B^2*x - ((g*
log(e) - g)*a^2*d^2 - (2*d^2*f*log(e) - c*d*g)*a*b)*B^2)*log(b*x + a) - 2*(
B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B^2*x
 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*
log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}+\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b}+\frac{2B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] 2*B*(-a*d+b*c)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+(b*
x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+2*B^2*(-a*d+b*c)*polylog(2,d*(b*x+a)/b
/(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2536, 2544, 2458, 2378, 2370, 2352}

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}+\frac{2B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (2*B*(b*c - a*d)*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/b + (
2*B^2*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} - \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}\,dx}{b}$$

$$= \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} - \frac{(2B^2(bc-ad)^2)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{bd}$$

$$= \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} - \frac{(2B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx, x, c+dx\right)}{bd^2}$$

$$= \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} + \frac{(2B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} + \frac{(2B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} + \frac{2B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.71

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2$$
$$+\frac{B\left(2ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2bc\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-aBd\left(\log(a+bx)\right.\right.}{}$$

[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]

[Out] x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*a*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)

Maple [F]

$$\int \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")

[Out] integral(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2, x)

Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] 2*(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)
/e)*A*B + A^2*x + B^2*((b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2
 - 2*(b*d*x*log(e) + (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d) + inte
grate(((log(e)^2 + 2*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*log(e)^2 +
 (log(e)^2 + 2*log(e))*a*b*d)*x + 2*(b^2*d*x^2*log(e) + a*b*c*log(e) + a^2*
d + (a*b*d*(log(e) + 2) + b^2*c*(log(e) - 1))*x)*log(b*x + a))/(b^2*d*x^2 +
 a*b*c + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 277

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{2B^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{2B^2\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+(A+B*ln(e*(b*x+a)
/(d*x+c)))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*(A+B*ln(e*(b
*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*(A+B*ln(e*(b*x+a)/(d*x
+c)))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+2*B^2*polylog(3,d*
(b*x+a)/b/(d*x+c))/g-2*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/g
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2554, 2404, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \frac{2B\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g} + \frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g} - \frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g} - \frac{2B^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/g
) + ((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - ((d*f - c*g)*(a + b*x))
/((b*f - a*g)*(c + d*x))])/g - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (2*B*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g +
(2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (2*B^2*PolyLog[3, ((d*f
 - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= (bc-ad)\text{Subst}\left(\int \left(\frac{d(A+B\log(ex))^2}{(bc-ad)g(b-dx)} + \frac{(-df+cg)(A+B\log(ex))^2}{(bc-ad)g(bf-ag-(df-cg)x)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}\\
&\quad + \frac{((-bc+ad)(df-cg))\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g}
\end{aligned}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{(2B)\text{Subst}\left(\int\frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g} - \frac{(2B)\text{Subst}\left(\int\frac{(A+B\log(ex))\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{(2B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g} - \frac{(2B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{2B^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{2B^2\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1348 vs. $2(277) = 554$.

Time = 0.37 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$$
$$= \frac{-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)+A^2\log(f+gx)-2AB\log\left(\frac{a}{b}+x\right)\log(f+gx)+B^2\log^2\left(\frac{a}{b}+x\right)}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] (-(B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f
- c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x]
+ B^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f + g*x] - 2*B^2
*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*Log[c/d + x]^2*Log[f + g*x] +
 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] - 2*B^2*Log[a/b + x]*Log[(
e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*B^2*Log[c/d + x]*Log[(e*(a + b*x))
/(c + d*x)]*Log[f + g*x] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[f + g*x]
+ 2*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[a/b + x]^2*Lo
g[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(e*(a + b*x))/(c + d*
x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(g*(c + d*x))/(
-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[(g*(c + d*x))/(-(d*
f) + c*g)]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[(g*(c + d*x))/(-(d*
f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f +
 g*x))/(b*f - a*g)] - B^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x
))]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Log[(d*(f + g*x))
/(d*f - c*g)] + 2*B^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*
g)] - B^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*Log[c/d + x
]*Log[(e*(a + b*x))/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*Log[a
/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] +
B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*
B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c
*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*f - c*g)] + B^2*Log[((b*f - a*g)*(c +
d*x))/((d*f - c*g)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(f + g*x))/((d*f - c*g
)*(a + b*x))] + 2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[((b*f - a*g
)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a
*g)] - 2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[((b*f - a*g)*(c + d*
x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*
B^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, (b*(c +
 d*x))/(d*(a + b*x))] + 2*B^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a +
 b*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))] + 2*B^2
*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*B^2*PolyLog[3, ((b*f - a*g)*(c
 + d*x))/((d*f - c*g)*(a + b*x))])/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(277) = 554$.

Time = 4.26 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.95

method	result
parts	$\frac{A^2\ln(gx+f)}{g} + B^2(ad-cb)\,e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d\left(\frac{be}{d}\cdots\right.}{\cdots}\right.}{eg(ad-cb)}\cdots\right.$
derivativedivides	$-\frac{e(ad-cb)\left(-d^2A^2\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B^2\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-d^2A^2\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B^2\cdots\right.}{\cdots}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] A^2*ln(g*x+f)/g+B^2*(a*d-b*c)*e*(-1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*polylog(2,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,1
/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))))+1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))
+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e
/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*
```

```
d-b*c)*e/d/(d*x+c)))))-2*B*A/d^2*(a*d-b*c)*e*(-d^2*(c*g-d*f)/e/g/(a*d-b*c)*
(dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f)
)/(c*g-d*f)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f))+d^3/e/g/(a*d-b*c)*(dil
og(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g - integrate(-(B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*
log(e) + 2*(B^2*log(e) + A*B)*log(b*x + a) - 2*(B^2*log(b*x + a) + B^2*log(
e) + A*B)*log(d*x + c))/(g*x + f), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{f+g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x), x)
```

3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+
b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

$= \{2554, 2355, 2354, 2438\}$

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \frac{2B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(bf-ag)(df-cg)} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)(bf-ag)} + \frac{2B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*f - a*g)*(f + g*x))
+ (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g
)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(
b*c - a*d)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b
*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)} - \frac{(2B(bc-ad))\text{Subst}\left(\int \frac{A+B\log(ex)}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bf-ag}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} - \frac{(2B^2(bc-ad))\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 402 vs. 2(196) = 392.

Time = 0.29 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$$

$$= \frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx} + \frac{B\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2(bc-ad)g\left(A+B\log\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)*
Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Log[a
```

```
+ b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b
*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f)
 + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2,
 (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^2}dx$$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 2*A*B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b
*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d
*x + c) + a*e/(d*x + c))/(g^2*x + f*g)) - B^2*(log(d*x + c)^2/(g^2*x + f*g)
 + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*log(b*x + a)^2
 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((g*log(e) - g)*d*x + c*g
*log(e) - d*f + (d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^3*x^3 + c*f^
2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - A^2/(g^2*x
+ f*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2, x)
```

3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 369

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx &= \frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2}\\ &\quad-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2}+\frac{B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}\\ &\quad+\frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\\ &\quad+\frac{B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\end{aligned}$$

```
[Out] B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)/
(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^2-1/2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*ln((g*x+f)/(d*x+c))/(-a*
g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a)
/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d
*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,(-c*g+d*f)*(b*x+a)/(-
a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$$
$$=\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}$$
$$+\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(bf-ag)^2(df-cg)^2}$$
$$-\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(f+gx)^2}+\frac{B^2(bc-ad)(-adg-bcg+2bdf)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$
$$+\frac{B^2g(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*
g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*g*(f + g*x)
^2) + (B^2*(b*c - a*d)^2*g*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^2*(d*f -
c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*
f - a*g)^2*(d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Poly
Log[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*
f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{(bf-ag-(df-cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B\text{Subst}\left(\int\left(\frac{b^2(A+B\log(ex))}{(bf-ag)^2x}+\frac{(bc-ad)^2g^2(A+B\log(ex))}{(bf-ag)(df-cg)(bf-ag-(df-cg)x)^2}+\frac{(bc-ad)g(-2bdf+bcg+adg)(A+B\log(ex))}{(bf-ag)^2(df-cg)(bf-ag-(df-cg)x)}\right)dx\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(b^2B)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g(bf-ag)^2} + \frac{(B(bc-ad)^2g)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)} - \frac{(B(bc-ad)(2bdf-bcg-adg))\text{Subst}\left(\int\frac{A+B\log(ex)}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)}$$

$$= \frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} - \frac{(B^2(bc-ad)^2g)\,\text{Subst}\left(\int\frac{1}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)} - \frac{(B^2(bc-ad)(2bdf-bcg-adg))\,\text{Subst}\left(\int\frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

$$= \frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\left(\frac{e}{\cdots}\right.\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*d)*
g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^2*(d*f
 - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d
^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*
d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c -
 a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x))/(-(b*f) +
a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g
)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^3}dx$$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="fricas")
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="maxima")
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))/(g^3*
x^2 + 2*f*g^2*x + f^2*g))*A*B - 1/2*B^2*(log(d*x + c)^2/(g^3*x^2 + 2*f*g^2*
x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*lo
g(b*x + a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - ((2*g*log(e) -
g)*d*x + 2*c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(
d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2
 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^3}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3, x)
```

3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 714

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = \frac{B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}$$
$$+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}$$
$$-\frac{B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df}{(bf}\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)+1/3*B^2*
(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B*(-a*d+
b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^3/(g*x
+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^3-1/3*B^
```

```
2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-
a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*
g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*
c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,(-c*g+d*f
)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$$
$$= \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bf-ag)^3}$$
$$-\frac{Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(f+gx)^3}$$
$$+\frac{2Bg(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}$$
$$+\frac{B^2g^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{B^2g^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2g(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x))
 + (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f -
 c*g)^3) - (B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*
f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*
```

```
x)])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*g*(
f + g*x)^3) - (B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*
g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - 2*a*d*g)*Lo
g[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b
*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)
*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f -
a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{(bf-ag-(df-cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B)\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))}{x(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B)\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex))}{(bf-ag)^3x} + \frac{(-bc+ad)^3g^3(A+B\log(ex))}{(bf-ag)(df-cg)^2(bf-ag-(df-cg)x)^3} + \frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(A+B\log(ex))}{(bf-ag)^2(df-cg)^2(bf-ag-(df-cg)x)^2}\right.\right.}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g(bf-ag)^3}$$
$$- \frac{(2B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^2}$$
$$+ \frac{(2B(bc-ad)^2g(3bdf-bcg-2adg))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)^2(df-cg)^2}$$
$$- \frac{(2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{A+B\log(ex)}{bf-ag+(-df+cg)x}\,d\right.}{3(bf-ag)^3(df-cg)^2}$$

$$= -\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+ \frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+ \frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{(B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^3}$$
$$- \frac{(2B^2(bc-ad)^2g(3bdf-bcg-2adg))\,\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^2}$$
$$- \frac{(2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\right.}{3(bf-ag)^3(df-cg)^3}$$

$$= -\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\mathrm{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{(B^2(bc-ad)^3g^2)\,\mathrm{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)}\right)dx,x,\frac{a+}{c+}\right.}{3(bf-ag)(df-cg)^3}$$

$$=\frac{B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}$$
$$-\frac{B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\mathrm{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bdf+bcg}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*((b*c - a*d)*g*
(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c -
 a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*(f + g*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*(b*c - a*d)*g*(a^2*
d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(
f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a
*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*Log[a + b*x] + (
-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + B*(b*c - a*d
)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f
 + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*
d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + b^3*B*(d*f - c*g)
^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d
)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b*f - a*g)^3*(f
+ g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^
2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*
x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)
])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f
+ g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^3))/(g*(f + g*x)^3)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^4}\,dx$$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
```

```
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(log(d
*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*
(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log(b*x + a)^2 + 6*(d*
g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((3*g*log(e) - g)*d*x + 3*c*g*log
(e) - d*f + 3*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^5*x^5 + c*f^4*
g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2*d*f^3*
g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 1/3*A^2/(g^4*x^3
 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^4}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4, x)
```

3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1159

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$-\frac{B^2(bc-ad)^3g^3(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$-\frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$
$$+\frac{B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,\mathrm{l}}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx}{(bf-ag)(c+dx}\right.}{2(bf-ag)^4(df-cg)^4}$$

```
[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-1/
6*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/4*B^2*(-
a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(
g*x+f)-1/6*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)
^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/(-
a*g+b*f)^4/(-c*g+d*f)^4+1/6*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*
c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)^
4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c
^2*g^2-4*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)
^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^
```

```
4-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*ln
((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*
d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-
a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6
*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(
-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d
^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(
2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*
g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 1159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules

used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^4}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$
$$+ \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4} - \frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B(bc-ad)g\left((6d^2f^2-4cdgf+c^2g^2)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$- \frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+ \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$- \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^2g\left((6d^2f^2-4cdgf+c^2g^2)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{2(bf-ag)^4(df-cg)^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\mathrm{PolyLog}\left(2,\frac{(df-cg)}{(bf-ag}\right.}{2(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$- \frac{B^2(bc-ad)^3g^3(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)} - \frac{B^2(bc-ad)^2g^3(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*(f +
 g*x)^2) - (B^2*(b*c - a*d)^3*g^3*(c + d*x))/(6*(b*f - a*g)^3*(d*f - c*g)^4
*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x))
/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) - (B^2*(b*c - a*d)^4*g^3*Log[(a
+ b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2
*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*
f - c*g)^4) + (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(6*(b*f - a*g)*(d*f - c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b
*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
4*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^
```

```
2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b
*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f
 + g*x)) + (b^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*g*(b*f - a*g)^4)
 - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*g*(f + g*x)^4) + (B^2*(b*c - a
*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*
(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g*x)/(c + d*x)])/(4*
(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*
d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[(f + g*x)/(c
 + d*x)])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g
^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 -
2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))^2}{(bf-ag-(df-cg)x)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{B\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex))}{x(bf-ag+(-df+cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{B\text{Subst}\left(\int\left(\frac{b^4(A+B\log(ex))}{(bf-ag)^4x}+\frac{(bc-ad)^4g^4(A+B\log(ex))}{(bf-ag)(df-cg)^3(bf-ag-(df-cg)x)^4}+\frac{(bc-ad)^3g^3(-4bdf+bcg+3adg)(A+B\log(ex))}{(bf-ag)^2(df-cg)^3(bf-ag-(df-cg)x)^3}+\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(b^4B)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{2g(bf-ag)^4}$$
$$+\frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bf-ag)(df-cg)^3}$$
$$-\frac{(B(bc-ad)^3g^2(4bdf-bcg-3adg))\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bf-ag)^2(df-cg)^3}$$
$$+\frac{(B(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)))\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(bf-ag+(-df+cg)\right.}{2(bf-ag)^3(df-cg)^3}$$
$$+\frac{(B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)))\,\text{Subst}\left(\int\frac{A+B}{bf-ag+}\right.}{2(bf-ag)^4(df-cg)^3}$$

$$= \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$

$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$

$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a}{c+}\right.\right.}{2(bf-ag)^4(df-cg)^4}$$

$$-\frac{(B^2(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{6(bf-ag)(df-cg)^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg))\,\text{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{4(bf-ag)^2(df-cg)^4}$$

$$-\frac{(B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)))\,\text{Subst}\left(\int\frac{1}{bf-ag+(-df+c\right.}{2(bf-ag)^4(df-cg)^3}$$

$$+\frac{(B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right))\,\text{Subst}\left(\int\frac{\log(}{}\right.}{2(bf-ag)^4(df-cg)^4}$$

$$=\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$
$$+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+}{c+d}\right.\right.}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right.}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{(B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\left(\frac{1}{(bf-ag)^3x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^3}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)^2}+\frac{}{(bf-ag)}\right.\right.}{6(bf-ag)(df-cg)^4}$$
$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg))\operatorname{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{df-}{(bf-ag)^2(bf-a}\right.\right.}{4(bf-ag)^2(df-cg)^4}$$

$$= -\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2} - \frac{B^2(bc-ad)^3g^3(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+ \frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)} - \frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+ \frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a\cdots}{c+\cdots}\right.\right.}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+ \frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$
$$+ \frac{B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} - \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a\cdots}{c\cdots}\right.\right.}{2(bf-ag)^4(df-cg)^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\mathrm{Li}_2\left(\frac{(df-cg)(a+bx\cdots}{(bf-ag)(c+dx\cdots}\right.}{2(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 1301, normalized size of antiderivative = 1.12

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx =$$
$$-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2bdf+\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*
d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 3*(
b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
```

```
x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)*g*(b*f - a*g)*(d*f
- c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*b^4*(d*f -
c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*d^
4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*
x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g
^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(
-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^3*(b*(d*f
- c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f
 + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*((b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] +
 d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g
 + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*
g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*
(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[
a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d*x] - 2*(b*c - a*d)*g*(
a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*(f + g*x)^2*Log[f + g*x]) + 3*b^4*B*(d*f - c*g)^4*(f + g*x)^3*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) - 3*B*d^4*(b*f - a*g)^4*(f + g*x)^3*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)]) - 6*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b
*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3
*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*L
og[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*
x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f - c*g)^4))/(g*(f + g*x)^4)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^5}dx$$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*
f^4*g*x + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
```

```
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - 1/4*B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f
^3*g^2*x + f^4*g) + 4*integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2
*(d*g*x + c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a)
- ((4*g*log(e) - g)*d*x + 4*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a)
)*log(d*x + c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2
*g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f
^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*
x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{{\left(f+g\,x\right)}^5}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5, x)
```

3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log\left(-\frac{x}{1-x}\right) - \frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x}$$

```
[Out] 2*ln(-x/(1-x))-(1+x)*ln((-1-x)/(1-x))/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2553, 2351, 31}

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log\left(-\frac{x}{1-x}\right) - \frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{x}$$

```
[In] Int[Log[(1 + x)/(-1 + x)]/x^2,x]
```

```
[Out] 2*Log[-(x/(1 - x))] - ((1 + x)*Log[-((1 + x)/(1 - x))])/x
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{\log(x)}{(-1-x)^2}\,dx, x, \frac{1+x}{-1+x}\right)\right)\\ &= -\frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x} - 2\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \frac{1+x}{-1+x}\right)\\ &= 2\log\left(-\frac{x}{1-x}\right) - \frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log(x) - \frac{\log\left(\frac{1+x}{-1+x}\right)}{x} - \log\left(1-x^2\right)$$

```
[In] Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]
[Out] 2*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\ln\left(\frac{x+1}{-1+x}\right)}{x}+2\ln\left(x\right)-\ln\left(x^{2}-1\right)$	29
parts	$-\frac{\ln\left(\frac{x+1}{-1+x}\right)}{x}-\ln\left(-1+x\right)+2\ln\left(x\right)-\ln\left(x+1\right)$	33
parallelrisch	$\frac{2\ln(x)x-2\ln(-1+x)x-x\ln\left(\frac{x+1}{-1+x}\right)-\ln\left(\frac{x+1}{-1+x}\right)}{x}$	43
derivativedivides	$2\ln\left(2+\frac{2}{-1+x}\right)-\frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46
default	$2\ln\left(2+\frac{2}{-1+x}\right)-\frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46

```
[In] int(ln((x+1)/(-1+x))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*ln((x+1)/(-1+x))+2*ln(x)-ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int\frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx=-\frac{x\log\left(x^2-1\right)-2\,x\log\left(x\right)+\log\left(\frac{x+1}{x-1}\right)}{x}$$

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="fricas")
[Out] -(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int\frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx=2\log\left(x\right)-\log\left(x^2-1\right)-\frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

```
[In] integrate(ln((1+x)/(-1+x))/x**2,x)
[Out] 2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = -\frac{\log\left(\frac{x+1}{x-1}\right)}{x} - \log\left(x+1\right) - \log\left(x-1\right) + 2\,\log\left(x\right)$$

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="maxima")
[Out] -log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(29) = 58.

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.94

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = \frac{2\,\log\left(\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}+1} - 2\,\log\left(\frac{|x+1|}{|x-1|}\right) + 2\,\log\left(\left|\frac{x+1}{x-1}+1\right|\right)$$

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="giac")
[Out] 2*log((((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/((((x + 1)/(x - 1) +
 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) + 1) - 2*log(abs(x + 1)/ab
s(x - 1)) + 2*log(abs((x + 1)/(x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\,\ln\left(x\right) - \ln\left(x^2-1\right) - \frac{\ln\left(\frac{x+1}{x-1}\right)}{x}$$

```
[In] int(log((x + 1)/(x - 1))/x^2,x)
[Out] 2*log(x) - log(x^2 - 1) - log((x + 1)/(x - 1))/x
```

3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 8.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(f+gx)^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 13.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)} \,d x$$

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

```
[In] int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A + B*log(e*(a + b*x)/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 17.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 166.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)^2}\,dx$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 27.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 21.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 38.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 7.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)} \,d x$$

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*
A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 10.97

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A
*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((4*b*d*g^2*x^3 + b*c*f^2 +
3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c -
a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x
)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral((g*x + f)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x +
 a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.02 (sec) , antiderivative size = 337, normalized size of antiderivative = 12.48

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$=\frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$
$$-\frac{\int\frac{acg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{adf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{bcf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2adgx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+}{B\,(ad-bc)}$$

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(
c + d*x))) - (Integral(a*c*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(a*d*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b*c*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*a*
d*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*c*g*x
/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*d*f*x/(A +
 B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(3*b*d*g*x**2/(A + B
*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 8.33

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b*d*g*x^2 + b*c*f +
 (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a)
 - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e
))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral(1/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d
*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.52

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{ac+adx+bcx+bdx^2}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{ad}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2bdx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B\,(ad-bc)}$$

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)
*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d/(A + B*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x) + Integral(b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))), x) + Integral(2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
)/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.14

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d
)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + int
egrate((2*b*d*x + b*c + a*d)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^
2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c)
)^2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 452, normalized size of antiderivative = 15.59

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) - a*d
*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)*
x + ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - ((b*c*g -
a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate((b*d*g*x^2 +
2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(
e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d
*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d*
f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B
^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2
 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 5.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x
+ A*B*f^2)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 688, normalized size of antiderivative = 23.72

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(e)
 - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d*g
^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d*f*
g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2
*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2 +
 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c)) - int
egrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3
- a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*
d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*
g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g -
 a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3
- a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d
*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3
)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B
^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 22.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 48.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.48

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(
(b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 921, normalized size of antiderivative = 31.76

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e
) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) -
a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) -
a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*lo
g(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g
^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*
f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*
f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)
*log(d*x + c)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g
- (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e) - a*d*g^4*l
og(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*log(e) - a*d*
f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log(e) - a*d*f^
4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2*log(e) -
a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g*
log(e) - a*d*f^3*g*log(e))*B^2)*x + ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f
*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*
f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*x + a) - ((b*c*g^
4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 -
 a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^
4)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \, dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2} \, dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 31.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \, dx = \int \frac{1}{(f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2} \, dx$$

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.262 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 357

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{5b^3d^3}$$
$$-\frac{2B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{10bd}-\frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}+\frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

```
[Out] 2/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/5*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2*
(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-2/15*B*(-a*d+b*c)*g^3*(-a*d*g-b
*c*g+5*b*d*f)*x^3/b^2/d^2-1/10*B*(-a*d+b*c)*g^4*x^4/b/d-2/5*B*(-a*g+b*f)^5*
ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+2/5*B*(-c*g
+d*f)^5*ln(d*x+c)/d^5/g
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2548, 84}

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= -\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{5b^3d^3}$$
$$+\frac{2Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-\right.}{5b^4d^4}$$
$$+\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-\frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$-\frac{2Bg^3x^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4x^4(bc-ad)}{10bd}+\frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(
10*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^
2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*
b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(5*b^3*d
^3) - (2*B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B
*(b*c - a*d)*g^4*x^4)/(10*b*d) - (2*B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g)
 + ((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) + (2*B*(d*f
 - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g} - \frac{(2B(bc-ad))\int\frac{(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$= \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g} - \frac{(2B(bc-ad))\int\left(\frac{g^2(-a^3d^3g^3+a^2bd^2g^2(5df-cg)-ab^2dg(10d^2f^2-5cdfg+c^2g^2)+b^3(10d^3f^3-10cd^2f^2g+5c^2dfg^2-c^3g^3))}{b^4d^4}+\right.}{}$$

$$= \frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2}{5b^4d^4}$$
$$- \frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{5b^3d^3}$$
$$- \frac{2B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}$$
$$- \frac{B(bc-ad)g^4x^4}{10bd} - \frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+ \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g} + \frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.79

$$\int(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{B(-bc+ad)g^2x(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2))+b^3(-12c^3g^3+6c^2dg^2(10f+g}{6b^4d^4}$$

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(6*b^4*d^4) - (2*B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (
f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(d*f - c*g)^5*Lo
g[c + d*x])/d^5)/(5*g)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.68

method	result
risch	$\frac{4g^2Bc^2f^2x}{d^2}+\frac{g^4Bax^4}{10b}-\frac{g^4Bcx^4}{10d}+2g^2Af^2x^3+\frac{2g^4Bc^2x^3}{15d^2}+2gAf^3x^2+\frac{g^4Ba^3x^2}{5b^3}-\frac{g^4Bc^3x^2}{5d^3}+$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^2*g^2*B*c^2*f^2*x+1/10/b*g^4*B*a*x^4-1/10/d*g^4*B*c*x^4+2*g^2*A*f^2*x^3
+2/15/d^2*g^4*B*c^2*x^3+2*g*A*f^3*x^2+1/5/b^3*g^4*B*a^3*x^2-1/5/d^3*g^4*B*c
^3*x^2+2/d^4*g^3*B*ln(d*x+c)*c^4*f-4/d^3*g^2*B*ln(d*x+c)*c^3*f^2+4/d^2*g*B*
ln(d*x+c)*c^2*f^3-2/b^4*g^3*B*ln(-b*x-a)*a^4*f+1/5*g^4*A*x^5-2/5/d^5*g^4*B*
ln(d*x+c)*c^5+1/5*(g*x+f)^5*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)-2/5/g*B*ln(-b*x-a
)*f^5+1/d^2*g^3*B*c^2*f*x^2-2/15/b^2*g^4*B*a^2*x^3+A*f^4*x+g^3*A*f*x^4+2/5/
g*B*ln(d*x+c)*f^5+4/b^3*g^2*B*ln(-b*x-a)*a^3*f^2+2/b^3*g^3*B*a^3*f*x-4/b^2*
g^2*B*a^2*f^2*x+4/b*g*B*a*f^3*x-2/d^3*g^3*B*c^3*f*x-4/d*g*B*c*f^3*x-2/5/b^4
*g^4*B*a^4*x+2/5/d^4*g^4*B*c^4*x-4/b^2*g*B*ln(-b*x-a)*a^2*f^3+2/3/b*g^3*B*a
*f*x^3-2/3/d*g^3*B*c*f*x^3-1/b^2*g^3*B*a^2*f*x^2+2/b*g^2*B*a*f^2*x^2-2/d*g^
2*B*c*f^2*x^2+2/5/b^5*g^4*B*ln(-b*x-a)*a^5+2/b*B*ln(-b*x-a)*a*f^4-2/d*B*ln(
d*x+c)*c*f^4
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.85

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{6\,Ab^5d^5g^4x^5+3\,(10\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4)x^4+4\,(15\,Ab^5d^5f^2g^2-5\,(Bb^5cd^4-Bab^4d^5)fg^3+}{}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 3*(10*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4*
d^5)*g^4)*x^4 + 4*(15*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*g
^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(10*A*b^5*d^5*f^3*g - 10*
(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g
^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 6*(5*A*b^5*d^5*f^4 - 20*(B*
```

```
b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 20*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2
 - 10*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + 2*(B*b^5*c^4*d - B*a^4*b*d^5)
*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5
*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*
c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4
*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b
^5*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(347) = 694$.

Time = 57.42 (sec) , antiderivative size = 1477, normalized size of antiderivative = 4.14

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**4*x**5/5 + 2*B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (2*B*a**5*c*d**4*g**4 - 10*B*a**
4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 20*B*a**2*b**3*c*d**4
*f**3*g + 2*B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g*
*2 - 10*a*b**3*f**3*g + 5*b**4*f**4)/b + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4
*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f*
*3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3
+ 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(2*B*a**5*d**5*
g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f**2*g**2 - 20*B*a**2*
b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**5*g**4 - 10*B*b**5*c
**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**5*c**2*d**3*f**3*g +
 10*B*b**5*c*d**4*f**4))/(5*b**5) - 2*B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10
*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (2*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 2
0*B*a**2*b**3*c*d**4*f**3*g + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4*c**4*d*f*g
**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f**3*g + 20*B
*a*b**4*c*d**4*f**4 - 2*B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d
**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4) + 2*B*b**5*c**2*(c**4*g**4
- 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4
)/d)/(2*B*a**5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f*
*2*g**2 - 20*B*a**2*b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**
5*g**4 - 10*B*b**5*c**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**
5*c**2*d**3*f**3*g + 10*B*b**5*c*d**4*f**4))/(5*d**5) + x**4*(A*f*g**3 + B*
a*g**4/(10*b) - B*c*g**4/(10*d)) + x**3*(2*A*f**2*g**2 - 2*B*a**2*g**4/(15*
```

```
b**2) + 2*B*a*f*g**3/(3*b) + 2*B*c**2*g**4/(15*d**2) - 2*B*c*f*g**3/(3*d))
+ x**2*(2*A*f**3*g + B*a**3*g**4/(5*b**3) - B*a**2*f*g**3/b**2 + 2*B*a*f**2
*g**2/b - B*c**3*g**4/(5*d**3) + B*c**2*f*g**3/d**2 - 2*B*c*f**2*g**2/d) +
x*(A*f**4 - 2*B*a**4*g**4/(5*b**4) + 2*B*a**3*f*g**3/b**3 - 4*B*a**2*f**2*g
**2/b**2 + 4*B*a*f**3*g/b + 2*B*c**4*g**4/(5*d**4) - 2*B*c**3*f*g**3/d**3 +
 4*B*c**2*f**2*g**2/d**2 - 4*B*c*f**3*g/d) + (B*f**4*x + 2*B*f**3*g*x**2 +
2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5)*log(e*(a + b*x)**2/(c +
 d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(343) = 686$.

Time = 0.23 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.39

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{5}\,Ag^4x^5 + Afg^3x^4 + 2\,Af^2g^2x^3 + 2\,Af^3gx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log\left(bx+a\right)}{b} - \frac{2\,c\log\left(dx-\right.}{d}\right.$$
$$+2\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log\left(bx+a\right)}{b^2} + \frac{2\,c^2\log\left(\right.}{d}\right.$$
$$+2\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log\left(bx+a\right)}{b^3} - \frac{2\,c^3\log\left(\right.}{d}\right.$$
$$+\frac{1}{3}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log\left(bx+a\right)}{b^4} + \frac{6\,c^4\log}{}\right.$$
$$+\frac{1}{30}\left(6\,x^5\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{12\,a^5\log\left(bx+a\right)}{b^5} - \frac{12\,c}{}\right.$$
$$+ Af^4x$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b^2*
e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2
*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*
f^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2
 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^3*g + 2*(x^3*log(b^2
*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^
2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^
2*g^2 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x +
a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/30*
```

```
(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a
^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)
/(b^4*d^4))*B*g^4 + A*f^4*x
```

Giac [F(-1)]

Timed out.

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 1403, normalized size of antiderivative = 3.93

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3
 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 3
0*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(
10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*
B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a
*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*
B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d
)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + x
^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d
*f^3*g + 20*A*b*c*f^3*g + 20*B*a*d*f^3*g - 20*B*b*c*f^3*g + 30*A*a*c*f^2*g^
2)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*
f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(5*b*d) +
 ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*
c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g
^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*
d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*
d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c
```

```
*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*
A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g
^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a
*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d)
- (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4
 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A
*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*b*d)) + (A*g^4*x^5)/5 + (log(a + b*
x)*((2*B*a^5*g^4)/5 + 2*B*a*b^4*f^4 - 4*B*a^2*b^3*f^3*g + 4*B*a^3*b^2*f^2*g
^2 - 2*B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(2*B*c^5*g^4 + 10*B*c*d^4*f^4 -
20*B*c^2*d^3*f^3*g + 20*B*c^3*d^2*f^2*g^2 - 10*B*c^4*d*f*g^3))/(5*d^5)
```

3.263 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 229

$$\begin{aligned}\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx &= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{2b^3d^3}\\ &\quad-\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{4b^2d^2}-\frac{B(bc-ad)g^3x^3}{6bd}-\frac{B(bf-ag)^4\log(a+bx)}{2b^4g}\\ &\quad+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{2d^4g}\end{aligned}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/6*B*(-a*d+b*c)*g^3*x^3/b/d-1/2*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+1/2*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/
g
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{2548, 84\}$

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= -\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{2b^3d^3} + \frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g} - \frac{B(bf-ag)^4\log(a+bx)}{2b^4g} - \frac{Bg^2x^2(bc-ad)(-adg-bcg+4bdf)}{4b^2d^2} - \frac{Bg^3x^3(bc-ad)}{6bd} + \frac{B(df-cg)^4\log(c+dx)}{2d^4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g
 - a*d*g)*x^2)/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(6*b*d) - (B*(b*f - a*
g)^4*Log[a + b*x])/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(2*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} - \frac{(B(bc-ad))\int\frac{(f+gx)^4}{(a+bx)(c+dx)}\,dx}{2g}$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} - \frac{(B(bc-ad))\int\left(\frac{g^2\left(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)\right)}{b^3d^3}+\frac{g^3(4bdf-bcg-adg)x}{b^2d^2}+\frac{g^4x^2}{bd}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+}\right.}{2g}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{2b^3d^3} \\
&\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{4b^2d^2}-\frac{B(bc-ad)g^3x^3}{6bd}-\frac{B(bf-ag)^4\log(a+bx)}{2b^4g} \\
&\quad +\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{2d^4g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95

$$
\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx
$$
$$
=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bdf-}{3b^4d^4}}{4g}
$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{g^2Bcf\,x^2}{d}-\frac{2g^2B\,a^2fx}{b^2}+\frac{3gBa\,f^2x}{b}+\frac{2g^2B\,c^2fx}{d^2}-\frac{2B\ln(-dx-c)c\,f^3}{d}+\frac{B\ln(-dx-c)f^4}{2g}+\frac{g^2Baf\,x^2}{b}-$
parts	$\frac{A(gx+f)^4}{4g}+\frac{B\left(-\left(-\frac{(dx+c)^4\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(\frac{1}{dx+c}\right)}{b^4}-\right.\right.\right.}{}$
derivativedivides	$-\frac{-\frac{A\left(\frac{g^3(dx+c)^4}{4}+\frac{3g\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)^2}{2}-g^2(cg-df)(dx+c)^3-\left(c^3g^3-3c^2df\,g^2+3c\,d^2f^2g-d^3f^3\right)(dx+c)\right)}{d^3}-\frac{B\left(-\left(\right.\right.}{}}{}$
default	$-\frac{-\frac{A\left(\frac{g^3(dx+c)^4}{4}+\frac{3g\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)^2}{2}-g^2(cg-df)(dx+c)^3-\left(c^3g^3-3c^2df\,g^2+3c\,d^2f^2g-d^3f^3\right)(dx+c)\right)}{d^3}-\frac{B\left(-\left(\right.\right.}{}}{}$
parallelrisch	$\frac{-24Bx\,a^2b^2d^4f\,g^2+36Bxa\,b^3d^4f^2g+36B\ln(bx+a)b^4c^2d^2f^2g+24B\ln(bx+a)a^3b\,d^4f\,g^2-36B\ln(bx+a)a^2b^2d^4f^2g-24}{}$

[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)

[Out] -1/d*g^2*B*c*f*x^2-2/b^2*g^2*B*a^2*f*x+3/b*g*B*a*f^2*x+2/d^2*g^2*B*c^2*f*x-2/d*B*ln(-d*x-c)*c*f^3+1/2/g*B*ln(-d*x-c)*f^4+1/b*g^2*B*a*f*x^2-3/d*g*B*c*f^2*x-1/2/g*B*ln(b*x+a)*f^4+1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2/d^4*g^3*B*ln(-d*x-c)*c^4+3/d^2*g*B*ln(-d*x-c)*c^2*f^2-1/6/d*g^3*B*c*x^3-1/4/b^2*g^3*B*a^2*x^2+1/4/d^2*g^3*B*c^2*x^2+A*f^3*x+1/2/b^3*g^3*B*a^3*x-1/2/d^3*g^3*B*c^3*x+2/b^3*g^2*B*ln(b*x+a)*a^3*f-3/b^2*g*B*ln(b*x+a)*a^2*f^2-2/d^3*g^2*B*ln(-d*x-c)*c^3*f-1/2/b^4*g^3*B*ln(b*x+a)*a^4+g^2*A*f*x^3+1/6/b*g^3*B*a*x^3+3/2*g*A*f^2*x^2+1/4*g^3*A*x^4+2/b*B*ln(b*x+a)*a*f^3

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(217) = 434$.

Time = 0.39 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.04

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{3\,Ab^4d^4g^3x^4+2\,(6\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3)x^3+3\,(6\,Ab^4d^4f^2g-4\,(Bb^4cd^3-Bab^3d^4)fg^2+(B}{}$$

[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 2*(6*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*d
^4)*g^3)*x^3 + 3*(6*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2 +
 (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(2*A*b^4*d^4*f^3 - 6*(B*b^4*c
*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*b^
4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^2*
g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*f^
3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x +
c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2 +
 4*B*b^4*d^4*f^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
+ c^2)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(211) = 422$.

Time = 8.17 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.36

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{Ag^3x^4}{4}$$

$$-\frac{Ba(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4(ag-2bf)\left(a^2g^2-2abfg+2b^2f\right.}{b}}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-}\right.}{2b^4}$$

$$+\frac{Bc(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+Bab^3c^4g^3-4Bab^3c^3dfg^2+6Bab^3c^2}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-4B}\right.}{2d^4}$$

$$+x^3\left(Afg^2+\frac{Bag^3}{6b}-\frac{Bcg^3}{6d}\right)+x^2\cdot\left(\frac{3Af^2g}{2}-\frac{Ba^2g^3}{4b^2}+\frac{Bafg^2}{b}+\frac{Bc^2g^3}{4d^2}-\frac{Bcfg^2}{d}\right)$$

$$+x\left(Af^3+\frac{Ba^3g^3}{2b^3}-\frac{2Ba^2fg^2}{b^2}+\frac{3Baf^2g}{b}-\frac{Bc^3g^3}{2d^3}+\frac{2Bc^2fg^2}{d^2}-\frac{3Bcf^2g}{d}\right)$$

$$+\left(Bf^3x+\frac{3Bf^2gx^2}{2}+Bfg^2x^3+\frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
```

```
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(6*b) - B*c*g**3/(6*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(4*b**2) + B*a
*f*g**2/b + B*c**2*g**3/(4*d**2) - B*c*f*g**2/d) + x*(A*f**3 + B*a**3*g**3/
(2*b**3) - 2*B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/b - B*c**3*g**3/(2*d**3) + 2
*B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/d) + (B*f**3*x + 3*B*f**2*g*x**2/2 + B*f
*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(217) = 434$.

Time = 0.23 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.72

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{4}Ag^3x^4 + Afg^2x^3 + \frac{3}{2}Af^2gx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a\log(bx+a)}{b}-\frac{2c\log(dx-}{d}\right.$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{2a^2\log(bx+a)}{b^2}+\frac{2c^2\log}{c}\right.$$
$$+\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(d}{d^3}\right.$$
$$+\frac{1}{12}\left(3x^4\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\mathrm{l}}{}\right.$$
$$+Af^3x$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d
*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^2*g + (x^3*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2
*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/12*(3*x^
4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
```

```
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Giac [A] (verification not implemented)

none

Time = 75.81 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.79

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\frac{1}{4}\,Ag^3x^4+\frac{(6\,Abdfg^2-Bbcg^3+Badg^3)x^3}{6\,bd}+\frac{1}{4}\left(Bg^3x^4+4\,Bfg^2x^3+6\,Bf^2gx^2+4\,Bf^3x\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{(6\,Ab^2d^2f^2g-4\,Bb^2cdfg^2+4\,Babd^2fg^2+Bb^2c^2g^3-Ba^2d^2g^3)x^2}{4\,b^2d^2}+\frac{(4\,Bab^3f^3-6\,Ba^2b^2f^2g+4\,Ba^3bfg^2-Ba^4g^3)\log(bx+a)}{2\,b^4}-\frac{(4\,Bcd^3f^3-6\,Bc^2d^2f^2g+4\,Bc^3dfg^2-Bc^4g^3)\log(-dx-c)}{2\,d^4}+\frac{(2\,Ab^3d^3f^3-6\,Bb^3cd^2f^2g+6\,Bab^2d^3f^2g+4\,Bb^3c^2dfg^2-4\,Ba^2bd^3fg^2-Bb^3c^3g^3+Ba^3d^3g^3)x}{2\,b^3d^3}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/4*A*g^3*x^4 + 1/6*(6*A*b*d*f*g^2 - B*b*c*g^3 + B*a*d*g^3)*x^3/(b*d) + 1/4
*(B*g^3*x^4 + 4*B*f*g^2*x^3 + 6*B*f^2*g*x^2 + 4*B*f^3*x)*log((b^2*e*x^2 + 2
*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(6*A*b^2*d^2*f^2*g - 4*B
*b^2*c*d*f*g^2 + 4*B*a*b*d^2*f*g^2 + B*b^2*c^2*g^3 - B*a^2*d^2*g^3)*x^2/(b^
2*d^2) + 1/2*(4*B*a*b^3*f^3 - 6*B*a^2*b^2*f^2*g + 4*B*a^3*b*f*g^2 - B*a^4*g
^3)*log(b*x + a)/b^4 - 1/2*(4*B*c*d^3*f^3 - 6*B*c^2*d^2*f^2*g + 4*B*c^3*d*f
*g^2 - B*c^4*g^3)*log(-d*x - c)/d^4 + 1/2*(2*A*b^3*d^3*f^3 - 6*B*b^3*c*d^2*
f^2*g + 6*B*a*b^2*d^3*f^2*g + 4*B*b^3*c^2*d*f*g^2 - 4*B*a^2*b*d^3*f*g^2 - B
*b^3*c^3*g^3 + B*a^3*d^3*g^3)*x/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.24

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{2\,A\,b\,d\,f^3+6\,A\,a\,c\,f\,g^2+6\,A\,a\,d\,f^2\,g+6\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{2\,b\,d}\right.$$

$$+\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f\,g^2 \cdots}{\cdots}\right)}{2\,b\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}\right.$$

$$-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f\,g^2+6\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{4\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{6\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{6\,b\,d}\right)$$

$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3\right)}{2\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3\right)}{2\,d^4}$$

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)
/2 + B*f*g^2*x^3) + x*((2*A*b*d*f^3 + 6*A*a*c*f*g^2 + 6*A*a*d*f^2*g + 6*A*b
*c*f^2*g + 6*B*a*d*f^2*g - 6*B*b*c*f^2*g)/(2*b*d) + ((2*a*d + 2*b*c)*((((2*
A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) -
```

```
(A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*b*c))/(2*b*d) - (2*A*a*c*g^3 +
6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g
^2)/(2*b*d) + (A*a*c*g^3)/(b*d)))/(2*b*d) - (a*c*((2*A*a*d*g^3 + 2*A*b*c*g^
3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c)
)/(2*b*d)))/(b*d)) - x^2*((((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*
g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*
b*c))/(4*b*d) - (2*A*a*c*g^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*
g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) + (A*a*c*g^3)/(2*b*d)) + x^3*((2
*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(6*b*d) -
 (A*g^3*(2*a*d + 2*b*c))/(6*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^
3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(2*b^4) + (log(c
+ d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*d*f*g^2))/(
2*d^4)
```

3.264 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 152

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{3bd} - \frac{2B(bf-ag)^3\log(a+bx)}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} + \frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

```
[Out] -2/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/3*B*(-a*d+b*c)*g^2*x
^2/b/d-2/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))/g+2/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{2548, 84\}$

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g} - \frac{2B(bf-ag)^3\log(a+bx)}{3b^3g} - \frac{2Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2} - \frac{Bg^2x^2(bc-ad)}{3bd} + \frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*
d)*g^2*x^2)/(3*b*d) - (2*B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*
x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) + (2*B*(d*f - c*g)^3*L
og[c + d*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int\frac{(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int\left(\frac{g^2(3bdf-bcg-adg)}{b^2d^2}+\frac{g^3x}{bd}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(-bc+ad)(c+dx)}\right)dx}{3g}$$

$$= -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{3bd} - \frac{2B(bf-ag)^3\log(a+bx)}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} + \frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) - \frac{B\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3}{b^3d^3}}{3g}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3))/(3
*g)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.78

method	result
risch	$\frac{(gx+f)^3 B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3g}+\frac{g^2Ax^3}{3}+gAf\,x^2+\frac{g^2Ba\,x^2}{3b}-\frac{g^2Bc\,x^2}{3d}+A\,f^2x-\frac{2g^2B\ln(dx+c)c^3}{3d^3}+\frac{2gB\ln}{}$
parts	$\frac{A(gx+f)^3}{3g}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)(g^2c\right.}{}$
derivativedivides	$-\frac{A\left(-\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2}+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln}{}\right.\right.\right.}{}$
default	$-\frac{A\left(-\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2}+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln}{}\right.\right.\right.}{}$
parallelrisch	$\frac{2Bx^2ab^2d^3g^2-2Bx^2b^3cd^2g^2+12Bxab^2d^3fg+6Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^3fg-4Bxa^2bd^3g^2-4Bc^3g^2b^3+4Ba^3d^3g^2-6B}{}$

[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)

[Out] 1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*A*x^3+g*A*f*x^2+1/3/b*g^2*B*a*x^2-1/3/d*g^2*B*c*x^2+A*f^2*x-2/3/d^3*g^2*B*ln(d*x+c)*c^3+2/d^2*g*B*ln(d*x+c)*c^2*f-2/d*B*ln(d*x+c)*c*f^2+2/3/g*B*ln(d*x+c)*f^3+2/3/b^3*g^2*B*ln(-b*x-a)*a^3-2/b^2*g*B*ln(-b*x-a)*a^2*f+2/b*B*ln(-b*x-a)*a*f^2-2/3/g*B*ln(-b*x-a)*f^3-2/3/b^2*g^2*B*a^2*x+2/b*g*B*a*f*x+2/3/d^2*g^2*B*c^2*x-2/d*g*B*c*f*x

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(142) = 284.

Time = 0.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{Ab^3d^3g^2x^3+(3\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2)x^2+(3\,Ab^3d^3f^2-6\,(Bb^3cd^2-Bab^2d^3)fg+2\,(Bb^3c^2d-}{}$$

[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")

[Out] 1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c^2

```
*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3
*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c
^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b^3*d
^3*f^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(
b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(139) = 278$.

Time = 3.27 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.55

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{Ag^2x^3}{3} + \frac{2Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+\frac{2Ba^2d^3(a^2g^2-3abfg+3b^2f^2)}{b}+2Bab^2c^3g^2-6Bab^2c^2dfg+12B\cdots}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg\cdots}\right)}{3b^3} - \frac{2Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd^2f^2-2Bab^2c(c^2g^2-3cdfg\cdots}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg+\cdots}\right)}{3d^3} + x^2\left(Afg+\frac{Bag^2}{3b}-\frac{Bcg^2}{3d}\right) + x\left(Af^2-\frac{2Ba^2g^2}{3b^2}+\frac{2Bafg}{b}+\frac{2Bc^2g^2}{3d^2}-\frac{2Bcfg}{d}\right) + \left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a*
*3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b*f
*g + 3*b**2*f**2)/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a
*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(2*
B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c*
*3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B*c*(c*
*2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6*B*a**2
*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a*b**2*
c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + 2*B*b**3
*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**2 - 6*B*a*
*2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*b**3*c**2*d
*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2/(3*b) - B*c
*g**2/(3*d)) + x*(A*f**2 - 2*B*a**2*g**2/(3*b**2) + 2*B*a*f*g/b + 2*B*c**2*
g**2/(3*d**2) - 2*B*c*f*g/d) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(
e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(142) = 284.

Time = 0.21 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.76

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{3}Ag^2x^3 + Afgx^2 + \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a\log(bx+a)}{b}-\frac{2c\log(dx+\ldots}{d}\right. + \left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{2a^2\log(bx+a)}{b^2}+\frac{2c^2\log(dx\ldots}{d^2}\right. + \frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(\ldots}{d^3}\right. + Af^2x$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*
log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*e*x^2/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a
*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a
*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(
b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Giac [A] (verification not implemented)

none

Time = 5.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{3}Ag^2x^3 + \frac{1}{3}\left(Bg^2x^3+3Bfgx^2+3Bf^2x\right)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{(3Abdfg-Bbcg^2+Badg^2)x^2}{3bd} + \frac{2(3Bab^2f^2-3Ba^2bfg+Ba^3g^2)\log(bx+a)}{3b^3} - \frac{2(3Bcd^2f^2-3Bc^2dfg+Bc^3g^2)\log(-dx-c)}{3d^3} + \frac{(3Ab^2d^2f^2-6Bb^2cdfg+6Babd^2fg+2Bb^2c^2g^2-2Ba^2d^2g^2)x}{3b^2d^2}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/3*A*g^2*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^2*e*x^2 +
2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g - B*b*c*g^
2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b*f*g + B*a^3*g^2)*
log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B*c^3*g^2)*log(-d*x
 - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 - 6*B*b^2*c*d*f*g + 6*B*a*b*d^2*f*g + 2*B*
b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.38

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$+x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$-x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right.$$
$$\left.-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+6\,B\,a\,d\,f\,g-6\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)$$
$$+\frac{\ln(a+b\,x)\,\left(2\,B\,a^3\,g^2-6\,B\,a^2\,b\,f\,g+6\,B\,a\,b^2\,f^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(2\,B\,c^3\,g^2-6\,B\,c^2\,d\,f\,g+6\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) + x^
2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(6
*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2
+ 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c)
)/(3*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*
f*g + 6*A*b*c*f*g + 6*B*a*d*f*g - 6*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d))
 + (log(a + b*x)*(2*B*a^3*g^2 + 6*B*a*b^2*f^2 - 6*B*a^2*b*f*g))/(3*b^3) - (
log(c + d*x)*(2*B*c^3*g^2 + 6*B*c*d^2*f^2 - 6*B*c^2*d*f*g))/(3*d^3) + (A*g^
2*x^3)/3
```

3.265 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=-\frac{B(bc-ad)gx}{bd}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

```
[Out] -B*(-a*d+b*c)*g*x/b/d-B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*(A+B*ln(
e*(b*x+a)^2/(d*x+c)^2))/g+B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2548, 84}

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}-\frac{Bgx(bc-ad)}{bd}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/(b*d)) - (B*(b*f - a*g)^2*Log[a + b*x])/(b^2*g) + ((f
 + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*g) + (B*(d*f - c*g)^
2*Log[c + d*x])/(d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}-\frac{(B(bc-ad))\int\frac{(f+gx)^2}{(a+bx)(c+dx)}\,dx}{g}\\
&= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}-\frac{(B(bc-ad))\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(-bc+ad)(c+dx)}\right)dx}{g}\\
&= -\frac{B(bc-ad)gx}{bd}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}\\
&\quad+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$

$$=\frac{-2Bd^2(bf-ag)^2\log(a+bx)+b\left(d(2B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\cdots\right.}{2b^2d^2g}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*d^2*(b*f - a*g)^2*Log[a + b*x] + b*(d*(2*B*(-(b*c) + a*d)*g^2*x + A*b
*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{A\,x^2 g}{2}+Afx-\frac{B\ln(bx+a)a^2 g}{b^2}+\frac{2B\ln(bx+a)af}{b}+\frac{B\ln(-dx-c)c^2 g}{d^2}-\frac{2B\ln(-$
parts	$A\left(\frac{1}{2}g\,x^2+fx\right)+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right.\right.\right.}{}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2f+2A\,b^2d^2fx-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)ab\,d^2f+2}{}$
derivativedivides	$-\frac{-\frac{A\left(-(cg-df)(dx+c)+\frac{g(dx+c)^2}{2}\right)}{d}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\right.}{b(ad-cb)}\right.\right.\right.}{}}{}$
default	$-\frac{-\frac{A\left(-(cg-df)(dx+c)+\frac{g(dx+c)^2}{2}\right)}{d}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\right.}{b(ad-cb)}\right.\right.\right.}{}}{}$

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*A*x^2*g+A*f*x-1/b^2*B*ln(b*
x+a)*a^2*g+2/b*B*ln(b*x+a)*a*f+1/d^2*B*ln(-d*x-c)*c^2*g-2/d*B*ln(-d*x-c)*c*
f+1/b*B*x*a*g-1/d*B*x*c*g
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+2\left(Ab^2d^2f-(Bb^2cd-Babd^2)g\right)x+2\left(2\,Babd^2f-Ba^2d^2g\right)\log\left(bx+a\right)-2\left(2\,Bb^2cdf-Bb^2c^2\right.}{2\,b^2d^2}$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(2
*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g)*
log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*e*x^2 + 2*a*b*e
*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(88) = 176$.

Time = 1.41 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{Agx^2}{2} - \frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{b^2}$$
$$+ \frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{d^2}$$
$$+ x\left(Af+\frac{Bag}{b}-\frac{Bcg}{d}\right) + \left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g - 2
*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c*g -
 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*
b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + x*(A*f + B*a*g/b - B*c*g/d) + (B*f*x
+ B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Agx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx-\right.}{d}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log}{}\right.$$
$$+Afx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b
```

```
 - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
 - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))
*B*g + A*f*x
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{2}Agx^2+\frac{1}{2}\left(Bgx^2+2Bfx\right)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{(Abdf-Bbcg+Badg)x}{bd} + \frac{(2Babf-Ba^2g)\log(bx+a)}{b^2}-\frac{(2Bcdf-Bc^2g)\log(-dx-c)}{d^2}$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/2*A*g*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(
d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f - B*b*c*g + B*a*d*g)*x/(b*d) + (2*B*a*
b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right) + x\left(\frac{A\,a\,d\,g+A\,b\,c\,g+A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{b\,d}-\frac{A\,g\,(a\,d+b\,c)}{b\,d}\right) + \frac{A\,g\,x^2}{2}-\frac{B\,a\,\ln(a+b\,x)\,(a\,g-2\,b\,f)}{b^2}+\frac{B\,c\,\ln(c+d\,x)\,(c\,g-2\,d\,f)}{d^2}$$

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*(B*f*x + (B*g*x^2)/2) + x*((A*a*d*g + A*b*
c*g + A*b*d*f + B*a*d*g - B*b*c*g)/(b*d) - (A*g*(a*d + b*c))/(b*d)) + (A*g*
x^2)/2 - (B*a*log(a + b*x)*(a*g - 2*b*f))/b^2 + (B*c*log(c + d*x)*(c*g - 2*
d*f))/d^2
```

3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)^2/(d*x+c)^2)/b-2*B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2536, 31}

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx=\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[In] Int[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= Ax + B\int \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)dx \\
&= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b} - \frac{(2B(bc-ad))\int \frac{1}{c+dx}\,dx}{b} \\
&= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b} - \frac{2B(bc-ad)\log(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

```
[In] Integrate[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result	s
risch	$Ax + Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) - \frac{2Bc\ln(dx+c)}{d} + \frac{2Ba\ln(-bx-a)}{b}$	5
parallelrisch	$\frac{B\left(2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)bd+4\ln(bx+a)ad-4\ln(bx+a)bc+2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)bc\right)}{2bd}+Ax$	7
default	$Ax - \frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1
parts	$Ax - \frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1
derivativedivides	$-\frac{-A(dx+c)+B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1

```
[In] int(A+B*ln(e*(b*x+a)^2/(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] A*x+B*x*ln(e*(b*x+a)^2/(d*x+c)^2)-2*B/d*c*ln(d*x+c)+2*B/b*a*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{Bbdx\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+Abdx+2\,Bad\log\left(bx+a\right)-2\,Bbc\log\left(dx+c\right)}{bd}$$

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="fricas")
[Out] (B*b*d*x*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A
*b*d*x + 2*B*a*d*log(b*x + a) - 2*B*b*c*log(d*x + c))/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(48) = 96$.

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.93

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = Ax + \frac{2Ba\log\left(x+\frac{\frac{2Ba^2d}{b}+2Bac}{2Bad+2Bbc}\right)}{b} - \frac{2Bc\log\left(x+\frac{2Bac+\frac{2Bbc^2}{d}}{2Bad+2Bbc}\right)}{d} + Bx\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

```
[In] integrate(A+B*ln(e*(b*x+a)**2/(d*x+c)**2),x)
```

```
[Out] A*x + 2*B*a*log(x + (2*B*a**2*d/b + 2*B*a*c)/(2*B*a*d + 2*B*b*c))/b - 2*B*c
*log(x + (2*B*a*c + 2*B*b*c**2/d)/(2*B*a*d + 2*B*b*c))/d + B*x*log(e*(a + b
*x)**2/(c + d*x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\left(x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+\frac{2\left(\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}\right)}{e}\right)B+Ax$$

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x + c
)/d)/e)*B + A*x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\left(2\,(bc-ad)\left(\frac{a\log(|bx+a|)}{b^2c-abd}-\frac{c\log(|dx+c|)}{bcd-ad^2}\right)+x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)\right)B+Ax$$

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (2*(b*c - a*d)*(a*log(abs(b*x + a))/(b^2*c - a*b*d) - c*log(abs(d*x + c))/(
b*c*d - a*d^2)) + x*log((b*x + a)^2*e/(d*x + c)^2))*B + A*x
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right) + \frac{2\,B\,a\,\ln\left(a+b\,x\right)}{b} - \frac{2\,B\,c\,\ln\left(c+d\,x\right)}{d}$$

```
[In] int(A + B*log((e*(a + b*x)^2)/(c + d*x)^2),x)
[Out] A*x + B*x*log((e*(a + b*x)^2)/(c + d*x)^2) + (2*B*a*log(a + b*x))/b - (2*B*
c*log(c + d*x))/d
```

3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx =& -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} \\ &+\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} \\ &+\frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} \\ &-\frac{2B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{2B\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}\end{aligned}$$

```
[Out] -2*B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*
ln(g*x+f)/g+2*B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-2*B*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+2*B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

$= \{2546, 2441, 2440, 2438\}$

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g} - \frac{2B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} - \frac{2B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g} + \frac{2B\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g} + \frac{2B\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] (-2*B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g + ((A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g + (2*B*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (2*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2
*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{(2bB)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(2Bd)\int\frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + (2B)\int\frac{\log\left(\frac{g(a+bx)}{-bf+ag}\right)}{f+gx}\,dx - (2B)\int\frac{\log\left(\frac{g(c+dx)}{-df+cg}\right)}{f+gx}\,dx$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} + \frac{(2B)\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bf+ag}\right)}{x}\,dx, x, f+gx\right)}{g} - \frac{(2B)\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{-df+cg}\right)}{x}\,dx, x, f+gx\right)}{g}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{2B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{2B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \frac{\left(A-2B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-2B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+2I}{g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] ((A - 2*B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2] + 2*B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - 2*B*PolyLog[2
, (b*(f + g*x))/(b*f - a*g)] + 2*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(144) = 288$.

Time = 3.37 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.75

method	result
parts	$\frac{A\ln(gx+f)}{g}+B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\mathrm{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right)}{g}\right.$
derivativedivides	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\mathrm{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\cdots\right.}{g}\right.}{}$
default	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(2ad-2cb)\left(\frac{\mathrm{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\cdots\right.}{g}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(g*x+f)/g+B*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(2
```

```
*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*ln(((a*
d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/g+(ln((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)*ln(e*
(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-2/(c*g-d*f)*(a*d-b*c)*(dilog((((c*g-d*f)
/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)+ln((c*g-d*f)/(d
*x+c)-g)*ln((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a
*d-b*c)))/g*(c*g-d*f))
```

Fricas [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\, dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{gx + f}\, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\, dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\, dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{gx + f}\, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + log(e))/(g*x + f), x) + A*
log(g*x + f)/g
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{gx + f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{f + g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x),x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x), x)
```

3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 90

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2554, 2351, 31}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)*(f + g*x)
) + (2*B*(b*c - a*d)*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (bc-ad)\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)} - \frac{(2B(bc-ad))\text{Subst}\left(\int \frac{1}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bf-ag}\\
&= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} + \frac{2B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)) + (2*B*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(90) = 180$.

Time = 0.67 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.72

method	result
derivativedivides	$-\frac{-\frac{d^2 A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{ag-bf}-\frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(ad-cb)\ln\left(\frac{cg}{dx+c}-\cdots\right)}{acg^2-adfg-bcfg+b\cdots}}{d}$
default	$-\frac{-\frac{d^2 A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{ag-bf}-\frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(ad-cb)\ln\left(\frac{cg}{dx+c}-\cdots\right)}{acg^2-adfg-bcfg+b\cdots}}{d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(gx+f)}-\frac{-2B\ln(-dx-c)adg^2x+2B\ln(-dx-c)bdfgx+2B\ln(gx+f)adg^2x-2B\ln(gx+f)bcg^2x+2B\ln(\cdots}{\cdots}$
parallelrisch	$\frac{4B\ln(gx+f)xabc^2fg+4B\ln(bx+a)xa^2cdfg-2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2cdfg+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)abcdf^2-4B\ln(bx+a)xab\cdots}{\cdots}$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^2*A/((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)+(-b*B*d/(a*g-b*f)*ln(e*(a*d/(d
*x+c)-b*c/(d*x+c)+b)^2/d^2)-B*d*(a*d-b*c)/(a*g-b*f)/(d*x+c)*ln(e*(a*d/(d*x+
c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)+2*B*d*(a*d-b*c)/(a*c*
g^2-a*d*f*g-b*c*f*g+b*d*f^2)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(90) = 180$.

Time = 3.50 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.10

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-2\left(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x\right)\log(bx+a)+2\left(Bbdf^2-\cdots\right.}{bdf^3g\cdots}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g +
(B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b*d
*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a
*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d*f^3*g + a*c
```

```
*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$= B\left(\frac{2\,b\log(bx+a)}{bfg-ag^2} - \frac{2\,d\log(dx+c)}{dfg-cg^2} + \frac{2\,(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{g^2x+fg}\right) - \frac{A}{g^2x+fg}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="maxima")
[Out] B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2*
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*e*
x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(90) = 180$.

Time = 0.52 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.04

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$= \left((bcg^2-adg^2)\left(\frac{(2\,bdf-bcg-adg)\log\left(\frac{\left|2\,bdfg-\frac{2\,bdf^2g}{gx+f}-bcg^2-adg^2+\frac{2\,bcfg^2}{gx+f}+\frac{2\,adfg^2}{gx+f}-\frac{2\,acg^3}{gx+f}-\left|-bcg^2+adg^2\right|\right|}{\left|2\,bdfg-\frac{2\,bdf^2g}{gx+f}-bcg^2-adg^2+\frac{2\,bcfg^2}{gx+f}+\frac{2\,adfg^2}{gx+f}-\frac{2\,acg^3}{gx+f}+\left|-bcg^2+adg^2\right|\right|}\right)}{(bdf^2g-bcfg^2-adfg^2+acg^3)\left|-bcg^2+adg^2\right|}-\frac{\log}{}\right.\right.$$
$$-\frac{A}{(gx+f)g}$$

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="giac")

[Out] ((b*c*g^2 - a*d*g^2)*((2*b*d*f - b*c*g - a*d*g)*log(abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) - abs(-b*c*g^2 + a*d*g^2))/abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) + abs(-b*c*g^2 + a*d*g^2)))/((b*d*f^2*g - b*c*f*g^2 - a*d*f*g^2 + a*c*g^3)*abs(-b*c*g^2 + a*d*g^2)) - log(abs(b*d - 2*b*d*f/(g*x + f) + b*d*f^2/(g*x + f)^2 + b*c*g/(g*x + f) + a*d*g/(g*x + f) - b*c*f*g/(g*x + f)^2 - a*d*f*g/(g*x + f)^2 + a*c*g^2/(g*x + f)^2))/(b*d*f^2*g^2 - b*c*f*g^3 - a*d*f*g^3 + a*c*g^4)) - log((b*x + a)^2*e/(d*x + c)^2)/((g*x + f)*g))*B - A/((g*x + f)*g)

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \frac{2\,B\,d\,\ln(c+d\,x)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{e\,a^2+2\,e\,a\,b\,x+e\,b^2\,x^2}{c^2+2\,c\,d\,x+d^2\,x^2}\right)}{x\,g^2+f\,g}-\frac{2\,B\,b\,\ln(a+b\,x)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{2\,B\,a\,d\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{2\,B\,b\,c\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^2,x)

[Out] (2*B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a^2*e + b^2*e*x^2 + 2*a*b*e*x)/(c^2 + d^2*x^2 + 2*c*d*x)))/(f*g + g^2*x) - (2*B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (2*B*a*d*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (2*B*b*c*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)

3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2} -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{g(df-cg)^2} +\frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{(bf-ag)^2(df-cg)^2}$$

```
[Out] -B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+b^2*B*ln(b*x+a)/g/(-a*g+b*f)^2+
1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^2-B*d^2*ln(d*x+c)/g/(-c*g+d*
f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2/(-c*g+d*f)^
2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{2548, 84\}$

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2} + \frac{b^2B\log(a+bx)}{g(bf-ag)^2} - \frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2} - \frac{Bd^2\log(c+dx)}{g(df-cg)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] -((B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*B*Log[a + b*x
])/(g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(2*g*(f + g
*x)^2) - (B*d^2*Log[c + d*x])/(g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f -
 b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)(f+gx)^2} - \frac{g^2(-2bdf+bc}{(bf-ag)^2(df-cg}\right.}{g}$$

$$
= -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B\log(a+bx)}{g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2} - \frac{Bd^2\log(c+dx)}{g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{(bf-ag)^2(df-cg)^2}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$
\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2} + 2B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)} + \frac{d^2\log(c+dx)}{-bc+ad} - \frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2) + 2*B*(b*c - a*d)*
((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f
 - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c
*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(176) = 352.

Time = 1.22 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.59

method	result
derivativedivides	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Bad^3g^2-Bbcd^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{b\cdots}{dx\cdots}\right.}{d^2}\cdots\right.}{\left(g^2a^2-2abfg+f^2b^2\right)(dx\cdots}}{\cdots}}{\cdots}$
default	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Bad^3g^2-Bbcd^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{b\cdots}{dx\cdots}\right.}{d^2}\cdots\right.}{\left(g^2a^2-2abfg+f^2b^2\right)(dx\cdots}}{\cdots}}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^3*A*(-1/(c*g-d*f)^2/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/2*g/(c*g-d*f)^2/
(c*g/(d*x+c)-f/(d*x+c)*d-g)^2)+((B*a*d^3*g^2-B*b*c*d^2*g^2)/g^2/(a*g-b*f)/(
d*x+c)^2+b^2*(c*g-d*f)*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(a*d/(d
*x+c)-b*c/(d*x+c)+b)^2/d^2)-(B*a*d^3*g^2-B*b*c*d^2*g^2)/g/(a*c*g^2-a*d*f*g-
b*c*f*g+b*d*f^2)/(d*x+c)+1/2*B*d*(a^2*d^2*g-2*a*b*d^2*f-b^2*c^2*g+2*b^2*c*d
*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^
2/d^2)-1/2*b^2*g*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln(e*(a*d/(d*x+c)-b*c/(d*x
+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-B*d*(a^2*d^2*g-2*a*b*d^2*f-b^2
*c^2*g+2*b^2*c*d*f)/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*
f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2
*d^2*f^4)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(173) = 346$.

Time = 47.45 (sec) , antiderivative size = 1036, normalized size of antiderivative = 5.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{Ab^2d^2f^4+Aa^2c^2g^4-2\left((A-B)b^2cd+(A+B)abd^2\right)f^3g+\left((A-2\,B)b^2c^2+4\,Aabcd+(A+2\,B)a^2d\right.}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*((A - B)*b^2*c*d + (A + B)*a*b*d^2)
*f^3*g + ((A - 2*B)*b^2*c^2 + 4*A*a*b*c*d + (A + 2*B)*a^2*d^2)*f^2*g^2 - 2*
((A - B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + 2*((B*b^2*c*d - B*a*b*d^2)*f^2*
g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - 2*(B
*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 -
 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*
f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + 2*(B*b^2*d^2*f^4 - 2*B*a*b*d^2
*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2
*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)
*x)*log(d*x + c) - 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*
d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g
^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*
g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*
b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2
 + B*a^2*c*d)*f*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2
+ (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 +
 (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*
```

```
d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(173) = 346$.

Time = 0.23 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.31

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{2\,d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{2\,(2\,(b^2cd-abd^2)f-(b^2c^2-a}{b^2d^2f^4+a^2c^2g^4-2\,(b^2cd+abd^2)f^3g+(b^2c^2+4}\right.$$
$$-\frac{A}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x
 + c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b
^2*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d +
 a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^
2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b
*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2
*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(173) = 346$.

Time = 0.50 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{Bb^3\log(|bx+a|)}{b^3f^2g-2\,ab^2fg^2+a^2bg^3} - \frac{Bd^3\log(|dx+c|)}{d^3f^2g-2\,cd^2fg^2+c^2dg^3} + \frac{(2\,Bb^2cdf-2\,Babd^2f-Bb^2c^2g+Ba^2d^2g)\log(gx+f)}{b^2d^2f^4-2\,b^2cdf^3g-2\,abd^2f^3g+b^2c^2f^2g^2+4\,abcdf^2g^2+a^2d^2f^2g^2-2\,abc^2fg^3-2\,a^2cdfg^3+a^2c^2g^4} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{2\,(g^3x^2+2\,fg^2x+f^2g)} - \frac{2\,Bbcg^2x-2\,Badg^2x+Abdf^2-Abcfg+2\,Bbcfg-Aadfg-2\,Badfg+Aac}{2\,(bdf^2g^3x^2-bcfg^4x^2-adfg^4x^2+acg^5x^2+2\,bdf^3g^2x-2\,bcf^2g^3x-2\,adf^2g^3x+2\,acfg^4x+bdf^4g-}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] B*b^3*log(abs(b*x + a))/(b^3*f^2*g - 2*a*b^2*f*g^2 + a^2*b*g^3) - B*d^3*log
(abs(d*x + c))/(d^3*f^2*g - 2*c*d^2*f*g^2 + c^2*d*g^3) + (2*B*b^2*c*d*f - 2
*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(g*x + f)/(b^2*d^2*f^4 - 2*b^2
*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^
2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*B*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^3*x^2 + 2*f*g^2
*x + f^2*g) - 1/2*(2*B*b*c*g^2*x - 2*B*a*d*g^2*x + A*b*d*f^2 - A*b*c*f*g +
2*B*b*c*f*g - A*a*d*f*g - 2*B*a*d*f*g + A*a*c*g^2)/(b*d*f^2*g^3*x^2 - b*c*f
*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x - 2*b*c*f^2*g^3*x
- 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2 - a*d*f^3*g^2 +
 a*c*f^2*g^3)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2-B\,b^2\,c^2)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g^2-2\,b^2\,c\,d\,f^3\,g} - \frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-2\,B\,a\,d\,f\,g+2\,B\,b\,c\,f\,g}{2\,(a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g)}-\frac{x\,(B\,a\,d\,g^2-B\,b\,c\,g^2)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{f^2\,g+2\,f\,g^2\,x+g^3\,x^2} + \frac{B\,b^2\,\ln(a+b\,x)}{a^2\,g^3-2\,a\,b\,f\,g^2+b^2\,f^2\,g} - \frac{B\,d^2\,\ln(c+d\,x)}{c^2\,g^3-2\,c\,d\,f\,g^2+d^2\,f^2\,g} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,g\,(f^2+2\,f\,g\,x+g^2\,x^2)}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*
f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2
*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - 2*B*a*d*f*g + 2*B
*b*c*f*g)/(2*(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)) - (x*(B*a*d*g^2 - B*b
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(f^2*g + g^3*x^2 + 2*f*g^
2*x) + (B*b^2*log(a + b*x))/(a^2*g^3 + b^2*f^2*g - 2*a*b*f*g^2) - (B*d^2*lo
g(c + d*x))/(c^2*g^3 + d^2*f^2*g - 2*c*d*f*g^2) - (B*log((e*(a + b*x)^2)/(c
 + d*x)^2))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x))
```

3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 277

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx &= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}\\ &\quad+\frac{2b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}-\frac{2Bd^3\log(c+dx)}{3g(df-cg)^3}\\ &\quad+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

```
[Out] -1/3*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-2/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+2/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^3-2/3*B*d^3*ln(d*x+
c)/g/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$
$$=\frac{2B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}+\frac{2b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{2B(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2}$$
$$-\frac{B(bc-ad)}{3(f+gx)^2(bf-ag)(df-cg)}-\frac{2Bd^3\log(c+dx)}{3g(df-cg)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] -1/3*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*
d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (
2*b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])/(3*g*(f + g*x)^3) - (2*B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) +
(2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*
c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral}=-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} + \frac{(2B(bc-ad))\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)(f+gx)^3} - \frac{g^2(-2bdf+\cdots}{(bf-ag)^2(df-\cdots}\right.}{3g}$$

$$= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{2b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} - \frac{2Bd^3\log(c+dx)}{3g(df-cg)^3} + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3} + 2B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2} + \frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3\log\cdots}{(bc-ad)(\cdots}\right.}{3g}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)*
(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*
g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*
d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*
(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. 2(268) = 536.

Time = 2.04 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.67

method	result	size
derivativedivides	Expression too large to display	1293
default	Expression too large to display	1293
risch	Expression too large to display	2444
parallelrisch	Expression too large to display	2946

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(d^4*A*(-1/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/3*g^2/(c*g-d*f)^3
/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3-g/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2)
+((c*g-d*f)*b^3*g*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)
*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/3*(2*B*a^2*d^4*g^4-4*B*a*b*d^4*f
*g^3-2*B*b^2*c^2*d^2*g^4+4*B*b^2*c*d^3*f*g^3)/g/(a^2*c^2*g^4-2*a^2*c*d*f*g^
3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2
*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)-1/3*(3*B*a^2*d^4*g^4-B*a*b*c*
d^3*g^4-5*B*a*b*d^4*f*g^3-2*B*b^2*c^2*d^2*g^4+5*B*b^2*c*d^3*f*g^3)/(a^2*g^2
-2*a*b*f*g+b^2*f^2)/g^3/(d*x+c)^3+1/3*(5*B*a^2*d^4*g^4-B*a*b*c*d^3*g^4-9*B*
a*b*d^4*f*g^3-4*B*b^2*c^2*d^2*g^4+9*B*b^2*c*d^3*f*g^3)/(c*g-d*f)/g^2/(a^2*g
^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2-1/3*B*d*(a^3*d^3*g^2-3*a^2*b*d^3*f*g+3*a*b^
2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^2*f^2)/(a^3*g^3-3*a^2*b*f*g
^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)
-1/3*b^3*g^2*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)*ln(e*(a*d/(d
*x+c)-b*c/(d*x+c)+b)^2/d^2)-(c^2*g^2-2*c*d*f*g+d^2*f^2)*b^3*B*d/(a^3*g^3-3*
a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+
b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3+2/3*B*d*(a^3*d^3*g^2-3*a^2*b*d^3*f
*g+3*a*b^2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^2*f^2)/(a^3*c^3*g^
6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9
*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*
f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c
^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(c*g/(d*x+c
)-f/(d*x+c)*d-g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(265) = 530$.

Time = 0.25 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.25

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$
$$= \frac{1}{3}\left(\frac{2\,b^3\log(bx+a)}{b^3f^3g - 3\,ab^2f^2g^2 + 3\,a^2bfg^3 - a^3g^4} - \frac{2\,d^3\log(dx+c)}{d^3f^3g - 3\,cd^2f^2g^2 + 3\,c^2dfg^3 - c^3g^4} + \frac{}{b^3d^3f^6 + a^3c^3g^6 - 3\,(b\right.$$
$$- \frac{A}{3\,(g^4x^3 + 3\,fg^3x^2 + 3\,f^2g^2x + f^3g)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="maxima")
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(265) = 530$.

Time = 0.84 (sec) , antiderivative size = 1359, normalized size of antiderivative = 4.91

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 2/3*B*b^4*log(abs(b*x + a))/(b^4*f^3*g - 3*a*b^3*f^2*g^2 + 3*a^2*b^2*f*g^3
- a^3*b*g^4) - 2/3*B*d^4*log(abs(d*x + c))/(d^4*f^3*g - 3*c*d^3*f^2*g^2 + 3
*c^2*d^2*f*g^3 - c^3*d*g^4) + 2/3*(3*B*b^3*c*d^2*f^2 - 3*B*a*b^2*d^3*f^2 -
3*B*b^3*c^2*d*f*g + 3*B*a^2*b*d^3*f*g + B*b^3*c^3*g^2 - B*a^3*d^3*g^2)*log(
g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d
*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 -
9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c
^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^
5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/3*B*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3
*g) - 1/3*(4*B*b^2*c*d*f*g^3*x^2 - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b^2*c^2*g^4*
x^2 + 2*B*a^2*d^2*g^4*x^2 + 9*B*b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2*f^2*g^2*x -
 5*B*b^2*c^2*f*g^3*x + 5*B*a^2*d^2*f*g^3*x + B*a*b*c^2*g^4*x - B*a^2*c*d*g^
4*x + A*b^2*d^2*f^4 - 2*A*b^2*c*d*f^3*g + 5*B*b^2*c*d*f^3*g - 2*A*a*b*d^2*f
^3*g - 5*B*a*b*d^2*f^3*g + A*b^2*c^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 + 4*A*a*
b*c*d*f^2*g^2 + A*a^2*d^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 2*A*a*b*c^2*f*g^3
 + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - B*a^2*c*d*f*g^3 + A*a^2*c^2*g^4)/(
b^2*d^2*f^4*g^4*x^3 - 2*b^2*c*d*f^3*g^5*x^3 - 2*a*b*d^2*f^3*g^5*x^3 + b^2*c
^2*f^2*g^6*x^3 + 4*a*b*c*d*f^2*g^6*x^3 + a^2*d^2*f^2*g^6*x^3 - 2*a*b*c^2*f*
g^7*x^3 - 2*a^2*c*d*f*g^7*x^3 + a^2*c^2*g^8*x^3 + 3*b^2*d^2*f^5*g^3*x^2 - 6
*b^2*c*d*f^4*g^4*x^2 - 6*a*b*d^2*f^4*g^4*x^2 + 3*b^2*c^2*f^3*g^5*x^2 + 12*a
*b*c*d*f^3*g^5*x^2 + 3*a^2*d^2*f^3*g^5*x^2 - 6*a*b*c^2*f^2*g^6*x^2 - 6*a^2*
c*d*f^2*g^6*x^2 + 3*a^2*c^2*f*g^7*x^2 + 3*b^2*d^2*f^6*g^2*x - 6*b^2*c*d*f^5
*g^3*x - 6*a*b*d^2*f^5*g^3*x + 3*b^2*c^2*f^4*g^4*x + 12*a*b*c*d*f^4*g^4*x +
 3*a^2*d^2*f^4*g^4*x - 6*a*b*c^2*f^3*g^5*x - 6*a^2*c*d*f^3*g^5*x + 3*a^2*c^
2*f^2*g^6*x + b^2*d^2*f^7*g - 2*b^2*c*d*f^6*g^2 - 2*a*b*d^2*f^6*g^2 + b^2*c
^2*f^5*g^3 + 4*a*b*c*d*f^5*g^3 + a^2*d^2*f^5*g^3 - 2*a*b*c^2*f^4*g^4 - 2*a^
2*c*d*f^4*g^4 + a^2*c^2*f^3*g^5)
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,(6\,B\,a^2\,b\,d^3\,f-6\,\cdots\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,\cdots}$$
$$-\frac{\frac{A\,a^2\,c^2\,g^4+A\,b^2\,d^2\,f^4+A\,a^2\,d^2\,f^2\,g^2+A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2-3\,B\,b^2\,c^2\,f^2\,g^2-2\,A\,a\,b\,c^2\,f\,g^3-2\,A\,a\,b\,d^2\,f^3\,g+B\,a\,b\,c^2\,f\,g^3-2\,A\,a^2\,c\,d\,f\,\cdots}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,\cdots}}{\cdots}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{3\,g\,\left(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3\right)}-\frac{2\,B\,b^3\,\ln(a+b\,x)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g}$$
$$+\frac{2\,B\,d^3\,\ln(c+d\,x)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^4,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b*d^3*f - 6*B*b^3*c^2*d*f) - g^2*(2*B*a^3*d^3 - 2
*B*b^3*c^3) - 6*B*a*b^2*d^3*f^2 + 6*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^
3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a
*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^
4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*
b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^
2*b*c^2*d*f^2*g^4) - ((A*a^2*c^2*g^4 + A*b^2*d^2*f^4 + A*a^2*d^2*f^2*g^2 +
A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 2*A*a*b*c^2
*f*g^3 - 2*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - 5*B*a*b*
d^2*f^3*g - 2*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 4*A*a
*b*c*d*f^2*g^2)/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2) + (2*x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a*b*d^2
*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 +
 b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*
b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*c^2*f*
g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c*d*f^2
*g^2))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a
*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*
c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) - (B*log((e
*(a + b*x)^2)/(c + d*x)^2))/(3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2))
 - (2*B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^
2*b*f*g^3) + (2*B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*
g^2 - 9*c^2*d*f*g^3)
```

3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 381

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx \\
&= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&\quad - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{2(bf-ag)^3(df-cg)^3(f+gx)} \\
&\quad + \frac{b^4B\log(a+bx)}{2g(bf-ag)^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} - \frac{Bd^4\log(c+dx)}{2g(df-cg)^4} \\
&\quad - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{2(bf-ag)^4(df-cg)^4}
\end{aligned}$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/4*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/2*B*(-a*d+b*c)*(a^2*d^
2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3/
(-c*g+d*f)^3/(g*x+f)+1/2*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*x
+a)^2/(d*x+c)^2))/g/(g*x+f)^4-1/2*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/2*B*(-a*
d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d
*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$$
$$=-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{2(f+gx)(bf-ag)^3(df-cg)^3}$$
$$-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+2d^2f^2))\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}+\frac{b^4B\log(a+bx)}{2g(bf-ag)^4}-\frac{B(bc-ad)(-adg-bcg+2bdf)}{4(f+gx)^2(bf-ag)^2(df-cg)^2}$$
$$-\frac{B(bc-ad)}{6(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4\log(c+dx)}{2g(df-cg)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] -1/6*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (
B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d
*f*g + c^2*g^2)))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a
+ b*x])/(2*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(4*g
*(f + g*x)^4) - (B*d^4*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(
2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*
c*d*f*g + c^2*g^2))*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)(f+gx)^4} - \frac{g^2(-2bdf+bcg\cdots}{(bf-ag)^2(df-cg)^2\cdots}\right.}{}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{2(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^4B\log(a+bx)}{2g(bf-ag)^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} - \frac{Bd^4\log(c+dx)}{2g(df-cg)^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{2(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4} + 2B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3} + \frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{g(a^2d^2g^2+abdg(-3df+cg)+b^2\cdots}{(bf-ag)^3(df-cg\cdots}\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4) + 2*B*(b*c - a*d)*
(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*
g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g
*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d
*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (
d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*
g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*L
og[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2298 vs. 2(370) = 740.

Time = 5.50 (sec) , antiderivative size = 2299, normalized size of antiderivative = 6.03

method	result	size
derivativedivides	Expression too large to display	2299
default	Expression too large to display	2299
risch	Expression too large to display	4452
parallelrisch	Expression too large to display	5619

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^5*A*(-1/4*g^3/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)^4-g^2/(c*g-d
*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3-3/2*g/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c
)*d-g)^2-1/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g))+((c*g-d*f)*b^4*g^2*B*d/
(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^4)/(d*x+c)*ln(
e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+(c*g-d*f)*(c^2*g^2-2*c*d*f*g+d^2*f^2)*
b^4*B*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^4)/(d*
x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/2*(B*a^3*d^5*g^6-3*B*a^2*b
*d^5*f*g^5+3*B*a*b^2*d^5*f^2*g^4-B*b^3*c^3*d^2*g^6+3*B*b^3*c^2*d^3*f*g^5-3*
B*b^3*c*d^4*f^2*g^4)/g/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a
^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^
3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d
^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^
2*f^5*g+b^3*d^3*f^6)/(d*x+c)+1/12*(11*B*a^3*d^5*g^6-2*B*a^2*b*c*d^4*g^6-31*
B*a^2*b*d^5*f*g^5-3*B*a*b^2*c^2*d^3*g^6+10*B*a*b^2*c*d^4*f*g^5+26*B*a*b^2*d
^5*f^2*g^4-6*B*b^3*c^3*d^2*g^6+21*B*b^3*c^2*d^3*f*g^5-26*B*b^3*c*d^4*f^2*g^
4)/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/g^4/(d*x+c)^4-1/6*(13*B*a^
3*d^5*g^6-B*a^2*b*c*d^4*g^6-38*B*a^2*b*d^5*f*g^5-3*B*a*b^2*c^2*d^3*g^6+8*B*
a*b^2*c*d^4*f*g^5+34*B*a*b^2*d^5*f^2*g^4-9*B*b^3*c^3*d^2*g^6+30*B*b^3*c^2*d
^3*f*g^5-34*B*b^3*c*d^4*f^2*g^4)/g^3/(a^3*c*g^4-a^3*d*f*g^3-3*a^2*b*c*f*g^3
+3*a^2*b*d*f^2*g^2+3*a*b^2*c*f^2*g^2-3*a*b^2*d*f^3*g-b^3*c*f^3*g+b^3*d*f^4)
/(d*x+c)^3+1/4*(7*B*a^3*d^5*g^6-21*B*a^2*b*d^5*f*g^5-B*a*b^2*c^2*d^3*g^6+2*
B*a*b^2*c*d^4*f*g^5+20*B*a*b^2*d^5*f^2*g^4-6*B*b^3*c^3*d^2*g^6+19*B*b^3*c^2
*d^3*f*g^5-20*B*b^3*c*d^4*f^2*g^4)/(c^2*g^2-2*c*d*f*g+d^2*f^2)/g^2/(a^3*g^3
-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2+1/4*B*d*(a^4*d^4*g^3-4*a^3*
b*d^4*f*g^2+6*a^2*b^2*d^4*f^2*g-4*a*b^3*d^4*f^3-b^4*c^4*g^3+4*b^4*c^3*d*f*g
^2-6*b^4*c^2*d^2*f^2*g+4*b^4*c*d^3*f^3)/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^
2*g^2-4*a*b^3*f^3*g+b^4*f^4)/(d*x+c)^4*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d
^2)-1/4*b^4*g^3*B*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+
b^4*f^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-3/2*(c^2*g^2-2*c*d*f*g+d^2
*f^2)*b^4*g*B*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*
f^4)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x
+c)*d-g)^4-1/2*B*d*(a^4*d^4*g^3-4*a^3*b*d^4*f*g^2+6*a^2*b^2*d^4*f^2*g-4*a*b
```

```
^3*d^4*f^3-b^4*c^4*g^3+4*b^4*c^3*d*f*g^2-6*b^4*c^2*d^2*f^2*g+4*b^4*c*d^3*f^
3)/(a^4*c^4*g^8-4*a^4*c^3*d*f*g^7+6*a^4*c^2*d^2*f^2*g^6-4*a^4*c*d^3*f^3*g^5
+a^4*d^4*f^4*g^4-4*a^3*b*c^4*f*g^7+16*a^3*b*c^3*d*f^2*g^6-24*a^3*b*c^2*d^2*
f^3*g^5+16*a^3*b*c*d^3*f^4*g^4-4*a^3*b*d^4*f^5*g^3+6*a^2*b^2*c^4*f^2*g^6-24
*a^2*b^2*c^3*d*f^3*g^5+36*a^2*b^2*c^2*d^2*f^4*g^4-24*a^2*b^2*c*d^3*f^5*g^3+
6*a^2*b^2*d^4*f^6*g^2-4*a*b^3*c^4*f^3*g^5+16*a*b^3*c^3*d*f^4*g^4-24*a*b^3*c
^2*d^2*f^5*g^3+16*a*b^3*c*d^3*f^6*g^2-4*a*b^3*d^4*f^7*g+b^4*c^4*f^4*g^4-4*b
^4*c^3*d*f^5*g^3+6*b^4*c^2*d^2*f^6*g^2-4*b^4*c*d^3*f^7*g+b^4*d^4*f^8)*ln(c*
g/(d*x+c)-f/(d*x+c)*d-g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1809 vs. $2(367) = 734$.

Time = 0.32 (sec) , antiderivative size = 1809, normalized size of antiderivative = 4.75

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="maxima")
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
```

```
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
c*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
)*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(367) = 734$.

Time = 3.18 (sec) , antiderivative size = 2159, normalized size of antiderivative = 5.67

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*B*b^4*c*d^3*f^3 - 4*B*a*b^3*d^4*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 6*B*a
^2*b^2*d^4*f^2*g + 4*B*b^4*c^3*d*f*g^2 - 4*B*a^3*b*d^4*f*g^2 - B*b^4*c^4*g^
```

3 + B*a^4*d^4*g^3)*log(abs(b*d - 2*b*d*f/(g*x + f) + b*d*f^2/(g*x + f)^2 + b*c*g/(g*x + f) + a*d*g/(g*x + f) - b*c*f*g/(g*x + f)^2 - a*d*f*g/(g*x + f)^2 + a*c*g^2/(g*x + f)^2))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 1/4*(2*B*b^5*c*d^4*f^4*g - 2*B*a*b^4*d^5*f^4*g - 4*B*b^5*c^2*d^3*f^3*g^2 + 4*B*a^2*b^3*d^5*f^3*g^2 + 6*B*b^5*c^3*d^2*f^2*g^3 - 6*B*a*b^4*c^2*d^3*f^2*g^3 + 6*B*a^2*b^3*c*d^4*f^2*g^3 - 6*B*a^3*b^2*d^5*f^2*g^3 - 4*B*b^5*c^4*d*f*g^4 + 4*B*a*b^4*c^3*d^2*f*g^4 - 4*B*a^3*b^2*c*d^4*f*g^4 + 4*B*a^4*b*d^5*f*g^4 + B*b^5*c^5*g^5 - B*a*b^4*c^4*d*g^5 + B*a^4*b*c*d^4*g^5 - B*a^5*d^5*g^5)*log(abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) - abs(-b*c*g^2 + a*d*g^2))/abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) + abs(-b*c*g^2 + a*d*g^2)))/((b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8)*abs(-b*c*g^2 + a*d*g^2)) - 1/2*(3*B*b^3*c*d^2*f^2*g - 3*B*a*b^2*d^3*f^2*g - 3*B*b^3*c^2*d*f*g^2 + 3*B*a^2*b*d^3*f*g^2 + B*b^3*c^3*g^3 - B*a^3*d^3*g^3)/((b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6)*(g*x + f)*g) - 1/4*B*log((b^2*e - 2*b^2*e*f/(g*x + f) + b^2*e*f^2/(g*x + f)^2 + 2*a*b*e*g/(g*x + f) - 2*a*b*e*f*g/(g*x + f)^2 + a^2*e*g^2/(g*x + f)^2)/(d^2 - 2*d^2*f/(g*x + f) + d^2*f^2/(g*x + f)^2 + 2*c*d*g/(g*x + f) - 2*c*d*f*g/(g*x + f)^2 + c^2*g^2/(g*x + f)^2))/((g*x + f)^4*g) - 1/4*(2*B*b^2*c*d*f*g^2 - 2*B*a*b*d^2*f*g^2 - B*b^2*c^2*g^3 + B*a^2*d^2*g^3)/((b^2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4)*(g*x + f)^2*g^2) - 1/4*A/((g*x + f)^4*g) - 1/6*(B*b*c*g^3 - B*a*d*g^3)/((b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2)*(g*x + f)^3*g^3)

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 2520, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^5,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3
*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f
^3 + 4*B*b^4*c*d^3*f^3))/(2*a^4*c^4*g^8 + 2*b^4*d^4*f^8 + 2*a^4*d^4*f^4*g^4
 + 2*b^4*c^4*f^4*g^4 + 12*a^2*b^2*c^4*f^2*g^6 + 12*a^2*b^2*d^4*f^6*g^2 + 12
*a^4*c^2*d^2*f^2*g^6 + 12*b^4*c^2*d^2*f^6*g^2 - 8*a^3*b*c^4*f*g^7 - 8*a*b^3
*d^4*f^7*g - 8*a^4*c^3*d*f*g^7 - 8*b^4*c*d^3*f^7*g - 8*a*b^3*c^4*f^3*g^5 -
8*a^3*b*d^4*f^5*g^3 - 8*a^4*c*d^3*f^3*g^5 - 8*b^4*c^3*d*f^5*g^3 + 32*a*b^3*
c*d^3*f^6*g^2 + 32*a*b^3*c^3*d*f^4*g^4 + 32*a^3*b*c*d^3*f^4*g^4 + 32*a^3*b*
c^3*d*f^2*g^6 - 48*a*b^3*c^2*d^2*f^5*g^3 - 48*a^2*b^2*c*d^3*f^5*g^3 - 48*a^
2*b^2*c^3*d*f^3*g^5 - 48*a^3*b*c^2*d^2*f^3*g^5 + 72*a^2*b^2*c^2*d^2*f^4*g^4
) - ((3*A*a^3*c^3*g^6 + 3*A*b^3*d^3*f^6 - 3*A*a^3*d^3*f^3*g^3 - 3*A*b^3*c^3
*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 9*A*a*b^2*c^3*f^2*
g^4 + 9*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 9*A*a^3*c*d^2*f^2*g^4
 + 31*B*a^2*b*d^3*f^4*g^2 + 9*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*d^2*f^2*g^4 -
 31*B*b^3*c^2*d*f^4*g^2 - 9*A*a^2*b*c^3*f*g^5 - 9*A*a*b^2*d^3*f^5*g + 2*B*a
^2*b*c^3*f*g^5 - 9*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g - 9*A*b^3*c*d^2
*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 27*A*a*b^2*c*d^2*f^4*
g^2 - 27*A*a*b^2*c^2*d*f^3*g^3 - 27*A*a^2*b*c*d^2*f^3*g^3 + 27*A*a^2*b*c^2*
d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^3*g^3)/(6*(a^3*c^
3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*
f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 +
 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*
a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^2*g^6 + 7*B*a^3*d
^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21*B*a^2*b*d^3*f^2*
g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*B*a*b^2*c^2*d*f*g
^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^
3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f
*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^
3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2
*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(B*a^2*b*
c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^3*c^3*f^2*g^4 - 3
4*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*c*d^2*f^4*g^2 - 3
8*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^2*f*g^5 + 12*B*a*
b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3*g^6 + b^3*d^3*f^
6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5
```

```
*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*
d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4
*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^
4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3*f^2*g^4 - 3*B*b^3
*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5))/(a^3*c^3*g^6 +
 b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*
b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4
 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^
2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c
^2*d*f^2*g^4))/(2*f^4*g + 2*g^5*x^4 + 8*f^3*g^2*x + 8*f*g^4*x^3 + 12*f^2*g^
3*x^2) + (B*b^4*log(a + b*x))/(2*a^4*g^5 + 2*b^4*f^4*g - 8*a*b^3*f^3*g^2 +
12*a^2*b^2*f^2*g^3 - 8*a^3*b*f*g^4) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/
(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) - (B*d^4*lo
g(c + d*x))/(2*c^4*g^5 + 2*d^4*f^4*g - 8*c*d^3*f^3*g^2 + 12*c^2*d^2*f^2*g^3
 - 8*c^3*d*f*g^4)
```

3.272 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 869

$$\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx \\
&= \frac{2B^2(bc-ad)^3g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{b^3d^3} + \frac{B^2(bc-ad)^2g^3(c+dx)^2}{3b^2d^4} \\
&- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^3} \\
&- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4} \\
&- \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4} \\
&- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} \\
&- \frac{B(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^4} \\
&+ \frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{b^4d^4} \\
&+ \frac{2B^2(bc-ad)^4g^3\log(c+dx)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{b^4d^4} \\
&+ \frac{2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{b^4d^4} \\
&- \frac{2B^2(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4d^4}
\end{aligned}$$

```
[Out] 2/3*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g+4*b
*d*f)*x/b^3/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4-B*(-a*d+b*c)*g*(
a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*
x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^4/d^3-1/2*B*(-a*d+b*c)*g^2*(-a*d*g-3
*b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d^4-1/3*B*(-a
*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4-1/4*(-a*g+b*f)^
4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))^2/g-B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2
*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(
(-a*d+b*c)/b/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/
b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b
^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*
d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*
a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4
-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules

used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{2B^2g^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{3b^4d^4} + \frac{2B^2g^3\log(c+dx)(bc-ad)^4}{3b^4d^4}$$

$$+ \frac{2B^2g^3x(bc-ad)^3}{3b^3d^3} + \frac{B^2g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^3}{b^4d^4}$$

$$+ \frac{B^2g^2(4bdf-3bcg-adg)\log(c+dx)(bc-ad)^3}{b^4d^4}$$

$$+ \frac{B^2g^3(c+dx)^2(bc-ad)^2}{3b^2d^4} + \frac{B^2g^2(4bdf-3bcg-adg)x(bc-ad)^2}{b^3d^3}$$

$$+ \frac{2B^2g\left((6d^2f^2-8cdgf+3c^2g^2)b^2-2adg(2df-cg)b+a^2d^2g^2\right)\log(c+dx)(bc-ad)^2}{b^4d^4}$$

$$- \frac{Bg^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)(bc-ad)}{3bd^4}$$

$$- \frac{Bg^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)(bc-ad)}{2b^2d^4}$$

$$- \frac{Bg\left((6d^2f^2-8cdgf+3c^2g^2)b^2-2adg(2df-cg)b+a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)(bc-ad}{b^4d^3}$$

$$- \frac{B(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(}{b^4d^4}$$

$$- \frac{2B^2(2bdf-bcg-adg)\left(-\left((2d^2f^2-2cdgf+c^2g^2)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-}{b^4d^4}$$

$$- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (2*B^2*(b*c - a*d)^3*g^3*x)/(3*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f -
 3*b*c*g - a*d*g)*x)/(b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(3*b^2
*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^
2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c + d*
x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b^2*d^4) - (B*(b*c - a*d)
*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^4) - ((b*
f - a*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^4*g) + ((f + g*
x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*g) - (B*(b*c - a*d)*(2*
b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*
d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/
(b*(c + d*x))])/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x
```

```
)])/(3*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a
 + b*x)/(c + d*x)])/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(3*b
^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/
(b^4*d^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) +
 b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(b^4*d^4) - (2*B^2*
(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2
*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
b^4*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^3\,(A+B\log(ex^2))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{B\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^4(A+B\log(ex^2))}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$

$$- \frac{B\text{Subst}\left(\int \left(\frac{(bf-ag)^4(A+B\log(ex^2))}{b^4x} + \frac{(bc-ad)^4g^4(A+B\log(ex^2))}{bd^3(b-dx)^4} + \frac{(bc-ad)^3g^3(4bdf-3bcg-adg)(A+B\log(ex^2))}{b^2d^3(b-dx)^3} + \right.\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^3} - \frac{(B(bf-ag)^4)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g} - \frac{(B(bc-ad)^3g^2(4bdf-3bcg-adg))\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^3} + \frac{(B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{A+B\log}{b-d}\right.}{b^4d^3} - \frac{(B(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2)))\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^2}\,dx,\right.}{b^3d^3}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+b}{(c+d}\right.\right.}{b^4d^3} - \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4} - \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4} - \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+}{(c+d}\right.\right.}{b^4d^4} + \frac{(2B^2(bc-ad)^4g^3)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^4} + \frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg))\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^4} + \frac{(2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{\log(1}{}\right.}{b^4d^4} + \frac{(2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2)))\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+}{c+}\right.}{b^4d^3}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a\cdot}{(c+}\right.\right.}{b^4d^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4}$$

$$-\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4}$$

$$-\frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a}{(c+}\right.\right.}{b^4d^4}$$

$$+\frac{2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{b^4d^4}$$

$$-\frac{2B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4d^4}$$

$$+\frac{(2B^2(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-3bcg-adg))\,\mathrm{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^4}$$

$$= \frac{2B^2(bc-ad)^3g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{b^3d^3}$$
$$+ \frac{B^2(bc-ad)^2g^3(c+dx)^2}{3b^2d^4}$$
$$- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+b}{(c+d}\right.\right.}{b^4d^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4}$$
$$- \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4}$$
$$- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+}{(c+d}\right.\right.}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log(c+dx)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{b^4d^4}$$
$$- \frac{2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4d^4}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 746, normalized size of antiderivative = 0.86

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(6Abd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+6Bd(bc-ad)g^2(a^2\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(6*A*b*d*(b*
c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d
*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f +
 c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x)^2
)/(c + d*x)^2] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A +
```

```
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2
 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c +
d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c +
 d*x] + 2*B*(b*c - a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*
a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 6*B*(b*c - a*d)*g^3*(-4*b*
d*f + b*c*g + a*d*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c
^2*Log[c + d*x])) - 6*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 6*b^4*B*(d*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))
/(4*g)
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2
+ 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. $2(852) = 1704$.

Time = 0.35 (sec) , antiderivative size = 2351, normalized size of antiderivative = 2.71

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^3 +
3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c
^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f^2*g + 2*(x^3*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/
(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^
2 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b
^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x
 - 1/3*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d
^3*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (12*c*d^3*f^3*log(e) - (3*g
^3*log(e) + 11*g^3)*c^4 + 12*(f*g^2*log(e) + 3*f*g^2)*c^3*d - 18*(f^2*g*log
(e) + 2*f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 2*(4*a*b^3*d^4*f^
3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^3 -
6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) +
1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 4*(a*b^3*d^4*g^3*log(e) + (3*d^4*f*g
^2*log(e)^2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - 2*((3*g^3*log(e) - 2*g^3)*a^
2*b^2*d^4 - 4*(3*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (9*d^4*f^2*g*log(e)^
2 - 12*c*d^3*f*g^2*log(e) + (3*g^3*log(e) + 2*g^3)*c^2*d^2)*b^4)*B^2*x^2 +
```

```
4*((3*g^3*log(e) - 5*g^3)*a^3*b*d^4 + (5*c*d^3*g^3 - 12*(f*g^2*log(e) - f*g
^2)*d^4)*a^2*b^2 + (18*d^4*f^2*g*log(e) - 24*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*a
*b^3 + (3*d^4*f^3*log(e)^2 - 18*c*d^3*f^2*g*log(e) - (3*g^3*log(e) + 5*g^3)
*c^3*d + 12*(f*g^2*log(e) + f*g^2)*c^2*d^2)*b^4)*B^2*x + 12*(B^2*b^4*d^4*g^
3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f
^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4
*g^3)*B^2)*log(b*x + a)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x
^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d
^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*B^2*b^4)*log(d*x + c)^2 + 4*(3*B^2*b^4*
d^4*g^3*x^4*log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*log(e) - c*d^3*g^3)*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*log(e) -
 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b
^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^
2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x - ((3*g^3*log(e) - 11*g^3)*a^4*d^4 + 2*(c*d
^3*g^3 - 6*(f*g^2*log(e) - 3*f*g^2)*d^4)*a^3*b - 3*(4*c*d^3*f*g^2 - c^2*d^2
*g^3 - 6*(f^2*g*log(e) - 2*f^2*g)*d^4)*a^2*b^2 - 6*(2*d^4*f^3*log(e) - 6*c*
d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*log(b*x + a) - 4*(3*B^
2*b^4*d^4*g^3*x^4*log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*log(e) - c*d^3*g
^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*lo
g(e) - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4
*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*
c^2*d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*
d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^
4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^4*d^4)
```

Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.273 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 542

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2} - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} - \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} + \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^3d^3} + \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{4B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} + \frac{8B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3} + \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2-4/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d
*f)*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^3/d^2-2/3*B*(-a*d+b*c)*g^2*(d
*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2
/g+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*
g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^3/
```

```
d^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/b^3/d^3+4/3*B^2*(-a*d+b*c)
^3*g^2*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln
(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$$
$$=\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b^3d^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$
$$-\frac{4Bg(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b^3d^2}$$
$$-\frac{(bf-ag)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b^3g}-\frac{2Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd^3}$$
$$+\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g}+\frac{8B^2g(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}$$
$$+\frac{4B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{4B^2g^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{4B^2g^2x(bc-ad)^2}{3b^2d^2}$$

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) - (4*B*(b*c - a*d)*g*(3*b*d*f - 2*b
*c*g - a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b^3*d^
2) - (2*B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(3*b*d^3) - ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)
/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*g)
 + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 -
 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c -
a*d)/(b*(c + d*x))])/(3*b^3*d^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(
c + d*x)])/(3*b^3*d^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b^3*d^3)
 + (8*B^2*(b*c - a*d)^2*g*(3*b*d*f - 2*b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*
d^3) + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2
*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^
3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
```

```
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)^2\,(A+B\log(ex^2))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^3\,(A+B\log(ex^2))}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B)\text{Subst}\left(\int \left(\frac{(bf-ag)^3\,(A+B\log(ex^2))}{b^3x} + \frac{(bc-ad)^3g^3\,(A+B\log(ex^2))}{bd^2(b-dx)^3} + \frac{(bc-ad)^2g^2(3bdf-2bcg-adg)\,(A+B\log(ex^2))}{b^2d^2(b-dx)^2}\right.}{3g}$$

$$= \frac{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^2} - \frac{(4B(bf-ag)^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^3g} - \frac{(4B(bc-ad)^2g(3bdf-2bcg-adg))\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2d^2} - \frac{(4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{b-dx}\,dx, x\right.}{3b^3d^2}$$

$$= -\frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2} - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} - \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} + \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log}{3b^3d^3} + \frac{(4B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd^3} + \frac{(8B^2(bc-ad)^2g(3bdf-2bcg-adg))\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^3d^2} - \frac{(8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x,\right.}{3b^3d^3}$$

$$= -\frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2}$$
$$-\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3}$$
$$-\frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g}$$
$$+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\right.}{3b^3d^3}$$
$$+\frac{8B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$
$$+\frac{(4B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}$$

$$=\frac{4B^2(bc-ad)^2g^2x}{3b^2d^2}$$
$$-\frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2}$$
$$-\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3}$$
$$-\frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g}$$
$$+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\right.}{3b^3d^3}$$
$$+\frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{4B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3}$$
$$+\frac{8B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.92

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{}}{}$$

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(2*A*b*d*(b*
c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f -
 b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + b^2*d^2*(b*c -
 a*d)*g^3*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f - a*g)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 4*B*(b*c - a*d)^2
*g^2*(-3*b*d*f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*(a^2*d^
2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B*d^3*(b*
f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*g)^3*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
```

```
+ 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*
e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. 2(521) = 1042.

Time = 0.33 (sec) , antiderivative size = 1458, normalized size of antiderivative = 2.69

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*e*x^2/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 -
2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^
2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 4/3*(2*a
^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2*log(e) + (g^2*l
og(e) + 3*g^2)*c^3 - 3*(f*g*log(e) + 2*f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b
^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f
^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^
3*g^2*x^3*log(e)^2 + (2*a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - 2*c*d^
2*g^2*log(e))*b^3)*B^2*x^2 - (4*(g^2*log(e) - g^2)*a^2*b*d^3 - 4*(3*d^3*f*g
*log(e) - 2*c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 12*c*d^2*f*g*log(e) +
4*(g^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3
*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a
^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*
```

```
g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3
)*log(d*x + c)^2 + 4*(B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*
f*g*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 +
 (3*d^3*f^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + ((g^2*log(e) -
 3*g^2)*a^3*d^3 + (c*d^2*g^2 - 3*(f*g*log(e) - 2*f*g)*d^3)*a^2*b + (3*d^3*f
^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - 4*(B^2*b^
3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*f*g*log(e) - c*d^2*g^2)*b^3)
*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 6*c*d^2
*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g
*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g
^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^3*d^3)
```

Giac [F]

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (f+g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 29, antiderivative size = 281

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx \\
&= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d} \\
&\quad -\frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} \\
&\quad +\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} \\
&\quad +\frac{4B^2(bc-ad)^2g\log(c+dx)}{b^2d^2}+\frac{4B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))^2/g+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+4*B^2*(-a*d+b*c)^2*g*ln(d*
x+c)/b^2/d^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/
(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b^2d^2}$$
$$-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b^2g}$$
$$-\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g}$$
$$+\frac{4B^2(bc-ad)(-adg-bcg+2bdf)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{4B^2g(bc-ad)^2\log(c+dx)}{b^2d^2}$$

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^
2*d) - ((b*f - a*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b^2*g)
 + ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*g) + (2*B*(b
*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
)*Log[(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (4*B^2*(b*c - a*d)^2*g*Log[c
+ d*x])/(b^2*d^2) + (4*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(bf-ag-(df-cg)x)\left(A+B\log\left(ex^2\right)\right)^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)$$

$$=\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g}-\frac{(2B)\text{Subst}\left(\int \frac{(bf-ag+(-df+cg)x)^2\left(A+B\log\left(ex^2\right)\right)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B)\text{Subst}\left(\int \left(\frac{(bf-ag)^2(A+B\log(ex^2))}{b^2x} + \frac{(bc-ad)^2g^2(A+B\log(ex^2))}{bd(b-dx)^2} + \frac{(bc-ad)g(2bdf-bcg-adg)(A+B\log(ex^2))}{b^2d(b-dx)}\right)\right.}{g}$$

$$= \frac{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad)^2g)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd} - \frac{(2B(bf-ag)^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^2)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2B(bc-ad)(2bdf-bcg-adg))\text{Subst}\left(\int \frac{A+B\log(ex^2)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d}$$

$$= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d} - \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} + \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} + \frac{(4B^2(bc-ad)^2g)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d} - \frac{(4B^2(bc-ad)(2bdf-bcg-adg))\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^2}$$

$$= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d} - \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} + \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{b^2d^2} + \frac{4B^2(bc-ad)(2bdf-bcg-adg)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4B\left(Abd(bc-ad)g^2x+Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+d^2(bf-ag)^2\log(a+bx)\left(A+B\log\right.\right.}{}}{}$$

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(A*b*d*(b*c
- a*d)*g^2*x + B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^
2] + d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
) - 2*B*(b*c - a*d)^2*g^2*Log[c + d*x] - b^2*(d*f - c*g)^2*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^2*d^2))/(2*g)
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2 + 2
*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(276) = 552$.

Time = 0.31 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.80

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{1}{2}A^2gx^2$$
$$+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log(d}{d^2}$$
$$+A^2fx-\frac{2\,(2\,acdg+(2\,cdf\log(e)-(g\log(e)+2\,g)c^2)b)B^2\log(dx+c)}{bd^2}$$
$$+\frac{4\,(2\,abd^2f-a^2d^2g-(2\,cdf-c^2g)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(2\,abd^2g\log(e)+\left(d^2f\log(e)^2-2\,cdg\log(e)\right)b^2\right)B^2x+4\,(B^2b^2d^2gx^2+2\,B^2b^2d}{}$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*A*B*f + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2
)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d
))*A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + 2*g)*c^2)
*b)*B^2*log(d*x + c)/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*
g)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*a
*b*d^2*g*log(e) + (d^2*f*log(e)^2 - 2*c*d*g*log(e))*b^2)*B^2*x + 4*(B^2*b^2
*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a
)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)
*log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g + (d^2*f*log(e
) - c*d*g)*b^2)*B^2*x - ((g*log(e) - 2*g)*a^2*d^2 - 2*(d^2*f*log(e) - c*d*g
```

```
)*a*b)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g + (d^
2*f*log(e) - c*d*g)*b^2)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x +
 (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.275 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{8B^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4*B*(-a*d+b*c)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d+8*B^2*(-a*d+b*c)*polylog(2,d*(
b*x+a)/b/(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{4B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b} + \frac{8B^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/b + (4*B*(b*c - a*d)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(
b*d) + (8*B^2*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} - \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx}\,dx}{b}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} - \frac{(8B^2(bc-ad)^2)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{bd}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} - \frac{(8B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx, x, c+dx\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{(8B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{(8B^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{8B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.71

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{4B\left(ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-bc\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-aBd\left(\log(a+bx)\right.\right.}{}$$

[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]

[Out] x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(a*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b*c*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)

Maple [F]

$$\int \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2, x)

Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x +
 c)/d)/e)*A*B + A^2*x + B^2*(4*(b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*
x + c)^2 - (b*d*x*log(e) + 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d
) + integrate(((log(e)^2 + 4*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*lo
g(e)^2 + (log(e)^2 + 4*log(e))*a*b*d)*x + 4*(b^2*d*x^2*log(e) + a*b*c*log(e
) + 2*a^2*d + (a*b*d*(log(e) + 4) + b^2*c*(log(e) - 2))*x)*log(b*x + a))/(b
^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 285

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{8B^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-4*B*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+4*B*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+8*
B^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-8*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/g
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2554, 2404, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \frac{4B\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g} + \frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g} - \frac{4B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g} - \frac{8B^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[(b*c - a*d)/(b*(c + d*x))
])/g) + ((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/g - (4*B*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (4*B*(A + B*Log[(e*(
a + b*x)^2)/(c + d*x)^2])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/g + (8*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (8*B^2*
PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{(b-dx)(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= (bc-ad)\text{Subst}\left(\int \left(\frac{d(A+B\log(ex^2))^2}{(bc-ad)g(b-dx)} + \frac{(-df+cg)(A+B\log(ex^2))^2}{(bc-ad)g(bf-ag-(df-cg)x)}\right)dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{g} + \frac{((-bc+ad)(df-cg))\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g}
\end{aligned}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{(4B)\text{Subst}\left(\int\frac{(A+B\log(ex^2))\log\left(1-\frac{dx}{b}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{g} - \frac{(4B)\text{Subst}\left(\int\frac{(A+B\log(ex^2))\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{(8B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{g} - \frac{(8B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{8B^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{8B^2\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1370 vs. 2(285) = 570.

Time = 0.42 (sec) , antiderivative size = 1370, normalized size of antiderivative = 4.81

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$$
$$=\frac{-4B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)+A^2\log(f+gx)-4AB\log\left(\frac{a}{b}+x\right)\log(f+gx)+4B^2\log^2\left(\frac{a}{b}+\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

```
[Out] (-4*B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f
 - c*g)*(a + b*x))]^2 + A^2*Log[f + g*x] - 4*A*B*Log[a/b + x]*Log[f + g*x]
+ 4*B^2*Log[a/b + x]^2*Log[f + g*x] + 4*A*B*Log[c/d + x]*Log[f + g*x] - 8*B
^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + 4*B^2*Log[c/d + x]^2*Log[f + g*
x] + 2*A*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] - 4*B^2*Log[a/b +
x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + 4*B^2*Log[c/d + x]*Log[(
e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + B^2*Log[(e*(a + b*x)^2)/(c + d*x
)^2]^2*Log[f + g*x] + 4*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a*g)] - 4
*B^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 4*B^2*Log[a/b + x]*Log
[(e*(a + b*x)^2)/(c + d*x)^2]*Log[(b*(f + g*x))/(b*f - a*g)] + 8*B^2*Log[a/
b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f - a*g)] - 4
*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 8
*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f -
c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] - 4*B^2*Log[((b*f - a*g)*(c
 + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 4*A*B*
Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] + 8*B^2*Log[a/b + x]*Log[c/d +
x]*Log[(d*(f + g*x))/(d*f - c*g)] - 4*B^2*Log[c/d + x]^2*Log[(d*(f + g*x))/
(d*f - c*g)] - 4*B^2*Log[c/d + x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[(d*(
f + g*x))/(d*f - c*g)] - 8*B^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g
)]*Log[(d*(f + g*x))/(d*f - c*g)] + 4*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]
^2*Log[(d*(f + g*x))/(d*f - c*g)] - 8*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]
*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*
f - c*g)] + 4*B^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Lo
g[((-(b*c) + a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))] + 4*B*(A + B*Log[(e*(
a + b*x)^2)/(c + d*x)^2] + 2*B*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a
+ b*x))])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] - 4*B*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2] + 2*B*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))])*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 8*B^2*Log[((b*f - a*g)*(c
 + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] +
 8*B^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, ((b*
f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*x
```

```
))/(d*(a + b*x))] - 8*B^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(
a + b*x))])/g
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{gx+f}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="fricas")
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="maxima")
[Out] A^2*log(g*x + f)/g - integrate(-(4*B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 4*(B^2*log(e) + A*B)*log(b*x + a) - 4*(2*B^2*log(b*x + a) + B^2*
log(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{f+g\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x),x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x), x)
```

3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 200

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{8B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*g+b*f)/(g*x+f)+4*B*(-a*d+b*c)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)/(-c*g+d*f)+8*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g
+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2554, 2355, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$$

$$=\frac{4B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)(bf-ag)}+\frac{8B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*f - a*g)*(f + g*
x)) + (4*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d
*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) +
(8*B^2*(b*c - a*d)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x
))])/((b*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^2))^2}{(bf-ag+(-df+cg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} - \frac{(4B(bc-ad))\text{Subst}\left(\int \frac{A+B\log(ex^2)}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bf-ag}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} - \frac{(8B^2(bc-ad))\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

$$= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{8B^2(bc-ad)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(200) = 400$.

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.04

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$$

$$= \frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx} + \frac{4B\left(b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+(bc-ad)g\left(A+\right.\right.}{}}{}$$

[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]

[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)) + (4*B*(b*(d*f - c*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - d*(b*f - a*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + (b*c - a*d)*g*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g

Maple [F]

$$\int \frac{\left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^2}dx$$

[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)

[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)

Fricas [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}{(gx+f)^2}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="fricas")

[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \text{Timed out}$$

[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="maxima
")
```

```
[Out] 2*A*B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2)
+ 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^
2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a
^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - B^2*(4*log(d*x + c)^2/(g^2
*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g)*log
(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*log(e) - 2
*g)*d*x + c*g*log(e) - 2*d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/
(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x),
x)) - A^2/(g^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2, x)
```

3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 381

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx &= \frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2}\\&-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{4B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}\\&+\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\\&+\frac{4B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\end{aligned}$$

```
[Out] 2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^2+4*B^2*(-a*d+b*c)^2*g*ln((g*x+
f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)^2/(-c*g+d*f)^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog
(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}dx \\
&=\frac{b^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)} \\
&+\frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(bf-ag)^2(df-cg)^2} \\
&-\frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(f+gx)^2} \\
&+\frac{4B^2(bc-ad)(-adg-bcg+2bdf)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} \\
&+\frac{4B^2g(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] (2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*
f - a*g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])^2)/(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/
(2*g*(f + g*x)^2) + (4*B^2*(b*c - a*d)^2*g*Log[(f + g*x)/(c + d*x)])/((b*f
- a*g)^2*(d*f - c*g)^2) + (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f -
a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (4*B^2*(b*c - a*d)*(2*b*d
*f - b*c*g - a*d*g)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*
x))])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)\left(A+B\log\left(ex^2\right)\right)^2}{(bf-ag-(df-cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$=-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{(2B)\text{Subst}\left(\int\frac{(b-dx)^2(A+B\log(ex^2))}{x(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{g}$$

$$=-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{(2B)\text{Subst}\left(\int\left(\frac{b^2(A+B\log(ex^2))}{(bf-ag)^2x}+\frac{(bc-ad)^2g^2(A+B\log(ex^2))}{(bf-ag)(df-cg)(bf-ag-(df-cg)x)^2}+\frac{(bc-ad)g(-2bdf+bcg+adg)(A+B\log(ex^2))}{(bf-ag)^2(df-cg)(bf-ag-(df-cg)x)}\right.\right.}{g}$$

$$=-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{(2b^2B)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g(bf-ag)^2}+\frac{(2B(bc-ad)^2g)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)}-\frac{(2B(bc-ad)(2bdf-bcg-adg))\text{Subst}\left(\int\frac{A+B\log(ex^2)}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)}$$

$$=\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}-\frac{(4B^2(bc-ad)^2g)\,\text{Subst}\left(\int\frac{1}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)}-\frac{(4B^2(bc-ad)(2bdf-bcg-adg))\,\text{Subst}\left(\int\frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

$$=\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{4B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}+\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}+\frac{4B^2(bc-ad)(2bdf-bcg-adg)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B(f+gx)\left((bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) + d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*
x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Pol
yLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])
))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^3}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b
*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*
d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f
^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
 c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - B^2*(2*log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x +
c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*lo
g(e) - g)*d*x + c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3
)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^3}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3, x)
```

3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 724

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx \\
&= \frac{4B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2} \\
&\quad + \frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad - \frac{4B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{8B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-}{(bf-}\right.}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}
$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-2/3*B*(-
a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(-c*g+d*f
)^3/(g*x+f)^2+4/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^
```

```
2/g/(g*x+f)^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3-4/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d
*f)^3+8/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(
-a*g+b*f)^3/(-c*g+d*f)^3+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)
+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-
c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+8/3*B^2*(-a*
d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))
*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$$
$$=\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{b^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bf-ag)^3}$$
$$-\frac{2Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(f+gx)^3}$$
$$+\frac{4Bg(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}$$
$$+\frac{4B^2g^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{4B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$-\frac{4B^2g^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{8B^2g(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x
)) - (2*B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (4*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3
*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x)^2)/(
```

```
c + d*x)^2])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])^2/(3*g*(f + g*x)^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)]
)/(3*(b*f - a*g)^3*(d*f - c*g)^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/
(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)^2*g*(3*b*d
*f - b*c*g - 2*a*d*g)*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g
)^3) + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f
^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 -
 ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f -
c*g)^3) + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*
d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f -
a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^2\left(A+B\log\left(ex^2\right)\right)^2}{(bf-ag-(df-cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B)\text{Subst}\left(\int \frac{(b-dx)^3\left(A+B\log\left(ex^2\right)\right)}{x(bf-ag+(-df+cg)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B)\text{Subst}\left(\int\left(\frac{b^3(A+B\log(ex^2))}{(bf-ag)^3x} + \frac{(-bc+ad)^3g^3(A+B\log(ex^2))}{(bf-ag)(df-cg)^2(bf-ag-(df-cg)x)^3} + \frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(A+B\log(ex^2}{(bf-ag)^2(df-cg)^2(bf-ag-(df-cg)x)^2}\right.\right.}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4b^3B)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3g(bf-ag)^3}$$
$$-\frac{(4B(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^2}$$
$$+\frac{(4B(bc-ad)^2g(3bdf-bcg-2adg))\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bf-ag)^2(df-cg)^2}$$
$$-\frac{(4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{bf-ag+(-df+cg)x}\,dx\right.}{3(bf-ag)^3(df-cg)^2}$$

$$= -\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{(4B^2(bc-ad)^3g^2)\,\text{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^3}$$
$$-\frac{(8B^2(bc-ad)^2g(3bdf-bcg-2adg))\,\text{Subst}\left(\int\frac{1}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^2}$$
$$-\frac{(8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right))\,\text{Subst}\left(\int\frac{\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,d\right.}{3(bf-ag)^3(df-cg)^3}$$

$$= -\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{8B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{(4B^2(bc-ad)^3g^2)\operatorname{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)}\right)dx,x,}{3(bf-ag)(df-cg)^3}$$

$$=\frac{4B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}-\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}$$
$$+\frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{4B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{8B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bdf+\dots}{\dots}}{\dots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
+ 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f
+ g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*b^3*(d*f - c*g)^3*(f +
g*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f -
 a*g)^3*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] -
 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3
*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*L
og[f + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d
*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Lo
g[f + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f -
 c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g
*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f
 + g*x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] -
2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 4*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] -
Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/
(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f
 - c*g)^3))/(g*(f + g*x)^3)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^4}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 2/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
```

```
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(4*log(
d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3
*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 12*(d*g*x + c*g)*log(b*x + a)^2 + 12*
(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((3*g*log(e) - 2*g)*d*x + 3*c*
g*log(e) - 2*d*f + 6*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^5*x^5 +
 c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2
*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 1/3*A^2/(
g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^4}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4, x)
```

3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1154

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=-\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$-\frac{2B^2(bc-ad)^3g^3(c+dx)}{3(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$
$$-\frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+\frac{2B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+\frac{2B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{lo}}{(bf-ag)^4(df-cg)^4}$$
$$-\frac{2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx}{(bf-ag)(c+dx}\right.}{(bf-ag)^4(df-cg)^4}$$

```
[Out] -1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-2/3
*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+B^2*(-a*d+b
*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f
)+1/3*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)
/(-c*g+d*f)^4/(g*x+f)^3-1/2*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2+B*
(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6
*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^4/(-c*g+d*f)^
3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^4-1/4*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^4-2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+
a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+
```

```
4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2/3*B^2*(-a*d+b*c)^4
*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B^2*(-a*d+b*c)^3*g^2*(-3
*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2*B^2*(
-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+
6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*
f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2
*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*g
+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 1154, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules

used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 b^4}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$
$$+ \frac{B(bc-ad)g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^3(f+gx)}$$
$$- \frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+ \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$
$$+ \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4} - \frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}$$
$$- \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+ \frac{2B^2(bc-ad)^2g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.}{(bf-ag)^4(df-cg)^4}$$
$$- \frac{2B^2(bc-ad)(2bdf-bcg-adg)\left(-\left(\left(2d^2f^2-2cdgf+c^2g^2\right)b^2\right)+2ad^2fgb-a^2d^2g^2\right)\mathrm{PolyLog}\left(2,\frac{(df-cg)}{(bf-ag)}\right.}{(bf-ag)^4(df-cg)^4}$$
$$+ \frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{(bf-ag)^3(df-cg)^4(f+gx)}$$
$$- \frac{2B^2(bc-ad)^3g^3(c+dx)}{3(bf-ag)^3(df-cg)^4(f+gx)} - \frac{B^2(bc-ad)^2g^3(c+dx)^2}{3(bf-ag)^2(df-cg)^4(f+gx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -1/3*(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*(f +
g*x)^2) - (2*B^2*(b*c - a*d)^3*g^3*(c + d*x))/(3*(b*f - a*g)^3*(d*f - c*g)^
4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)
)/((b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3*(c + d*x)^3*
(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)^4*(f +
 g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*
x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*
d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/((b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A + B*Log[(e*(a +
```

```
b*x)^2)/(c + d*x)^2])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(
c + d*x)^2])^2/(4*g*(f + g*x)^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(
c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*
f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4
) + (2*B^2*(b*c - a*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*
f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g
*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(3*a
^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2
))*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 -
2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*
f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4)
 - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^
2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x
))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{(b-dx)^3\left(A+B\log\left(ex^2\right)\right)^2}{(bf-ag-(df-cg)x)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{B\text{Subst}\left(\int \frac{(b-dx)^4\left(A+B\log\left(ex^2\right)\right)}{x(bf-ag+(-df+cg)x)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{B\text{Subst}\left(\int\left(\frac{b^4(A+B\log(ex^2))}{(bf-ag)^4x} + \frac{(bc-ad)^4g^4(A+B\log(ex^2))}{(bf-ag)(df-cg)^3(bf-ag-(df-cg)x)^4} + \frac{(bc-ad)^3g^3(-4bdf+bcg+3adg)(A+B\log(ex^2}{(bf-ag)^2(df-cg)^3(bf-ag-(df-cg)x)^3}\right.\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(b^4B)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{g(bf-ag)^4}$$
$$+ \frac{(B(bc-ad)^4g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)(df-cg)^3}$$
$$- \frac{(B(bc-ad)^3g^2(4bdf-bcg-3adg))\,\text{Subst}\left(\int\frac{A+B\log(ex^2)}{(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)^3}$$
$$+ \frac{(B(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)))\,\text{Subst}\left(\int\frac{A+B\log(ex^2}{(bf-ag+(-df+cg}\right.}{(bf-ag)^3(df-cg)^3}$$
$$+ \frac{(B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)))\,\text{Subst}\left(\int\frac{A+}{bf-ag}\right.}{(bf-ag)^4(df-cg)^3}$$

$$= \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$

$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+b}{(c+dx}\right.\right.}{(bf-ag)^4(df-cg)^3(f+gx)}$$

$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+}{(c+d}\right.\right.}{(bf-ag)^4(df-cg)^4}$$

$$-\frac{(2B^2(bc-ad)^4g^3)\,\mathrm{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bf-ag)(df-cg)^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg))\,\mathrm{Subst}\left(\int\frac{1}{x(bf-ag+(-df+cg)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)^2(df-cg)^4}$$

$$-\frac{(2B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2)))\,\mathrm{Subst}\left(\int\frac{1}{bf-ag+(-df+cg}\right.}{(bf-ag)^4(df-cg)^3}$$

$$+\frac{(2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right))\,\mathrm{Subst}\left(\int\frac{\log(1}{}\right.}{(bf-ag)^4(df-cg)^4}$$

$$= \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$

$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$

$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a\cdots}{(c+\cdots}\right.\right.}{(bf-ag)^4(df-cg)^3(f+gx)}$$

$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$

$$+\frac{2B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$

$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a\cdots}{(c+\cdots}\right.\right.}{(bf-ag)^4(df-cg)^4}$$

$$-\frac{2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+\cdots}{(bf-ag)(c+\cdots}\right.}{(bf-ag)^4(df-cg)^4}$$

$$-\frac{(2B^2(bc-ad)^4g^3)\operatorname{Subst}\left(\int\left(\frac{1}{(bf-ag)^3x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^3}+\frac{df-cg}{(bf-ag)^2(bf-ag-(df-cg)x)^2}+\frac{\cdots}{(bf-\cdots}\right.\right.}{3(bf-ag)(df-cg)^4}$$

$$+\frac{(B^2(bc-ad)^3g^2(4bdf-bcg-3adg))\operatorname{Subst}\left(\int\left(\frac{1}{(bf-ag)^2x}+\frac{df-cg}{(bf-ag)(bf-ag-(df-cg)x)^2}+\frac{df\cdots}{(bf-ag)^2(bf-\cdots}\right.\right.}{(bf-ag)^2(df-cg)^4}$$

$$= -\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3(bf-ag)^2(df-cg)^4(f+gx)^2} - \frac{2B^2(bc-ad)^3g^3(c+dx)}{3(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+ \frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+ \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+ \frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+b}{(c+d}\right.\right.}{(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+ \frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(bf-ag)^4} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$
$$- \frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4} + \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4} - \frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+ \frac{2B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a+}{(c+d}\right.\right.}{(bf-ag)^4(df-cg)^4}$$
$$- \frac{2B^2(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\operatorname{Li}_2\left(\frac{(df-cg)(a+bx}{(bf-ag)(c+dx}\right.}{(bf-ag)^4(df-cg)^4}$$

Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 1317, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx =$$
$$-\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{2B(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2bc}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*(2*(b*
c - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*
```

```
g)*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*(b*c - a*d)*g*(b*
f - a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]) - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(
-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f +
g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 12*B*(b*c -
a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log
[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 6*B*(b*c - a*d)*g*(2*b*d*f - b*c*
g - a*d*g)*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f -
c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] +
(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + 2*B*(b*c
 - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a
*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b
^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2
*Log[c + d*x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2
*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*Log[f + g*x]) + 6*b^4*B*(d*
f - c*g)^4*(f + g*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*B*d^4*(b*f -
a*g)^4*(f + g*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Lo
g[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 12*B*(b*c - a*d)*g*
(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 -
 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Lo
g[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b
*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f -
 c*g)^4))/(g*(f + g*x)^4)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^5}dx$$

[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)

[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*g*x
 + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="maxima
")
```

```
[Out] 1/6*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
+ 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d^
4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f*
g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4*
(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b^
2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d
^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d
^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
```

```
a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2*
g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b*
c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c^
3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2*
d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^3
*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d
 + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g^
4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3*
(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3*
(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^3
)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*c
^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^3
*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^
2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^
4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*e*x^2/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g
^2*x + f^4*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g)
*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((2*g*log(
e) - g)*d*x + 2*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g^4 + c*f*g
^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3*g^3)*x^2
+ (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*
g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{\left(f+g\,x\right)^5}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5, x)
```

3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(
d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 21.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(f+gx)^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral((f + g*x)**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*
b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 6.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral((f + g*x)/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e
*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))
), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

```
[In] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral(1/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A), x)
```

Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral(1/(A + B*log(e*(a + b*x)**2/(c + d*x)**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 7.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \operatorname{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3}\left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{\left(gx+f\right)^{3}\left(B\log\left(\frac{\left(bx+a\right)^{2}e}{\left(dx+c\right)^{2}}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d
^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 12.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(
d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 10.35

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
[Out] -1/2*(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f
*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*
x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c
- a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b*d*g^2*x^3
+ b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2
*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/(2*(b*c - a*d)*B^2*log(b*x
+ a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d
*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.59

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
[Out] integral((g*x + f)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 26.30 (sec) , antiderivative size = 729, normalized size of antiderivative = 25.14

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$= \frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$
$$-\frac{\int \frac{acg}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{adf}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+}{}$$

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a
+ b*x)**2/(c + d*x)**2)) - (Integral(a*c*g/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(a*d*f/(A + B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f/(A + B*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*d*g*x/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*c*g*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*f*x/
(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x
+ d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*
b*d*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(
2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.86

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g
)*a)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b*d*g*x
^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(2*(b*c - a*d)*B^
2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*lo
g(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.26

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
[Out] integral(1/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.94 (sec) , antiderivative size = 333, normalized size of antiderivative = 14.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{ac+adx+bcx+bdx^2}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{\int \frac{ad}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{bc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \cdots}{2B\,(ad-bc)}$$

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B
**2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c/(A + B*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) +
 b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.57

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b
*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^
2) + integrate(1/2*(2*b*d*x + b*c + a*d)/(2*(b*c - a*d)*B^2*log(b*x + a) -
2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e)
)*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2
+ 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.68

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) -
 a*d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B
^2)*x + 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - 2*((
b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate(1/2*(
b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (
b*c*f^2*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*
log(e) - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*l
og(e) - a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*
g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 -
a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*l
og(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 8.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 690, normalized size of antiderivative = 22.26

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*lo
g(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a
*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*
d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*
g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 - a*d*g^2)*B
^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c
)) - integrate(-1/2*(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)
/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (
b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*
g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*(
(b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x
+ 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b
*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b
*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*
g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 32.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.61

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x
^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2
+ 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 924, normalized size of antiderivative = 29.81

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*l
og(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e
) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e
) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*
g*log(e) - a*d*f^2*g*log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b
*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
- a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^
2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f
^3)*B^2)*log(d*x + c)) - integrate(1/2*(b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a
 + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e)
 - a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*l
og(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log
(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g
^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B +
 (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + 2*((b*c*g^4 - a*d*g^4)*B^2*
x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2
*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*x +
 a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 +
6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (
b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 48.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.293 $\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 365

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(10d^2g^2-5cdgh+c^2h^2)-b^3(10d^3g^3-10cd^2g^2h+5c^2}{5b^4d^4}$$

$$-\frac{B(bc-ad)h^2\left(a^2d^2h^2-abdh(5dg-ch)+b^2(10d^2g^2-5cdgh+c^2h^2)\right)nx^2}{10b^3d^3}$$

$$-\frac{B(bc-ad)h^3(5bdg-bch-adh)nx^3}{15b^2d^2}-\frac{B(bc-ad)h^4nx^4}{20bd}-\frac{B(bg-ah)^5n\log(a+bx)}{5b^5h}$$

$$+\frac{B(dg-ch)^5n\log(c+dx)}{5d^5h}+\frac{(g+hx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h}$$

```
[Out] 1/5*B*(-a*d+b*c)*h*(a^3*d^3*h^3-a^2*b*d^2*h^2*(-c*h+5*d*g)+a*b^2*d*h*(c^2*h
^2-5*c*d*g*h+10*d^2*g^2)-b^3*(-c^3*h^3+5*c^2*d*g*h^2-10*c*d^2*g^2*h+10*d^3*
g^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*h^2*(a^2*d^2*h^2-a*b*d*h*(-c*h+5*d*g)+b
^2*(c^2*h^2-5*c*d*g*h+10*d^2*g^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*h^3*(-a*
d*h-b*c*h+5*b*d*g)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*h^4*n*x^4/b/d-1/5*B*(-a*
h+b*g)^5*n*ln(b*x+a)/b^5/h+1/5*B*(-c*h+d*g)^5*n*ln(d*x+c)/d^5/h+1/5*(h*x+g)
^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 84}

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= -\frac{Bh^2nx^2(bc-ad)\left(a^2d^2h^2-abdh(5dg-ch)+b^2(c^2h^2-5cdgh+10d^2g^2)\right)}{10b^3d^3}$$
$$+\frac{Bhnx(bc-ad)\left(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(c^2h^2-5cdgh+10d^2g^2)-(b^3(-c^3h^3+5c^2dgh^2-\right.}{5b^4d^4}$$
$$+\frac{(g+hx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bg-ah)^5\log(a+bx)}{5b^5h}$$
$$-\frac{Bh^3nx^3(bc-ad)(-adh-bch+5bdg)}{15b^2d^2}-\frac{Bh^4nx^4(bc-ad)}{20bd}+\frac{Bn(dg-ch)^5\log(c+dx)}{5d^5h}$$

```
[In] Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)*h*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) + a*b^2*d*h*(10
*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2*g^2*h + 5*c^2*
d*g*h^2 - c^3*h^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*h^2*(a^2*d^2*h^2 - a*
b*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*n*x^2)/(10*b^
3*d^3) - (B*(b*c - a*d)*h^3*(5*b*d*g - b*c*h - a*d*h)*n*x^3)/(15*b^2*d^2) -
 (B*(b*c - a*d)*h^4*n*x^4)/(20*b*d) - (B*(b*g - a*h)^5*n*Log[a + b*x])/(5*b
^5*h) + (B*(d*g - c*h)^5*n*Log[c + d*x])/(5*d^5*h) + ((g + h*x)^5*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/(5*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^5 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h} - \frac{(B(bc-ad)n)\int \frac{(g+hx)^5}{(a+bx)(c+dx)}\,dx}{5h}$$

$$= \frac{(g+hx)^5 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h} - \frac{(B(bc-ad)n)\int \left(\frac{h^2\left(-a^3d^3h^3+a^2bd^2h^2(5dg-ch)-ab^2dh\left(10d^2g^2-5cdgh+c^2h^2\right)+b^3\left(10d^3g^3-10cd^2g^2h+5c^2dgh^2-c^3h^3\right)\right)}{b^4d^4}\right.}{}$$

$$= \frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(10d^2g^2-5cdgh+c^2h^2)-b^3(10d^3g^3-10cd^2g}{5b^4d^4}$$
$$- \frac{B(bc-ad)h^2(a^2d^2h^2-abdh(5dg-ch)+b^2(10d^2g^2-5cdgh+c^2h^2))\,nx^2}{10b^3d^3}$$
$$- \frac{B(bc-ad)h^3(5bdg-bch-adh)nx^3}{15b^2d^2} - \frac{B(bc-ad)h^4nx^4}{20bd}$$
$$- \frac{B(bg-ah)^5 n\log(a+bx)}{5b^5h} + \frac{B(dg-ch)^5 n\log(c+dx)}{5d^5h}$$
$$+ \frac{(g+hx)^5 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.27

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{bdx(12Ab^4d^4(5g^4+10g^3hx+10g^2h^2x^2+5gh^3x^3+h^4x^4)+B(bc-ad)hn(12a^3d^3h^3-6a^2bd^2h^2(10dg-}{}$$

```
[In] Integrate[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(12*A*b^4*d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 + h
^4*x^4) + B*(b*c - a*d)*h*n*(12*a^3*d^3*h^3 - 6*a^2*b*d^2*h^2*(10*d*g - 2*c
*h + d*h*x) + 2*a*b^2*d*h*(6*c^2*h^2 - 3*c*d*h*(10*g + h*x) + d^2*(60*g^2 +
 15*g*h*x + 2*h^2*x^2)) - b^3*(-12*c^3*h^3 + 6*c^2*d*h^2*(10*g + h*x) - 2*c
*d^2*h*(60*g^2 + 15*g*h*x + 2*h^2*x^2) + d^3*(120*g^3 + 60*g^2*h*x + 20*g*h
^2*x^2 + 3*h^3*x^3)))) + 12*a^2*B*d^5*h*(-10*b^3*g^3 + 10*a*b^2*g^2*h - 5*a
^2*b*g*h^2 + a^3*h^3)*n*Log[a + b*x] - 12*b^4*B*(-5*a*d^5*g^4 + b*c*(5*d^4*
g^4 - 10*c*d^3*g^3*h + 10*c^2*d^2*g^2*h^2 - 5*c^3*d*g*h^3 + c^4*h^4))*n*Log
[c + d*x] + 12*b^4*B*d^5*(5*a*g^4 + b*x*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^
2 + 5*g*h^3*x^3 + h^4*x^4))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b^5*d^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. 2(351) = 702.

Time = 85.50 (sec) , antiderivative size = 1170, normalized size of antiderivative = 3.21

method	result	size
parallelrisch	Expression too large to display	1170
risch	Expression too large to display	2612

```
[In] int((h*x+g)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(12*B*ln(b*x+a)*a^6*c*d^5*h^4*n^2-12*B*ln(b*x+a)*a*b^5*c^6*h^4*n^2+12*
A*x^5*a*b^5*c*d^5*h^4*n+12*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^6*h^4*n+12
*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*h^4*n+60*A*x^4*a*b^5*c*d^5*g
*h^3*n+20*B*x^3*a^2*b^4*c*d^5*g*h^3*n^2-20*B*x^3*a*b^5*c^2*d^4*g*h^3*n^2+12
0*A*x^3*a*b^5*c*d^5*g^2*h^2*n-30*B*x^2*a^3*b^3*c*d^5*g*h^3*n^2+60*B*x^2*a^2
*b^4*c*d^5*g^2*h^2*n^2+30*B*x^2*a*b^5*c^3*d^3*g*h^3*n^2-60*B*x^2*a*b^5*c^2*
d^4*g^2*h^2*n^2+120*A*x^2*a*b^5*c*d^5*g^3*h*n+60*B*x*ln(e*(b*x+a)^n/((d*x+c
)^n))*a*b^5*c*d^5*g^4*n+60*B*x*a^4*b^2*c*d^5*g*h^3*n^2-120*B*x*a^3*b^3*c*d^
5*g^2*h^2*n^2+120*B*x*a^2*b^4*c*d^5*g^3*h*n^2-60*B*x*a*b^5*c^4*d^2*g*h^3*n^
2+120*B*x*a*b^5*c^3*d^3*g^2*h^2*n^2-120*B*x*a*b^5*c^2*d^4*g^3*h*n^2-60*B*ln
(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^5*d*g*h^3*n+120*B*ln(e*(b*x+a)^n/((d*x+c)
^n))*a*b^5*c^4*d^2*g^2*h^2*n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^3*d^
3*g^3*h*n-60*B*ln(b*x+a)*a^5*b*c*d^5*g*h^3*n^2+120*B*ln(b*x+a)*a^4*b^2*c*d^
5*g^2*h^2*n^2-120*B*ln(b*x+a)*a^3*b^3*c*d^5*g^3*h*n^2+60*B*ln(b*x+a)*a*b^5*
c^5*d*g*h^3*n^2-120*B*ln(b*x+a)*a*b^5*c^4*d^2*g^2*h^2*n^2+120*B*ln(b*x+a)*a
*b^5*c^3*d^3*g^3*h*n^2+3*B*x^4*a^2*b^4*c*d^5*h^4*n^2-3*B*x^4*a*b^5*c^2*d^4*
h^4*n^2-4*B*x^3*a^3*b^3*c*d^5*h^4*n^2+4*B*x^3*a*b^5*c^3*d^3*h^4*n^2+6*B*x^2
*a^4*b^2*c*d^5*h^4*n^2-6*B*x^2*a*b^5*c^4*d^2*h^4*n^2-12*B*x*a^5*b*c*d^5*h^4
*n^2+12*B*x*a*b^5*c^5*d*h^4*n^2+60*A*x*a*b^5*c*d^5*g^4*n+60*B*ln(e*(b*x+a)^
n/((d*x+c)^n))*a*b^5*c^2*d^4*g^4*n+60*B*ln(b*x+a)*a^2*b^4*c*d^5*g^4*n^2-60*
B*ln(b*x+a)*a*b^5*c^2*d^4*g^4*n^2+60*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^
5*c*d^5*g*h^3*n+120*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*g^2*h^2*n
+120*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*g^3*h*n)/a/c/d^5/n/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(351) = 702.

Time = 0.34 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.21

$$\int (g+hx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{12\,Ab^5d^5h^4x^5+3\,(20\,Ab^5d^5gh^3-(Bb^5cd^4-Bab^4d^5)h^4n)x^4+4\,(30\,Ab^5d^5g^2h^2-(5\,(Bb^5cd^4-Bab^4d^5)gh^3}{}$$

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*A*b^5*d^5*h^4*x^5 + 3*(20*A*b^5*d^5*g*h^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*h^4*n)*x^4 + 4*(30*A*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)
*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*(20*A*b^5*d^5*g^3*
h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*
d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n)*x^2 + 12*(5*A*b^5*d^5*
g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h - 10*(B*b^5*c^2*d^3 - B*a^2*b^3
*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g*h^3 - (B*b^5*c^4*d - B*
a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 +
10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x
 + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*h + 10*B*a^3*b^2*d^5*g^2*h^2 -
 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(b*x + a) - 12*(B*b^5*d^5*h^4*n
*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*
g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*c*d^4*g^4 - 10*B*b^5*c^2*d^3*g
^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4*d*g*h^3 + B*b^5*c^5*h^4)*n)*l
og(d*x + c) + 12*(B*b^5*d^5*h^4*x^5 + 5*B*b^5*d^5*g*h^3*x^4 + 10*B*b^5*d^5*
g^2*h^2*x^3 + 10*B*b^5*d^5*g^3*h*x^2 + 5*B*b^5*d^5*g^4*x)*log(e))/(b^5*d^5)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.84

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{1}{5}\,Bh^4x^5\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{5}\,Ah^4x^5 + Bgh^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Agh^3x^4 + 2\,Bg^2h^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2\,Ag^2h^2x^3 + 2\,Bg^3hx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2\,Ag^3hx^2 + Bg^4x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ag^4x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Bg^4}{e} - \frac{2\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bg^3h}{e} + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bg^2h^2}{e} - \frac{\left(\frac{6\,a^4en\log(bx+a)}{b^4} - \frac{6\,c^4en\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2en-ab^2d^3en)x^3-3\,(b^3c^2den-a^2bd^3en)x^2+6\,(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bgh^3}{6\,e} + \frac{\left(\frac{12\,a^5en\log(bx+a)}{b^5} - \frac{12\,c^5en\log(dx+c)}{d^5} - \frac{3\,(b^4cd^3en-ab^3d^4en)x^4-4\,(b^4c^2d^2en-a^2b^2d^4en)x^3+6\,(b^4c^3den-a^3bd^4en)x^2-12\,(b^4c^4en-a^4}{b^4d^4}\right.}{60\,e}$$

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*h^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*g^3*h*x^2 + B*g^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*g^4*x
 + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^4/e - 2*(a^2*e*n*log(b
*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*g^3
*h/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d
*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*g^2
*h^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*
(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2
 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*g*h^3/e + 1/60*(12*a^5*e*n
*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3*e*n - a*b^3
*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*(b^4*c^3*d*e*
n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b^4*d^4))*B*h^4
/e
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 1434, normalized size of antiderivative = 3.93

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((g + h*x)^4*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
[Out] x*((5*A*b*d*g^4 + 20*A*a*d*g^3*h + 20*A*b*c*g^3*h + 30*A*a*c*g^2*h^2 + 10*B
*a*d*g^3*h*n - 10*B*b*c*g^3*h*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*g*h^
3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2
*n - 10*B*b*c*g^2*h^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b
*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*
d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h
^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)
/(5*b*d) + (A*a*c*h^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 +
 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*
c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b
*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*
b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g
*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a
*c*h^4)/(b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^4*x^5)/5 +
B*g^4*x + 2*B*g^2*h^2*x^3 + 2*B*g^3*h*x^2 + B*g*h^3*x^4) + x^4*((5*A*a*d*h^
4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(20*b*d) - (A
*h^4*(5*a*d + 5*b*c))/(20*b*d)) - x^3*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*
b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5
*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c
*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(15*b*d) + (
A*a*c*h^4)/(3*b*d)) + x^2*((20*A*a*c*g*h^3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*
h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2*n - 10*B*b*c*g^2*h^2*n)/(10*b*d)
+ ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h
^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b
*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^
2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a*c*h^4)/(b*d)))/(1
```

```
0*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n -
B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + (A*h^4*
x^5)/5 + (log(a + b*x)*((B*a^5*h^4*n)/5 + B*a*b^4*g^4*n + 2*B*a^3*b^2*g^2*h
^2*n - B*a^4*b*g*h^3*n - 2*B*a^2*b^3*g^3*h*n))/b^5 - (log(c + d*x)*(B*c^5*h
^4*n + 5*B*c*d^4*g^4*n + 10*B*c^3*d^2*g^2*h^2*n - 5*B*c^4*d*g*h^3*n - 10*B*
c^2*d^3*g^3*h*n))/(5*d^5)
```

3.294 $\int (g+hx)^3 (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 236

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= -\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2-4cdgh+c^2h^2))\,nx}{4b^3d^3} - \frac{B(bc-ad)h^2(4bdg-bch-adh)nx^2}{8b^2d^2} - \frac{B(bc-ad)h^3nx^3}{12bd} - \frac{B(bg-ah)^4n\log(a+bx)}{4b^4h} + \frac{B(dg-ch)^4n\log(c+dx)}{4d^4h} + \frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h}$$

```
[Out] -1/4*B*(-a*d+b*c)*h*(a^2*d^2*h^2-a*b*d*h*(-c*h+4*d*g)+b^2*(c^2*h^2-4*c*d*g*
h+6*d^2*g^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*h^2*(-a*d*h-b*c*h+4*b*d*g)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*h^3*n*x^3/b/d-1/4*B*(-a*h+b*g)^4*n*ln(b*x+a)/b^4
/h+1/4*B*(-c*h+d*g)^4*n*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*(A+B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{2548, 84\}$

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= -\frac{Bhnx(bc-ad)\left(a^2d^2h^2-abdh(4dg-ch)+b^2(c^2h^2-4cdgh+6d^2g^2)\right)}{4b^3d^3} + \frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h} - \frac{Bn(bg-ah)^4\log(a+bx)}{4b^4h} - \frac{Bh^2nx^2(bc-ad)(-adh-bch+4bdg)}{8b^2d^2} - \frac{Bh^3nx^3(bc-ad)}{12bd} + \frac{Bn(dg-ch)^4\log(c+dx)}{4d^4h}$$

```
[In] Int[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*d^2*g^2
 - 4*c*d*g*h + c^2*h^2))*n*x)/(b^3*d^3) - (B*(b*c - a*d)*h^2*(4*b*d*g - b*c
*h - a*d*h)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*h^3*n*x^3)/(12*b*d) - (B*(b
*g - a*h)^4*n*Log[a + b*x])/(4*b^4*h) + (B*(d*g - c*h)^4*n*Log[c + d*x])/(4
*d^4*h) + ((g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule 2548

Rubi steps

$$\text{integral} = \frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h} - \frac{(B(bc-ad)n)\int \frac{(g+hx)^4}{(a+bx)(c+dx)}\,dx}{4h}$$
$$= \frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h} - \frac{(B(bc-ad)n)\int \left(\frac{h^2\left(a^2d^2h^2-abdh(4dg-ch)+b^2\left(6d^2g^2-4cdgh+c^2h^2\right)\right)}{b^3d^3} + \frac{h^3(4bdg-bch-adh)x}{b^2d^2} + \frac{h^4x^2}{bd} + \frac{(bg-ah)}{b^3(bc-ad)(a}\right.}{4h}$$

$$= -\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2-4cdgh+c^2h^2))\,nx}{4b^3d^3} - \frac{B(bc-ad)h^2(4bdg-bch-adh)nx^2}{8b^2d^2} - \frac{B(bc-ad)h^3nx^3}{12bd} - \frac{B(bg-ah)^4n\log(a+bx)}{4b^4h} + \frac{B(dg-ch)^4n\log(c+dx)}{4d^4h} + \frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.33

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{bdx(6Ab^3d^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)-B(bc-ad)hn(6a^2d^2h^2-3abdh(8dg-2ch+dhx)+b^2(6c^2}{}$$

```
[In] Integrate[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(6*A*b^3*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - B*(b*c -
a*d)*h*n*(6*a^2*d^2*h^2 - 3*a*b*d*h*(8*d*g - 2*c*h + d*h*x) + b^2*(6*c^2*h^
2 - 3*c*d*h*(8*g + h*x) + 2*d^2*(18*g^2 + 6*g*h*x + h^2*x^2)))) - 6*a^2*B*d
^4*h*(6*b^2*g^2 - 4*a*b*g*h + a^2*h^2)*n*Log[a + b*x] + 6*b^3*B*(4*a*d^4*g^
3 + b*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*n*Log[c + d
*x] + 6*b^3*B*d^4*(4*a*g^3 + b*x*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3
))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b^4*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(224) = 448$.

Time = 33.09 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.17

method	result
parallelrisch	$\frac{12B\,a^2b^2c\,d^3g\,h^2n^2-12Ba\,b^3c^2d^2g\,h^2n^2-36Aa\,b^3c\,d^3g^2hn-6B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^4c^4h^3n+24B\ln\left(e(bx+a)^n(dx+c)^{-}\right.}{}$
risch	Expression too large to display

```
[In] int((h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(12*B*a^2*b^2*c*d^3*g*h^2*n^2-12*B*a*b^3*c^2*d^2*g*h^2*n^2-36*A*a*b^3*
c*d^3*g^2*h*n-12*B*x^2*b^4*c*d^3*g*h^2*n^2+6*A*x^4*b^4*d^4*h^3*n+24*A*x*b^4
*d^4*g^3*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^4*h^3*n-6*B*ln(b*x+a)*a^4*
d^4*h^3*n^2+6*B*ln(b*x+a)*b^4*c^4*h^3*n^2-24*A*a*b^3*d^4*g^3*n-24*A*b^4*c*d
^3*g^3*n-24*B*x*a^2*b^2*d^4*g*h^2*n^2+36*B*x*a*b^3*d^4*g^2*h*n^2+24*B*x*b^4
```

```
*c^2*d^2*g*h^2*n^2-36*B*x*b^4*c*d^3*g^2*h*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^
n))*b^4*c^3*d*g*h^2*n-36*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^2*d^2*g^2*h*n+
24*B*ln(b*x+a)*a^3*b*d^4*g*h^2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*g^2*h*n^2-24*
B*ln(b*x+a)*b^4*c^3*d*g*h^2*n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*g^2*h*n^2+24*B*x
^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g*h^2*n+36*B*x^2*ln(e*(b*x+a)^n/((d*
x+c)^n))*b^4*d^4*g^2*h*n+12*B*x^2*a*b^3*d^4*g*h^2*n^2+6*B*x^4*ln(e*(b*x+a)^
n/((d*x+c)^n))*b^4*d^4*h^3*n+2*B*x^3*a*b^3*d^4*h^3*n^2-2*B*x^3*b^4*c*d^3*h^
3*n^2+24*A*x^3*b^4*d^4*g*h^2*n-3*B*x^2*a^2*b^2*d^4*h^3*n^2+3*B*x^2*b^4*c^2*
d^2*h^3*n^2+36*A*x^2*b^4*d^4*g^2*h*n+24*B*ln(b*x+a)*a*b^3*d^4*g^3*n^2+24*B*
x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g^3*n+6*B*x*a^3*b*d^4*h^3*n^2-6*B*x*b
^4*c^3*d*h^3*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c*d^3*g^3*n-24*B*ln(b
*x+a)*b^4*c*d^3*g^3*n^2-6*B*a^4*d^4*h^3*n^2+6*B*b^4*c^4*h^3*n^2-3*B*a^3*b*c
*d^3*h^3*n^2+24*B*a^3*b*d^4*g*h^2*n^2-36*B*a^2*b^2*d^4*g^2*h*n^2+3*B*a*b^3*
c^3*d*h^3*n^2-24*B*b^4*c^3*d*g*h^2*n^2+36*B*b^4*c^2*d^2*g^2*h*n^2)/b^4/d^4/
n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(224) = 448$.

Time = 0.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.42

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{6\,Ab^4d^4h^3x^4+2\,(12\,Ab^4d^4gh^2-(Bb^4cd^3-Bab^3d^4)h^3n)x^3+3\,(12\,Ab^4d^4g^2h-(4\,(Bb^4cd^3-Bab^3d^4)gh^2-}{}$$

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*A*b^4*d^4*h^3*x^4 + 2*(12*A*b^4*d^4*g*h^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*h^3*n)*x^3 + 3*(12*A*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*g*
h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*A*b^4*d^4*g^3 - (6
*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g*h^
2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^
4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*a*
b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4*g*h^2 - B*a^4*d^4*h^3)*
n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^
4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*c*d^3*g^3 - 6*B*b^4*c^2*
d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)*log(d*x + c) + 6*(B*b^4
*d^4*h^3*x^4 + 4*B*b^4*d^4*g*h^2*x^3 + 6*B*b^4*d^4*g^2*h*x^2 + 4*B*b^4*d^4*
g^3*x)*log(e))/(b^4*d^4)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(224) = 448$.

Time = 0.22 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.98

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{1}{4}Bh^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4}Ah^3x^4 + Bgh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Agh^2x^3 + \frac{3}{2}Bg^2hx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2}Ag^2hx^2 + Bg^3x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ag^3x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Bg^3}{e} - \frac{3\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bg^2h}{2\,e} + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bgh^2}{2\,e} - \frac{\left(\frac{6\,a^4en\log(bx+a)}{b^4} - \frac{6\,c^4en\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2en-ab^2d^3en)x^3-3\,(b^3c^2den-a^2bd^3en)x^2+6\,(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bh^3}{24\,e}$$

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
[Out] 1/4*B*h^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*g^2*h*x^2 + B*g^3*x*log((b*x + a)^n*e/(d*x + c)^
n) + A*g^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^3/e - 3/2*
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*g^2*h/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x +
c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x
)/(b^2*d^2))*B*g*h^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d
*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a
^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*h^3/e
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.25

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,g^3+12\,A\,a\,c\,g\,h^2+12\,A\,a\,d\,g^2\,h+12\,A\,b\,c\,g^2\,h+6\,B\,a\,d\,g^2\,h\,n-6\,B\,b\,c\,g^2\,h\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\,A}{}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,g^3\,x+\frac{3\,B\,g^2\,h\,x^2}{2}+B\,g\,h^2\,x^3+\frac{B\,h^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\,A\,b\,c\,g\,h^2+12\,A\,b\,d\,g^2\,h+4\,B\,a\,d\,g\,h^2\,n-4\,B\,b\,c\,g\,h^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,h^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{12\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,h^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,h^3-4\,B\,n\,a^3\,b\,g\,h^2+6\,B\,n\,a^2\,b^2\,g^2\,h-4\,B\,n\,a\,b^3\,g^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,h^3-4\,B\,n\,c^3\,d\,g\,h^2+6\,B\,n\,c^2\,d^2\,g^2\,h-4\,B\,n\,c\,d^3\,g^3\right)}{4\,d^4}$$

```
[In] int((g + h*x)^3*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x*((4*A*b*d*g^3 + 12*A*a*c*g*h^2 + 12*A*a*d*g^2*h + 12*A*b*c*g^2*h + 6*B*a*
d*g^2*h*n - 6*B*b*c*g^2*h*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*h^3 + 4
*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*h^3 + 12*A*a*d
```

```
*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*
n)/(4*b*d) + (A*a*c*h^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*h^3 + 4*A*b*c*h^3
 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^3*x^4)/4 +
B*g^3*x + (3*B*g^2*h*x^2)/2 + B*g*h^2*x^3) - x^2*((((4*A*a*d*h^3 + 4*A*b*c*
h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d +
 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*h^3 + 12*A*a*d*g*h^2
+ 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*n)/(8*b
*d) + (A*a*c*h^3)/(2*b*d)) + x^3*((4*A*a*d*h^3 + 4*A*b*c*h^3 + 12*A*b*d*g*h
^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(12*b*d) - (A*h^3*(4*a*d + 4*b*c))/(12*b*d)
) + (A*h^3*x^4)/4 - (log(a + b*x)*(B*a^4*h^3*n - 4*B*a*b^3*g^3*n - 4*B*a^3*
b*g*h^2*n + 6*B*a^2*b^2*g^2*h*n))/(4*b^4) + (log(c + d*x)*(B*c^4*h^3*n - 4*
B*c*d^3*g^3*n - 4*B*c^3*d*g*h^2*n + 6*B*c^2*d^2*g^2*h*n))/(4*d^4)
```

3.295 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 158

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{B(bc-ad)h^2nx^2}{6bd} - \frac{B(bg-ah)^3n\log(a+bx)}{3b^3h} + \frac{B(dg-ch)^3n\log(c+dx)}{3d^3h} + \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}$$

```
[Out] -1/3*B*(-a*d+b*c)*h*(-a*d*h-b*c*h+3*b*d*g)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*h^2
*n*x^2/b/d-1/3*B*(-a*h+b*g)^3*n*ln(b*x+a)/b^3/h+1/3*B*(-c*h+d*g)^3*n*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 84}

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h} - \frac{Bn(bg-ah)^3\log(a+bx)}{3b^3h} - \frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2} - \frac{Bh^2nx^2(bc-ad)}{6bd} + \frac{Bn(dg-ch)^3\log(c+dx)}{3d^3h}$$

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(b^2*d^2) - (B*(b*c -
a*d)*h^2*n*x^2)/(6*b*d) - (B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (B*(
```

```
d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(3*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}-\frac{(B(bc-ad)n)\int\frac{(g+hx)^3}{(a+bx)(c+dx)}\,dx}{3h}\\
&= \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}\\
&\quad -\frac{(B(bc-ad)n)\int\left(\frac{h^2(3bdg-bch-adh)}{b^2d^2}+\frac{h^3x}{bd}+\frac{(bg-ah)^3}{b^2(bc-ad)(a+bx)}+\frac{(dg-ch)^3}{d^2(-bc+ad)(c+dx)}\right)dx}{3h}\\
&= -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{B(bc-ad)h^2nx^2}{6bd}-\frac{B(bg-ah)^3n\log(a+bx)}{3b^3h}\\
&\quad +\frac{B(dg-ch)^3n\log(c+dx)}{3d^3h}+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.29

$$\int(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx$$
$$=\frac{2a^2Bd^3h(-3bg+ah)n\log(a+bx)+b(dx(B(bc-ad)hn(-6bdg+2bch+2adh-bdhx)+2Ab^2d^2(3g^2+3$$

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (2*a^2*B*d^3*h*(-3*b*g + a*h)*n*Log[a + b*x] + b*(d*x*(B*(b*c - a*d)*h*n*(-
6*b*d*g + 2*b*c*h + 2*a*d*h - b*d*h*x) + 2*A*b^2*d^2*(3*g^2 + 3*g*h*x + h^2
*x^2)) - 2*b*B*(-3*a*d^3*g^2 + b*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log
[c + d*x] + 2*b*B*d^3*(3*a*g^2 + b*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]))/(6*b^3*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(148) = 296$.

Time = 11.35 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.94

method	result
parallelrisch	$\frac{-2B\,b^3c^3h^2n^2+2B\,a^3d^3h^2n^2+6Bxa\,b^2d^3gh\,n^2-6Bx\,b^3c\,d^2gh\,n^2-6B\ln(bx+a)a^2b\,d^3gh\,n^2+6B\,x^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b}{}$
risch	Expression too large to display

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*B*b^3*c^3*h^2*n^2+2*B*a^3*d^3*h^2*n^2-6*B*ln(e*(b*x+a)^n/((d*x+c)^n
))*b^3*c^2*d*g*h*n+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g*h*n+6*B*x*
a*b^2*d^3*g*h*n^2-6*B*x*b^3*c*d^2*g*h*n^2-6*B*ln(b*x+a)*a^2*b*d^3*g*h*n^2+6
*B*ln(b*x+a)*b^3*c^2*d*g*h*n^2-6*A*a*b^2*c*d^2*g*h*n+B*a^2*b*c*d^2*h^2*n^2-
6*B*a^2*b*d^3*g*h*n^2-B*a*b^2*c^2*d*h^2*n^2+6*B*b^3*c^2*d*g*h*n^2+6*B*ln(e*
(b*x+a)^n/((d*x+c)^n))*b^3*c*d^2*g^2*n+2*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*
b^3*d^3*h^2*n+B*x^2*a*b^2*d^3*h^2*n^2-B*x^2*b^3*c*d^2*h^2*n^2+6*A*x^2*b^3*d
^3*g*h*n+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g^2*n-2*B*x*a^2*b*d^3*h^
2*n^2+2*B*x*b^3*c^2*d*h^2*n^2+6*B*ln(b*x+a)*a*b^2*d^3*g^2*n^2-6*B*ln(b*x+a)
*b^3*c*d^2*g^2*n^2-6*A*a*b^2*d^3*g^2*n-6*A*b^3*c*d^2*g^2*n+2*B*ln(e*(b*x+a)
^n/((d*x+c)^n))*b^3*c^3*h^2*n+2*A*x^3*b^3*d^3*h^2*n+6*A*x*b^3*d^3*g^2*n+2*B
*ln(b*x+a)*a^3*d^3*h^2*n^2-2*B*ln(b*x+a)*b^3*c^3*h^2*n^2)/b^3/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(148) = 296$.

Time = 0.32 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.31

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3h^2x^3 + (6\,Ab^3d^3gh - (Bb^3cd^2 - Bab^2d^3)h^2n)x^2 + 2\,(3\,Ab^3d^3g^2 - (3\,(Bb^3cd^2 - Bab^2d^3)gh - (Bb^3}{}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*A*b^3*d^3*h^2*x^3 + (6*A*b^3*d^3*g*h - (B*b^3*c*d^2 - B*a*b^2*d^3)*h
^2*n)*x^2 + 2*(3*A*b^3*d^3*g^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*g*h - (B*b^
3*c^2*d - B*a^2*b*d^3)*h^2)*n)*x + 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h
*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*a*b^2*d^3*g^2 - 3*B*a^2*b*d^3*g*h + B*a
^3*d^3*h^2)*n)*log(b*x + a) - 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h*n*x^
2 + 3*B*b^3*d^3*g^2*n*x + (3*B*b^3*c*d^2*g^2 - 3*B*b^3*c^2*d*g*h + B*b^3*c^
3*h^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*h^2*x^3 + 3*B*b^3*d^3*g*h*x^2 + 3*B*b
^3*d^3*g^2*x)*log(e))/(b^3*d^3)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.86

$$\begin{aligned}
&\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx \\
&= \frac{1}{3}Bh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{3}Ah^2x^3+Bghx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aghx^2 \\
&\quad + Bg^2x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ag^2x+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg^2}{e} \\
&\quad - \frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bgh}{e} \\
&\quad + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3}-\frac{2\,c^3en\log(dx+c)}{d^3}-\frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bh^2}{6\,e}
\end{aligned}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*h^2*x^3 + B*g*h*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*g*h*x^2 + B*g^2*x*log((b*x + a)^n*e/(d*x +
 c)^n) + A*g^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^2/e -
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*g*h/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)
/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/
(b^2*d^2))*B*h^2/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(148) = 296$.

Time = 46.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \frac{1}{3}\left(Bh^2\log(e)+Ah^2\right)x^3+\frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(bx+a)$$
$$-\frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(dx+c)$$
$$-\frac{(Bbch^2n-Badh^2n-6\,Bbdgh\log(e)-6\,Abdgh)x^2}{6\,bd}$$
$$+\frac{(3\,Bab^2g^2n-3\,Ba^2bghn+Ba^3h^2n)\log(bx+a)}{3\,b^3}$$
$$-\frac{(3\,Bcd^2g^2n-3\,Bc^2dghn+Bc^3h^2n)\log(-dx-c)}{3\,d^3}$$
$$-\frac{(3\,Bb^2cdghn-3\,Babd^2ghn-Bb^2c^2h^2n+Ba^2d^2h^2n-3\,Bb^2d^2g^2\log(e)-3\,Ab^2d^2g^2)x}{3\,b^2d^2}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/3*(B*h^2*log(e) + A*h^2)*x^3 + 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2
*n*x)*log(b*x + a) - 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2*n*x)*log(d*
x + c) - 1/6*(B*b*c*h^2*n - B*a*d*h^2*n - 6*B*b*d*g*h*log(e) - 6*A*b*d*g*h)
*x^2/(b*d) + 1/3*(3*B*a*b^2*g^2*n - 3*B*a^2*b*g*h*n + B*a^3*h^2*n)*log(b*x
+ a)/b^3 - 1/3*(3*B*c*d^2*g^2*n - 3*B*c^2*d*g*h*n + B*c^3*h^2*n)*log(-d*x -
 c)/d^3 - 1/3*(3*B*b^2*c*d*g*h*n - 3*B*a*b*d^2*g*h*n - B*b^2*c^2*h^2*n + B*
a^2*d^2*h^2*n - 3*B*b^2*d^2*g^2*log(e) - 3*A*b^2*d^2*g^2)*x/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.35

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{6\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,g^2\,x+B\,g\,h\,x^2+\frac{B\,h^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{3\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d}\right.$$
$$-\frac{3\,A\,a\,c\,h^2+3\,A\,b\,d\,g^2+6\,A\,a\,d\,g\,h+6\,A\,b\,c\,g\,h+3\,B\,a\,d\,g\,h\,n-3\,B\,b\,c\,g\,h\,n}{3\,b\,d}$$
$$\left.+\frac{A\,a\,c\,h^2}{b\,d}\right)+\frac{A\,h^2\,x^3}{3}+\frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,h^2-3\,B\,n\,a^2\,b\,g\,h+3\,B\,n\,a\,b^2\,g^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(B\,n\,c^3\,h^2-3\,B\,n\,c^2\,d\,g\,h+3\,B\,n\,c\,d^2\,g^2\right)}{3\,d^3}$$

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x^2*((3*A*a*d*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/
(6*b*d) - (A*h^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x)^n)/(c + d*x)^
n)*((B*h^2*x^3)/3 + B*g^2*x + B*g*h*x^2) - x*(((3*a*d + 3*b*c)*((3*A*a*d*h^
2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/(3*b*d) - (A*h^2
*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*h^2 + 3*A*b*d*g^2 + 6*A*a*d*
g*h + 6*A*b*c*g*h + 3*B*a*d*g*h*n - 3*B*b*c*g*h*n)/(3*b*d) + (A*a*c*h^2)/(b
*d)) + (A*h^2*x^3)/3 + (log(a + b*x)*(B*a^3*h^2*n + 3*B*a*b^2*g^2*n - 3*B*a
^2*b*g*h*n))/(3*b^3) - (log(c + d*x)*(B*c^3*h^2*n + 3*B*c*d^2*g^2*n - 3*B*c
^2*d*g*h*n))/(3*d^3)
```

3.296 $\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 116

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)hnx}{2bd}-\frac{B(bg-ah)^2n\log(a+bx)}{2b^2h}+\frac{B(dg-ch)^2n\log(c+dx)}{2d^2h}+\frac{(g+hx)^2\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h}$$

```
[Out] -1/2*B*(-a*d+b*c)*h*n*x/b/d-1/2*B*(-a*h+b*g)^2*n*ln(b*x+a)/b^2/h+1/2*B*(-c*
h+d*g)^2*n*ln(d*x+c)/d^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/
h
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2548, 84}

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{(g+hx)^2\left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bg-ah)^2\log(a+bx)}{2b^2h}-\frac{Bhnx(bc-ad)}{2bd}+\frac{Bn(dg-ch)^2\log(c+dx)}{2d^2h}$$

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*h*n*x)/(b*d) - (B*(b*g - a*h)^2*n*Log[a + b*x])/(2*b^2*
h) + (B*(d*g - c*h)^2*n*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*(A + B*Log[(
e*(a + b*x)^n)/(c + d*x)^n]))/(2*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h} - \frac{(B(bc-ad)n)\int\frac{(g+hx)^2}{(a+bx)(c+dx)}\,dx}{2h} \\
&= \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h} \\
&\quad - \frac{(B(bc-ad)n)\int\left(\frac{h^2}{bd}+\frac{(bg-ah)^2}{b(bc-ad)(a+bx)}+\frac{(dg-ch)^2}{d(-bc+ad)(c+dx)}\right)dx}{2h} \\
&= -\frac{B(bc-ad)hnx}{2bd} - \frac{B(bg-ah)^2n\log(a+bx)}{2b^2h} + \frac{B(dg-ch)^2n\log(c+dx)}{2d^2h} \\
&\quad + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int(g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{-a^2Bd^2hn\log(a+bx)+bB(2ad^2g+bc(-2dg+ch))\,n\log(c+dx)+bd(x(B(-bc+ad)hn+Abd(2g+hx$$
$$2b^2d^2$$

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (-(a^2*B*d^2*h*n*Log[a + b*x]) + b*B*(2*a*d^2*g + b*c*(-2*d*g + c*h))*n*Log
[c + d*x] + b*d*(x*(B*(-(b*c) + a*d)*h*n + A*b*d*(2*g + h*x)) + B*d*(2*a*g
+ b*x*(2*g + h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b^2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(108) = 216$.

Time = 3.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.20

method	result
parallelrisch	$\frac{-B\ln(bx+a)a^2d^2hn+B\ln(bx+a)abcdhn-2B\ln(dx+c)ab\,d^2gn-4B\ln(dx+c)b^2cdgn-B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)abcdh+2B\cdots}{\cdots}$
risch	Expression too large to display

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-B*ln(b*x+a)*a^2*d^2*h*n+B*ln(b*x+a)*a*b*c*d*h*n-2*B*ln(d*x+c)*a*b*d^2
*g*n-4*B*ln(d*x+c)*b^2*c*d*g*n-B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*h+2*B*
ln(b*x+a)*b^2*c*d*g*n-B*a^2*d^2*h*n+B*b^2*c^2*h*n-A*a*b*c*d*h+A*b^2*d^2*h*x
^2+4*B*ln(b*x+a)*a*b*d^2*g*n+B*a*b*d^2*h*n*x-B*b^2*c*d*h*n*x-2*B*ln(e*(b*x+
a)^n/((d*x+c)^n))*a*b*d^2*g-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*c*d*g+B*ln(
d*x+c)*b^2*c^2*h*n+B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*h+2*B*x*ln(e*(
b*x+a)^n/((d*x+c)^n))*b^2*d^2*g+2*A*x*b^2*d^2*g-B*ln(d*x+c)*a*b*c*d*h*n-2*A
*a*b*d^2*g-2*A*b^2*c*d*g)/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.66

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{Ab^2d^2hx^2+(2\,Ab^2d^2g-(Bb^2cd-Babd^2)hn)x+(Bb^2d^2hnx^2+2\,Bb^2d^2gnx+(2\,Babd^2g-Ba^2d^2h)n)\,\mathrm{lo}\cdots}{\cdots}$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*h*x^2 + (2*A*b^2*d^2*g - (B*b^2*c*d - B*a*b*d^2)*h*n)*x + (B
*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*a*b*d^2*g - B*a^2*d^2*h)*n)*log
(b*x + a) - (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*b^2*c*d*g - B*b^2
*c^2*h)*n)*log(d*x + c) + (B*b^2*d^2*h*x^2 + 2*B*b^2*d^2*g*x)*log(e))/(b^2*
d^2)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\begin{aligned}\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= \frac{1}{2}Bhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}Ahx^2+Bgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agx\\ &\quad+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bh}{2\,e}\end{aligned}$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
[Out] 1/2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*h*x^2 + B*g*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A*g*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d
)*B*g/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e
*n - a*d*e*n)*x/(b*d))*B*h/e
```

Giac [A] (verification not implemented)

none

Time = 2.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\begin{aligned}\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= \frac{1}{2}(Bh\log(e)+Ah)x^2+\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log(bx+a)\\ &\quad-\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log(dx+c)-\frac{(Bbchn-Badhn-2\,Bbdg\log(e)-2\,Abdg)x}{2\,bd}\\ &\quad+\frac{(2\,Babgn-Ba^2hn)\log(bx+a)}{2\,b^2}-\frac{(2\,Bcdgn-Bc^2hn)\log(-dx-c)}{2\,d^2}\end{aligned}$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*(B*h*log(e) + A*h)*x^2 + 1/2*(B*h*n*x^2 + 2*B*g*n*x)*log(b*x + a) - 1/2
*(B*h*n*x^2 + 2*B*g*n*x)*log(d*x + c) - 1/2*(B*b*c*h*n - B*a*d*h*n - 2*B*b*
d*g*log(e) - 2*A*b*d*g)*x/(b*d) + 1/2*(2*B*a*b*g*n - B*a^2*h*n)*log(b*x + a
)/b^2 - 1/2*(2*B*c*d*g*n - B*c^2*h*n)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,h\,x^2}{2}+B\,g\,x\right)+x\left(\frac{2\,A\,a\,d\,h+2\,A\,b\,c\,h+2\,A\,b\,d\,g+B\,a\,d\,h\,n-B\,b\,c\,h\,n}{2\,b\,d}-\frac{A\,h\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,h\,n-2\,B\,a\,b\,g\,n\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,h\,n-2\,B\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,h\,x^2}{2}$$

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*g*x + (B*h*x^2)/2) + x*((2*A*a*d*h + 2*
A*b*c*h + 2*A*b*d*g + B*a*d*h*n - B*b*c*h*n)/(2*b*d) - (A*h*(2*a*d + 2*b*c)
)/(2*b*d)) - (log(a + b*x)*(B*a^2*h*n - 2*B*a*b*g*n))/(2*b^2) + (log(c + d*
x)*(B*c^2*h*n - 2*B*c*d*g*n))/(2*d^2) + (A*h*x^2)/2
```

3.297 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{b}$$

```
[Out] A*x-B*(-a*d+b*c)*n*ln(d*x+c)/b/d+B*(b*x+a)*ln(e*(b*x+a)^n/((d*x+c)^n))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2536, 31}

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = \frac{B(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[In] Int[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= Ax + B\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx\\ &= Ax + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{(B(bc-ad)n)\int\frac{1}{c+dx}\,dx}{b}\\ &= Ax - \frac{B(bc-ad)n\log(c+dx)}{bd} + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = Ax - \frac{B(bc-ad)n\log(c+dx)}{bd} + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

```
[In] Integrate[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

method	result
default	$Ax+B\left(\ln\left(e(bx+a)^n\left(dx+c\right)^{-n}\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$
parts	$Ax+B\left(\ln\left(e(bx+a)^n\left(dx+c\right)^{-n}\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$
parallelrisch	$\frac{B\left(\ln(bx+a)ad\,n^2-\ln(bx+a)bc\,n^2+x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)bdn+\ln\left(e(bx+a)^n(dx+c)^{-n}\right)bcn\right)}{bdn}+Ax$
risch	$Ax-Bx\ln\left((dx+c)^n\right)-\frac{iB\pi x\,\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)^3}{2}+\frac{iB\pi x\,\mathrm{csgn}(i(bx+a)^n)\,\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)^2}{2}$

```
[In] int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] A*x+B*(ln(e*(b*x+a)^n/((d*x+c)^n))*x+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)+
a/(a*d-b*c)/b*ln(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\ dx = \frac{Bbdx\log(e) + Abdx + (Bbdnx + Badn)\log(bx+a) - (Bbdnx + Bbcn)\log(dx+c)}{bd}$$

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")
```

```
[Out] (B*b*d*x*log(e) + A*b*d*x + (B*b*d*n*x + B*a*d*n)*log(b*x + a) - (B*b*d*n*x
 + B*b*c*n)*log(d*x + c))/(b*d)
```

Sympy [F(-2)]

Exception generated.

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\ dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\ dx = Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ax + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)B}{e}$$

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

```
[Out] B*x*log((b*x + a)^n*e/(d*x + c)^n) + A*x + (a*e*n*log(b*x + a)/b - c*e*n*lo
g(d*x + c)/d)*B/e
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\left(nx\log(bx+a)-nx\log(dx+c)+\frac{an\log(bx+a)}{b}-\frac{cn\log(-dx-c)}{d}+x\log(e)\right)B$$
$$+Ax$$

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

```
[Out] (n*x*log(b*x + a) - n*x*log(d*x + c) + a*n*log(b*x + a)/b - c*n*log(-d*x -
c)/d + x*log(e))*B + A*x
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)$$
$$+\frac{B\,a\,n\,\ln(a+b\,x)}{b}-\frac{B\,c\,n\,\ln(c+d\,x)}{d}$$

```
[In] int(A + B*log((e*(a + b*x)^n)/(c + d*x)^n),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x)^n)/(c + d*x)^n) + (B*a*n*log(a + b*x))/b - (B*c*
n*log(c + d*x))/d
```

3.298 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} + \frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log(g+hx)}{h} - \frac{Bn\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{Bn\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}$$

```
[Out] -B*n*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h+B*n*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(h*x+g)/h-B*n*polylog(2,b*(h*
x+g)/(-a*h+b*g))/h+B*n*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

$= \{2546, 2441, 2440, 2438\}$

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = \frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h} - \frac{Bn\,\text{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{Bn\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h} + \frac{Bn\,\text{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} + \frac{Bn\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{h}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] -((B*n*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) + (B*n*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + ((A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])*Log[g + h*x])/h - (B*n*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h +
(B*n*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
```

```
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log(g+hx)}{h} \\
&\quad - \frac{(bBn)\int\frac{\log(g+hx)}{a+bx}\,dx}{h} + \frac{(Bdn)\int\frac{\log(g+hx)}{c+dx}\,dx}{h} \\
&= -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} + \frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log(g+hx)}{h} \\
&\quad + (Bn)\int\frac{\log\left(\frac{h(a+bx)}{-bg+ah}\right)}{g+hx}\,dx - (Bn)\int\frac{\log\left(\frac{h(c+dx)}{-dg+ch}\right)}{g+hx}\,dx \\
&= -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} + \frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log(g+hx)}{h} \\
&\quad + \frac{(Bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bg+ah}\right)}{x}\,dx,x,g+hx\right)}{h} \\
&\quad - \frac{(Bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{-dg+ch}\right)}{x}\,dx,x,g+hx\right)}{h} \\
&= -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} + \frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log(g+hx)}{h} \\
&\quad - \frac{Bn\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{Bn\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{h}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$$
$$= \frac{(A + B(-n\log(a+bx) + n\log(c+dx) + \log\left(e(a+bx)^n(c+dx)^{-n}\right)))\log(g+hx) + Bn\Big(\log(a+bx)\log}{h}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))*Log[g + h*x] + B*n*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + Pol
yLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - B*n*(Log[c + d*x]*Log[(d*(g + h*x)
)/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]))/h
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.52 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.52

method	result
risch	$\frac{\left(-iB\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)+iB\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)^2-iB\pi\,\mathrm{csgn}(i(b\right.}{}$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-I*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)+I*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*
(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*
(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*cs
gn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)+2*A)*ln(h*x+g)/h+B*ln((b*x+a)^n)*ln(h
*x+g)/h-B/h*n*dilog(((h*x+g)*b+a*h-b*g)/(a*h-b*g))-B/h*n*ln(h*x+g)*ln(((h*x
+g)*b+a*h-b*g)/(a*h-b*g))-B*ln((d*x+c)^n)*ln(h*x+g)/h+B/h*n*dilog((d*(h*x+g
)+c*h-d*g)/(c*h-d*g))+B/h*n*ln(h*x+g)*ln((d*(h*x+g)+c*h-d*g)/(c*h-d*g))
```

Fricas [F]

$$\int \frac{A + B \log \left(e(a+bx)^n (c+dx)^{-n}\right)}{g + hx} \, dx = \int \frac{B \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx + g} \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="fricas")
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B \log \left(e(a+bx)^n (c+dx)^{-n}\right)}{g + hx} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A + B \log \left(e(a+bx)^n (c+dx)^{-n}\right)}{g + hx} \, dx = \int \frac{B \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx + g} \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="maxima")
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(h*x + g), x)
+ A*log(h*x + g)/h
```

Giac [F]

$$\int \frac{A + B \log \left(e(a+bx)^n (c+dx)^{-n}\right)}{g + hx} \, dx = \int \frac{B \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx + g} \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{g + hx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{g + h\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x), x)
```

3.299 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{bBn\log(a+bx)}{h(bg-ah)} - \frac{Bdn\log(c+dx)}{h(dg-ch)} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)} + \frac{B(bc-ad)n\log(g+hx)}{(bg-ah)(dg-ch)}$$

```
[Out] b*B*n*ln(b*x+a)/h/(-a*h+b*g)-B*d*n*ln(d*x+c)/h/(-c*h+d*g)+(-A-B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/h/(h*x+g)+B*(-a*d+b*c)*n*ln(h*x+g)/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = -\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)} + \frac{Bn(bc-ad)\log(g+hx)}{(bg-ah)(dg-ch)} + \frac{bBn\log(a+bx)}{h(bg-ah)} - \frac{Bdn\log(c+dx)}{h(dg-ch)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] (b*B*n*Log[a + b*x])/(h*(b*g - a*h)) - (B*d*n*Log[c + d*x])/(h*(d*g - c*h))
 - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)) + (B*(b*c - a*d)*
n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(g+hx)}\,dx}{h}\\
&= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}\\
&\quad+\frac{(B(bc-ad)n)\int\left(\frac{b^2}{(bc-ad)(bg-ah)(a+bx)}+\frac{d^2}{(bc-ad)(-dg+ch)(c+dx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)}\right)dx}{h}\\
&= \frac{bBn\log(a+bx)}{h(bg-ah)}-\frac{Bdn\log(c+dx)}{h(dg-ch)}\\
&\quad-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}+\frac{B(bc-ad)n\log(g+hx)}{(bg-ah)(dg-ch)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx\\
=\frac{-\frac{A}{g+hx}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}+\frac{Bn(b(dg-ch)\log(a+bx)+(-bdg+adh)\log(c+dx)+(bc-ad)h\log(g+hx))}{(bg-ah)(dg-ch)}}{h}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] (-(A/(g + h*x)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x) + (B*n*(b*
(d*g - c*h)*Log[a + b*x] + (-(b*d*g) + a*d*h)*Log[c + d*x] + (b*c - a*d)*h*
Log[g + h*x]))/((b*g - a*h)*(d*g - c*h)))/h
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(122) = 244.

Time = 8.52 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.07

method	result
parallelrisch	$\frac{Axabcd\,g^2n - Ax\,a^2cdghn + B\ln(bx+a)a^2cd\,g^2n^2 - B\ln(bx+a)ab\,c^2g^2n^2 - B\ln(hx+g)a^2cd\,g^2n^2 + B\ln(hx+g)ab\,c^2g^2n^2 - B\ln\left(e\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (A*x*a*b*c*d*g^2*n-A*x*a^2*c*d*g*h*n+B*ln(b*x+a)*a^2*c*d*g^2*n^2-B*ln(b*x+a
)*a*b*c^2*g^2*n^2-B*ln(h*x+g)*a^2*c*d*g^2*n^2+B*ln(h*x+g)*a*b*c^2*g^2*n^2-B
*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c^2*g*h*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*
b*c^2*g^2*n-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c*d*g*h*n+B*x*ln(e*(b*x+a)^
n/((d*x+c)^n))*a*b*c*d*g^2*n+B*ln(b*x+a)*x*a^2*c*d*g*h*n^2-B*ln(b*x+a)*x*a*
b*c^2*g*h*n^2-B*ln(h*x+g)*x*a^2*c*d*g*h*n^2+B*ln(h*x+g)*x*a*b*c^2*g*h*n^2-A
*x*a*b*c^2*g*h*n+A*x*a^2*c^2*h^2*n)/(a*h-b*g)/(h*x+g)/n/(c*h-d*g)/a/c/g
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(120) = 240.

Time = 3.58 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx =$$
$$-\frac{Abdg^2 + Aach^2 - (Abc + Aad)gh - ((Bbdgh - Bbch^2)nx + (Badgh - Bach^2)n)\log(bx+a) + ((Bbdg}{bdg^3h + a}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] -(A*b*d*g^2 + A*a*c*h^2 - (A*b*c + A*a*d)*g*h - ((B*b*d*g*h - B*b*c*h^2)*n*
x + (B*a*d*g*h - B*a*c*h^2)*n)*log(b*x + a) + ((B*b*d*g*h - B*a*d*h^2)*n*x
+ (B*b*c*g*h - B*a*c*h^2)*n)*log(d*x + c) - ((B*b*c - B*a*d)*h^2*n*x + (B*b
*c - B*a*d)*g*h*n)*log(h*x + g) + (B*b*d*g^2 + B*a*c*h^2 - (B*b*c + B*a*d)*
g*h)*log(e))/(b*d*g^3*h + a*c*g*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 +
a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{\left(\frac{ben\log(bx+a)}{bgh-ah^2} - \frac{den\log(dx+c)}{dgh-ch^2} - \frac{(bcen-aden)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)B}{e} - \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x+gh} - \frac{A}{h^2x+gh}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="maxima
")
[Out] (b*e*n*log(b*x + a)/(b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) -
(b*c*e*n - a*d*e*n)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*B
/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A/(h^2*x + g*h)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{Bb^2n\log(|bx+a|)}{b^2gh-abh^2} - \frac{Bd^2n\log(|-dx-c|)}{d^2gh-cdh^2} - \frac{Bn\log(bx+a)}{h^2x+gh} + \frac{Bn\log(dx+c)}{h^2x+gh} + \frac{(Bbcn-Badn)\log(hx+g)}{bdg^2-bcgh-adgh+ach^2} - \frac{B\log(e)+A}{h^2x+gh}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="giac")
[Out] B*b^2*n*log(abs(b*x + a))/(b^2*g*h - a*b*h^2) - B*d^2*n*log(abs(-d*x - c))/
(d^2*g*h - c*d*h^2) - B*n*log(b*x + a)/(h^2*x + g*h) + B*n*log(d*x + c)/(h^
2*x + g*h) + (B*b*c*n - B*a*d*n)*log(h*x + g)/(b*d*g^2 - b*c*g*h - a*d*g*h
+ a*c*h^2) - (B*log(e) + A)/(h^2*x + g*h)
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,h^2-d\,g\,h} - \frac{\ln\left(g+h\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{h\,\left(g+h\,x\right)} - \frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,h^2-b\,g\,h} - \frac{A}{x\,h^2+g\,h}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^2,x)
```

```
[Out] (B*d*n*log(c + d*x))/(c*h^2 - d*g*h) - (log(g + h*x)*(B*a*d*n - B*b*c*n))/(
a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (B*log((e*(a + b*x)^n)/(c + d*x)^n
))/(h*(g + h*x)) - (B*b*n*log(a + b*x))/(a*h^2 - b*g*h) - A/(g*h + h^2*x)
```

3.300 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} \, dx = -\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2} - \frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2} + \frac{B(bc-ad)(2bdg-bch-adh)n\log(g+hx)}{2(bg-ah)^2(dg-ch)^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+1/2*b^2*B*n*ln(b*x+a)/h/(
-a*h+b*g)^2-1/2*B*d^2*n*ln(d*x+c)/h/(-c*h+d*g)^2+1/2*(-A-B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h/(h*x+g)^2+1/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln(h*x+g
)/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} \, dx = -\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2} + \frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2} - \frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)} + \frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)}{2(bg-ah)^2(dg-ch)^2} - \frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)) + (b^2*B*n*Log[a
 + b*x])/(2*h*(b*g - a*h)^2) - (B*d^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) -
 (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h*(g + h*x)^2) + (B*(b*c - a*d
)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(g+hx)^2}\,dx}{2h}$$

$$= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)(bg-ah)^2(a+bx)}-\frac{d^3}{(bc-ad)(-dg+ch)^2(c+dx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)^2}-\frac{h^2(-2bdg+bc}{(bg-ah)^2(dg-c}\right.}{2h}$$

$$= -\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{B(bc-ad)(2bdg-bch-adh)n\log(g+hx)}{2(bg-ah)^2(dg-ch)^2}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx =$$

$$-\frac{\frac{A}{(g+hx)^2}+\frac{B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}+B(bc-ad)n\left(-\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}+\frac{\frac{d^2\log(c+dx)}{bc-ad}+\frac{h\left(\frac{(bg-ah)(dg-ch)}{g+hx}+(-2bdg+bch+adh\right)}{(bg-ah)^2}}{(dg-ch)^2}\right)}{2h}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] -1/2*(A/(g + h*x)^2 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2 + B*
(b*c - a*d)*n*(-((b^2*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)^2)) + ((d^2*Lo
g[c + d*x])/(b*c - a*d) + (h*(((b*g - a*h)*(d*g - c*h))/(g + h*x) + (-2*b*d
*g + b*c*h + a*d*h)*Log[g + h*x]))/(b*g - a*h)^2)/(d*g - c*h)^2))/h
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. 2(184) = 368.

Time = 28.81 (sec) , antiderivative size = 1385, normalized size of antiderivative = 7.25

method	result	size
parallelrisch	Expression too large to display	1385
risch	Expression too large to display	4925

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*B*ln(h*x+g)*b^3*c*d^2*g^3*h^2*n-B*ln(b*x+a)*x^2*b^3*c^2*d*h^5*n-B*
ln(b*x+a)*x^2*b^3*d^3*g^2*h^3*n+B*ln(d*x+c)*x^2*a^2*b*d^3*h^5*n-B*ln(b*x+a)
*b^3*d^3*g^4*h*n+B*ln(d*x+c)*b^3*d^3*g^4*h*n+B*x*a^2*b*d^3*g*h^4*n+B*x*a*b^
2*c^2*d*h^5*n-B*x*a*b^2*d^3*g^2*h^3*n-B*x*b^3*c^2*d*g*h^4*n+B*x*b^3*c*d^2*g
^2*h^3*n-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c*d^2*g*h^4-2*B*ln(e*(b*x+a)
^n/((d*x+c)^n))*a*b^2*c^2*d*g*h^4+4*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*c*d
^2*g^2*h^3+B*ln(d*x+c)*x^2*b^3*d^3*g^2*h^3*n-B*ln(h*x+g)*x^2*a^2*b*d^3*h^5*
n+B*ln(h*x+g)*x^2*b^3*c^2*d*h^5*n-2*B*ln(b*x+a)*x*b^3*d^3*g^3*h^2*n+2*B*ln(
d*x+c)*x*b^3*d^3*g^3*h^2*n-B*ln(b*x+a)*b^3*c^2*d*g^2*h^3*n+2*B*ln(b*x+a)*b^
3*c*d^2*g^3*h^2*n+B*ln(d*x+c)*a^2*b*d^3*g^2*h^3*n-2*B*ln(d*x+c)*a*b^2*d^3*g
^3*h^2*n-B*ln(h*x+g)*a^2*b*d^3*g^2*h^3*n+2*B*ln(h*x+g)*a*b^2*d^3*g^3*h^2*n+
B*ln(h*x+g)*b^3*c^2*d*g^2*h^3*n+2*B*ln(h*x+g)*x^2*a*b^2*d^3*g*h^4*n-2*B*ln(
h*x+g)*x^2*b^3*c*d^2*g*h^4*n-2*B*ln(b*x+a)*x*b^3*c^2*d*g*h^4*n+4*B*ln(b*x+a
)*x*b^3*c*d^2*g^2*h^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*g*h^4*n-4*B*ln(d*x+c)*x*a
*b^2*d^3*g^2*h^3*n-2*B*ln(h*x+g)*x*a^2*b*d^3*g*h^4*n+4*B*ln(h*x+g)*x*a*b^2*
```

```
d^3*g^2*h^3*n+2*B*ln(h*x+g)*x*b^3*c^2*d*g*h^4*n-4*B*ln(h*x+g)*x*b^3*c*d^2*g
^2*h^3*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g^4*h+A*b^3*d^3*g^4*h+B*a^2*
b*d^3*g^2*h^3*n-B*a*b^2*d^3*g^3*h^2*n-B*b^3*c^2*d*g^2*h^3*n+B*b^3*c*d^2*g^3
*h^2*n-2*A*a^2*b*c*d^2*g*h^4-2*A*a*b^2*c^2*d*g*h^4+4*A*a*b^2*c*d^2*g^2*h^3-
B*x*a^2*b*c*d^2*h^5*n+2*B*ln(b*x+a)*x^2*b^3*c*d^2*g*h^4*n-2*B*ln(d*x+c)*x^2
*a*b^2*d^3*g*h^4*n+A*a^2*b*c^2*d*h^5+A*a^2*b*d^3*g^2*h^3-2*A*a*b^2*d^3*g^3*
h^2+A*b^3*c^2*d*g^2*h^3-2*A*b^3*c*d^2*g^3*h^2-B*a^2*b*c*d^2*g*h^4*n+B*a*b^2
*c^2*d*g*h^4*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c^2*d*h^5+B*ln(e*(b*x+a)
^n/((d*x+c)^n))*a^2*b*d^3*g^2*h^3-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*d^3
*g^3*h^2+B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^2*d*g^2*h^3-2*B*ln(e*(b*x+a)^n
/((d*x+c)^n))*b^3*c*d^2*g^3*h^2)/(c^2*h^2-2*c*d*g*h+d^2*g^2)/(a^2*h^2-2*a*b
*g*h+b^2*g^2)/(h*x+g)^2/b/d/h^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(181) = 362$.

Time = 52.00 (sec) , antiderivative size = 1127, normalized size of antiderivative = 5.90

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = -\frac{Ab^2d^2g^4 + Aa^2c^2h^4 - 2\,(Ab^2cd + Aabd^2)g^3h + (Ab^2c^2 + 4\,Aabcd + Aa^2d^2)g^2h^2 - 2\,(Aabc^2 + Aa^2cd)gh^3}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(A*b^2*d^2*g^4 + A*a^2*c^2*h^4 - 2*(A*b^2*c*d + A*a*b*d^2)*g^3*h + (A*
b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*g^2*h^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*g*h^
3 + ((B*b^2*c*d - B*a*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3 + (B*a
*b*c^2 - B*a^2*c*d)*h^4)*n*x + ((B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*c^2
- B*a^2*d^2)*g^2*h^2 + (B*a*b*c^2 - B*a^2*c*d)*g*h^3)*n - ((B*b^2*d^2*g^2*h
^2 - 2*B*b^2*c*d*g*h^3 + B*b^2*c^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h - 2*B*b^
2*c*d*g^2*h^2 + B*b^2*c^2*g*h^3)*n*x + (2*B*a*b*d^2*g^3*h - B*a^2*c^2*h^4 -
 (4*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*n)*lo
g(b*x + a) + ((B*b^2*d^2*g^2*h^2 - 2*B*a*b*d^2*g*h^3 + B*a^2*d^2*h^4)*n*x^2
 + 2*(B*b^2*d^2*g^3*h - 2*B*a*b*d^2*g^2*h^2 + B*a^2*d^2*g*h^3)*n*x + (2*B*b
^2*c*d*g^3*h - B*a^2*c^2*h^4 - (B*b^2*c^2 + 4*B*a*b*c*d)*g^2*h^2 + 2*(B*a*b
*c^2 + B*a^2*c*d)*g*h^3)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*g*h^
3 - (B*b^2*c^2 - B*a^2*d^2)*h^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*g^2*h
^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*g^3*h
- (B*b^2*c^2 - B*a^2*d^2)*g^2*h^2)*n)*log(h*x + g) + (B*b^2*d^2*g^4 + B*a^2
*c^2*h^4 - 2*(B*b^2*c*d + B*a*b*d^2)*g^3*h + (B*b^2*c^2 + 4*B*a*b*c*d + B*a
^2*d^2)*g^2*h^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*log(e))/(b^2*d^2*g^6*h +
 a^2*c^2*g^2*h^5 - 2*(b^2*c*d + a*b*d^2)*g^5*h^2 + (b^2*c^2 + 4*a*b*c*d + a
```

```
^2*d^2)*g^4*h^3 - 2*(a*b*c^2 + a^2*c*d)*g^3*h^4 + (b^2*d^2*g^4*h^3 + a^2*c^
2*h^7 - 2*(b^2*c*d + a*b*d^2)*g^3*h^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^2
*h^5 - 2*(a*b*c^2 + a^2*c*d)*g*h^6)*x^2 + 2*(b^2*d^2*g^5*h^2 + a^2*c^2*g*h^
6 - 2*(b^2*c*d + a*b*d^2)*g^4*h^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^3*h^4
 - 2*(a*b*c^2 + a^2*c*d)*g^2*h^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(181) = 362.

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$
$$=\frac{\left(\frac{b^2en\log(bx+a)}{b^2g^2h-2\,abgh^2+a^2h^3}-\frac{d^2en\log(dx+c)}{d^2g^2h-2\,cdgh^2+c^2h^3}-\frac{\left(2\,abd^2egn-a^2d^2ehn-\left(2\,cdegn-c^2ehn\right)b^2\right)\log(hx+g)}{\left(d^2g^2h^2-2\,cdgh^3+c^2h^4\right)a^2-2\left(d^2g^3h-2\,cdg^2h^2+c^2gh^3\right)ab+\left(d^2g^4-2\,cdg^3h+c^2g^2h^2\right)b^2}\right.}{2\,e}$$
$$-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}-\frac{A}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log
(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*
e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g
*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*
g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g
*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*
b)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^
2*h) - 1/2*A/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(181) = 362.

Time = 0.52 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \frac{Bb^3n\log\left(|bx+a|\right)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)} - \frac{Bd^3n\log\left(|dx+c|\right)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)} - \frac{Bn\log\left(bx+a\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} + \frac{Bn\log\left(dx+c\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} + \frac{\left(2\,Bb^2cdgn-2\,Babd^2gn-Bb^2c^2hn+Ba^2d^2hn\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4-2\,b^2cdg^3h-2\,abd^2g^3h+b^2c^2g^2h^2+4\,abcdg^2h^2+a^2d^2g^2h^2-2\,abc^2gh^3-2\,a^2cdgh^3+a^2c^2h^4\right)} - \frac{Bbch^2nx-Badh^2nx+Bbcghn-Badghn+Bbdg^2\log\left(e\right)-Bbcgh\log\left(e\right)-Badgh\log\left(e\right)+Bach^2\log\left(e\right)+\cdots}{2\left(bdg^2h^3x^2-bcgh^4x^2-adgh^4x^2+ach^5x^2+2\,bdg^3h^2x-2\,bcg^2h^3x-2\,adg^2h^3x+2\,acgh^4x+\cdots\right)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b^3*n*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) - 1/2
*B*d^3*n*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*B*
n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g*n - 2*B*a*b*d^2*g*n - B*b^2*c^2
*h*n + B*a^2*d^2*h*n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*a*b*d
^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*a*b*c^
2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*d*h^2*n
*x + B*b*c*g*h*n - B*a*d*g*h*n + B*b*d*g^2*log(e) - B*b*c*g*h*log(e) - B*a*
d*g*h*log(e) + B*a*c*h^2*log(e) + A*b*d*g^2 - A*b*c*g*h - A*a*d*g*h + A*a*c
*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*
d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h
 - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \frac{\ln\left(g+h\,x\right)\,\left(h\,\left(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n\right)-2\,B\,a\,b\,d^2\,g\,n+2\,B\,b^2\,c\,d\,g\,n\right)}{2\,a^2\,c^2\,h^4-4\,a^2\,c\,d\,g\,h^3+2\,a^2\,d^2\,g^2\,h^2-4\,a\,b\,c^2\,g\,h^3+8\,a\,b\,c\,d\,g^2\,h^2-4\,a\,b\,d^2\,g^3\,h+2\,b^2\,c^2\,g^2\,h^2-4\,b^2\,c\,d\,g^3\,h+\cdots} - \frac{\frac{A\,a\,c\,h^2+A\,b\,d\,g^2-A\,a\,d\,g\,h-A\,b\,c\,g\,h-B\,a\,d\,g\,h\,n+B\,b\,c\,g\,h\,n}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{x\,\left(B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2} - \frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{2\,h\,\left(g^2+2\,g\,h\,x+h^2\,x^2\right)} + \frac{B\,b^2\,n\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h} - \frac{B\,d^2\,n\,\ln\left(c+d\,x\right)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^3,x)
```

```
[Out] (log(g + h*x)*(h*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*g*n + 2*B*b^2*c*
d*g*n))/(2*a^2*c^2*h^4 + 2*b^2*d^2*g^4 + 2*a^2*d^2*g^2*h^2 + 2*b^2*c^2*g^2*
h^2 - 4*a*b*c^2*g*h^3 - 4*a*b*d^2*g^3*h - 4*a^2*c*d*g*h^3 - 4*b^2*c*d*g^3*h
 + 8*a*b*c*d*g^2*h^2) - ((A*a*c*h^2 + A*b*d*g^2 - A*a*d*g*h - A*b*c*g*h - B
*a*d*g*h*n + B*b*c*g*h*n)/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (x*(B*a
*d*h^2*n - B*b*c*h^2*n))/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h))/(2*g^2*h
+ 2*h^3*x^2 + 4*g*h^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(2*h*(g^2 +
 h^2*x^2 + 2*g*h*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*
a*b*g*h^2) - (B*d^2*n*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h^2)
```

3.301 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 284

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = -\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2}-\frac{B(bc-ad)(2bdg-bch-adh)n}{3(bg-ah)^2(dg-ch)^2(g+hx)} + \frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}-\frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3} + \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log(g+hx)}{3(bg-ah)^3(dg-ch)^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2-1/3*B*(-a*d+b*c)*(-a*d*
h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*B*n*ln(b*x+a)/
h/(-a*h+b*g)^3-1/3*B*d^3*n*ln(d*x+c)/h/(-c*h+d*g)^3+1/3*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/h/(h*x+g)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^3/(-c*h+d*g)^
3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{2548, 84\}$

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$$
$$= \frac{Bn(bc-ad)\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)}{3(bg-ah)^3(dg-ch)^3}$$
$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}+\frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}$$
$$-\frac{Bn(bc-ad)(-adh-bch+2bdg)}{3(g+hx)(bg-ah)^2(dg-ch)^2}$$
$$-\frac{Bn(bc-ad)}{6(g+hx)^2(bg-ah)(dg-ch)}-\frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (B*(b*c - a*
d)*(2*b*d*g - b*c*h - a*d*h)*n)/(3*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) +
 (b^3*B*n*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (B*d^3*n*Log[c + d*x])/(3*h*(
d*g - c*h)^3) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h*(g + h*x)^3)
+ (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(3*(b*g - a*h)^3*(d*g - c*h)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(g+hx)^3}\,dx}{3h}$$
$$= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}$$
$$+\frac{(B(bc-ad)n)\int\left(\frac{b^4}{(bc-ad)(bg-ah)^3(a+bx)}+\frac{d^4}{(bc-ad)(-dg+ch)^3(c+dx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)^3}-\frac{h^2(-2bdg-}{(bg-ah)^2(dg-}\right.}{3h}$$

$$= -\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2} - \frac{B(bc-ad)(2bdg-bch-adh)n}{3(bg-ah)^2(dg-ch)^2(g+hx)} + \frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3} - \frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3} + \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log(g+hx)}{3(bg-ah)^3(dg-ch)^3}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = -\frac{\frac{2A}{(g+hx)^3} + \frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} + B(bc-ad)n\left(\frac{h}{(bg-ah)(dg-ch)(g+hx)^2} - \frac{2h(-2bdg+bch+adh)}{(bg-ah)^2(dg-ch)^2(g+hx)} - \frac{2b^3\log(a+bx}{(bc-ad)(bg-ah}\right.}{6h}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] -1/6*((2*A)/(g + h*x)^3 + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^
3 + B*(b*c - a*d)*n*(h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (2*h*(-2*b*d
*g + b*c*h + a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) - (2*b^3*Log[a
 + b*x])/((b*c - a*d)*(b*g - a*h)^3) + (2*d^3*Log[c + d*x])/((b*c - a*d)*(d
*g - c*h)^3) - (2*h*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2
- 3*c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))/h
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3285 vs. 2(275) = 550.

Time = 88.44 (sec) , antiderivative size = 3286, normalized size of antiderivative = 11.57

method	result	size
parallelrisch	Expression too large to display	3286
risch	Expression too large to display	9645

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-6*B*ln(b*x+a)*x*a^3*b*d^4*g^2*h^6*n^2+18*B*ln(b*x+a)*x*a^2*b^2*d^4*g
^3*h^5*n^2+15*B*x*a*b^3*c^2*d^2*g^2*h^6*n^2-6*B*x^2*a^2*b^2*c*d^3*g*h^7*n^2
+6*B*x^2*a*b^3*c^2*d^2*g*h^7*n^2+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*d^
4*g^2*h^6*n-18*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*g^3*h^5*n+18*B*x
*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*g^4*h^4*n+6*B*x*a^3*b*c*d^3*g*h^7*n^
2-15*B*x*a^2*b^2*c*d^3*g^2*h^6*n^2-6*B*x*a*b^3*c^3*d*g*h^7*n^2-6*B*x^3*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*g*h^7*n+6*B*x^3*ln(e*(b*x+a)^n/((d*x+c)
```

^n))*a*b^3*d^4*g^2*h^6*n-6*B*ln(b*x+a)*x^3*a*b^3*d^4*g^2*h^6*n^2-6*B*ln(b*x
+a)*x^3*b^4*c^2*d^2*g*h^7*n^2+6*B*ln(b*x+a)*x^3*b^4*c*d^3*g^2*h^6*n^2-18*B*
ln(b*x+a)*x*a*b^3*d^4*g^4*h^4*n^2+6*B*ln(b*x+a)*x*b^4*c^3*d*g^2*h^6*n^2-18*
B*ln(b*x+a)*x*b^4*c^2*d^2*g^3*h^5*n^2+18*B*ln(b*x+a)*x*b^4*c*d^3*g^4*h^4*n^
2+6*B*ln(h*x+g)*x*a^3*b*d^4*g^2*h^6*n^2-18*B*ln(h*x+g)*x*a^2*b^2*d^4*g^3*h^
5*n^2+18*B*ln(h*x+g)*x*a*b^3*d^4*g^4*h^4*n^2-6*B*ln(h*x+g)*x*b^4*c^3*d*g^2*
h^6*n^2+18*B*ln(h*x+g)*x*b^4*c^2*d^2*g^3*h^5*n^2+6*B*x^2*ln(e*(b*x+a)^n/((d
*x+c)^n))*a^3*b*d^4*g*h^7*n-18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^
4*g^2*h^6*n+18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*g^3*h^5*n-2*B*ln
(b*x+a)*a^3*b*d^4*g^3*h^5*n^2+6*B*ln(b*x+a)*x^3*a^2*b^2*d^4*g*h^7*n^2-18*B*
ln(h*x+g)*x*b^4*c*d^3*g^4*h^4*n^2-6*B*ln(b*x+a)*x^2*a^3*b*d^4*g*h^7*n^2+18*
B*ln(b*x+a)*x^2*a^2*b^2*d^4*g^2*h^6*n^2-18*B*ln(b*x+a)*x^2*a*b^3*d^4*g^3*h^
5*n^2+6*B*ln(b*x+a)*x^2*b^4*c^3*d*g*h^7*n^2-18*B*ln(b*x+a)*x^2*b^4*c^2*d^2*
g^2*h^6*n^2+18*B*ln(b*x+a)*x^2*b^4*c*d^3*g^3*h^5*n^2+6*B*ln(h*x+g)*x^2*a^3*
b*d^4*g*h^7*n^2-18*B*ln(h*x+g)*x^2*a^2*b^2*d^4*g^2*h^6*n^2+18*B*ln(h*x+g)*x
^2*a*b^3*d^4*g^3*h^5*n^2-6*B*ln(h*x+g)*x^2*b^4*c^3*d*g*h^7*n^2+18*B*ln(h*x+
g)*x^2*b^4*c^2*d^2*g^2*h^6*n^2-18*B*ln(h*x+g)*x^2*b^4*c*d^3*g^3*h^5*n^2-6*B
*ln(h*x+g)*x^3*a^2*b^2*d^4*g*h^7*n^2+6*B*ln(h*x+g)*x^3*a*b^3*d^4*g^2*h^6*n^
2+6*B*ln(h*x+g)*x^3*b^4*c^2*d^2*g*h^7*n^2-6*B*ln(h*x+g)*x^3*b^4*c*d^3*g^2*h
^6*n^2-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*c^2*d^2*g*h^7*n+6*B*ln(e*(b*x+
a)^n/((d*x+c)^n))*a^3*b*c*d^3*g^2*h^6*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2
*b^2*c^3*d*g*h^7*n+18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*c^2*d^2*g^2*h^6
*n-18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*c*d^3*g^3*h^5*n+6*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*a*b^3*c^3*d*g^2*h^6*n-18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b
^3*c^2*d^2*g^3*h^5*n+18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*c*d^3*g^4*h^4*n
+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*c^3*d*h^8*n-2*B*ln(e*(b*x+a)^n/((d*x
+c)^n))*b^4*c^3*d*g^3*h^5*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^2*d^2*g^4
*h^4*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c*d^3*g^5*h^3*n+2*B*x^3*ln(e*(b*
x+a)^n/((d*x+c)^n))*a^3*b*d^4*h^8*n-2*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4
*d^4*g^3*h^5*n-3*B*a^3*b*d^4*g^3*h^5*n^2+8*B*a^2*b^2*d^4*g^4*h^4*n^2-5*B*a*
b^3*d^4*g^5*h^3*n^2+3*B*b^4*c^3*d*g^3*h^5*n^2-8*B*b^4*c^2*d^2*g^4*h^4*n^2+5
*B*b^4*c*d^3*g^5*h^3*n^2+2*A*a^3*b*c^3*d*h^8*n-2*A*a^3*b*d^4*g^3*h^5*n+6*A*
a^2*b^2*d^4*g^4*h^4*n-6*A*a*b^3*d^4*g^5*h^3*n-2*A*b^4*c^3*d*g^3*h^5*n+6*A*b
^4*c^2*d^2*g^4*h^4*n-6*A*b^4*c*d^3*g^5*h^3*n+2*A*b^4*d^4*g^6*h^2*n-6*A*a^3*
b*c^2*d^2*g*h^7*n+6*A*a^3*b*c*d^3*g^2*h^6*n-6*A*a^2*b^2*c^3*d*g*h^7*n+18*A*
a^2*b^2*c^2*d^2*g^2*h^6*n-18*A*a^2*b^2*c*d^3*g^3*h^5*n+6*A*a*b^3*c^3*d*g^2*
h^6*n-18*A*a*b^3*c^2*d^2*g^3*h^5*n+18*A*a*b^3*c*d^3*g^4*h^4*n+2*B*ln(h*x+g)
*x^3*a^3*b*d^4*h^8*n^2-2*B*ln(h*x+g)*x^3*b^4*c^3*d*h^8*n^2-5*B*x*a^3*b*d^4*
g^2*h^6*n^2+B*x*a^2*b^2*c^3*d*h^8*n^2+14*B*x*a^2*b^2*d^4*g^3*h^5*n^2-9*B*x*
a*b^3*d^4*g^4*h^4*n^2+5*B*x*b^4*c^3*d*g^2*h^6*n^2-14*B*x*b^4*c^2*d^2*g^3*h^
5*n^2+9*B*x*b^4*c*d^3*g^4*h^4*n^2-B*a^3*b*c^2*d^2*g*h^7*n^2+4*B*a^3*b*c*d^3
*g^2*h^6*n^2+B*a^2*b^2*c^3*d*g*h^7*n^2-9*B*a^2*b^2*c*d^3*g^3*h^5*n^2-4*B*a*
b^3*c^3*d*g^2*h^6*n^2+9*B*a*b^3*c^2*d^2*g^3*h^5*n^2+6*B*ln(b*x+a)*a^2*b^2*d
^4*g^4*h^4*n^2-6*B*ln(b*x+a)*a*b^3*d^4*g^5*h^3*n^2+2*B*ln(b*x+a)*b^4*c^3*d*
g^3*h^5*n^2-6*B*ln(b*x+a)*b^4*c^2*d^2*g^4*h^4*n^2-6*B*x^2*ln(e*(b*x+a)^n/((

```
d*x+c)^n))*b^4*d^4*g^4*h^4*n+2*B*x^2*a^3*b*c*d^3*h^8*n^2-2*B*x^2*a^3*b*d^4*
g*h^7*n^2+6*B*x^2*a^2*b^2*d^4*g^2*h^6*n^2-2*B*x^2*a*b^3*c^3*d*h^8*n^2-4*B*x
^2*a*b^3*d^4*g^3*h^5*n^2+2*B*x^2*b^4*c^3*d*g*h^7*n^2-6*B*x^2*b^4*c^2*d^2*g^
2*h^6*n^2+4*B*x^2*b^4*c*d^3*g^3*h^5*n^2-6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b
^4*d^4*g^5*h^3*n-B*x*a^3*b*c^2*d^2*h^8*n^2+6*B*ln(b*x+a)*b^4*c*d^3*g^5*h^3*
n^2+2*B*ln(h*x+g)*a^3*b*d^4*g^3*h^5*n^2-6*B*ln(h*x+g)*a^2*b^2*d^4*g^4*h^4*n
^2+6*B*ln(h*x+g)*a*b^3*d^4*g^5*h^3*n^2-2*B*ln(h*x+g)*b^4*c^3*d*g^3*h^5*n^2+
6*B*ln(h*x+g)*b^4*c^2*d^2*g^4*h^4*n^2-6*B*ln(h*x+g)*b^4*c*d^3*g^5*h^3*n^2-2
*B*ln(b*x+a)*x^3*a^3*b*d^4*h^8*n^2+2*B*ln(b*x+a)*x^3*b^4*c^3*d*h^8*n^2)/(a^
3*h^3-3*a^2*b*g*h^2+3*a*b^2*g^2*h-b^3*g^3)/(h*x+g)^3/(c^3*h^3-3*c^2*d*g*h^2
+3*c*d^2*g^2*h-d^3*g^3)/b/d/h^3/n
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="fricas
")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(272) = 544$.

Time = 0.26 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.24

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \frac{\left(\frac{2\,b^3en\log(bx+a)}{b^3g^3h-3\,ab^2g^2h^2+3\,a^2bgh^3-a^3h^4}-\frac{2\,d^3en\log(dx+c)}{d^3g^3h-3\,cd^2g^2h^2+3\,c^2dgh^3-c^3h^4}+\frac{2\,(3\,ab^2d^3eg^2n-3\,a^2b\cdots}{(d^3g^3h^3-3\,cd^2g^2h^4+3\,c^2dgh^5-c^3h^6)a^3-3\,(d^3g^4h^2-3\,cd^2g^3h^3+\cdots}\cdots\right.}{\cdots} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)}-\frac{A}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b^3*e*n*log(b*x + a)/(b^3*g^3*h - 3*a*b^2*g^2*h^2 + 3*a^2*b*g*h^3 -
a^3*h^4) - 2*d^3*e*n*log(d*x + c)/(d^3*g^3*h - 3*c*d^2*g^2*h^2 + 3*c^2*d*g*
h^3 - c^3*h^4) + 2*(3*a*b^2*d^3*e*g^2*n - 3*a^2*b*d^3*e*g*h*n + a^3*d^3*e*h
^2*n - (3*c*d^2*e*g^2*n - 3*c^2*d*e*g*h*n + c^3*e*h^2*n)*b^3)*log(h*x + g)/
((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4
*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h
- 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d
^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - ((3*d^2*e*g*h*n - c*d*e*h^
2*n)*a^2 - (5*d^2*e*g^2*n - c^2*e*h^2*n)*a*b + (5*c*d*e*g^2*n - 3*c^2*e*g*h
*n)*b^2 - 2*(2*a*b*d^2*e*g*h*n - a^2*d^2*e*h^2*n - (2*c*d*e*g*h*n - c^2*e*h
^2*n)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5
*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^
2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*
d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^
2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2
- 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h
^3)*b^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^4*x^3 + 3*g*h^3*
x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g
^3*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(272) = 544$.

Time = 1.14 (sec) , antiderivative size = 1530, normalized size of antiderivative = 5.39

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^4*n*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) - 1/3*B*d^4*n*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n - 3*B*
b^3*c^2*d*g*h*n + 3*B*a^2*b*d^3*g*h*n + B*b^3*c^3*h^2*n - B*a^3*d^3*h^2*n)*
log(h*x + g)/(b^3*d^3*g^6 - 3*b^3*c*d^2*g^5*h - 3*a*b^2*d^3*g^5*h + 3*b^3*c
^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 + 3*a^2*b*d^3*g^4*h^2 - b^3*c^3*g^3*h^
3 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 - a^3*d^3*g^3*h^3 + 3*a*b
^2*c^3*g^2*h^4 + 9*a^2*b*c^2*d*g^2*h^4 + 3*a^3*c*d^2*g^2*h^4 - 3*a^2*b*c^3*
g*h^5 - 3*a^3*c^2*d*g*h^5 + a^3*c^3*h^6) - 1/6*(4*B*b^2*c*d*g*h^3*n*x^2 - 4
*B*a*b*d^2*g*h^3*n*x^2 - 2*B*b^2*c^2*h^4*n*x^2 + 2*B*a^2*d^2*h^4*n*x^2 + 9*
```

```
B*b^2*c*d*g^2*h^2*n*x - 9*B*a*b*d^2*g^2*h^2*n*x - 5*B*b^2*c^2*g*h^3*n*x + 5
*B*a^2*d^2*g*h^3*n*x + B*a*b*c^2*h^4*n*x - B*a^2*c*d*h^4*n*x + 5*B*b^2*c*d*
g^3*h*n - 5*B*a*b*d^2*g^3*h*n - 3*B*b^2*c^2*g^2*h^2*n + 3*B*a^2*d^2*g^2*h^2
*n + B*a*b*c^2*g*h^3*n - B*a^2*c*d*g*h^3*n + 2*B*b^2*d^2*g^4*log(e) - 4*B*b
^2*c*d*g^3*h*log(e) - 4*B*a*b*d^2*g^3*h*log(e) + 2*B*b^2*c^2*g^2*h^2*log(e)
 + 8*B*a*b*c*d*g^2*h^2*log(e) + 2*B*a^2*d^2*g^2*h^2*log(e) - 4*B*a*b*c^2*g*
h^3*log(e) - 4*B*a^2*c*d*g*h^3*log(e) + 2*B*a^2*c^2*h^4*log(e) + 2*A*b^2*d^
2*g^4 - 4*A*b^2*c*d*g^3*h - 4*A*a*b*d^2*g^3*h + 2*A*b^2*c^2*g^2*h^2 + 8*A*a
*b*c*d*g^2*h^2 + 2*A*a^2*d^2*g^2*h^2 - 4*A*a*b*c^2*g*h^3 - 4*A*a^2*c*d*g*h^
3 + 2*A*a^2*c^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d
^2*g^3*h^5*x^3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*
h^6*x^3 - 2*a*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b
^2*d^2*g^5*h^3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*
c^2*g^3*h^5*x^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^
2*g^2*h^6*x^2 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6
*h^2*x - 6*b^2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x +
12*a*b*c*d*g^4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*
d*g^3*h^5*x + 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a
*b*d^2*g^6*h^2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*
a*b*c^2*g^4*h^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.17

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \frac{B\,d^3\,n\,\ln\left(c+d\,x\right)}{3\,c^3\,h^4-9\,c^2\,d\,g\,h^3+9\,c\,d^2\,g^2\,h^2-3\,d^3\,g^3\,h}$$
$$-\frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(B\,a^3\,d^3\,n-B\,b^3\,c^3\right.\right.}{3\,a^3\,c^3\,h^6-9\,a^3\,c^2\,d\,g\,h^5+9\,a^3\,c\,d^2\,g^2\,h^4-3\,a^3\,d^3\,g^3\,h^3-9\,a^2\,b\,c^3\,g\,h^5+27\,a^2\,b\,c^2\,d\,g^2\,h^4-27\,a^2\,b\,c\,d^2\,g^3}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{3\,h\,\left(g^3+3\,g^2\,h\,x+3\,g\,h^2\,x^2+h^3\,x^3\right)}-\frac{B\,b^3\,n\,\ln\left(a+b\,x\right)}{3\,a^3\,h^4-9\,a^2\,b\,g\,h^3+9\,a\,b^2\,g^2\,h^2-3\,b^3\,g^3\,h}$$
$$-\frac{\frac{2\,A\,a^2\,c^2\,h^4+2\,A\,b^2\,d^2\,g^4+2\,A\,a^2\,d^2\,g^2\,h^2+2\,A\,b^2\,c^2\,g^2\,h^2+3\,B\,a^2\,d^2\,g^2\,h^2\,n-3\,B\,b^2\,c^2\,g^2\,h^2\,n-4\,A\,a\,b\,c^2\,g\,h^3-4\,A\,a\,b\,d^2\,g^3\,h-4\,A\,a^2\,c\,d\,g\,h^3-4}{2\,\left(a^2\,c^2\,h^4-2\,a^2\,c\,d\,g\,h^3+a^2\,d^2\,g^2\,h^2-2\,a\,b\,c^2\,g\,h^3+4\,a\,b\,c\,d\,g^2\,h^2-2\,a\,b\,d^2\,g^3\,h+b^2}}{}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^4,x)
```

```
[Out] (B*d^3*n*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d
*g*h^3) - (log(g + h*x)*(h^2*(B*a^3*d^3*n - B*b^3*c^3*n) - h*(3*B*a^2*b*d^3
*g*n - 3*B*b^3*c^2*d*g*n) + 3*B*a*b^2*d^3*g^2*n - 3*B*b^3*c*d^2*g^2*n))/(3*
a^3*c^3*h^6 + 3*b^3*d^3*g^6 - 3*a^3*d^3*g^3*h^3 - 3*b^3*c^3*g^3*h^3 - 9*a^2
*b*c^3*g*h^5 - 9*a*b^2*d^3*g^5*h - 9*a^3*c^2*d*g*h^5 - 9*b^3*c*d^2*g^5*h +
9*a*b^2*c^3*g^2*h^4 + 9*a^2*b*d^3*g^4*h^2 + 9*a^3*c*d^2*g^2*h^4 + 9*b^3*c^2
*d*g^4*h^2 + 27*a*b^2*c*d^2*g^4*h^2 - 27*a*b^2*c^2*d*g^3*h^3 - 27*a^2*b*c*d
```

```
^2*g^3*h^3 + 27*a^2*b*c^2*d*g^2*h^4) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))
/(3*h*(g^3 + h^3*x^3 + 3*g^2*h*x + 3*g*h^2*x^2)) - (B*b^3*n*log(a + b*x))/(
3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - ((2*A*a^2*c^2*
h^4 + 2*A*b^2*d^2*g^4 + 2*A*a^2*d^2*g^2*h^2 + 2*A*b^2*c^2*g^2*h^2 + 3*B*a^2
*d^2*g^2*h^2*n - 3*B*b^2*c^2*g^2*h^2*n - 4*A*a*b*c^2*g*h^3 - 4*A*a*b*d^2*g^
3*h - 4*A*a^2*c*d*g*h^3 - 4*A*b^2*c*d*g^3*h + 8*A*a*b*c*d*g^2*h^2 + B*a*b*c
^2*g*h^3*n - 5*B*a*b*d^2*g^3*h*n - B*a^2*c*d*g*h^3*n + 5*B*b^2*c*d*g^3*h*n)
/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*
c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d
*g^2*h^2)) + (x*(B*a*b*c^2*h^4*n - B*a^2*c*d*h^4*n + 5*B*a^2*d^2*g*h^3*n -
5*B*b^2*c^2*g*h^3*n - 9*B*a*b*d^2*g^2*h^2*n + 9*B*b^2*c*d*g^2*h^2*n))/(2*(a
^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*c^2*g*
h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d*g^2*h
^2)) + (x^2*(B*a^2*d^2*h^4*n - B*b^2*c^2*h^4*n - 2*B*a*b*d^2*g*h^3*n + 2*B*
b^2*c*d*g*h^3*n))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^
2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3
*h + 4*a*b*c*d*g^2*h^2))/(3*g^3*h + 3*h^4*x^3 + 9*g^2*h^2*x + 9*g*h^3*x^2)
```

3.302 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5} dx$

Optimal result

Integrand size = 31, antiderivative size = 389

$$\begin{aligned}
&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5} dx \\
&= -\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3} - \frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2} \\
&\quad - \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n}{4(bg-ah)^3(dg-ch)^3(g+hx)} \\
&\quad + \frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4} - \frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4} \\
&\quad - \frac{B(bc-ad)(2bdg-bch-adh)\left(2abd^2gh-a^2d^2h^2-b^2(2d^2g^2-2cdgh+c^2h^2)\right)n\log(g+hx)}{4(bg-ah)^4(dg-ch)^4}
\end{aligned}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^3-1/8*B*(-a*d+b*c)*(-a*d
*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n/(-a*h+
b*g)^3/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*B*n*ln(b*x+a)/h/(-a*h+b*g)^4-1/4*B*d^4*
n*ln(d*x+c)/h/(-c*h+d*g)^4+1/4*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h/(h*x+g)
^4-1/4*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*(2*a*b*d^2*g*h-a^2*d^2*h^2-b^2*(
c^2*h^2-2*c*d*g*h+2*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^4/(-c*h+d*g)^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 84}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$$
$$= -\frac{Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)}{4(g+hx)(bg-ah)^3(dg-ch)^3}$$
$$-\frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)\left(-a^2d^2h^2+2abd^2gh-(b^2(c^2h^2-2cdgh+2d^2g^2))\right)}{4(bg-ah)^4(dg-ch)^4}$$
$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}+\frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4}$$
$$-\frac{Bn(bc-ad)(-adh-bch+2bdg)}{8(g+hx)^2(bg-ah)^2(dg-ch)^2}-\frac{Bn(bc-ad)}{12(g+hx)^3(bg-ah)(dg-ch)}-\frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) - (B*(b*c - a
*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(8*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2
) - (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2))*n)/(4*(b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*B
*n*Log[a + b*x])/(4*h*(b*g - a*h)^4) - (B*d^4*n*Log[c + d*x])/(4*h*(d*g - c
*h)^4) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*h*(g + h*x)^4) - (B*(b
*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^2*h^2 - b^2*(2*d
^2*g^2 - 2*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(4*(b*g - a*h)^4*(d*g - c*h)
^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(g+hx)^4}\,dx}{4h}$$

$$= -\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4} + \frac{(B(bc-ad)n)\int\left(\frac{b^5}{(bc-ad)(bg-ah)^4(a+bx)} - \frac{d^5}{(bc-ad)(-dg+ch)^4(c+dx)} + \frac{h^2}{(bg-ah)(dg-ch)(g+hx)^4} - \frac{h^2(-2bdg+b}{(bg-ah)^2(dg-c}\right.}{}$$

$$= -\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3} - \frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2} - \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n}{4(bg-ah)^3(dg-ch)^3(g+hx)} + \frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4} - \frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4} - \frac{B(bc-ad)(2bdg-bch-adh)\left(2abd^2gh-a^2d^2h^2-b^2(2d^2g^2-2cdgh+c^2h^2)\right)n\log(g+hx)}{4(bg-ah)^4(dg-ch)^4}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = -\frac{\frac{A}{(g+hx)^4} + \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4} - B(bc-ad)n\left(-\frac{h}{3(bg-ah)(dg-ch)(g+hx)^3} + \frac{h(-2bdg+bch+adh)}{2(bg-ah)^2(dg-ch)^2(g+hx)^2} - \frac{h(a^2d^2h^2}{}\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] -1/4*(A/(g + h*x)^4 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4 - B*
(b*c - a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (h*(-2*b*d*g
+ b*c*h + a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*d^2
*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))/((b
*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*g
 - a*h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(-2*b*d*g
+ b*c*h + a*d*h)*(-2*a*b*d^2*g*h + a^2*d^2*h^2 + b^2*(2*d^2*g^2 - 2*c*d*g*h
 + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/h
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5230 vs. $2(378) = 756$.

Time = 233.55 (sec) , antiderivative size = 5231, normalized size of antiderivative = 13.45

method	result	size
parallelrisch	Expression too large to display	5231
risch	Expression too large to display	16077

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="fricas
")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(375) = 750$.

Time = 0.36 (sec) , antiderivative size = 1912, normalized size of antiderivative = 4.92

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxima
")
```

[Out] 1/24*(6*b^4*e*n*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*h^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*e*n*log(d*x + c)/(d^4*g^4*h - 4*c*d^3*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4*e*g^3*n - 6*a^2*b^2*d^4*e*g^2*h*n + 4*a^3*b*d^4*e*g*h^2*n - a^4*d^4*e*h^3*n - (4*c*d^3*e*g^3*n - 6*c^2*d^2*e*g^2*h*n + 4*c^3*d*e*g*h^2*n - c^4*e*h^3*n)*b^4)*log(h*x + g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6 - 4*c^3*d*g*h^7 + c^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2*d^2*g^3*h^5 - 4*c^3*d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d^3*g^5*h^3 + 6*c^2*d^2*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 - 4*(d^4*g^7*h - 4*c*d^3*g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4*g^3*h^5)*a*b^3 + (d^4*g^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g^5*h^3 + c^4*g^4*h^4)*b^4) - ((11*d^3*e*g^2*h^2*n - 7*c*d^2*e*g*h^3*n + 2*c^2*d*e*h^4*n)*a^3 - (31*d^3*e*g^3*h*n - 15*c*d^2*e*g^2*h^2*n + 2*c^3*e*h^4*n)*a^2*b + (26*d^3*e*g^4*n - 15*c^2*d*e*g^2*h^2*n + 7*c^3*e*g*h^3*n)*a*b^2 - (26*c*d^2*e*g^4*n - 31*c^2*d*e*g^3*h*n + 11*c^3*e*g^2*h^2*n)*b^3 + 6*(3*a*b^2*d^3*e*g^2*h^2*n - 3*a^2*b*d^3*e*g*h^3*n + a^3*d^3*e*h^4*n - (3*c*d^2*e*g^2*h^2*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*b^3)*x^2 + 3*((5*d^3*e*g*h^3*n - c*d^2*e*h^4*n)*a^3 - 3*(5*d^3*e*g^2*h^2*n - c*d^2*e*g*h^3*n)*a^2*b + (14*d^3*e*g^3*h*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*a*b^2 - (14*c*d^2*e*g^3*h*n - 15*c^2*d*e*g^2*h^2*n + 5*c^3*e*g*h^3*n)*b^3)*x)/((d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^3 - 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a^2*b + 3*(d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*a*b^2 - (d^3*g^9 - 3*c*d^2*g^8*h + 3*c^2*d*g^7*h^2 - c^3*g^6*h^3)*b^3 + ((d^3*g^3*h^6 - 3*c*d^2*g^2*h^7 + 3*c^2*d*g*h^8 - c^3*h^9)*a^3 - 3*(d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^8)*a^2*b + 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a*b^2 - (d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*b^3)*x^3 + 3*((d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^8)*a^3 - 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^2*b + 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a*b^2 - (d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*b^3)*x^2 + 3*((d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^3 - 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^2*b + 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a*b^2 - (d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*b^3)*x))*B/e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) - 1/4*A/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3325 vs. 2(375) = 750.

Time = 2.55 (sec) , antiderivative size = 3325, normalized size of antiderivative = 8.55

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*n*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) - 1/4*B*d^5*n*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^
3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6*B*b^4
*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n - 4*B*a^
3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(b^4*d^4*
g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^2 + 16*a*
b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 - 24*a*b^3*
c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3 + b^4*c^4*
g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 + 16*a^3*b*c*
d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b^2*c^3*d*g^3*
h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*b^2*c^4*g^2*h^
6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b*c^4*g*h^7 - 4*
a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h^4*n*x^3 - 18*B*
a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*B*a^2*b*d^3*g*h^5
*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 + 60*B*b^3*c*d^2*g^3
*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2*d*g^2*h^4*n*x^2 +
63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*x^2 + 9*B*a*b^2*c^2*d*g
*h^5*n*x^2 - 9*B*a^2*b*c*d^2*g*h^5*n*x^2 - 21*B*a^3*d^3*g*h^5*n*x^2 - 3*B*a
*b^2*c^3*h^6*n*x^2 + 3*B*a^3*c*d^2*h^6*n*x^2 + 68*B*b^3*c*d^2*g^4*h^2*n*x -
 68*B*a*b^2*d^3*g^4*h^2*n*x - 76*B*b^3*c^2*d*g^3*h^3*n*x + 76*B*a^2*b*d^3*g
^3*h^3*n*x + 26*B*b^3*c^3*g^2*h^4*n*x + 24*B*a*b^2*c^2*d*g^2*h^4*n*x - 24*B
*a^2*b*c*d^2*g^2*h^4*n*x - 26*B*a^3*d^3*g^2*h^4*n*x - 10*B*a*b^2*c^3*g*h^5*
n*x + 10*B*a^3*c*d^2*g*h^5*n*x + 2*B*a^2*b*c^3*h^6*n*x - 2*B*a^3*c^2*d*h^6*
n*x + 26*B*b^3*c*d^2*g^5*h*n - 26*B*a*b^2*d^3*g^5*h*n - 31*B*b^3*c^2*d*g^4*
h^2*n + 31*B*a^2*b*d^3*g^4*h^2*n + 11*B*b^3*c^3*g^3*h^3*n + 15*B*a*b^2*c^2*
d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n - 11*B*a^3*d^3*g^3*h^3*n - 7*B*a*b
^2*c^3*g^2*h^4*n + 7*B*a^3*c*d^2*g^2*h^4*n + 2*B*a^2*b*c^3*g*h^5*n - 2*B*a^
3*c^2*d*g*h^5*n + 6*B*b^3*d^3*g^6*log(e) - 18*B*b^3*c*d^2*g^5*h*log(e) - 18
*B*a*b^2*d^3*g^5*h*log(e) + 18*B*b^3*c^2*d*g^4*h^2*log(e) + 54*B*a*b^2*c*d^
2*g^4*h^2*log(e) + 18*B*a^2*b*d^3*g^4*h^2*log(e) - 6*B*b^3*c^3*g^3*h^3*log(
e) - 54*B*a*b^2*c^2*d*g^3*h^3*log(e) - 54*B*a^2*b*c*d^2*g^3*h^3*log(e) - 6*
B*a^3*d^3*g^3*h^3*log(e) + 18*B*a*b^2*c^3*g^2*h^4*log(e) + 54*B*a^2*b*c^2*d
```

```
*g^2*h^4*log(e) + 18*B*a^3*c*d^2*g^2*h^4*log(e) - 18*B*a^2*b*c^3*g*h^5*log(
e) - 18*B*a^3*c^2*d*g*h^5*log(e) + 6*B*a^3*c^3*h^6*log(e) + 6*A*b^3*d^3*g^6
 - 18*A*b^3*c*d^2*g^5*h - 18*A*a*b^2*d^3*g^5*h + 18*A*b^3*c^2*d*g^4*h^2 + 5
4*A*a*b^2*c*d^2*g^4*h^2 + 18*A*a^2*b*d^3*g^4*h^2 - 6*A*b^3*c^3*g^3*h^3 - 54
*A*a*b^2*c^2*d*g^3*h^3 - 54*A*a^2*b*c*d^2*g^3*h^3 - 6*A*a^3*d^3*g^3*h^3 + 1
8*A*a*b^2*c^3*g^2*h^4 + 54*A*a^2*b*c^2*d*g^2*h^4 + 18*A*a^3*c*d^2*g^2*h^4 -
 18*A*a^2*b*c^3*g*h^5 - 18*A*a^3*c^2*d*g*h^5 + 6*A*a^3*c^3*h^6)/(b^3*d^3*g^
6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 - 3*a*b^2*d^3*g^5*h^6*x^4 + 3*b^3*c^2*d
*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*x^4 + 3*a^2*b*d^3*g^4*h^7*x^4 - b^3*c^
3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8*x^4 - 9*a^2*b*c*d^2*g^3*h^8*x^4 - a^3
*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^9*x^4 + 9*a^2*b*c^2*d*g^2*h^9*x^4 + 3*
a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g*h^10*x^4 - 3*a^3*c^2*d*g*h^10*x^4 + a
^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x^3 - 12*b^3*c*d^2*g^6*h^5*x^3 - 12*a*b
^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5*h^6*x^3 + 36*a*b^2*c*d^2*g^5*h^6*x^3
+ 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^3*g^4*h^7*x^3 - 36*a*b^2*c^2*d*g^4*h^7
*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4*a^3*d^3*g^4*h^7*x^3 + 12*a*b^2*c^3*g^
3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^3 + 12*a^3*c*d^2*g^3*h^8*x^3 - 12*a^2*
b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*h^9*x^3 + 4*a^3*c^3*g*h^10*x^3 + 6*b^3
*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h^4*x^2 - 18*a*b^2*d^3*g^7*h^4*x^2 + 18
*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d^2*g^6*h^5*x^2 + 18*a^2*b*d^3*g^6*h^5*
x^2 - 6*b^3*c^3*g^5*h^6*x^2 - 54*a*b^2*c^2*d*g^5*h^6*x^2 - 54*a^2*b*c*d^2*g
^5*h^6*x^2 - 6*a^3*d^3*g^5*h^6*x^2 + 18*a*b^2*c^3*g^4*h^7*x^2 + 54*a^2*b*c^
2*d*g^4*h^7*x^2 + 18*a^3*c*d^2*g^4*h^7*x^2 - 18*a^2*b*c^3*g^3*h^8*x^2 - 18*
a^3*c^2*d*g^3*h^8*x^2 + 6*a^3*c^3*g^2*h^9*x^2 + 4*b^3*d^3*g^9*h^2*x - 12*b^
3*c*d^2*g^8*h^3*x - 12*a*b^2*d^3*g^8*h^3*x + 12*b^3*c^2*d*g^7*h^4*x + 36*a*
b^2*c*d^2*g^7*h^4*x + 12*a^2*b*d^3*g^7*h^4*x - 4*b^3*c^3*g^6*h^5*x - 36*a*b
^2*c^2*d*g^6*h^5*x - 36*a^2*b*c*d^2*g^6*h^5*x - 4*a^3*d^3*g^6*h^5*x + 12*a*
b^2*c^3*g^5*h^6*x + 36*a^2*b*c^2*d*g^5*h^6*x + 12*a^3*c*d^2*g^5*h^6*x - 12*
a^2*b*c^3*g^4*h^7*x - 12*a^3*c^2*d*g^4*h^7*x + 4*a^3*c^3*g^3*h^8*x + b^3*d^
3*g^10*h - 3*b^3*c*d^2*g^9*h^2 - 3*a*b^2*d^3*g^9*h^2 + 3*b^3*c^2*d*g^8*h^3
+ 9*a*b^2*c*d^2*g^8*h^3 + 3*a^2*b*d^3*g^8*h^3 - b^3*c^3*g^7*h^4 - 9*a*b^2*c
^2*d*g^7*h^4 - 9*a^2*b*c*d^2*g^7*h^4 - a^3*d^3*g^7*h^4 + 3*a*b^2*c^3*g^6*h^
5 + 9*a^2*b*c^2*d*g^6*h^5 + 3*a^3*c*d^2*g^6*h^5 - 3*a^2*b*c^3*g^5*h^6 - 3*a
^3*c^2*d*g^5*h^6 + a^3*c^3*g^4*h^7)
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 2570, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^5,x)
```

```
[Out] ((x*(13*B*a^3*d^3*g^2*h^4*n - 13*B*b^3*c^3*g^2*h^4*n - B*a^2*b*c^3*h^6*n +
B*a^3*c^2*d*h^6*n + 5*B*a*b^2*c^3*g*h^5*n - 5*B*a^3*c*d^2*g*h^5*n + 34*B*a*
```

b^2*d^3*g^4*h^2*n - 38*B*a^2*b*d^3*g^3*h^3*n - 34*B*b^3*c*d^2*g^4*h^2*n + 3
8*B*b^3*c^2*d*g^3*h^3*n - 12*B*a*b^2*c^2*d*g^2*h^4*n + 12*B*a^2*b*c*d^2*g^2
*h^4*n))/(3*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3
- 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g
^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*
b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b
*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (6*A*a^3*c^3*h^6 + 6*A*b^3*d^3*g
^6 - 6*A*a^3*d^3*g^3*h^3 - 6*A*b^3*c^3*g^3*h^3 + 18*A*a*b^2*c^3*g^2*h^4 + 1
8*A*a^2*b*d^3*g^4*h^2 + 18*A*a^3*c*d^2*g^2*h^4 + 18*A*b^3*c^2*d*g^4*h^2 - 1
1*B*a^3*d^3*g^3*h^3*n + 11*B*b^3*c^3*g^3*h^3*n - 18*A*a^2*b*c^3*g*h^5 - 18*
A*a*b^2*d^3*g^5*h - 18*A*a^3*c^2*d*g*h^5 - 18*A*b^3*c*d^2*g^5*h + 2*B*a^2*b
*c^3*g*h^5*n - 26*B*a*b^2*d^3*g^5*h*n - 2*B*a^3*c^2*d*g*h^5*n + 26*B*b^3*c*
d^2*g^5*h*n + 54*A*a*b^2*c*d^2*g^4*h^2 - 54*A*a*b^2*c^2*d*g^3*h^3 - 54*A*a^
2*b*c*d^2*g^3*h^3 + 54*A*a^2*b*c^2*d*g^2*h^4 - 7*B*a*b^2*c^3*g^2*h^4*n + 31
*B*a^2*b*d^3*g^4*h^2*n + 7*B*a^3*c*d^2*g^2*h^4*n - 31*B*b^3*c^2*d*g^4*h^2*n
 + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n)/(6*(a^3*c^3*h^6
 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*
a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h
^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*
b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b
*c^2*d*g^2*h^4)) + (x^3*(B*a^3*d^3*h^6*n - B*b^3*c^3*h^6*n - 3*B*a^2*b*d^3*
g*h^5*n + 3*B*b^3*c^2*d*g*h^5*n + 3*B*a*b^2*d^3*g^2*h^4*n - 3*B*b^3*c*d^2*g
^2*h^4*n))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 -
 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^
5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b
^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*
c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x^2*(B*a*b^2*c^3*h^6*n - B*a^3*c*
d^2*h^6*n + 7*B*a^3*d^3*g*h^5*n - 7*B*b^3*c^3*g*h^5*n + 20*B*a*b^2*d^3*g^3*
h^3*n - 21*B*a^2*b*d^3*g^2*h^4*n - 20*B*b^3*c*d^2*g^3*h^3*n + 21*B*b^3*c^2*
d*g^2*h^4*n - 3*B*a*b^2*c^2*d*g*h^5*n + 3*B*a^2*b*c*d^2*g*h^5*n))/(2*(a^3*c
^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^
5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3
*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2
+ 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9
*a^2*b*c^2*d*g^2*h^4)))/(4*g^4*h + 4*h^5*x^4 + 16*g^3*h^2*x + 16*g*h^4*x^3
+ 24*g^2*h^3*x^2) + (log(g + h*x)*(h*(6*B*a^2*b^2*d^4*g^2*n - 6*B*b^4*c^2*d
^2*g^2*n) - h^2*(4*B*a^3*b*d^4*g*n - 4*B*b^4*c^3*d*g*n) + h^3*(B*a^4*d^4*n
- B*b^4*c^4*n) - 4*B*a*b^3*d^4*g^3*n + 4*B*b^4*c*d^3*g^3*n))/(4*a^4*c^4*h^8
 + 4*b^4*d^4*g^8 + 4*a^4*d^4*g^4*h^4 + 4*b^4*c^4*g^4*h^4 + 24*a^2*b^2*c^4*g
^2*h^6 + 24*a^2*b^2*d^4*g^6*h^2 + 24*a^4*c^2*d^2*g^2*h^6 + 24*b^4*c^2*d^2*g
^6*h^2 - 16*a^3*b*c^4*g*h^7 - 16*a*b^3*d^4*g^7*h - 16*a^4*c^3*d*g*h^7 - 16*
b^4*c*d^3*g^7*h - 16*a*b^3*c^4*g^3*h^5 - 16*a^3*b*d^4*g^5*h^3 - 16*a^4*c*d^
3*g^3*h^5 - 16*b^4*c^3*d*g^5*h^3 + 64*a*b^3*c*d^3*g^6*h^2 + 64*a*b^3*c^3*d*
g^4*h^4 + 64*a^3*b*c*d^3*g^4*h^4 + 64*a^3*b*c^3*d*g^2*h^6 - 96*a*b^3*c^2*d^
2*g^5*h^3 - 96*a^2*b^2*c*d^3*g^5*h^3 - 96*a^2*b^2*c^3*d*g^3*h^5 - 96*a^3*b*

```
c^2*d^2*g^3*h^5 + 144*a^2*b^2*c^2*d^2*g^4*h^4) - (B*log((e*(a + b*x)^n)/(c
+ d*x)^n))/(4*h*(g^4 + h^4*x^4 + 4*g^3*h*x + 4*g*h^3*x^3 + 6*g^2*h^2*x^2))
+ (B*b^4*n*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a
^2*b^2*g^2*h^3 - 16*a^3*b*g*h^4) - (B*d^4*n*log(c + d*x))/(4*c^4*h^5 + 4*d^
4*g^4*h - 16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)
```

3.303 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 570

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$

$$= \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3h^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3h^2n^2\log(c+dx)}{3b^3d^3}$$

$$+ \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log(c+dx)}{3b^3d^3}$$

$$- \frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2}$$

$$- \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3}$$

$$+ \frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+b\right.\right.}{3b^3d^3}$$

$$- \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h}$$

$$+ \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h}$$

$$+ \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*h^2*n^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln((b*x
+a)/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2
*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^2*ln(d*x+c)/b^3/d^3-2/3*B*(-a*d+
b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b^3/d^2-1/3*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*
d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n
```

```
)))/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^3/h+1/3*
(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+2/3*B^2*(-a*d+b*c)*(a^2*d^2
*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2573, 2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$

$$= \frac{2Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+\right.\right.}{3b^3d^3}$$

$$+\frac{2B^2n^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

$$-\frac{2Bhn(a+bx)(bc-ad)(-adh-2bch+3bdg)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b^3d^2}$$

$$-\frac{(bg-ah)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3b^3h}$$

$$-\frac{Bh^2n(c+dx)^2(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3bd^3}$$

$$+\frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3h}$$

$$+\frac{2B^2hn^2(bc-ad)^2\log(c+dx)(-adh-2bch+3bdg)}{3b^3d^3}$$

$$+\frac{B^2h^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2h^2n^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2h^2n^2x(bc-ad)^2}{3b^2d^2}$$

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*h^2*n^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[
(a + b*x)/(c + d*x)])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[c + d*x]
)/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[
c + d*x])/(3*b^3*d^3) - (2*B*(b*c - a*d)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a
 + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b^3*d^2) - (B*(b*c - a
*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) +
 (2*B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3
*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(3*b^3*d^3) - ((b*g - a*h)^3*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(3*b^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(3*h) + (2*B^2*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h
```

```
) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
```

```
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(bg-ah-(dg-ch)x)^2\left(A+B\log\left(ex^n\right)\right)^2}{(b-dx)^4}dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h} - \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int\frac{(bg-ah+(-dg+ch)x)^3(A+B\log(ex^n))}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h} - \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int\left(\frac{(bg-ah)^3(A+B\log(ex^n))}{b^3x}+\frac{(bc-ad)^3h^3(A+B\log(ex^n))}{bd^2(b-dx)^3}+\frac{(bc-ad)^2h^2(3bdg-2bch-adh)(A}{b^2d^2(b-dx)^2}\right.\right.}{3h}\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h}$$

$$-\operatorname{Subst}\left(\frac{(2B(bc-ad)^3h^2n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\operatorname{Subst}\left(\frac{(2B(bg-ah)^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\operatorname{Subst}\left(\frac{(2B(bc-ad)^2h(3bdg-2bch-adh)n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e\ \ldots +bx)^n(c+dx)^{-n}\right)$$

$$-\operatorname{Subst}\left(\frac{(2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n)\operatorname{Subst}\left(\int\frac{A+B\log\ldots}{b-d\ldots}\right.}{3b^3d^2}\ \ldots +bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\right.}{3b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h}$$
$$+\text{Subst}\left(\frac{(B^2(bc-ad)^3h^2n^2)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{(2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2)\,\text{Subst}\left(\int\frac{\log(}{}\right.}{3b^3d^3}\right.$$
$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log(c+dx)}{3b^3d^3} - \frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2} - \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} + \frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log(e\right.}{3b^3d^3} - \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h} + \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h} + \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3} + \text{Subst}\left(\frac{(B^2(bc-ad)^3h^2n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd^3}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3h^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3h^2n^2\log(c+dx)}{3b^3d^3} + \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log(c+dx)}{3b^3d^3} - \frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2} - \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} + \frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log(e\right.}{3b^3d^3} - \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h} + \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h} + \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.59

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \frac{-aB^2d^3(3b^2g^2-3abgh+a^2h^2)\,n^2\log^2(a+bx)+Bn\log(a+bx)\left(2b^3Bc(3d^2g^2-3cdgh+c^2h^2)\,n\log(c-\right.}{}$$

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*Lo
g[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] + 2*
B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b^2*g
^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h) + 2*b
^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*g*h +
a^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + b*(-(b^2*B^2*c*(3*d^2*g^2 - 3
*c*d*g*h + c^2*h^2)*n^2*Log[c + d*x]^2) + B*n*Log[c + d*x]*(-2*A*b^2*c*(3*d
^2*g^2 - 3*c*d*g*h + c^2*h^2) + B*(2*a^2*c*d^2*h^2 - 3*b^2*c^2*h*(-2*d*g +
c*h) + a*b*d*(-6*d^2*g^2 - 6*c*d*g*h + c^2*h^2))*n - 2*b^2*B*c*(3*d^2*g^2 -
 3*c*d*g*h + c^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(a^2*B*d^2*h^2*
n*(-2*A + B*n)*x + a*b*B*n*(A*d^2*(-6*g^2 + 6*g*h*x + h^2*x^2) - 2*B*n*(3*d
^2*g^2 + c^2*h^2 + c*d*h*(-3*g + h*x))) + b^2*x*(B^2*c^2*h^2*n^2 + A^2*d^2*
(3*g^2 + 3*g*h*x + h^2*x^2) + A*B*c*h*n*(2*c*h - d*(6*g + h*x))) + B*(-2*a^
2*B*d^2*h^2*n*x + a*b*B*d^2*n*(-6*g^2 + 6*g*h*x + h^2*x^2) + b^2*x*(B*c*h*n
*(-6*d*g + 2*c*h - d*h*x) + 2*A*d^2*(3*g^2 + 3*g*h*x + h^2*x^2)))*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + b^2*B^2*d^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h +
a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(3*b^3*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 90.91 (sec) , antiderivative size = 8443, normalized size of antiderivative = 14.81

method	result	size
risch	Expression too large to display	8443

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (hx+g)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*h^2*x^2 + 2*A^2*g*h*x + A^2*g^2 + (B^2*h^2*x^2 + 2*B^2*g*h*x +
 B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h^2*x^2 + 2*A*B*g*h*x +
 A*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. $2(549) = 1098$.

Time = 0.76 (sec) , antiderivative size = 1671, normalized size of antiderivative = 2.93

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*h^2*x^3 + 2*A*B*g*
h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*h*x^2 + 2*A*B*g^2*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A^2*g^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x
+ c)/d)*A*B*g^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
+ (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*g*h/e + 1/3*(2*a^3*e*n*log(b*x + a)/b^3
- 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^
2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*h^2/e + 1/3*(2*a^2*c*d^2*h^2*n^2 - (
6*c*d^2*g*h*n^2 - c^2*d*h^2*n^2)*a*b - (6*c*d^2*g^2*n*log(e) + (3*h^2*n^2 +
```

```
 2*h^2*n*log(e))*c^3 - 6*(g*h*n^2 + g*h*n*log(e))*c^2*d)*b^2)*B^2*log(d*x +
 c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^
2*n^2 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*b^3)*(log(b*x + a
)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B
^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*h^2*x^3*log(e)^2 + 2*(3*c*d^2*g^2*n^2 - 3*c
^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*g^
2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*
h^2*n*log(e) - (c*d^2*h^2*n*log(e) - 3*d^3*g*h*log(e)^2)*b^3)*B^2*x^2 - (3*
a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^2*log(b*x + a)
^2 + ((h^2*n^2 - 2*h^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*h^2*n^2 - 3*d^3*g*h*n
*log(e))*a*b^2 - (6*c*d^2*g*h*n*log(e) - 3*d^3*g^2*log(e)^2 - (h^2*n^2 + 2*
h^2*n*log(e))*c^2*d)*b^3)*B^2*x - ((3*h^2*n^2 - 2*h^2*n*log(e))*a^3*d^3 - (
c*d^2*h^2*n^2 + 6*(g*h*n^2 - g*h*n*log(e))*d^3)*a^2*b + 2*(3*c*d^2*g*h*n^2
- c^2*d*h^2*n^2 - 3*d^3*g^2*n*log(e))*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^
3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log((b*x + a)^n)^2
 + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log(
(d*x + c)^n)^2 + (2*B^2*b^3*d^3*h^2*x^3*log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d
*g*h*n + c^3*h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n
- 6*d^3*g*h*log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n -
 (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d
^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*log(b*x + a))*log((b*x +
a)^n) - (2*B^2*b^3*d^3*h^2*x^3*log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n +
c^3*h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g
*h*log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2
*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*g^2*n
- 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*h^2*
x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d
*x + c)^n))/(b^3*d^3)
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Timed out}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int (g+h\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 dx$$

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.304 $\int (g+hx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 294

$$\begin{aligned}
&\int (g+hx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2hn^2\log(c+dx)}{b^2d^2} \\
&\quad - \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d} \\
&\quad + \frac{B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&\quad - \frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2h} \\
&\quad + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} \\
&\quad + \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

```
[Out] B^2*(-a*d+b*c)^2*h*n^2*ln(d*x+c)/b^2/d^2-B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e
*(b*x+a)^n/((d*x+c)^n)))/b^2/d+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a
*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^2/d^2-1/2*(-a*h+b*g)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))^2/h+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*polylog(2,d
*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2573, 2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\frac{Bn(bc-ad)(-adh-bch+2bdg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}$$
$$-\frac{(bg-ah)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2h}$$
$$-\frac{Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d}$$
$$+\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2h}$$
$$+\frac{B^2n^2(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$
$$+\frac{B^2hn^2(bc-ad)^2\log(c+dx)}{b^2d^2}$$

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/(b^2*d^2) - (B*(b*c - a*d)*h*n*(a +
b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^2*d) + (B*(b*c - a*d)*(2*
b*d*g - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a +
b*x)^n)/(c + d*x)^n]))/(b^2*d^2) - ((b*g - a*h)^2*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(2*b^2*h) + ((g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(2*h) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(bg-ah-(dg-ch)x)\left(A+B\log\left(ex^n\right)\right)^2}{(b-dx)^3}dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} - \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \frac{(bg-ah+(-dg+ch)x)^2(A+B\log(ex^n))}{x(b-dx)^2}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} - \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \left(\frac{(bg-ah)^2(A+B\log(ex^n))}{b^2x} + \frac{(bc-ad)^2h^2(A+B\log(ex^n))}{bd(b-dx)^2} + \frac{(bc-ad)h(2bdg-bch-adh)(A+B\log}{b^2d(b-dx)}\right.}{h} \right.$$

$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} - \text{Subst}\left(\frac{(B(bc-ad)^2hn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(B(bg-ah)^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(B(bc-ad)(2bdg-bch-adh)n)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d} + \frac{B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} - \frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2h} + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} + \text{Subst}\left(\frac{(B^2(bc-ad)^2hn^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(B^2(bc-ad)(2bdg-bch-adh)n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{B^2(bc-ad)^2hn^2\log(c+dx)}{b^2d^2} - \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d} + \frac{B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} - \frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2h} + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} + \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.61

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \frac{aB^2d^2(-2bg+ah)n^2\log^2(a+bx)-2Bn\log(a+bx)\left(b^2Bc(-2dg+ch)n\log(c+dx)-B(bc-ad)(-2bd\right.}{}$$

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (a*B^2*d^2*(-2*b*g + a*h)*n^2*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*(b^2*B*c*
(-2*d*g + c*h)*n*Log[c + d*x] - B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n*
Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(A*(-2*b*d*g + a*d*h) + B*(-2*b*d*g +
b*c*h - a*d*h)*n + B*d*(-2*b*g + a*h)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) +
b*(b*B^2*c*(-2*d*g + c*h)*n^2*Log[c + d*x]^2 + 2*B*n*Log[c + d*x]*(A*b*c*(-
2*d*g + c*h) + B*(b*c^2*h - a*d*(2*d*g + c*h))*n + b*B*c*(-2*d*g + c*h)*Log
[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(A*b*x*(2*A*d*g - 2*B*c*h*n + A*d*h*x) +
 2*a*B*n*(-2*A*d*g - 2*B*d*g*n + B*c*h*n + A*d*h*x) + 2*B*(a*B*d*n*(-2*g +
h*x) + b*x*(2*A*d*g - B*c*h*n + A*d*h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ b*B^2*d*x*(2*g + h*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(b*c -
 a*d)*(-2*b*d*g + b*c*h + a*d*h)*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])/(2*b^2*d^2)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

Fricas [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(289) = 578$.

Time = 0.71 (sec) , antiderivative size = 903, normalized size of antiderivative = 3.07

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= ABhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2hx^2+2\,ABgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2gx$$
$$+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABg}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)ABh}{e}$$
$$-\frac{(acdhn^2+(2\,cdgn\log(e)-(hn^2+hn\log(e))c^2)b)B^2\log(dx+c)}{bd^2}$$
$$+\frac{(2\,abd^2gn^2-a^2d^2hn^2-(2\,cdgn^2-c^2hn^2)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}$$
$$+\frac{B^2b^2d^2hx^2\log(e)^2+2\,(2\,cdgn^2-c^2hn^2)B^2b^2\log(bx+a)\log(dx+c)-(2\,cdgn^2-c^2hn^2)B^2b^2\log(dx-}{}$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")

[Out] A*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*A*B*g*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^2*g*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*g/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*h/e - (a*c*d*h*n^2 + (2*c*d*g*n*log(e) - (
h*n^2 + h*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^2 - a
^2*d^2*h*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/
2*(B^2*b^2*d^2*h*x^2*log(e)^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(b*x
 + a)*log(d*x + c) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c)^2 - (2*
a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*n*log(e) -
 (c*d*h*n*log(e) - d^2*g*log(e)^2)*b^2)*B^2*x + 2*((h*n^2 - h*n*log(e))*a^2
*d^2 - (c*d*h*n^2 - 2*d^2*g*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*d^2*
h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*h*x^2 + 2*B^2*
b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n
- c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))
*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a))*log((b*x + a)
^n) - 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x +
 c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n
 - a^2*d^2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*
log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
[Out] integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)^2\,dx$$

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.305 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx = \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd} + \frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{b} + \frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] 2*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+2*B^2*(-a*d+b*c)*n^2*pol
ylog(2,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx = \frac{2Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{bd} + \frac{(a+bx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{b} + \frac{2B^2n^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (2*B*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n]))/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)^2)/b + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b
*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} - \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{c+dx}\,dx}{b}$$

$$= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} - \frac{(2B^2(bc-ad)^2n^2)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{bd}$$

$$= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} - \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx, x, c+dx\right)}{bd^2}$$

$$= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} + \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} + \frac{(2B^2(bc-ad)^2n^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx, x, \frac{1}{c+dx}\right)}{bd^2}$$

$$= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b} + \frac{2B^2(bc-ad)n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$

$$=\frac{A^2bdx-2AB(bc-ad)n\log(c+dx)+2ABd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2d(a+bx)\log^2\left(e(a\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (A^2*b*d*x - 2*A*B*(b*c - a*d)*n*Log[c + d*x] + 2*A*B*d*(a + b*x)*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^
2 + B^2*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*
x))/(-(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d
)/(b*c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*d)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 2240, normalized size of antiderivative = 16.35

method	result	size
risch	Expression too large to display	2240

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*n^2*B^2*c/d+(-2*x*B^2*ln((b*x+a)^n)-B*(-I*B*Pi*b*d*x*csgn(I*e)*csgn(I*(b
*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*b*d*x*csgn(I*e)
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a
)^n/((d*x+c)^n))^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*b*d*
x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*
d*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)*b*d*x-2*B*ln(d*x+c)*b*c*n+2
*B*a*d*n*ln(b*x+a)+2*A*b*d*x)/b/d)*ln((d*x+c)^n)+B^2/b*ln((b*x+a)^n)^2*a+x*
B^2*ln((d*x+c)^n)^2-2*B^2/b*ln((b*x+a)^n)*a*n+1/4*x*(-I*B*Pi*csgn(I*e)*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I*e)*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^
n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-
I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3
+2*B*ln(e)+2*A)^2+x*B^2*ln((b*x+a)^n)^2+B*(-I*B*Pi*csgn(I*e)*csgn(I*(b*x+a)
```

```
^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I*e)*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csg
n(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)
+2*A)*(ln((b*x+a)^n)*x-b*n*(x/b-a/b^2*ln(b*x+a)))+2*B^2*a*n^2/b+2*B^2*ln(e)
*x*n-I*n*x*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((
d*x+c)^n))-n^2*B^2*c/d*ln(d*x+c)^2-2/d*n*B*c*ln(d*x+c)*A+2*n^2*B^2*a/b*ln(b
*x+a)*ln((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))-I*n*x*B^2*Pi*csgn(I*(b*x+a)^n/((d
*x+c)^n))^3-I*n*x*B^2*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I/d*n*c*ln(d*x+c
)*B^2*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)+I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))+2/d*n^2*B^2*c*dilog((a*d-c*b+b*(d*x+c))/(a*d-b*c))+
2*n^2*B^2/b*a*ln(a*d-c*b+b*(d*x+c))+2*n^2*B^2*a/b*dilog((-a*d+c*b+d*(b*x+a)
)/(-a*d+b*c))+I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I/d*
n*c*ln(d*x+c)*B^2*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*n*x*B^2*Pi*csgn(I*
e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2/d*n^2*B^
2*c*ln(d*x+c)*ln((a*d-c*b+b*(d*x+c))/(a*d-b*c))-2/d*n*B^2*ln((b*x+a)^n)*c*l
n(d*x+c)-2/d*n*c*ln(d*x+c)*B^2*ln(e)+I*n*x*B^2*Pi*csgn(I*e)*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2+I*n*x*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2+I*n*x*B^2*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*n
*x*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+2
*B*A*x*n-I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
^2-I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2-I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n
))^2-I/d*n*c*ln(d*x+c)*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2
```

Fricas [F]

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\int \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d*
x + c)^n) + A^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")
[Out] 2*A*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x +
a)*log(d*x + c) - b*c*n^2*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x
*log((d*x + c)^n)^2 + 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*lo
g(e))*log((b*x + a)^n) - 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x
*log((b*x + a)^n) + b*d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2
*d*x^2*log(e)^2 + a*b*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*lo
g(e) + log(e)^2)*a*b*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b
*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + 2*(a*e*n*log(b*x +
a)/b - c*e*n*log(d*x + c)/d)*A*B/e
```

Giac [F]

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.306 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 301

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} \\ &\quad+\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad-\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\ &\quad+\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h}-\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}\end{aligned}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-2*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/h+2*B*n*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/h+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/h-2*B^2*n^2*polylog(3,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2573, 2553, 2404, 2354, 2421, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \frac{2Bn\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h} + \frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h} - \frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h} - \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2)/h) + ((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/h - (2*B*n*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/h + (2*B*n*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*
h)*(c + d*x))])/h + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/h -
 (2*B^2*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g+hx}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)(bg-ah-(dg-ch)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc - ad)\text{Subst}\left(\int \left(\frac{d(A+B\log(ex^n))^2}{(bc-ad)h(b-dx)} + \frac{(-dg+ch)(A+B\log(ex^n))^2}{(bc-ad)h(bg-ah-(dg-ch)x)}\right)dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\Bigg)$$

$$= \text{Subst}\left(\frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) + \text{Subst}\left(\frac{((-bc+ad)(dg-ch))\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{bg-ah+(-dg+ch)x}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \operatorname{Subst}\left(\frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \operatorname{Subst}\left(\frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{(-dg+ch)x}{bg-ah}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \frac{2B^2n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} - \frac{2B^2n^2\operatorname{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1082 vs. 2(301) = 602.

Time = 0.26 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.59

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx$$
$$=\frac{(A+B(-n\log(a+bx)+n\log(c+dx)+\log(e(a+bx)^n(c+dx)^{-n})))^2\log(g+hx)+2Bn(A+B(-n\log}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))^2*Log[g + h*x] + 2*B*n*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*
h)] + PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - 2*A*B*n*(Log[c + d*x]*Log
[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]) - 2
*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^
n])*(Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))
/(-(d*g) + c*h)]) + B^2*n^2*(Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g - a*h)]
+ 2*Log[a + b*x]*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*PolyLog[3, (h
*(a + b*x))/(-(b*g) + a*h)]) + B^2*n^2*(Log[c + d*x]^2*Log[(d*(g + h*x))/(d
*g - c*h)] + 2*Log[c + d*x]*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*Po
lyLog[3, (h*(c + d*x))/(-(d*g) + c*h)]) - 2*B^2*n^2*(Log[a + b*x]*Log[c + d
*x]*Log[(b*(g + h*x))/(b*g - a*h)] + (Log[(h*(c + d*x))/(-(d*g) + c*h)]*(-2
*Log[a + b*x] + Log[(h*(c + d*x))/(-(d*g) + c*h)])*(Log[(b*(g + h*x))/(b*g
- a*h)] - Log[(d*(g + h*x))/(d*g - c*h)]))/2 + Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-Log[(b*(g + h*
x))/(b*g - a*h)] + Log[(d*(g + h*x))/(d*g - c*h)]) + (Log[((b*g - a*h)*(c +
 d*x))/((d*g - c*h)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*(a + b*x))] + Log[
(b*(g + h*x))/(b*g - a*h)] - Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c*h)*(a
 + b*x))]))/2 + (Log[c + d*x] - Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
 + b*x))])*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + (Log[a + b*x] + Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])*PolyLog[2, (h*(c + d*x))/(
-(d*g) + c*h)] + Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))] - PolyLog[2, ((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))]) - PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - PolyLo
g[3, (h*(c + d*x))/(-(d*g) + c*h)] - PolyLog[3, (b*(c + d*x))/(d*(a + b*x))
] + PolyLog[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]))/h
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{hx+g}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{g+hx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{hx+g}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h*x + g), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{g+hx}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{g+hx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{hx+g}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^2*log(h*x + g)/h + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n)
- 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(h*x + g),
x)
```

Giac [F]

$$\int \frac{(A + B\log (e(a + bx)^n(c + dx)^{-n}))^2}{g + hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{hx + g}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="giac")

[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log (e(a + bx)^n(c + dx)^{-n}))^2}{g + hx}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{g + h\,x}\,dx$$

[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x),x)

[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x), x)

3.307 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 208

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} + \frac{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} + \frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)/(h*x+g)+2*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/(-a*h+b*g)/(-c*h+d*g)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*h+d*g)*(b*
x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

$= \{2573, 2553, 2355, 2354, 2438\}$

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx$$
$$= \frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)(dg-ch)} + \frac{(a+bx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)(bg-ah)} + \frac{2B^2n^2(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*g - a*h)*(g + h*
x)) + (2*B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (
(d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
+ (2*B^2*(b*c - a*d)*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c
 + d*x))])/((b*g - a*h)*(d*g - c*h))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(g+hx)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(bg-ah+(-dg+ch)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} - \text{Subst}\left(\frac{(2B(bc-ad)n)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{bg-ah+(-dg+ch)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg-ah}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} + \frac{2B(bc-ad)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} - \text{Subst}\left(\frac{(2B^2(bc-ad)n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bg-ah)(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$
= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} + \frac{2B(bc-ad)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} + \frac{2B^2(bc-ad)n^2\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3460 vs. $2(208) = 416$.

Time = 0.56 (sec) , antiderivative size = 3460, normalized size of antiderivative = 16.63

$$
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] (-(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*n
*Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a + b*
x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b*B^
2*c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c*h^2
*n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n*Log[
c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c + d*x]
+ 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Log[a + b
*x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*
d*h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*x]^2 + a*
B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 + a*B^2*d*h
^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/
(-(d*g) + c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g)
+ c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)
] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + b*
B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g)
+ c*h)]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - 2*b*B^2
*c*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))
]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*
x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*
x))] + 2*a*B^2*d*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)
*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x)
```

)/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*
a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d
*x))/((d*g - c*h)*(a + b*x))] + b*B^2*c*g*h*n^2*Log[((b*g - a*h)*(c + d*x))
/((d*g - c*h)*(a + b*x))]^2 - a*B^2*d*g*h*n^2*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 + b*B^2*c*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - a*B^2*d*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - 2*A*b*B*d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ 2*A*b*B*c*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*a*A*B*d*g*h*Log[(e*(a
+ b*x)^n)/(c + d*x)^n] - 2*a*A*B*c*h^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2
*b*B^2*d*g^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*g*
h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b*B^2*d*g*h*n*x*Log[a
 + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*h^2*n*x*Log[a + b*x]*L
og[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*d*g^2*n*Log[c + d*x]*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + 2*a*B^2*d*g*h*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] - 2*b*B^2*d*g*h*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + 2*a*B^2*d*h^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b*B^2*
d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + b*B^2*c*g*h*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^2 + a*B^2*d*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - a*B^2*c*h
^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*A*b*B*d*g^2*n*Log[(b*(g + h*x))/(
b*g - a*h)] + 2*A*b*B*c*g*h*n*Log[(b*(g + h*x))/(b*g - a*h)] - 2*A*b*B*d*g*
h*n*x*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*c*h^2*n*x*Log[(b*(g + h*x))/
(b*g - a*h)] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)
] - 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2
*d*g*h*n^2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*a*B^2*d*h^2*n^
2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n
^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B
^2*d*g*h*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a
*h)] + 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*
x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[(b*
(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*
Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g*h*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*h^2*n*x*Log[(e*(a + b*x
)^n)/(c + d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*d*g^2*n*Log[(d*(
g + h*x))/(d*g - c*h)] - 2*a*A*B*d*g*h*n*Log[(d*(g + h*x))/(d*g - c*h)] + 2
*A*b*B*d*g*h*n*x*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*A*B*d*h^2*n*x*Log[(d*
(g + h*x))/(d*g - c*h)] - 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(d*(g + h*x))/
(d*g - c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)
] - 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*B
^2*d*h^2*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*
n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*
B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*
h)] + 2*b*B^2*d*g*h*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x
))/(d*g - c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log
[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)

```
^n]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*g*h*n*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g*h*n*x*Log[(e*(a + b
*x)^n)/(c + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*h^2*n*x*Log[
(e*(a + b*x)^n)/(c + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*B^2*(b*c -
a*d)*h*n^2*(g + h*x)*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*B^2*(b*c
- a*d)*h*n^2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*b*B^2*c
*g*h*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*a*B^2*d*g*h*n^2*PolyLo
g[2, (b*(c + d*x))/(d*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*PolyLog[2, (b*(c + d*x))/(d*(a +
 b*x))])/(h*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{2}}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{(g+hx)^{2}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="fric
as")
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{(g+hx)^{2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B^2*(log((d*x + c)^n)^2/(h^2*x + g*h) + integrate(-(d*h*x*log(e)^2 + c*h*l
og(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x*log(e) + c*h*log(e))*
log((b*x + a)^n) + 2*(d*g*n + (h*n - h*log(e))*d*x - c*h*log(e) - (d*h*x +
c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2
+ c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2)*x), x)) + 2*(b*e*n*log(b*x + a)/(b*g*h
 - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*
x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A*B/e - 2*A*B*log((b*x + a)
^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x + g*h)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2, x)
```

3.308 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3} dx$

Optimal result

Integrand size = 33, antiderivative size = 393

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx \\
&= \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad + \frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(bg-ah)^2} \\
&\quad - \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2} + \frac{B^2(bc-ad)^2hn^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{B(bc-ad)(2bdg-bch-adh)n(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

```
[Out] B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*h+b*g)^2/(-c
*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(-a*h+b*g)^2-
1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(h*x+g)^2+B^2*(-a*d+b*c)^2*h*n^2*
ln((h*x+g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)
*n^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*
g)^2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2573, 2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \frac{b^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{2h(bg-ah)^2} + \frac{Bhn(a+bx)(bc-ad)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(g+hx)(bg-ah)^2(dg-ch)} + \frac{Bn(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)^2(dg-ch)^2} - \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{2h(g+hx)^2} + \frac{B^2n^2(bc-ad)(-adh-bch+2bdg)\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} + \frac{B^2hn^2(bc-ad)^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*
g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n])^2)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/
(2*h*(g + h*x)^2) + (B^2*(b*c - a*d)^2*h*n^2*Log[(g + h*x)/(c + d*x)])/((b*
g - a*h)^2*(d*g - c*h)^2) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*(A +
 B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g
- a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d
*g - b*c*h - a*d*h)*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c
+ d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(g+hx)^3}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(b-dx)\left(A+B\log\left(ex^n\right)\right)^2}{(bg-ah-(dg-ch)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2} + \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x(bg-ah+(-dg+ch)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2} + \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))}{(bg-ah)^2x} + \frac{(bc-ad)^2h^2(A+B\log(ex^n))}{(bg-ah)(dg-ch)(bg-ah-(dg-ch)x)^2} + \frac{(bc-ad)h(-2bdg+bch+adh)(A\cdots}{(bg-ah)^2(dg-ch)(bg-ah-(\cdots}\right.\right.}{h} \cdots + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2}$$

$$+\,\text{Subst}\left(\frac{(b^2Bn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{h(bg-ah)^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{(B(bc-ad)^2hn)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(bg-ah+(-dg+ch)x)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)(dg-ch)},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(B(bc-ad)(2bdg-bch-adh)n)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{bg-ah+(-dg+ch)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)}$$

$$+\,\frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(bg-ah)^2}-\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2}$$

$$+\frac{B(bc-ad)(2bdg-bch-adh)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$

$$-\text{Subst}\left(\frac{(B^2(bc-ad)^2hn^2)\,\text{Subst}\left(\int\frac{1}{bg-ah+(-dg+ch)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(B^2(bc-ad)(2bdg-bch-adh)n^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2},e\left(\frac{a+bx}{c+dx}\right)^n,\ldots+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)}$$
$$+ \frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(bg-ah)^2}$$
$$- \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2} + \frac{B^2(bc-ad)^2hn^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2}$$
$$+ \frac{B(bc-ad)(2bdg-bch-adh)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$
$$+ \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13182 vs. $2(393) = 786$.

Time = 5.23 (sec) , antiderivative size = 13182, normalized size of antiderivative = 33.54

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^2}{(hx+g)^3}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="fric
as")
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B^2*(log((d*x + c)^n)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 2*integrate(-(
d*h*x*log(e)^2 + c*h*log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x
*log(e) + c*h*log(e))*log((b*x + a)^n) + (d*g*n + (h*n - 2*h*log(e))*d*x -
2*c*h*log(e) - 2*(d*h*x + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^4*x
^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 + (d*g
^3*h + 3*c*g^2*h^2)*x), x)) + (b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^
2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (
2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x +
g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^
2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n
- a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3
)*a - (d*g^2*h - c*g*h^2)*b)*x))*A*B/e - A*B*log((b*x + a)^n*e/(d*x + c)^n)
/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*A^2/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^3}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3, x)
```

3.309 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 875

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = -\frac{B^3(bc-ad)^3h^2n^3\log(c+dx)}{b^3d^3}$$
$$+\frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}$$
$$-\frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$
$$-\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx\right.\right.}{b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$-\frac{B^2(bc-ad)^3h^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\left(A+B\log\left(e(a+bx)^n(c+dx\right.\right.}{b^3d^3}$$
$$+\frac{B^3(bc-ad)^3h^2n^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$

```
[Out] -B^3*(-a*d+b*c)^3*h^2*n^3*ln(d*x+c)/b^3/d^3+B^2*(-a*d+b*c)^2*h^2*n^2*(b*x+a
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^3/d^2-2*B^2*(-a*d+b*c)^2*h*(-a*d*h-2*
b*c*h+3*b*d*g)*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))/b^3/d^3-B*(-a*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))^2/b^3/d^2-1/2*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))^2/b/d^3+B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))^3/b^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h-B^2*(-
a*d+b*c)^3*h^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+
```

```
a))/b^3/d^3-2*B^3*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^3*polylog(2,d*(
b*x+a)/b/(d*x+c))/b^3/d^3+2*B^2*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+B^3*(-a*d+b*c)^3*h^2*n^3*polylog(2,b
*(d*x+c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
^3/d^3
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = -\frac{B^3h^2n^3\log(c+dx)(bc-ad)^3}{b^3d^3}$$
$$-\frac{B^2h^2n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(bc-ad)^3}{b^3d^3}$$
$$+\frac{B^3h^2n^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)(bc-ad)^3}{b^3d^3}$$
$$+\frac{B^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(bc-ad)^2}{b^3d^2}$$
$$-\frac{2B^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)(bc-ad)^2}{b^3d^3}$$
$$-\frac{2B^3h(3bdg-2bch-adh)n^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^2}{b^3d^3}$$
$$-\frac{Bh^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)}{2bd^3}$$
$$-\frac{Bh(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)}{b^3d^2}$$
$$+\frac{B\left((3d^2g^2-3cdhg+c^2h^2)b^2-adh(3dg-ch)b+a^2d^2h^2\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx\right.\right.}{b^3d^3}$$
$$+\frac{2B^2\left((3d^2g^2-3cdhg+c^2h^2)b^2-adh(3dg-ch)b+a^2d^2h^2\right)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\mathrm{Poly}}{b^3d^3}$$
$$-\frac{2B^3\left((3d^2g^2-3cdhg+c^2h^2)b^2-adh(3dg-ch)b+a^2d^2h^2\right)n^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)}{b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((B^3*(b*c - a*d)^3*h^2*n^3*Log[c + d*x])/(b^3*d^3)) + (B^2*(b*c - a*d)^2*
h^2*n^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^2) - (2*
B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[(b*c - a*d)/(b*(c +
 d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^3) - (B*(b*c - a*d
)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(b^3*d^2) - (B*(b*c - a*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])^2)/(2*b*d^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*
h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/
(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b^3*d^3) - ((b*
g - a*h)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b^3*h) + ((g + h*
```

```
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*h) - (B^2*(b*c - a*d)^3
*h^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*
(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)
*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) + (2*B^2*(b*c - a*d
)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h
^2))*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/
(b*(c + d*x))])/(b^3*d^3) + (B^3*(b*c - a*d)^3*h^2*n^3*PolyLog[2, (b*(c + d
*x))/(d*(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*
(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^3*PolyLog[3, (d*(a
 + b*x))/(b*(c + d*x))])/(b^3*d^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(bg-ah-(dg-ch)x)^2 (A+B\log(ex^n))^3}{(b-dx)^4} dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h} - \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \frac{(bg-ah+(-dg+ch)x)^3(A+B\log(ex^n))^2}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h} - \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \left(\frac{(bg-ah)^3(A+B\log(ex^n))^2}{b^3x} + \frac{(bc-ad)^3h^3(A+B\log(ex^n))^2}{bd^2(b-dx)^3} + \frac{(bc-ad)^2h^2(3bdg-2bch-adh)(A}{b^2d^2(b-dx)^2}\right.\right.}{h}\right.$$

$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$

$$- \operatorname{Subst}\left(\frac{\left(B(bc-ad)^3h^2n\right)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$- \operatorname{Subst}\left(\frac{\left(B(bg-ah)^3n\right)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\operatorname{Subst}\left(\frac{\left(B(bc-ad)^2h(3bdg-2bch-adh)n\right)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^2}, e\left(\frac{a+bx}{c+dx}\right)^n, \ldots + bx)^n(c+dx)^{-n}\right)$$

$$-\operatorname{Subst}\left(\frac{\left(B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\right)\operatorname{Subst}\left(\int \frac{(A+B\log(\ldots}{b-d\ldots}\right.}{b^3d^2} \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\right.\right.}{b^3d^3}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$-\text{Subst}\left(\frac{(bg-ah)^3\text{Subst}\left(\int x^2\,dx,x,A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{\left(B^2(bc-ad)^3h^2n^2\right)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{bd^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$+\text{Subst}\left(\frac{\left(2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\right)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3d^2},e\left(\frac{a+bx}{c+dx}\right)^n\right.$$
$$\left.+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{\left(2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\right)\text{Subst}\left(\int\frac{(A+}{}\right.}{b^3d^3}\right.$$
$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$=$$

$$-\frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$

$$-\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$

$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$

$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(\right.\right.}{b^3d^3}$$

$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$

$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$

$$+\frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2(A+B\log\left(e(a+bx)^n(c\right.}{b^3d^3}$$

$$+\text{Subst}\left(\frac{(B^2(bc-ad)^3h^2n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{(B^2(bc-ad)^3h^2n^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{(2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e\ \ldots\ +bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3)\,\text{Subst}\left(\int\frac{\text{Li}_2(\ldots}{x}\right.}{b^3d^3}\ \ldots\ +bx)^n(c+dx)^{-n}\right)$$

$$= \frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}$$
$$-\frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$
$$-\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e\right.\right.}{b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$-\frac{B^2(bc-ad)^3h^2n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2(A+B\log\left(e(a+bx)^n\right.}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+\text{Subst}\left(\frac{(B^3(bc-ad)^3h^2n^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3d^3},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(B^3(bc-ad)^3h^2n^3)\,\text{Subst}(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx})}{b^3d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{B^3(bc-ad)^3h^2n^3\log(c+dx)}{b^3d^3}$$
$$+ \frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}$$
$$- \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$
$$- \frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$- \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+ \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log(e(\right.}{b^3d^3}$$
$$- \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+ \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$- \frac{B^2(bc-ad)^3h^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$- \frac{2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\left(A+B\log\left(e(a+bx)^n(c\right.\right.}{b^3d^3}$$
$$+ \frac{B^3(bc-ad)^3h^2n^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$- \frac{2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7279 vs. 2(875) = 1750.

Time = 2.67 (sec) , antiderivative size = 7279, normalized size of antiderivative = 8.32

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int (hx+g)^2 \left(A+B\ln \left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
[Out] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (hx+g)^2\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
[Out] integral(A^3*h^2*x^2 + 2*A^3*g*h*x + A^3*g^2 + (B^3*h^2*x^2 + 2*B^3*g*h*x +
 B^3*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*h^2*x^2 + 2*A*B^2*g*h
*x + A*B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h^2*x^2 + 2*A^2
*B*g*h*x + A^2*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (hx+g)^2\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

[Out] A^2*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*A^2*B*g^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*A^2*B*g^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*g*h/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*h^2/e - 1/6*(2*(B^3*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((d*x + c)^n)^3 + 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*log(d*x + c) - 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*log(b*x + a) - 2*(B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^3*d^3*g*h + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*log(e))*b^3)*B^3)*x^2 - 2*(3*A*B^2*b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^3)*x - 2*(B^3*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^3*d^3) - integrate(-(B^3*b^3*c*d^2*g^2*log(e)^3 + 3*A*B^2*b^3*c*d^2*g^2*log(e)^2 + (B^3*b^3*d^3*h^2*log(e)^3 + 3*A*B^2*b^3*d^3*h^2*log(e)^2)*x^3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x + a)^n)^3 + (3*(2*d^3*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*A*B^2*b^3 + (2*d^3*g*h*log(e)^3 + c*d^2*h^2*log(e)^3)*B^3*b^3)*x^2 + 3*(B^3*b^3*c*d^2*g^2*log(e) + A*B^2*b^3*c*d^2*g^2 + (B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 + ((2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (2*d^3*g*h*log(e) + c*d^2*h^2*log(e))*B^3*b^3)*x^2 + ((d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d^3*g^2*log(e) + 2*c*d^2*g*h*log(e))*B^3*b^3)*x)*log((b*x + a)^n)^2 + (3*(d^3*g^2*log(e)^2 + 2*c*d^2*g*h*log(e)^2)*A*B^2*b^3 + (d^3*g^2*log(e)^3 + 2*c*d^2*g*h*log(e)^3)*B^3*b^3)*x + 3*(B^3*b^3*c*d^2*g^2*log(e)^2 + 2*A*B^2*b^3*c*d^2*g^2*log(e) + (B^3*b^3*d^3*h^2*log(e)^2 + 2*A*B^2*b^3*d^3*h^2*log(e))*x^3 + (2*(2*d^3*g*h*log(e) + c*d^2*h^2*log(e))*A*B^2*b^3 + (2*d^3*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*B^3*b^3)*x^2 + (2*(d^3*g^2*log(e) + 2*c*d^2*g*h*log(e))*A*B^2*b^3 + (d^3*g^2*log(e)^2 + 2*c*d^2*g*h*log(e)^2)*B^3*b^3)*x)*log((b*x + a)^n) - (3*B^3*b^3*c*d^2*g^2*log(e)^2 + 6*A*B^2*b^3*c*d^2*g^2*log(e) - 2*(3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^3*b^3*log(d*x + c) + 2*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^3*log(b*x + a) + (2*(h^2*n + 3*h^2*log(e))*A*B^2*b^3*d^3 + (2*h^2*n*log(e) + 3*h^2*log(e)^2)*B^3*b^3*d^3)*x^3 + (6*(c*d^2*h^2*log(e) + (g*h*n + 2*g*h*log(e))*d^3)*A*B^2*b^3 + (a*b^2*d^3*h^2*n^2 - ((h^2*n^2 - 3*h^2*log(e)^2)*c*d^2 - 6*(g*h*n*log(e) + g*h*log(e)^2)*d^3)*b^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x + a)^n)^2 + (6*(2*c*d^2*g*h*log(e) + (g^2*n + g^2*log(e))*d^3)*A*B^2*b^3 + (6*a*b^2*d^3*g*h*n^2 - 2*a^2*b*d^3*h^2*n^2 + (2*c^2*d*h^2*n^2 - 6*(g*h*n^2 - g*h*log(e)^2)*c*d^2 + 3*(2*g^2*n*log(e) + g^2*log(e)^2)*d^3)*b^3)*B^3)*x + 2*(3*B^3*b^3*c*d^2*g^2*log(e) + 3*A*B^2*b^3*c*d^2*g^2 + (3*A*B^2*b^3*d^3*h^2 + (h^2*n + 3*h^2*log(e))*B^3*b^3*d^3)*x^3 + 3*((2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (c*d^2*h^2*log(e) + (g*h*n + 2*g*h*log(e))*d^3)*B^3*b^3)*x^2 + 3*((d

```
^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (2*c*d^2*g*h*log(e) + (g^2*n + g^2*log(e)
)*d^3)*B^3*b^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3*x + b^3*c*d
^2), x)
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Timed out}$$

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (g+h\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 \,dx$$

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

```
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.310 $\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 466

$$\begin{aligned}
&\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx \\
&= -\frac{3B^2(bc-ad)^2hn^2\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&\quad - \frac{3B(bc-ad)hn(a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d} \\
&\quad + \frac{3B(bc-ad)(2bdg-bch-adh)n\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d^2} \\
&\quad - \frac{(bg-ah)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b^2h} \\
&\quad + \frac{(g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h} \\
&\quad - \frac{3B^3(bc-ad)^2hn^3 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad + \frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad - \frac{3B^3(bc-ad)(2bdg-bch-adh)n^3 \operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*h*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*
x+c)^n)))/b^2/d^2-3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))^2/b^2/d+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/d^2-1/2*(-a*h+b*g)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))^3/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c
```

```
)^n)))^3/h-3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2+
3*B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2-3*B^3*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2355, 2354, 2438, 2421, 6724}

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}-\frac{3B^2hn^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}+\frac{3Bn(bc-ad)(-adh-bch+2bdg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2d^2}-\frac{(bg-ah)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2b^2h}-\frac{3Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2d}+\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2h}-\frac{3B^3n^3(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}-\frac{3B^3hn^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*B^2*(b*c - a*d)^2*h*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]))/(b^2*d^2) - (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B
*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b^2*d) + (3*B*(b*c - a*d)*(2*b*d*g
 - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n])^2)/(2*b^2*d^2) - ((b*g - a*h)^2*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^3)/(2*b^2*h) + ((g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^3)/(2*h) - (3*B^3*(b*c - a*d)^2*h*n^3*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(b^2*d^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*
```

```
x))])/(b^2*d^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(bg-ah-(dg-ch)x)\left(A+B\log\left(ex^n\right)\right)^3}{(b-dx)^3}dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h}$$
$$-\text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{(bg-ah+(-dg+ch)x)^2(A+B\log(ex^n))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h}$$
$$-\text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \left(\frac{(bg-ah)^2(A+B\log(ex^n))^2}{b^2x} + \frac{(bc-ad)^2h^2(A+B\log(ex^n))^2}{bd(b-dx)^2} + \frac{(bc-ad)h(2bdg-bch-adh)(A+B}{b^2d(b-dx)}\right.\right.}{2h}\right.$$
$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h}$$
$$-\text{Subst}\left(\frac{(3B(bc-ad)^2hn)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(3B(bg-ah)^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(3B(bc-ad)(2bdg-bch-adh)n)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2d}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d} + \frac{3B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d^2} + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h} - \text{Subst}\left(\frac{(3(bg-ah)^2)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) + \text{Subst}\left(\frac{(3B^2(bc-ad)^2hn^2)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) - \text{Subst}\left(\frac{(3B^2(bc-ad)(2bdg-bch-adh)n^2)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2d^2}, e\left(\frac{a}{c}\right. \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B^2(bc-ad)^2hn^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2}$$
$$-\frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d}$$
$$+\frac{3B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d^2}$$
$$-\frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b^2h}$$
$$+\frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h}$$
$$+\frac{3B^2(bc-ad)(2bdg-bch-adh)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$
$$+\operatorname{Subst}\left(\frac{(3B^3(bc-ad)^2hn^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\operatorname{Subst}\left(\frac{(3B^3(bc-ad)(2bdg-bch-adh)n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{3B^2(bc-ad)^2hn^2\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{b^2d^2} - \frac{3B(bc-ad)hn(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2b^2d} + \frac{3B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2b^2d^2} - \frac{(bg-ah)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2b^2h} + \frac{(g+hx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2h} - \frac{3B^3(bc-ad)^2hn^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} + \frac{3B^2(bc-ad)(2bdg-bch-adh)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} - \frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3890 vs. 2(466) = 932.

Time = 0.94 (sec) , antiderivative size = 3890, normalized size of antiderivative = 8.35

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-12*A*b^2*B^2*c*d*g*n^2 - 12*a*A*b*B^2*d^2*g*n^2 + 12*a*A*b*B^2*c*d*h*n^2
+ 6*a*b*B^3*c*d*h*n^3 - 6*a^2*B^3*d^2*h*n^3 + 2*A^3*b^2*d^2*g*x - 3*A^2*b^2
*B*c*d*h*n*x + 3*a*A^2*b*B*d^2*h*n*x + A^3*b^2*d^2*h*x^2 + 6*a*A^2*b*B*d^2*
g*n*Log[a + b*x] - 3*a^2*A^2*B*d^2*h*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*h*n^2
*Log[a + b*x] + 6*a^2*A*B^2*d^2*h*n^2*Log[a + b*x] + 12*b^2*B^3*c*d*g*n^3*L
og[a + b*x] + 12*a*b*B^3*d^2*g*n^3*Log[a + b*x] - 12*a*b*B^3*c*d*h*n^3*Log[
a + b*x] - 6*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]^2 + 3*a^2*A*B^2*d^2*h*n^2*Log
[a + b*x]^2 + 3*a*b*B^3*c*d*h*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*h*n^3*Log[
a + b*x]^2 + 2*a*b*B^3*d^2*g*n^3*Log[a + b*x]^3 - a^2*B^3*d^2*h*n^3*Log[a +
 b*x]^3 - 6*A^2*b^2*B*c*d*g*n*Log[c + d*x] + 3*A^2*b^2*B*c^2*h*n*Log[c + d*
x] + 6*A*b^2*B^2*c^2*h*n^2*Log[c + d*x] - 6*a*A*b*B^2*c*d*h*n^2*Log[c + d*x
] - 12*b^2*B^3*c*d*g*n^3*Log[c + d*x] - 12*a*b*B^3*d^2*g*n^3*Log[c + d*x] +
 12*a*b*B^3*c*d*h*n^3*Log[c + d*x] + 12*A*b^2*B^2*c*d*g*n^2*Log[a + b*x]*Lo
g[c + d*x] + 12*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]*Log[c + d*x] - 6*A*b^2*B^2
*c^2*h*n^2*Log[a + b*x]*Log[c + d*x] - 6*a^2*A*B^2*d^2*h*n^2*Log[a + b*x]*L
og[c + d*x] - 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[c + d*x] + 6*a*b*B^3*c*d
*h*n^3*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c*d*g*n^3*Log[a + b*x]^2*Log[c
```

+ d*x] - 12*a*b*B^3*d^2*g*n^3*Log[a + b*x]^2*Log[c + d*x] + 3*b^2*B^3*c^2*h*n^3*Log[a + b*x]^2*Log[c + d*x] + 6*a^2*B^3*d^2*h*n^3*Log[a + b*x]^2*Log[c + d*x] - 12*a*A*b*B^2*d^2*g*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 6*a^2*A*B^2*d^2*h*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 12*a*b*B^3*d^2*g*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 6*a^2*B^3*d^2*h*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 6*A*b^2*B^2*c*d*g*n^2*Log[c + d*x]^2 + 3*A*b^2*B^2*c^2*h*n^2*Log[c + d*x]^2 + 3*b^2*B^3*c^2*h*n^3*Log[c + d*x]^2 - 3*a*b*B^3*c*d*h*n^3*Log[c + d*x]^2 + 12*b^2*B^3*c*d*g*n^3*Log[a + b*x]*Log[c + d*x]^2 + 6*a*b*B^3*d^2*g*n^3*Log[a + b*x]*Log[c + d*x]^2 - 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[c + d*x]^2 - 3*a^2*B^3*d^2*h*n^3*Log[a + b*x]*Log[c + d*x]^2 - 6*b^2*B^3*c*d*g*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 6*a*b*B^3*d^2*g*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + 3*b^2*B^3*c^2*h*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + 3*a^2*B^3*d^2*h*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 2*b^2*B^3*c*d*g*n^3*Log[c + d*x]^3 + b^2*B^3*c^2*h*n^3*Log[c + d*x]^3 - 12*A*b^2*B^2*c*d*g*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*A*b^2*B^2*c^2*h*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12*a*b*B^3*c*d*h*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*a^2*B^3*d^2*h*n^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*b^2*B^3*c*d*g*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*a*b*B^3*d^2*g*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*b^2*B^3*c^2*h*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*a^2*B^3*d^2*h*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 12*b^2*B^3*c*d*g*n^3*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12*b^2*B^3*c*d*g*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a*b*B^3*d^2*g*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a*b*B^3*c*d*h*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*A^2*b^2*B*d^2*g*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*A*b^2*B^2*c*d*h*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*A*b*B^2*d^2*h*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*A^2*b^2*B*d^2*h*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a*A*b*B^2*d^2*g*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*A*B^2*d^2*h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*c*d*h*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a^2*B^3*d^2*h*n^2*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*d^2*g*n^2*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*a^2*B^3*d^2*h*n^2*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*A*b^2*B^2*c*d*g*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*A*b^2*B^2*c^2*h*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^2*B^3*c^2*h*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b*B^3*c*d*h*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*b^2*B^3*c*d*g*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a*b*B^3*d^2*g*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^2*B^3*c^2*h*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*B^3*d^2*h*n^2*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a*b*B^3*d^2*g*n^2*Log

```
[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n
] + 6*a^2*B^3*d^2*h*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]*Log[
(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^2*B^3*c*d*g*n^2*Log[c + d*x]^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 3*b^2*B^3*c^2*h*n^2*Log[c + d*x]^2*Log[(e*(a + b*
x)^n)/(c + d*x)^n] - 12*b^2*B^3*c*d*g*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b
*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^2*B^3*c^2*h*n^2*Log[a + b
*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*A*b
^2*B^2*d^2*g*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 3*b^2*B^3*c*d*h*n*x*Log
[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b*B^3*d^2*h*n*x*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2 + 3*A*b^2*B^2*d^2*h*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 +
6*a*b*B^3*d^2*g*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 3*a^2*B
^3*d^2*h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*b^2*B^3*c*d*
g*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^2*B^3*c^2*h*n*Log
[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*b^2*B^3*d^2*g*x*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]^3 + b^2*B^3*d^2*h*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^
n]^3 + 6*B^2*n^2*(-2*A*b^2*c*d*g + A*b^2*c^2*h + b^2*B*c^2*h*n - 2*a*b*B*c*
d*h*n + a^2*B*d^2*h*n + a*B*d^2*(2*b*g - a*h)*n*Log[a + b*x] + b^2*B*c*(-2*
d*g + c*h)*n*Log[c + d*x] - 2*b^2*B*c*d*g*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ b^2*B*c^2*h*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)] + 6*B^2*n^2*(a*B*d^2*(2*b*g - a*h)*n*Log[a + b*x] + b^2*B*c*(
-2*d*g + c*h)*n*Log[c + d*x] + a*d^2*(-2*b*g + a*h)*(A + B*Log[(e*(a + b*x)
^n)/(c + d*x)^n]))*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*b^2*B^3*c*d*g
*n^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 12*a*b*B^3*d^2*g*n^3*PolyLo
g[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*b^2*B^3*c^2*h*n^3*PolyLog[3, (d*(a +
 b*x))/(-(b*c) + a*d)] + 6*a^2*B^3*d^2*h*n^3*PolyLog[3, (d*(a + b*x))/(-(b*
c) + a*d)] + 12*b^2*B^3*c*d*g*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 1
2*a*b*B^3*d^2*g*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 6*b^2*B^3*c^2*h
*n^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 6*a^2*B^3*d^2*h*n^3*PolyLog[3,
 (b*(c + d*x))/(b*c - a*d)])/(2*b^2*d^2)
```

Maple [F]

$$\int (hx+g)\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}dx$$

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (hx+g)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*h*x + A^3*g + (B^3*h*x + B^3*g)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*h*x + A*B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h*
x + A^2*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (hx+g)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*h*x^2 + 3*A^2*B*g*
x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*x + 3*(a*e*n*log(b*x + a)/b - c*e*
n*log(d*x + c)/d)*A^2*B*g/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*h/e - 1/2*((B^3*b^2*d^2*h*
x^2 + 2*B^3*b^2*d^2*g*x)*log((d*x + c)^n)^3 + 3*((2*c*d*g*n - c^2*h*n)*B^3*
b^2*log(d*x + c) - (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^3*log(b*x + a) - (B^3*b^
2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 - (2*A*B^2*b^2*d^2*g + (a*b*d^2*h*n -
 (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^3)*x - (B^3*b^2*d^2*h*x^2 + 2*B^3*b^2*d^
```

```
2*g*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^2*d^2) - integrate(-(B^3*b^
2*c*d*g*log(e)^3 + 3*A*B^2*b^2*c*d*g*log(e)^2 + (B^3*b^2*d^2*h*x^2 + B^3*b^
2*c*d*g + (d^2*g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^3 + (B^3*b^2*d^2*h*lo
g(e)^3 + 3*A*B^2*b^2*d^2*h*log(e)^2)*x^2 + 3*(B^3*b^2*c*d*g*log(e) + A*B^2*
b^2*c*d*g + (B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 + ((d^2*g + c*d*h)
*A*B^2*b^2 + (d^2*g*log(e) + c*d*h*log(e))*B^3*b^2)*x)*log((b*x + a)^n)^2 +
 (3*(d^2*g*log(e)^2 + c*d*h*log(e)^2)*A*B^2*b^2 + (d^2*g*log(e)^3 + c*d*h*l
og(e)^3)*B^3*b^2)*x + 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b^2*c*d*g*log(e)
+ (B^3*b^2*d^2*h*log(e)^2 + 2*A*B^2*b^2*d^2*h*log(e))*x^2 + (2*(d^2*g*log(e
) + c*d*h*log(e))*A*B^2*b^2 + (d^2*g*log(e)^2 + c*d*h*log(e)^2)*B^3*b^2)*x)
*log((b*x + a)^n) - 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b^2*c*d*g*log(e) -
(2*c*d*g*n^2 - c^2*h*n^2)*B^3*b^2*log(d*x + c) + (2*a*b*d^2*g*n^2 - a^2*d^2
*h*n^2)*B^3*log(b*x + a) + ((h*n + 2*h*log(e))*A*B^2*b^2*d^2 + (h*n*log(e)
+ h*log(e)^2)*B^3*b^2*d^2)*x^2 + (B^3*b^2*d^2*h*x^2 + B^3*b^2*c*d*g + (d^2*
g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^2 + (2*(c*d*h*log(e) + (g*n + g*log(
e))*d^2)*A*B^2*b^2 + (a*b*d^2*h*n^2 - ((h*n^2 - h*log(e)^2)*c*d - (2*g*n*lo
g(e) + g*log(e)^2)*d^2)*b^2)*B^3)*x + (2*B^3*b^2*c*d*g*log(e) + 2*A*B^2*b^2
*c*d*g + ((h*n + 2*h*log(e))*B^3*b^2*d^2 + 2*A*B^2*b^2*d^2*h)*x^2 + 2*((d^2
*g + c*d*h)*A*B^2*b^2 + (c*d*h*log(e) + (g*n + g*log(e))*d^2)*B^3*b^2)*x)*l
og((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2*x + b^2*c*d), x)
```

Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
[Out] integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)^3\,dx$$

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.311 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 203

$$\begin{aligned}\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx &= \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd} \\ &\quad+\frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} \\ &\quad+\frac{6B^2(bc-ad)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd} \\ &\quad-\frac{6B^3(bc-ad)n^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}\end{aligned}$$

```
[Out] 3*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
^2/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+6*B^2*(-a*d+b*c)*n^2*(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d-6*B^3*(
-a*d+b*c)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{2536, 2573, 2551, 2354, 2421, 6724\}$

$$\int \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \frac{6B^2n^2(bc-ad)\,\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd}$$
$$+ \frac{3Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{bd}$$
$$+ \frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b} - \frac{6B^3n^3(bc-ad)\,\text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (3*B*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^3)/b + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d) - (6*B^3*(b*c - a*d)*n^3*Pol
yLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} - \frac{(3B(bc-ad)n)\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{c+dx}\,dx}{b}$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} - \frac{(3B(bc-ad)n)\text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} - \frac{(3B(bc-ad)n)\text{Subst}\left(\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

$$= \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} - \frac{(3B(bc-ad)n)\text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$= \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} + \frac{6B^2(bc-ad)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd} - \frac{(3B(bc-ad)n)\text{Subst}\left(\frac{(2B^2n^2)\text{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{d},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

$$= \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd} + \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{b} + \frac{6B^2(bc-ad)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd} - \frac{6B^3(bc-ad)n^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.86

$$\int\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$

$$= \frac{A^3bdx-3A^2B(bc-ad)n\log(c+dx)+3A^2Bd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB^2d(a+bx)\log^2}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (A^3*b*d*x - 3*A^2*B*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*B*d*(a + b*x)*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 3*A*B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c +
d*x)^n]^2 + B^3*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B^2*(b
*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*c + b
*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B^3*(b*c - a*d)*n
*(Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*n*L
og[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2
*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/(b*d)
```

Maple [F]

$$\int \left(A+B\ln \left(e(bx+a)^{n} (dx+c)^{-n}\right)\right)^{3}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
[Out] integral(B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/(
d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")
[Out] 3*A^2*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*x + 3*(a*e*n*log(b*x + a)/b
- c*e*n*log(d*x + c)/d)*A^2*B/e - (B^3*b*d*x*log((d*x + c)^n)^3 - 3*(B^3*a*
d*n*log(b*x + a) - B^3*b*c*n*log(d*x + c) + B^3*b*d*x*log((b*x + a)^n) + (B
^3*b*d*log(e) + A*B^2*b*d)*x)*log((d*x + c)^n)^2)/(b*d) - integrate(-(B^3*b
*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)
^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e) + A*B^2*b*d)*x)*log((b
*x + a)^n)^2 + (B^3*b*d*log(e)^3 + 3*A*B^2*b*d*log(e)^2)*x + 3*(B^3*b*c*log
```

```
(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e))*x)*log
((b*x + a)^n) - 3*(2*B^3*a*d*n^2*log(b*x + a) - 2*B^3*b*c*n^2*log(d*x + c)
+ B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*x + B^3*b*c)*log((b*x +
a)^n)^2 + ((2*n*log(e) + log(e)^2)*B^3*b*d + 2*A*B^2*b*d*(n + log(e)))*x +
2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*(n + log(e)) + A*B^2*b*d)*x)*log((
b*x + a)^n))*log((d*x + c)^n))/(b*d*x + b*c), x)
```

Giac [F]

$$\int (A + B\log (e(a+bx)^n(c+dx)^{-n}))^3 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3 \, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A + B\log (e(a+bx)^n(c+dx)^{-n}))^3 \, dx = \int \left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 \, dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.312 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 425

$$\begin{aligned}
\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{h} \\
&\quad + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad - \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad - \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad - \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}
\end{aligned}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,d*(b*x+a)/b/(d*x+c))/h+3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,d*(b*x+
```

```
a)/b/(d*x+c))/h-6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-6*B^3*n^3*polylog(4,d*(b*x+a)/b/(d*x+c)
)/h+6*B^3*n^3*polylog(4,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2573, 2553, 2404, 2354, 2421, 2430, 6724}

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx \\
&= -\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h} \\
&+\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h} \\
&+\frac{3Bn\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h} \\
&+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h} \\
&-\frac{3Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h} \\
&-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h} \\
&+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}-\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{h}
\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
3)/h) + ((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/h - (3*B*n*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/h + (3*B*n*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g
- a*h)*(c + d*x))])/h + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])
*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/h - (6*B^2*n^2*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c +
 d*x))])/h - (6*B^3*n^3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/h + (6*B^3
*n^3*PolyLog[4, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{g+hx}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{(b-dx)(bg-ah-(dg-ch)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc-ad)\text{Subst}\left(\int \left(\frac{d(A+B\log(ex^n))^3}{(bc-ad)h(b-dx)} + \frac{(-dg+ch)(A+B\log(ex^n))^3}{(bc-ad)h(bg-ah-(dg-ch)x)}\right) dx, x, \frac{a+bx}{c+dx}\right), e\left(\ldots + bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{((-bc+ad)(dg-ch))\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{bg-ah+(-dg+ch)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{h}$$

$$+ \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}$$

$$+ \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{h}$$
$$+ \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}$$
$$- \frac{3Bn(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h}$$
$$+ \frac{3Bn(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}$$
$$+ \operatorname{Subst}\left(\frac{(6B^2n^2)\operatorname{Subst}\left(\int\frac{(A+B\log(ex^n))\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\operatorname{Subst}\left(\frac{(6B^2n^2)\operatorname{Subst}\left(\int\frac{(A+B\log(ex^n))\operatorname{Li}_2\left(-\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{h} + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} - \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \operatorname{Subst}\left(\frac{(6B^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right) + \operatorname{Subst}\left(\frac{(6B^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{h} + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} - \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} - \frac{6B^3n^3\operatorname{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{h} + \frac{6B^3n^3\operatorname{Li}_4\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}$$

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x), x]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3}{hx+g}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h*x + g), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^3*log(h*x + g)/h - integrate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)
^n)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) +
A*B^2)*log((b*x + a)^n)^2 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*l
og((d*x + c)^n)^2 + 3*(B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)
^n) - 3*(B^3*log((b*x + a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2
*(B^3*log(e) + A*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(h*x + g), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{g+h\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x), x)
```

3.313 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 302

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx &= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)}\\ &+ \frac{3B(bc-ad)n(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}\\ &+ \frac{6B^2(bc-ad)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}\\ &- \frac{6B^3(bc-ad)n^3\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}\end{aligned}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*h+b*g)/(h*x+g)+3*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)
/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+6*B^2*(-a*d+b*c)*n^2*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c
*h+d*g)-6*B^3*(-a*d+b*c)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c
))/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2573, 2553, 2355, 2354, 2421, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx$$
$$= \frac{6B^2n^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)(dg-ch)}$$
$$+\frac{3Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(bg-ah)(dg-ch)}$$
$$+\frac{(a+bx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)(bg-ah)}$$
$$-\frac{6B^3n^3(bc-ad)\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*g - a*h)*(g + h*
x)) + (3*B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 -
 ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)
) + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog
[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c
*h)) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*
h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(g+hx)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc - ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{(bg-ah+(-dg+ch)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)} - \text{Subst}\left(\frac{(3B(bc-ad)n)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{bg-ah+(-dg+ch)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg-ah}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)} + \frac{3B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} - \text{Subst}\left(\frac{(6B^2(bc-ad)n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bg-ah)(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)} + \frac{3B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} + \frac{6B^2(bc-ad)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\text{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} - \text{Subst}\left(\frac{(6B^3(bc-ad)n^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bg-ah)(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)} \\
&\quad + \frac{3B(bc-ad)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\
&\quad + \frac{6B^2(bc-ad)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\
&\quad - \frac{6B^3(bc-ad)n^3\operatorname{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx = \int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

Maple [F]

$$
\int \frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^3}{(hx+g)^2}dx
$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

Fricas [F]

$$
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx
$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fric
as")
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 + 2
*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="maxi
ma")
[Out] B^3*log((d*x + c)^n)^3/(h^2*x + g*h) + 3*(b*e*n*log(b*x + a)/(b*g*h - a*h^2
) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*x + g)/(
(d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e
/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((B^3*c*h*log(e)
^3 + 3*A*B^2*c*h*log(e)^2 + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 3*(B
^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n
)^2 + 3*(A*B^2*c*h - (d*g*n - c*h*log(e))*B^3 - ((h*n - h*log(e))*B^3*d - A
*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)^n)^2 +
(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 3*(B^3*c*h*log(e)^2 + 2*A*B^2
*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x + a)^n) -
 3*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h)*log((b*x
+ a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*c*h*log(e) +
 A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))*log((d*x + c
)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2
)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="giac
")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g+h\,x)^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2, x)
```

3.314 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 629

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx \\
&= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad+\frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(bg-ah)^2}-\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} \\
&\quad+\frac{3B^2(bc-ad)^2hn^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B(bc-ad)(2bdg-bch-adh)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B^3(bc-ad)^2hn^3\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad-\frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

```
[Out] 3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)
^2/(-c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(-a*h+b
*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(h*x+g)^2+3*B^2*(-a*d+b*c)^
2*h*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g
)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g
```

```
)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/
(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^2*(-a*d+b*c
)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2-3*B^3*(-a*d+
b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*
x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \frac{b^2(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{2h(bg-ah)^2} + \frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)^2(dg-ch)^2} + \frac{3B^2hn^2(bc-ad)^2\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)^2(dg-ch)^2} + \frac{3Bhn(a+bx)(bc-ad)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{2(g+hx)(bg-ah)^2(dg-ch)} + \frac{3Bn(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{2(bg-ah)^2(dg-ch)^2} - \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{2h(g+hx)^2} + \frac{3B^3hn^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} - \frac{3B^3n^3(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

```
[Out] (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/
(2*(b*g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n])^3)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^3/(2*h*(g + h*x)^2) + (3*B^2*(b*c - a*d)^2*h*n^2*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h
)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a + b*
x))/((b*g - a*h)*(c + d*x))])/(2*(b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b*c
 - a*d)^2*h*n^3*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
```

```
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d
*h)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*B^3*(
b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))
/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(g+hx)^3}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((bc - ad)\text{Subst}\left(\int \frac{(b-dx)\left(A+B\log\left(ex^n\right)\right)^3}{(bg-ah-(dg-ch)x)^3}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} + \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x(bg-ah+(-dg+ch)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} + \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{(bg-ah)^2x} + \frac{(bc-ad)^2h^2(A+B\log(ex^n))^2}{(bg-ah)(dg-ch)(bg-ah-(dg-ch)x)^2} + \frac{(bc-ad)h(-2bdg+bch+adh)(}{(bg-ah)^2(dg-ch)(bg-ah}\right.\right.}{2h} \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} + \text{Subst}\left(\frac{(3b^2Bn)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2h(bg-ah)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$+\text{Subst}\left(\frac{(3B(bc-ad)^2hn)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(bg-ah+(-dg+ch)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bg-ah)(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$-\text{Subst}\left(\frac{(3B(bc-ad)(2bdg-bch-adh)n)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{bg-ah+(-dg+ch)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bg-ah)^2(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)}$$
$$- \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2}$$
$$+ \frac{3B(bc-ad)(2bdg-bch-adh)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2}$$
$$+ \text{Subst}\left(\frac{(3b^2)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2h(bg-ah)^2}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(3B^2(bc-ad)^2hn^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{bg-ah+(-dg+ch)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\text{Subst}\left(\frac{(3B^2(bc-ad)(2bdg-bch-adh)n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2}, e \ldots + bx)^n(c+dx)^{-n}\right)$$

$$= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)}$$
$$+\frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(bg-ah)^2}-\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2}$$
$$+\frac{3B^2(bc-ad)^2hn^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$
$$+\frac{3B(bc-ad)(2bdg-bch-adh)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2}$$
$$+\frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}$$
$$-\operatorname{Subst}\left(\frac{\left(3B^3(bc-ad)^2hn^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$
$$-\operatorname{Subst}\left(\frac{\left(3B^3(bc-ad)(2bdg-bch-adh)n^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{(-dg+ch)x}{bg-ah}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$
\begin{aligned}
&= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)} \\
&+ \frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(bg-ah)^2} - \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} \\
&+ \frac{3B^2(bc-ad)^2hn^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&+ \frac{3B(bc-ad)(2bdg-bch-adh)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2} \\
&+ \frac{3B^3(bc-ad)^2hn^3\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&+ \frac{3B^2(bc-ad)(2bdg-bch-adh)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&- \frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\operatorname{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx = \int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3}{(hx+g)^3}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 + 3
*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*B^3*log((d*x + c)^n)^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 3/2*(b^2*e*n*log
(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g
^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e
*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3
 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A^2*B/e -
 3/2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1
/2*A^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(2*B^3*c*h*log(e)^3 +
6*A*B^2*c*h*log(e)^2 + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 6*(B^3*
c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n)^2
```

```
 + 3*(2*A*B^2*c*h - (d*g*n - 2*c*h*log(e))*B^3 - ((h*n - 2*h*log(e))*B^3*d
- 2*A*B^2*d*h)*x + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)^
n)^2 + 2*(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 6*(B^3*c*h*log(e)^2
+ 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x
+ a)^n) - 6*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h)*
log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*c*h
*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))*log
((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^
2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g+h\,x)^3}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):

  #print ("Enter grade_antiderivative for sagemath")
  #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

  leaf_count_result  = leaf_count(result)
  leaf_count_optimal = leaf_count(optimal)

  #print("leaf_count_result=",leaf_count_result)
  #print("leaf_count_optimal=",leaf_count_optimal)

  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)

  if str(result).find("Integral") != -1:
    grade = "F"
    grade_annotation =""
  else:
    if expnType_result <= expnType_optimal:
      if result.has(I):
        if optimal.has(I): #both result and optimal complex
          if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =""
          else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
        else: #result contains complex but optimal is not
          grade = "C"
          grade_annotation ="Result contains complex when optimal does not."
      else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
          grade = "A"
          grade_annotation =""
        else:
          grade = "B"
          grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
    else:
      grade = "C"
      grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```